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Section C

Subject Differential Eq.

Submmited Ma'am SHOMAILA

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Ans. 01:

Fourier series;

$$f(t) = 1+t \; ; \; -\pi \leq t \leq \pi$$

Sol:

using the formula;

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cdot \cos nx + \sum_{n=1}^{\infty} b_n \cdot \sin nx \quad - (A)$$

Here;

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt$$

$$'' = \frac{1}{2} \int_{-\pi}^{\pi} (1+t)\, dt$$

$$'' = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$'' = \frac{1}{2\pi} \left(\pi - (-\pi) \right) + \frac{\pi^2}{2} - \left(-\frac{\pi^2}{2} \right)$$

$$'' = \frac{1}{2\pi} \left(2\pi + \cancel{2} \right) \left(\frac{\pi}{\cancel{2}} \right))$$

$$\Rightarrow \boxed{a_0 = \frac{1}{2\pi} \left(2\pi + \pi^2 \right)}$$

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Now;

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt \cdot dt$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t)\cos nt \cdot dt$$

$$'' = \frac{1}{\pi}\left((1+t)\cdot\frac{\sin nt}{n}\Big|_{-\pi}^{\pi} - \int\left(\frac{\sin nt}{n}\cdot\frac{d}{dt}(1+t)\right)\right)$$

$$'' = \frac{1}{\pi}\left((1+t)\frac{\sin nt}{n} - \frac{\cos nt}{n^2}\Big|_{-\pi}^{\pi}\right)$$

$$'' = \frac{-1}{\pi n^2}(\cos n\pi - \cos n(-\pi))$$

$$'' = \frac{-1}{\pi n^2}(-1-(-1))$$

$$'' = \frac{-1}{\pi n^2}(-1+1)$$

$$'' = \frac{-1}{\pi n^2}(0)$$

$$= \boxed{a_n = 0}$$

Now;

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cdot \sin nt \cdot dt.$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t)\sin nt \cdot dt.$$

$$'' = \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi} \sin nt - \int (\sin nt \cdot \frac{d}{dt}(1+t)dt\right)$$

$$'' = \frac{1}{\pi}\left((1+t)(-\cos nt)\Big|_{-\pi}^{\pi} - \int \left(-\frac{\cos nt}{n}(1)\right)\right)$$

$$'' = \frac{1}{\pi}\left(\frac{-(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} + \left(\frac{\cancel{\sin nt}^{\,0}}{n^2}\Big|_{-\pi}^{\pi}\right)\right)$$

$$'' = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - ((1+\pi)\cos t nt\right)$$

$$'' = \frac{-1}{n\pi}\left(\cancel{\cos n\pi} + \pi\cos n\pi - \cancel{\cos n\pi} + \pi \cos n\pi\right)$$

$$'' = \frac{-1}{n\cancel{\pi}}(2\cancel{\pi}\cos n\pi) \quad \left\{\cos n\pi = \frac{(-1)^{n+1}}{n}\right\}$$

$$b_n = \frac{-2}{n}(1-)^{n+1}$$

BY putting values in eq $(\bar{A})$

$$f(t) = \frac{1}{2\pi}(2\pi+\pi^2) + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin t$$

Required solution:

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"Ans, 02"

$$A = \begin{vmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix}$$

Eigen values =?

Sol:- $\boxed{\text{step} = 01}$

$(A - \lambda I)X = 0$ A = given matrix

$\boxed{\text{step} = 02}$

= The characteristics equation is given by;

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

= $\boxed{\text{Step \#3}}$

$$\lambda^3 - \left|\begin{matrix}\text{sum of} \\ \text{Diagonal element}\end{matrix}\right| \lambda^2 \left|\begin{matrix}\text{sum of} \\ \text{Diagonal} \\ \text{minor}\end{matrix}\right| \lambda - |A| = 0 \quad \text{— (B)}$$

Sum of Diagonal elements $= 1+1+2 = 4$

Sum of Diagonal minors =

$$\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$$

$= (-6) + (2) + (1)$

$= -6 + 2 + 1$

$= -3$

By putting values in eq (B)

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{— (C)}$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$= 1(2-8) - 0 + 1(6-0)$

$= -6+6$

$= 0$

BY putting values in (c);

$\lambda^3 - 4\lambda - 3\lambda - 0 = 0$

$\lambda^3 - 4\lambda - 3\lambda = 0$

$\lambda(\lambda^2 - 4\lambda - 3) = 0$

$\lambda = 0$

$\lambda^2 = 4\lambda - 3 = 0$

using Quadratic Formula:

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$
$b = -4$
$c = -3$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4+\sqrt{28}}{28}, \quad \lambda = \frac{4-\sqrt{28}}{28}$$

Thus we have eigen valus;

$$\lambda = \left(0, \frac{4+\sqrt{28}}{2}, \frac{4-\sqrt{28}}{2}\right)$$ Required Solution

Ans#03:-

$$5x+4z+2m=3$$
$$x-y+2z+m=1$$
$$4x+y+2z=1$$
$$x+y+z+m=0$$

Sol:-

;MATRIX;

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$R_4-R_2 \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

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$$-\frac{1}{5}R3 \quad \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\begin{array}{l} 5R3 \\ 5R4 \end{array} \quad \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right]$$

$$\begin{array}{l} 5R1 \\ \\ \\ 5R3 \end{array} \quad \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 6 & 1 \end{array}\right]$$

$$\begin{array}{l} \frac{1}{5}R1 \\ 5R2 \end{array} \quad \left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 8/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right]$$

$$R3-R2 \quad \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right]$$

$$\begin{array}{l} R3 \Leftrightarrow R4 \\ \frac{1}{7} R3 \\ \frac{1}{3} R4 \end{array} \quad \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$-5R2 \quad \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 1 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\frac{5}{4}R1 \quad \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 36/21 \\ 0 & 0 & 1 & 0 & 11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

Now we have:,

$(x, y, z, m) = \left(3/4, \ 31/21, \ -11/21, \ 1/3\right)$

$x = 3/4$

$y = 31/21$

$z = -11/21$

$m = 1/3$

Required sol:.

Ans:- 4;

verify that;

$u(x,t) = \sin(x+2t)$ is a solution of one-dimensional equation:.

Sol:- Given that:

$$u(x,t) = \sin(x+2t)$$

Differentiat w.r.t x partially

$$\frac{\partial u}{\partial u} = \frac{\partial}{\partial x}\sin(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)\frac{\partial}{\partial x}(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)(1+0)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \cos(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t) \cdot \frac{\partial}{\partial x}(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)(1+0)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)$$

and, $u(x,t) = \sin(x+2t)$

Differentutite w.r.t "t"

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t} \sin(x+2t)$$

$$\frac{\partial u}{\partial t} = \cos(x+2t) \quad (0+2)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial t^2} = (2) - \sin(x+2t) \quad (0+2)$$

$$\Rightarrow \boxed{\frac{\partial^2 u}{\partial t^2} = -4\sin(x+2t)}$$

We know that one-dimensional have equations is;

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$-4\sin(x+2t) = c^2[-\sin(x+2t)]$$

$$-4\sin(x+2t) = c^2 \sin(x+2t)$$

$$-4\sin(x+2t) = c^2 \sin(x+2t)$$

$$-4\sin(x+2t) + c^2 \sin(x+2t) = 0$$

For the orbitary constant $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$-4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0$$

Then it will be verified for orbitary constant

$$c = 2$$