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Class Time: TUE: 08:00 to 11:00
Semester: 2nd spring BS (CS).
Subject: Discrete Structure.
Instructor: Mr. Saifullah Jan.
Mid-Term Assignment # 01
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Q 1. A: Let p statement = "DATAENDFLAG is off". q = "ERROR is equal to 0".
 r = "SUM is less than 1000".

Express the following sentences in symbolic notation.

(a) DATAENDFLAG is off, ERR equals 0, and SUM is less than 1000.

Symbolic Notation:

p, q, r

(b) DATAENDFLAG is off but ERROR is not equal to 0.

Symbolic Notation:

$p \wedge \neg q$

(c) DATAENDFLAG is off, however ERROR is not 0 or SUM is greater than or equal to 1000.

Symbolic notation:

$$PV(\neg q \vee \neg r)$$

(cd) DATAENDFLAG is on and ERROR equals 0 but SUM is greater than or equal to 1000.

Symbolic notation:

$$\neg PA(q \wedge \neg r)$$

(e) Either DATAENDFLAG is on or it is the case that both, ERROR equals 0 and SUM is less than 1000.

Symbolic notation:

$$\neg PV(q \wedge r)$$

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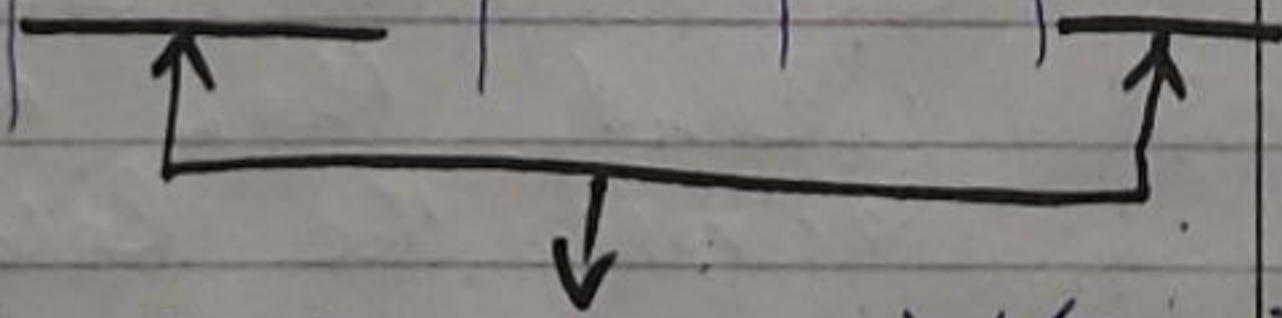
Q1 B:

Show that $p \vee q \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

Soln

$p \vee q \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

1	2	3	4	5	6	7	8
p	q	r	$p \vee q$	$p \vee q \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T



Hence proved that $p \vee q \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

The truth table shows that column 5 and column 8 are logically equal to each other.

Q Q

Q2 (A) Write the converse, inverse and contrapositive of the following.

(a) If Howard can swim across the lake, then Howard can swim to the island.

Sol:

Given Statement:

p = Howard can swim across the lake.

q = Howard can swim to the island.

(i) Converse: $q \rightarrow p$.

If Howard can swim to the island, then Howard can swim across the lake.

(ii) Inverse: $\neg p \rightarrow \neg q$.

If Howard can not swim across the lake, then Howard can not swim to island.

(iii) Contrapositive: $\neg q \rightarrow \neg p$.

If Howard can not swim to the island, then Howard can not swim across the lake.

P.T.O

Q2 (A) (b) If today is Easter, then tomorrow is Monday.

$P =$ Today is Easter.

$Q =$ Tomorrow is Monday.

(i)

Converse: $Q \rightarrow P$.

If tomorrow is Monday, then today is Easter.

(ii)

Inverse: $\neg P \rightarrow \neg Q$.

If today is not Easter, then tomorrow is not Monday.

(iii)

Contrapositive: $\neg Q \rightarrow \neg P$.

If tomorrow is not Monday, then today is not Easter.

d a

Q2:

B:

use the truth tables to determine whether the argument forms are valid. Indicate which columns represent the premises and which columns represent the conclusion.

(a)

P
 $P \rightarrow Q$
 $\neg Q \vee R$
 $\therefore R$

P	Q	R	$P \rightarrow Q$	$\neg Q \vee R$	$\neg Q$	R
T	T	T	T	T	F	T
T	T	F	T	F	F	T
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	F	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

The argument is valid and column 7 represent the premises

$P \cdot T \cdot 0$

a a

Q 2 (B)

(b)

$$p \wedge \neg r$$

$$p \vee \neg q$$

$$\neg q \rightarrow \neg p$$

$$\therefore \neg r$$

P	q	$\neg q$	r	$\neg r$	$p \wedge \neg r$	$p \wedge \neg r \rightarrow \neg r$	$\neg q \rightarrow \neg p$
T	T	F	T	F	T	F	T
T	T	F	F	T	T	T	T
T	F	T	T	F	F	T	T
T	F	T	F	T	F	T	T
F	T	F	T	F	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	F	F	T	F
F	F	T	F	T	F	T	F

The argument is valid but column 5 indicate the conclusion.

2 2

Q3:

In the back of an old cupboard you discover a note signed by a pirate famous for his his bizarre sense of humor and love of logical puzzles. In the note he wrote that he had hidden treasure somewhere on the property. He listed five true statements (a-e below) and challenged the reader to use them to figure out the location of the treasure.

- (a) If the house is next to a lake, then the treasure is not in the kitchen.
- (b) If the tree in the front yard is an elm, then the treasure is in the kitchen.
- (c) This house is next to a lake.
- (d) The tree in the front yard is an elm or the treasure is buried under the flagpole.
- (e) If the tree in the back yard is an oak, then the treasure is in the garage.

where is the treasure hidden?

P.T.O

9.

To solve this qns first know about the rules of inference.

Modus ponens

$$\begin{array}{l} p \rightarrow q \\ p \end{array}$$

$$\therefore q$$

Modus tollens

$$p \rightarrow q$$

$$\neg q$$

$$\therefore \neg p$$

Elimination

$$p \vee q$$

$$\neg p$$

$$\therefore q$$

Now know about this.

$\neg p$, not p

$p \vee q$, p or q

$p \wedge q$, p and q

$p \rightarrow q$, if p then q

$p \leftrightarrow q$, p if and only if q .

P.T.O.

lot. $p =$ This house is next to a lake.

$q =$ The treasure is in the kitchen.

$r =$ The tree in the front yard is an oak elm.

✓ $s =$ The treasure is buried under the flagpole.

$t =$ The tree in the back yard is an oak.

$u =$ The treasure is in the garage.

Steps

Reasons.

(1)	$p \rightarrow \neg r$	premise
(2)	$r \rightarrow q$	premise
(3)	p	premise
(4)	$r \vee s$	premise
(5)	$t \rightarrow u$	premise
(6)	$\neg r \leftarrow \begin{array}{l} (p \rightarrow r) \\ \therefore q \end{array}$	modus ponens of (1) and (3)
(7)	$\neg r \leftarrow \begin{array}{l} (\neg r \\ r \rightarrow q) \\ \therefore \neg r \end{array}$	modus tollens of (2) and (6)
(8)	s	Elimination of (4) and (7)

Hence we proved in step (8) that s is true and thus the treasure is buried under the flagpole.

Q.E.D.