Date: ___________

Name:- Muhammad Daniyal

ID NO:- 17011

Subject:- Differential Equations

Program:- BS(CS)

Instructor:- Latif jan.

Final - Term

Assignment

"Question no 1"

"part a"

Define 2nd order homogenous / non homogenous differential equation along with example?

Answer:-

The nonhomogenous differential equation of this type has the form
$y''+py'+qy=f(x)$

where p, q are constant number (that can be both as real as complex number) for each equations we can write the relation homogenous or complementary equation.

$$y''+py'+qy=0$$

For example:-

$$y''+y=0 \qquad y(0)=3.0$$
$$y'(0)=-0.5$$

Solution:-

The function $\cos x$, $\sin x$ are solution of the ODES

$$y=C_1\cos x+C_2\cos x$$

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This will Turn out to be a general Solution as defined below.
we need the derivative $y' = -C_1 \sin x$.
From this and the initial values are obtain. Since $\cos 0 = 1$ and $\sin 0 = 0$

$$y(0) = C_1 = 3.0 \quad \text{and} \quad y'(0) = C_2 = -0.5$$

$$y = 3.0 \cos x - 0.5 \sin x$$

$$y \, C_1 \cos x + C_2 (k \cos x) = C \cos x$$

where

$$C = C_1 + C_2 k.$$

Answer.

MASTER NOTES

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"Question no 1"

"Part B"

Solve the following 2nd order Linear homogenous and non-homogenous equations.

1) $4y''-6y'+7y=0$

Solution:-

$$4y''-6y'+7y=0$$

First finding r

So

$$4\lambda^2-6\lambda+7=0$$

$$\lambda=\frac{6\pm\sqrt{36-112}}{8} \Rightarrow \lambda=\frac{6\pm\sqrt{-76}}{8}$$

$$\lambda=\frac{6}{8}+\frac{2\sqrt{19}}{8} \Rightarrow \lambda\ \frac{3}{4}\pm\frac{\sqrt{19}}{4}i$$

$$\lambda_1=\frac{3}{4}+\frac{\sqrt{19}}{4}i, \quad \lambda_2=\frac{3}{4}-\frac{\sqrt{19}}{4}$$

So it has the Complex Conjugate

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$Q_1(x) = e^{\lambda_1 x} \quad \cos \lambda_1 (x)$

$Q_2(x) = e^{\lambda_2 x} \quad \sin \lambda_2 (x)$

$y = C_1 e^{3/4 x} \quad \cos 2\sqrt{19}/4 (x) + e^{3x/4}$

$\sin \frac{\sqrt{19}}{4} i (x)$

Answer.

"part 2"

$y'' - 4y' - 12y = 3e^\wedge(5x)$

Solution:-

$y'' - 4y' - 12y = 3e^\wedge(5x)$

The characteristic equation and its roots

$y^2 - 4y - 12 = (y-6)(y+6) = 0$

$y_1 = -2 \quad , \quad y_2 = 6$

The Complementry Solution is:

$y(t) = c_1 e^{-2t} + C_2 e^{6t}$

Answer.

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"Question no 2"

Solve the following IVP for the 2nd order linear equations.

i) $16y'' - 40y' + 25y = 0 \quad y(0)=3 \; y'(0)=\frac{-9}{4}$

Solution:-

$$16y'' - 40y' + 25y = 0 \qquad y(0)=3, \; y'(0)=\frac{-9}{4}$$

$$16y^2 - 40y + 25 = (4y-5)^2 = 0$$

$$y(t) = C_1 e^{\frac{5t}{4}} + C_2 t e^{5t/4}$$

$$y'(t) = \frac{5}{4} C_1 e^{\frac{5t}{4}} + C_2 e^{5t/4} + \frac{5}{4} C_2 t e^{5t/4}$$

$$3 = y(0) = C_1$$

$$-\frac{9}{4} = y'(0) = \frac{5}{4} C_1 + C_2$$

$$y(t) = 3e^{5t/4} - 6te^{5t/4}$$

Answer.

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"Question no 2"

"Part II"

$\Rightarrow y'' + 14y' + 49y = 0 \qquad y(-4) = -1$
$\qquad\qquad\qquad\qquad\qquad y'(-4) = 5$

Solution:-

$$y'' + 14y' + 49y = 0 \qquad y(-4) = -1$$
$$y'(-4) = 5$$

$r^2 - 4r - 12 = (r-6)(r+2) = 0 \Rightarrow r_1 = (-2), r_2 = 6$

$$y_c(t) = c_1 e^{-2t} + ce^{6t}$$

$$y_p(t) = Ae^{5t}$$

$\Rightarrow 25Ae^{5t} - 4(5Ae^{5t}) - 12(Ae^{5t}) - 3e^{5t}$

$\Rightarrow \quad -7Ae^{5t} = 3e^{5t}$

$\Rightarrow \quad -7A = 3 \Rightarrow A = \frac{-3}{7}$

$$y_p(t) = \frac{-3}{7} e^{5t}$$

Answer.

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"Question no 2"

"Part III"

$\Rightarrow y'' - 4y' + 9y = 0$ $\qquad y(0) = 0$

$y'(0) = -8$

Solution:-

$y'' - 4y' + 9y = 0$ $\qquad y(0) = 0$

$y'(0) = -8$

$\Rightarrow r^2 - 4r + 9 = 0$

$y(t) = C_1 e^{2t} \cos(\sqrt{5}t) + C_2 e^{2t} \sin(\sqrt{5}t)$

$\Rightarrow \quad 0 = y(0) = C_1$

$y(t) = C_2 e^{2t} \sin(\sqrt{5}t)$

$y(t) = 2c_2 e^{2t} \sin(\sqrt{5}t) + \sqrt{5}\, C_2 e^{2t} \cos(\sqrt{5}t)$

$-8 = y^2(0) = \sqrt{5}C_2 \Rightarrow C_2 = \frac{-8}{\sqrt{5}}$

$y(t) = -\frac{8}{\sqrt{5}} e^{2t} \sin(\sqrt{5}t)$

Answer

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"Question no 2"

"Part IV"

$y'' - 8y' + 17y = 0 \qquad y(0) = 4.$
$y'(0) = -1$

Solution:-

$y'' - 8y' + 17y = 0 \qquad y(0) = 4$
$y'(0) = -1$

$r^2 - 8r + 17 = 0$

$y(t) = C_1 e^{4t} \cos(t) + C_2 e^{4t} \sin(t)$

$\Rightarrow y'(t) = 4C_1 e^{4t} \cos(t) - C_1 e^{4t} \sin(t)$
$+ 4C_2 e^{4t} \sin(t) + C_2 e^{4t} \cos(t)$

$\Rightarrow -4 = y(0) = C_1$
$-1 = y'(0) = 4C_1 + C_2$

$y(t) = -4e^{4t} \cos(t) + 15e^{4t} \sin(t)$

Answer.

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"Question no 3"

Define Laplace Transform along with example?

Answer:-

Laplace Transform:-

Laplace Transform is an integral Transform that converts a function of a real variable t (often time) to a function of a complex variable s. The transform has many applications in Science and engineering because it is a tool for solving differential equations. In particular it performs a Tranform differential equation into algebric equations and Convolution into multiplication.

For example:-

Let $F(t) = 1$ when $t \geq 0$, Find $F(s)$

Solution:

From (1) we obtain by Integration

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$$\mathcal{L}(f) = \mathcal{L}(1) = \int_0^\infty e^{-st}\,dt = -\frac{1}{s} e^{-st}\Big|_0^\infty = \frac{1}{s}$$

Such an integral is Called an Improper Integral and by definition. is evaluated according to the rule.

$$\int_0^\infty e^{-st} f(t)\,dt = \lim_{T \to \infty} \int_0^T e^{-st} f(t)\,dt.$$

Hence our Convenient notation means

$$\int_0^\infty e^{-st}\,dt = \lim_{T \to \infty} \left[-\frac{1}{s} e^{-st} \right]_0^T = \lim_{T \to \infty}$$

$$\left[-\frac{1}{s} e^{-sT} + \frac{1}{s} e^{0} \right] = \frac{1}{s} \qquad (s > 0)$$

proved.

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"Question no 3"

"part 1"

Find the Laplace transform of the given function.

$f(t) = 6(e^{\wedge}-5t) + e^{\wedge}3t + 5(t^{\wedge}3) - 9$

Solution:-

$f(t) = 6(e^{\wedge}-5t) + e^{\wedge}3t + 5(t^{\wedge}3) - 9$

$F(s) = 6\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\frac{3!}{s^{3+1}} - 9\frac{1}{s}$

$= \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}$

Ans.

"Part 2"

$g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

Solution:-

$g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

$G = 4\frac{s}{s^2+4^2} - 9\frac{4}{s^2+4^2} + 2\frac{s}{s^2+10^2}$

$$= \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}$$

Answer.

"part 3"

h(t)= e^3t+ Cos(6t)-e^(3t) Cos (6t)

Solution:-

h(t) = e^ 3t + Cos (6t) - e^(3t) Cos (6t)

$$H = \frac{1}{s-3} + \frac{3}{s^2+6^2} - \frac{s-3}{(s-3)^2+(6)^2}$$

$$= \frac{1}{s-3} + \frac{s}{s^2+36} - \frac{s-3}{(s-3)^2+36}$$

Answer.

"Question no 4"

"part 1"

Solve the following IVP using laplace transform.

$$y'' - 10y' + 9y = 5t \qquad y(0) = -1, \quad y'(0) = 2$$

Solution:-

$$y'' - 10y' + 9y = 5t \qquad y(0) = -1, \quad y'(0) = 2$$

$$\mathcal{L}\{y''\} - 10\,\mathcal{L}\{y'\} + 9\{y\} = \mathcal{L}\{5t\}$$

$$s^2 y(s) - sy(0) - y' - 10(sy(s) - y(0)) + 9y(s) = \frac{5}{s^2}$$

$$(s^2 - 10s + 9)\, y(s) + s - 12 = \frac{5}{s^2}$$

$$y(s) = \frac{5}{s^2(s-9)(s-1)} + \frac{12-s}{(s-9)(s-1)}$$

$$y(s) = \frac{5 + 12s^2 - s^3}{s^2(s-9)(s-1)}$$

$$y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-9} + \frac{D}{s-1}$$

$$= 5 + 12s^2 - s^3 = As(s-9)(s-1) + B(s-9)(s-1) + Cs^2(s-1) + Ds^2(s-9)$$

$s = 0 \qquad 5 = 9B \quad \Rightarrow \quad B = 5/9$

$s = 1 \qquad 16 = -8D \quad \Rightarrow \quad D = -2$

$s = 9 \qquad 248 = 648C \quad \Rightarrow \quad C = 31/81$

$s = 2 \qquad 45 = -14A + \frac{4345}{81} \quad \Rightarrow \quad A = 50/81$

$$y(s) = \frac{\frac{50}{81}}{s} + \frac{\frac{5}{9}}{s^2} + \frac{\frac{31}{81}}{s-9} - \frac{2}{s-1}$$

$$y(t) = \frac{50}{81} + \frac{5}{9}t + \frac{31}{81}e^{9t} - 2e^{t}$$

Answer

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"Question no 4"

"Part 2"

$$y'' - 6y' + 15y = 2\sin(3t) \qquad \begin{matrix} y(0) = -1 \\ y'(0) = -4 \end{matrix}$$

Solution:-

$$y'' - 6y' + 15y = 2\sin(3t) \qquad \begin{matrix} y(0) = -1 \\ y'(0) = -4 \end{matrix}$$

$$\mathcal{L}\{y''\} - 6\mathcal{L}\{y'\} + 15\,\mathcal{L}\{y\} = 2\sin \mathcal{L}\{3t\}$$

$$= s^2 y(s) - s y(0) - y'(0) - 6(s y(s)) - y(0)) + 15y(3) - 2\,\frac{3}{s^2+9}$$

$$y(s) = \frac{-s^3 + 2s^2 - 9s + 24}{(s^2+9)(s^2-6s+15)}$$

$$y(s) = \frac{As+B}{s^2+9} + \frac{Cs+D}{s^2-6s+15}$$

$$= s^3 + 2s^2 - 9s + 24 = (As+B)(s^2-6s+15) + ((s+D) + (s^2+9))$$

$$= (A+C)s^3 + (-6A+B+D)s^2 + (15A-6B+9C)s + 15B + 9D$$

$s^3 = \quad A + C = 1 \quad \Rightarrow A = 1/10$

$s^2 = \quad -6A + B + D = 2 \quad \Rightarrow B = 1/10$

$s^1 = \quad 15A - 6B + 9C = -9 \quad \Rightarrow C = -11/10$

$s^0 = \quad 15B + 9D = 24 \quad \Rightarrow D = 5/2$

$$y(s) = 1/10 \left(\frac{s+1}{s^2+9} + \frac{-11s + 25}{s^2 - 6s + 15} \right)$$

$$= \frac{1}{10} \left(\frac{s+1}{s^2+9} + \frac{-11(s-3+3)+25}{(s-3)^2+6} \right.$$

$$= \frac{1}{10} \left(\frac{3}{s^2+9} + \frac{1\,3/3}{s^2+9} \quad \frac{-11(s-3)}{(s-3)^2+6} - \frac{8\frac{\sqrt{6}}{\sqrt{6}}}{(s-3)^2+6} \right.$$

$$y(t) = \frac{1}{10} \left(\cos(3t) + \frac{1}{3} \sin(3t) - 11e^{3t} \cos(\sqrt{6}t) \right.$$

$$- \frac{8}{\sqrt{6}} e^{3t} \sin(\sqrt{6}t)$$

Answer.