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Q1 Consider the following vectors R^3:

$$V_1 = \begin{pmatrix} ID1 \\ ID2 \\ ID3 \end{pmatrix}, V_2 = \begin{pmatrix} ID2 \\ ID3 \\ ID4 \end{pmatrix}, V_3 = \begin{pmatrix} ID3 \\ ID4 \\ ID5 \end{pmatrix}$$

ID = 16419

$$V_1 = \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}, V_2 = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix}, V_3 = \begin{pmatrix} 4 \\ 1 \\ 9 \end{pmatrix}$$

Solve the system and find if these vectors are linearly independent.

Ans. SOLUTION:. These vectors will be linearly independent if $x_1v_1 + x_2v_2 + x_3v_3 = 0$.

So,

$$\begin{bmatrix} 1 & 6 & 4 \\ 6 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Writing in augmented form, (ID: 16419)

$$\left[\begin{array}{ccc|c} 1 & 6 & 4 & 0 \\ 6 & 4 & 1 & 0 \\ 4 & 1 & 9 & 0 \end{array}\right] \xrightarrow[R_3 = R_3 - 4R_1]{R_2 = R_2 - 6R_1} \left[\begin{array}{ccc|c} 1 & 6 & 4 & 0 \\ 0 & -32 & -23 & 0 \\ 0 & -23 & -7 & 0 \end{array}\right]$$

So writing in equation form,

$$x_1 + 6x_2 + 4x_3 = 0 \quad \text{— (i)}$$
$$-32x_2 - 23x_3 = 0 \quad \text{— (ii)}$$
$$-23x_2 - 7x_3 = 0 \quad \text{— (iii)}$$

by solving (ii) and (iii), we get

$x_2 = 0$, $x_3 = 0$.

Putting in equation (i)

$$x_1 + 0 + 0 = 0$$
$$x_1 = 0.$$

Therefore

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

ID: 16419

So these vectors are linearly independent.

Q3. What are the four main things we need to define for a vector space? Which of the following is a vector space over R? For those that are not vector spaces, modify one part of the definition to make it into a vector space.

a. V = { 2x2 matrices with entries in R },
usual matrix addition, and

$$k.\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & b \\ kc & d \end{pmatrix} \text{ for } k \in R$$

(Ans. 3) First of all, The four main ingredients are (1) a set V of vectors, (2) a number field k (usually k = R), (3) a rule for adding vectors (vector addition) and (4) a way to multiply vectors by a number to produce a new vector (scalar multiplication). They should obey ten rules.

ID: 16419

Ans 3(a) This is not a vector space. Notice that distributivity of scalar multiplication requires $2u = (1+1) = u+u$ for any vector u but

$$2.\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2a & b \\ 2c & d \end{pmatrix}$$

which does not equal

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2a & 2b \\ 2c & 2d \end{pmatrix}.$$

This could be repaired by taking

$$k.\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}.$$

(b.) $V = \{$Polynomials with complex coefficients of degrees $\leq 3\}$ with usual addition and scalar multiplication of polynomials.

Ans. This is a vector space.
This has two parts, i.e (i) scalar and (ii) usual addition.

(i) Scalar,
In this case, the necessary condition is one, i.e the scalar which will be multiplied with matrix, should also belong to the field.

(ii) Usual addition,

ID: 16419

Since, in matrix addition, corresponding entities are added, so all these vector would be added if their complex co-efficient of degrees ≤ 3.

All ten vector space rules are satisfied.

Q4. Determinants: Let $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a 2×2 matrix.

a. For which values of det M does M have an inverse?

Ans. M will have an inverse whenever, $\det M = ad - bc \neq 0$.

b. Write down all 2×2 bit matrices with determinant 1. (Remember bits are either 0 or 1 and $1 + 1 = 0$ in bits).

Ans. So, all such 2×2 bit matrices are: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$.

Their determinant is 1.

ID : 16419

(c) Write down 2x2 bit matrices with determinant 0.

Ans. Bit matrices (2x2) with 0 determinant are:

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

(d) Compute det A for below 3 x 3 matrix.

$$A = \begin{bmatrix} ID1 & ID1 & ID1 \\ ID2 & ID3 & ID2 \\ ID4 & ID1 & ID5 \end{bmatrix}$$

Ans SOLUTION:-

ID = 16419

So,

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 6 & 4 & 6 \\ 1 & 1 & 9 \end{pmatrix}$$

$$|A| = (-1)^{1+1} \cdot 1 \begin{vmatrix} 4 & 6 \\ 1 & 9 \end{vmatrix} + (-1)^{1+2} \cdot 1 \begin{vmatrix} 6 & 6 \\ 1 & 9 \end{vmatrix}$$

$$+ (-1)^{1+3} \cdot 1 \begin{vmatrix} 6 & 4 \\ 1 & 1 \end{vmatrix}$$

ID: 16419

So, $1(36-6)-1(54-6)+1(6-4)=30-48+2=-16.$

So determinant $=-16.$