Subject :: Advance Fluid Mechanics

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Q 1 (a)

Define Drag With its Components. Write down the equations for friction Drag Co-efficent both in laminar & turbulent boundary layer.

Ans A body which is wholy Immersed in a homogenous fluid may be subjected two kind of forces araising from relative motion b/w body & fluid. These forces are termed as drag & lift. depending on forces parallel or right angle to motion

Drag Force are Submerged body Can have 2 Components.

(1) Pressure Drag :- It is equal to the Integration of Componets in direction of motion of all pressure forces exerted on surface of body.

$$F_P = C_P \cdot \rho \frac{v^2}{2} \cdot A$$

(Where C_p depend on shape)

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(2) Friction Drag :- It is equal to integration of Component of all Shear Stress along the Surface in direction of motion.

$$Fp = Cf \cdot \int \frac{v^2}{2} (Bc)$$

(CF depend on velocity)

Friction Drag of Boundary layer :-

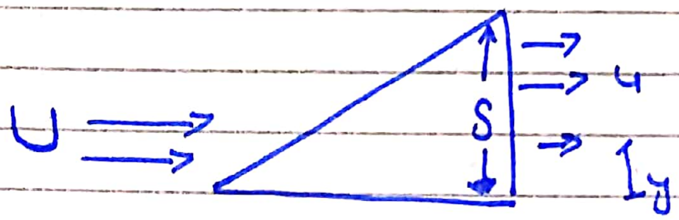
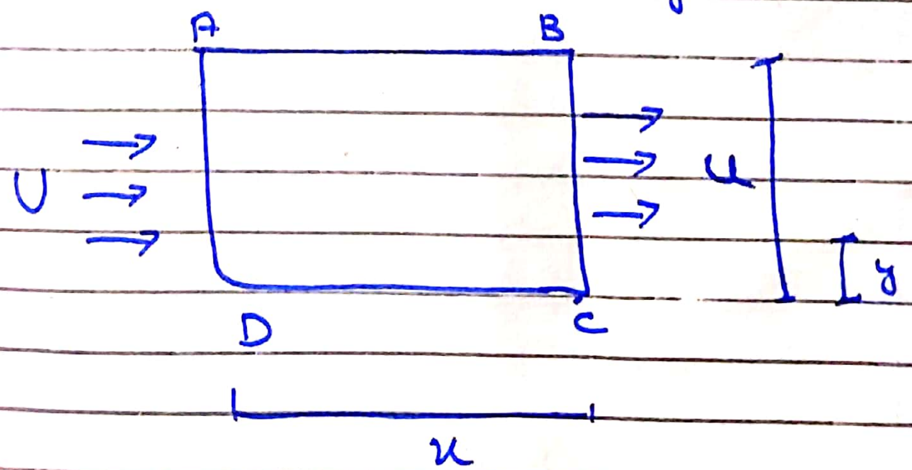


Figure Shows growth of Boundary layer along one Side of Smooth plate in Steady Flow of incompressible Fluid Consider volume Where δ is the thickness of boundary layer & U is the undisturbed velocity.



As we have $\Sigma Fx = 0$
Where

$$F_x = \frac{\Delta P}{\Delta t} = \frac{\Delta mv}{\Delta t} \quad \because m = \rho v$$

$$F_x = \frac{\Delta \rho \cdot vol \cdot v}{\Delta t} = \Delta \rho Q v$$

$$F_x = \Delta \rho Q v$$

$-F_x$ = rate of change of $BC + AB - AD$

$$AD = \rho U (U \delta B)$$

$$BC = \rho B (u^2 dy)$$

$$AB = \rho U (U B \delta) - B \int_0^{\delta} u\, dy$$

$$F_x = \rho B \int_0^{\delta} u (U - u)\, dy$$

Integration on b/s —— (1)

$$F_x = \rho B U^2 \delta \alpha$$

Where α is a function of boundary layer velocity distribution

Now to find Shear Stress

$$\tau = \frac{F_x}{A} = \frac{dF_x}{B\,dx} = \frac{dF_x}{B\,dx}$$

$$\tau_0 = \frac{\rho B U^2 \alpha\, d\delta}{B\,dx} = \rho U^2 \alpha \frac{d\delta}{dx}$$

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$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$

Laminar boundary layer:-

$$\frac{u}{U} = f\left(\frac{y}{\delta}\right) \quad \text{--- (1)}$$

$$\frac{y}{\delta} = \eta \Rightarrow y = \delta\eta$$

$$dy = \delta d\eta \quad \text{--- (2)}$$

$$\frac{u}{U} = f(\eta)$$

$$du = U df(\eta) \quad \text{--- (3)}$$

For laminar flow

$$\tau_0 = \mu \frac{du}{dy} \quad (4)$$

$$\tau_0 = \mu U \frac{df(\eta)}{\delta d(\eta)}$$

$$\tau_0 = \frac{\mu U B}{\delta} \quad (5)$$

As we have $\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$

Compare both.

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$$\frac{\delta u^2 \alpha \, d\delta}{dx} = \frac{\mu U B}{\delta}$$

$$\delta \, d\delta = \frac{\mu B \, dx}{\rho U \alpha}$$

Int on Both side

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} x + C$$

$\because C = 0$

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}}$$

$\because R_x = \frac{U x \rho}{\mu}$

$B = 1.63$, $\alpha = 0.135$

$$\delta = \frac{4.91 x}{\sqrt{R_x}} \quad \text{—— (6)}$$

Where (R_x) is local Reynold no:-

As we have

$$\tau_0 = \frac{\mu U B}{\delta}$$

Put eq (6) in (5)

$$\tau_o = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

Now

$$F_f = B \int_0^L \tau_o \, dx$$

Where $\tau_o = 0.332 \frac{\mu U}{x} \sqrt{R_x}$

$$R_x = \frac{\cancel{x} U \rho}{\mu}$$

then put values

$$F_f = 0.664 \sqrt{\rho \mu L U^3}$$

$$F_f = C_F \rho \frac{v^2}{2} BL$$

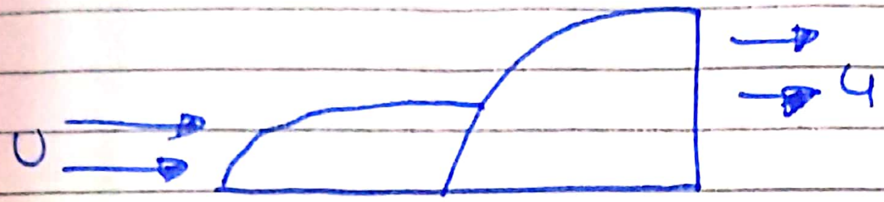
equation on b/s

$$C_F = 1.328 \sqrt{\frac{\mu}{\rho L U}}$$

$$\boxed{C_F = \frac{1.328}{\sqrt{R_x}}}$$

for laminar $R < 500,000$

Turbulent Boundary layer :-



The fig shows that velocity distribution of boundary layer which is steeper near walls & flatter through out reminder of layer.
The shear stress is greater in turbulent than in laminar.
thus

$$\tau_o = \rho f \frac{V^2}{8}$$

Where V is the average velocity to obtain relation b/w average & max we have.

$$\frac{V}{U_{max}} = \frac{1}{1 + 1.33\sqrt{f}} \qquad \because f = 0.023$$

$$\frac{V}{U_{max}} = \frac{1}{1 + 1.33\sqrt{0.023}}$$

$$U = 1.235 V$$

$$V = \frac{U}{1.235}$$

& $$F = \frac{0.316}{(R_h)^{1/4}} \qquad \because R_h = \frac{\rho D V}{\nu}$$

$$D = 2\delta$$

$$\tau_0 = f \rho \frac{v^2}{8}$$

$$\tau_0 = \frac{0.316}{\left(\left(\frac{D}{\nu}\right)\left(\frac{u}{1.235}\right)\right)^{1/4}} \cdot \frac{\rho}{8}\left(\frac{u}{1.235}\right)^2$$

$$\tau_0 = \frac{0.023 \rho u^2}{\left(\frac{2\delta}{\nu}\right)^{1/4}} \quad -(1)$$

As we have general equ

$$\tau_0 = \rho u^2 \alpha \frac{d\delta}{dx} \quad -(2)$$

equ (1) & (2)

$x = 0 \quad , \quad \delta = 0$

$$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\frac{\nu}{ux}\right)^{1/5} \cdot x$$

$\alpha = 0.0972$

$$\boxed{\delta = \frac{0.377}{(R_x)^{1/5}} \cdot x}$$

$$\tau_0 = 0.0587 \rho \frac{V^2}{2}\left(\frac{\nu}{Ux}\right)^{1/5}$$

Now

$$F_f = B\int_0^L \tau_0\, dx$$

$$F_f = 0.0735 \rho \frac{U^2}{2}\left(\frac{\nu}{UL}\right)^{1/5} \cdot BL$$

$$F_f = C_f \cdot \rho \frac{V^2}{2} BL$$

equating b/s

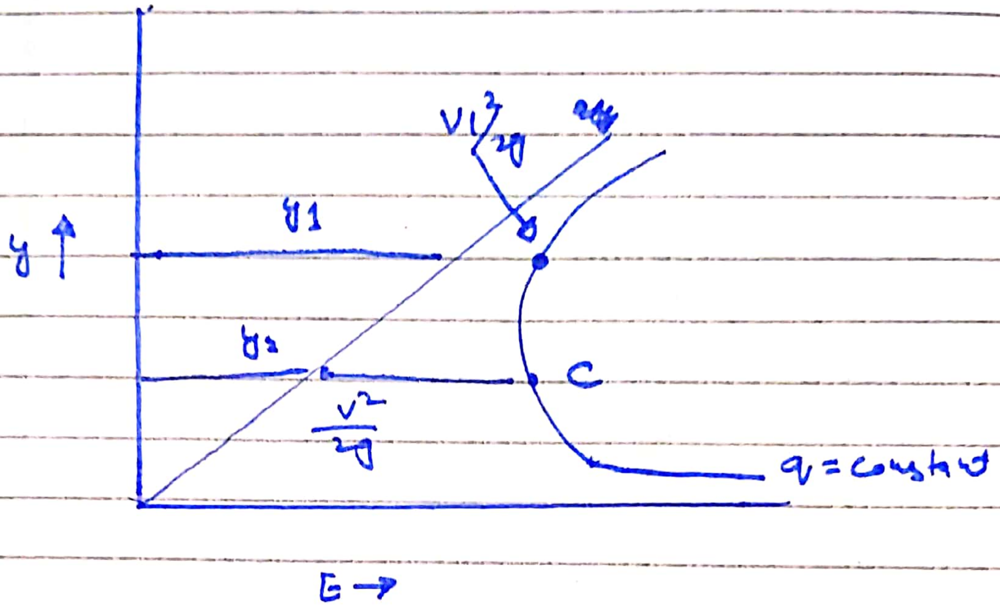
$$C_F = \frac{0.0735}{(R)^{1/5}} \qquad (5000{,}00 < R < 10^7)$$

For $R > 10^7$

$$\boxed{C_F = \frac{0.455}{(\log R)^{2.52}}}$$

Q 1 (b)

Derive equation for Critical depth Critical velocity of rectangular Section of a Channel.



This is Specific energy eqn:-

for particular q, there will be two kind of possible values of y for given E. The eqn is Cubic With three roots with third being negative giving no values Thus two alternative depths represents two totally different flow regimes - slow & deep on upper portion & fast & shallow on lower portion.

Point represent dividing point between two regime of flow

Thus for given "q", value of E is minimum & flow at this point is Critical flow Depth of flow at this point is critical depth y_c & velocity at this point is Critical velocity.

Thus relation of critical depth can be found as

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

For minimum Specific energy

$$\frac{dE}{dy} = 0$$

$$\frac{dE}{dy} = 1 - \frac{\cancel{2}}{\cancel{2}g}\left(\frac{q^2}{y^3}\right)$$

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3}$$

$$1 = \frac{q^2}{gy^3} = q^2 = gy^3$$

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$ Critical depth

As $q = vy$, $V_c^2 = gy^3$

OR $\boxed{V_c = \sqrt{g y_c}}$ critical velocity

$$y_c = \frac{V_c^2}{g}$$

Now

$$\frac{y_c}{2} = \frac{V_c^2}{2g}$$

$$E_{min} = y_c + \frac{v_c^2}{2g} = y_c + \frac{y_c}{2}$$

$$\frac{3}{2} y_c \quad \text{OR} \quad y_{cr} = \frac{2}{3} \text{Constant}$$

	Subcritical	Critical	Super Critical
Depth of Flow	$y > y_c$	$y = y_c$	$y < y_c$
velocity Slop	$V < V_c$ mild slop $S_0 < S_c$	$V = V_c$ critical Slop	$V > V_c$

Q 2

Given Data :-

Depth of Rectangular Channel $(d) = ?$

Flow rate $(Q) = 3.5\ m^3/sec$

Slop of Bed $(S_o) = 0.0008$

$n = 0.0219$

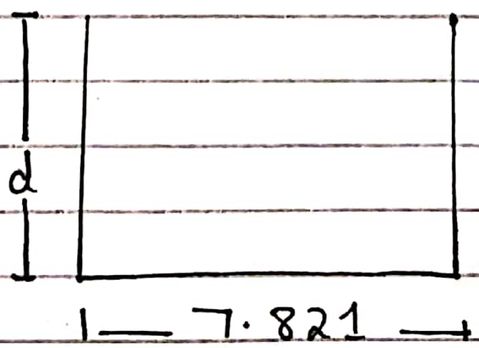
Width of bed $= 7821\ mm$

$= 7.821\ m$

Critical depth $= ?$

Flow Subcritical or Super Critical $= ?$

Sol:-



Area $= 7.821 \times d$

$= 7.821d$

Parameter $= d + 7.821 + d$

$= 7.821 + 2d$

Hydraulic Radius (Rh) = A/p

$$A/p = \frac{7.821\,d}{7.821 + 2d}$$

By using manning equ:-

$$Q = \frac{1}{n} A Rh^{2/3} \cdot (S_o)^{1/2} \quad — \ast$$

Put value in (A) equ

$$3.5 = \frac{0}{0.0219} \times 7.821\,d \times \left(\frac{7.821\,d}{2d + 7.821}\right) —$$

$$\times (0.008)^{1/2}$$

$$\boxed{d = 0.55 \ m}$$

$$\text{Area} = 7.821\,(0.55)$$

$$= 4.30\,m^2$$

$$\text{Parameter} = 7.821 + 2\,(0.55)$$

$$= 8.921\,m$$

$$\text{Hydraulic Radius (Rh)} = \frac{4.37}{8.921}$$

$$= 0.489\,m$$

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Finding Critical depth:-

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

As $q = Q/B$

$$= \frac{3.5}{7.821}$$

$$= 0.447 \, m^2/sec$$

$$\Rightarrow y_{cr} = \left(\frac{(0.447)^2}{9.81}\right)^{1/3}$$

$$= 0.27$$

$y > y_{cr}$ (So flow is sub critical)

$0.55 > 0.27$

Q 3

"Solution"

Given data :-

Friction Drag $(F_D) = ?$

Width $(B) = 200mm = 0.2m$

Length $(L) = 800mm = 0.8m$

undisturbed velocity $(U) = 5m/sec$

Specific gravity $(S) = 0.89$

kinematic viscosity $(\nu) = 0.98 \times 10^{-4} m^2/sec$

Sol:-

Check whether flow is laminar or not By Reynold number

$$R = \frac{DV}{\nu}$$

For Smooth plate

$D = L \quad , \quad V = U$

So

$$R = \frac{LU}{\nu}$$

$$= \frac{0.8 \times 5}{0.93 \times 10^{-4}} = 43010$$

$43010 < 500,000 \rightarrow$ Laminar

By using Formula

$$F_f = C_f \cdot \rho \cdot \frac{V^2}{2} \cdot BL$$

Where

$$C_f = \frac{1.328}{\sqrt{R}} = \frac{1.328}{\sqrt{43010}}$$

$$C_f = 0.0064$$

$$S = \frac{\rho_{oil}}{\rho_{water}} \Rightarrow 0.89 = \frac{\rho_{oil}}{1000}$$

$$\rho_{oil} = 0.89 \times 1000$$

$$\rho_{oil} = 890 kg/m^3$$

$$F_f = C_f \cdot \rho \cdot \frac{U^2}{2} \cdot BL$$

$$F_f = 0.0064 \times 890 \times \frac{(5)^2}{2} \times 0.2 \times 0.8$$

$$\boxed{F_f = 11.39 N}$$