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COURSE TITLE:- Digital signal proccessing.

Q1(a):- Consider the following analog signal

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

i. Determine the minimum sampling rate required to avoid aliasing.

ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discreate-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discreate time signal.

iii. What is the analoge signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?

Sol:- (i)

$$f_s \geq 2f_{max}$$

$$f_1 = \frac{100\pi}{2\pi} = 50\ Hz$$

$$f_2 = \frac{200\pi}{2\pi} = 100\ Hz$$

So $f_s = 2 \times f_{max}$

$f_s = 2 \times 100$

$f_s = 200\ Hz$

(b) $f_s = 100\ Hz$

$\frac{100}{2} = 50\ Hz$

this is the maximum frequvenee which is representing by the sampled signal.

$$x_a(n) = 3\cos 2\pi\left(\frac{50}{100}\right)n + 4\sin 2\pi\left(\frac{100}{100}\right)n$$

$$= 3\cos 2\pi\left(\frac{5}{10}\right)n + 4\sin 2\pi n$$

(iii)

folding frequency of Sampled signal

$f_{fold} = F_s/2 = \frac{100}{2}$

$f_{fold} = 50Hz$

and the frequency of original signal

$f_1 = 50HZ$ $f_2 = 100HZ$

this frequency is either equal or greater

then the folding frequency.

Hence for ideal interpolation we can construct

the orignal signal

$x_a(t) = 3\cos 100\pi t + 4 \sin 200\pi t$

The orignal signal is constructed because we

use sampling frequency at Nyquist Rate

So the orignal signal is Constructed.

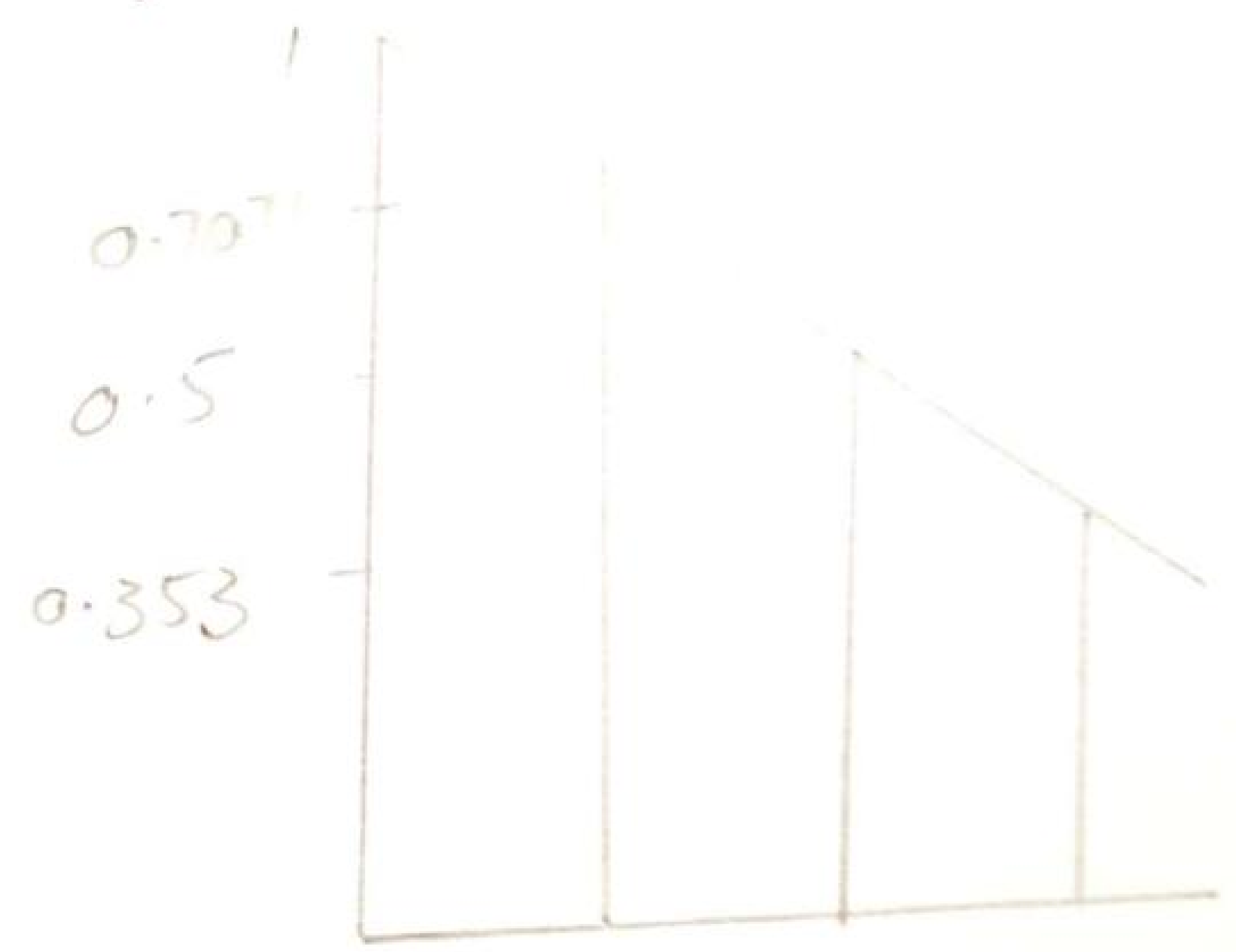
Q1(b):- Consider a discreate time signal which is given by

$$x(n) = \begin{cases} 0.5^n & n \geq 0 \\ 0, & n < 0 \end{cases}$$

This is signal is sampled at the rate $F_s = 2Hz$

i. Draw the sampled signal.

x_n	0.5
0	1
0.5	0.7071
1	0.5
1.5	0.353



T = 0.5 sec

ii.

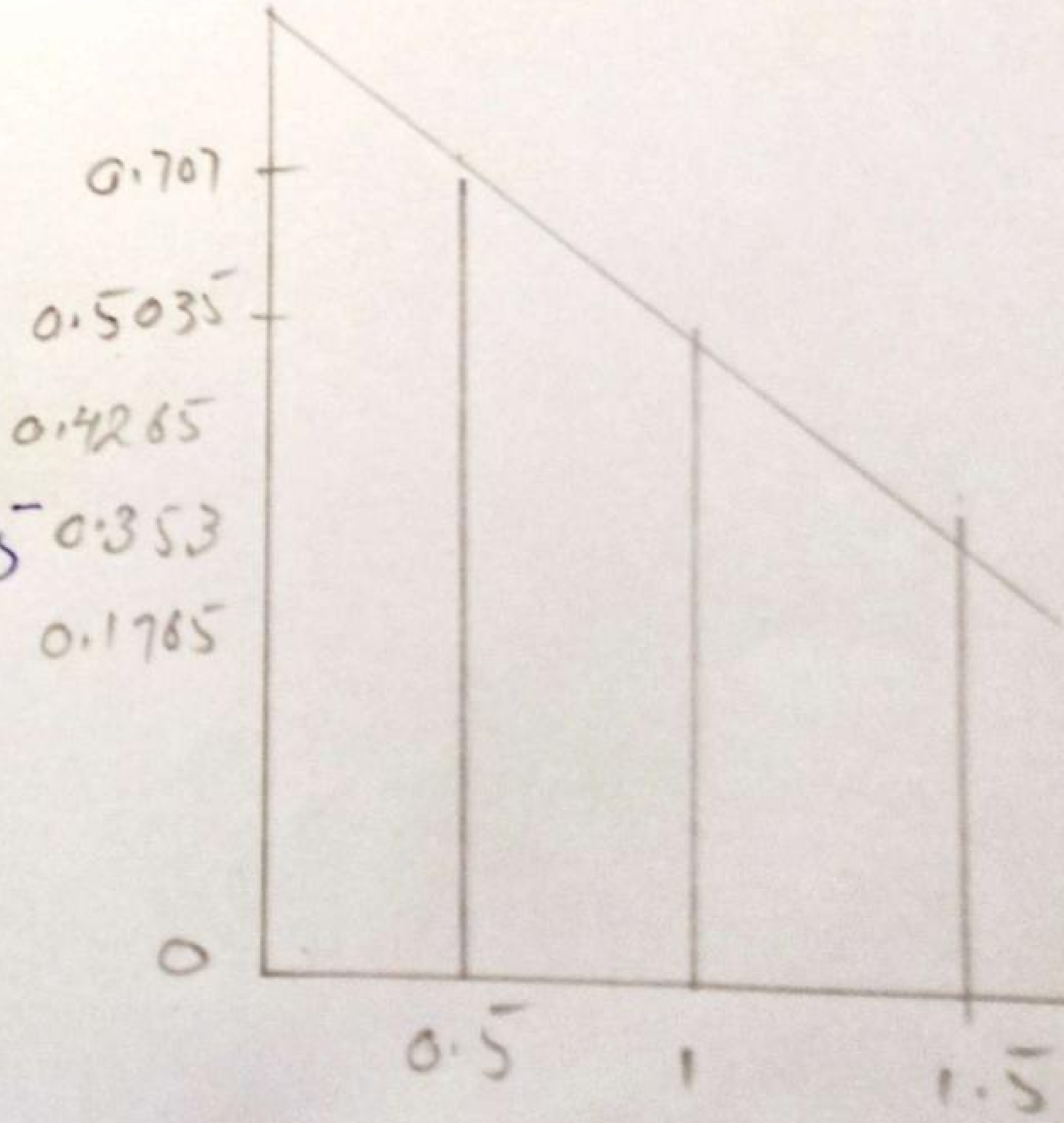
$L = 2^n$

$n = \text{bits} = 3$

$L = 2^3 = 8$ levels

$$\text{Resolution} = \frac{x_{max} - x_{min}}{L}$$

$$= \frac{1-0}{8} = 0.125$$



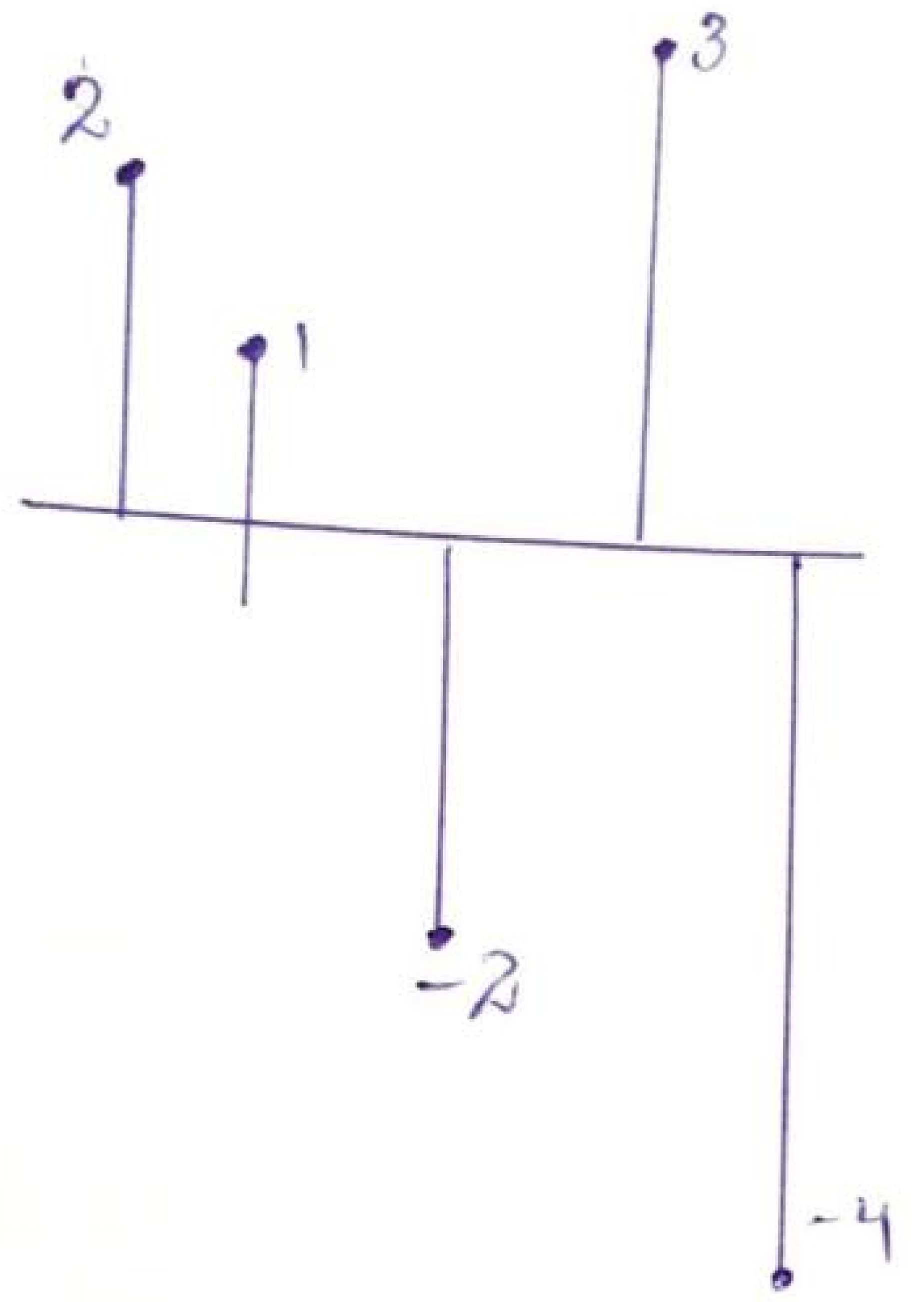
iii.

	Discreat time signal	Truation	Reading	errors
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.2	-0.1

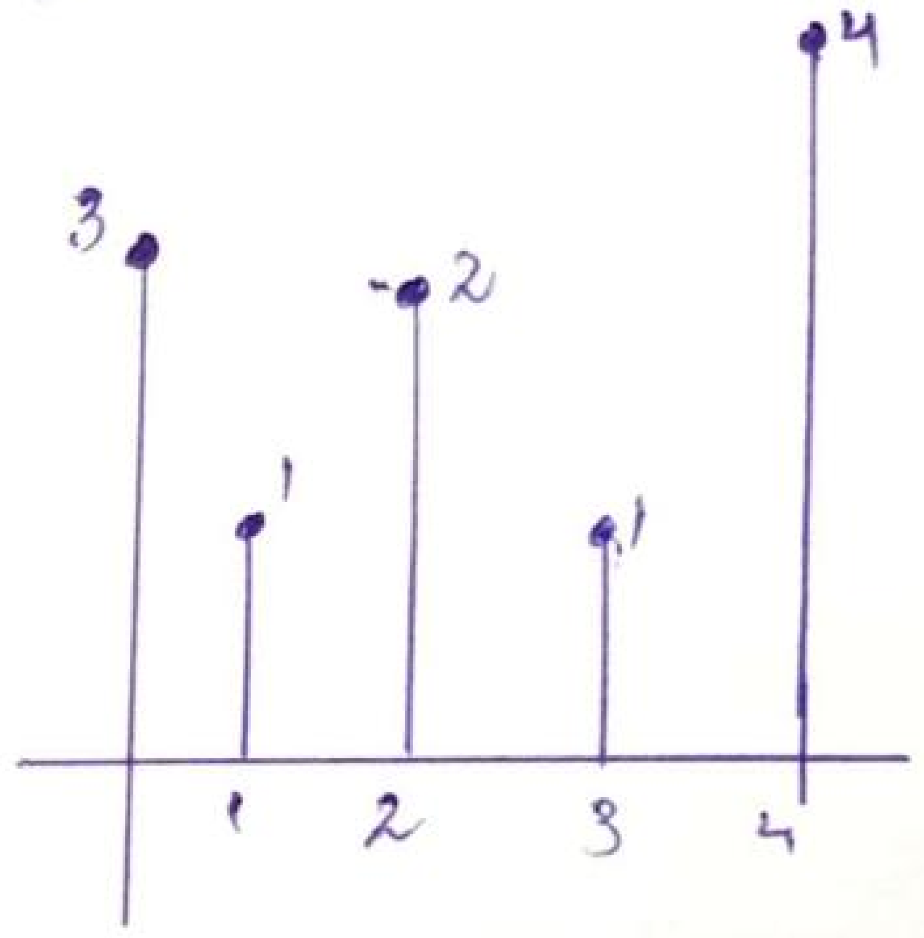
Q2:-(a):- Determine the response of the system to the following input signal with given impulse response

$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}, \quad h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

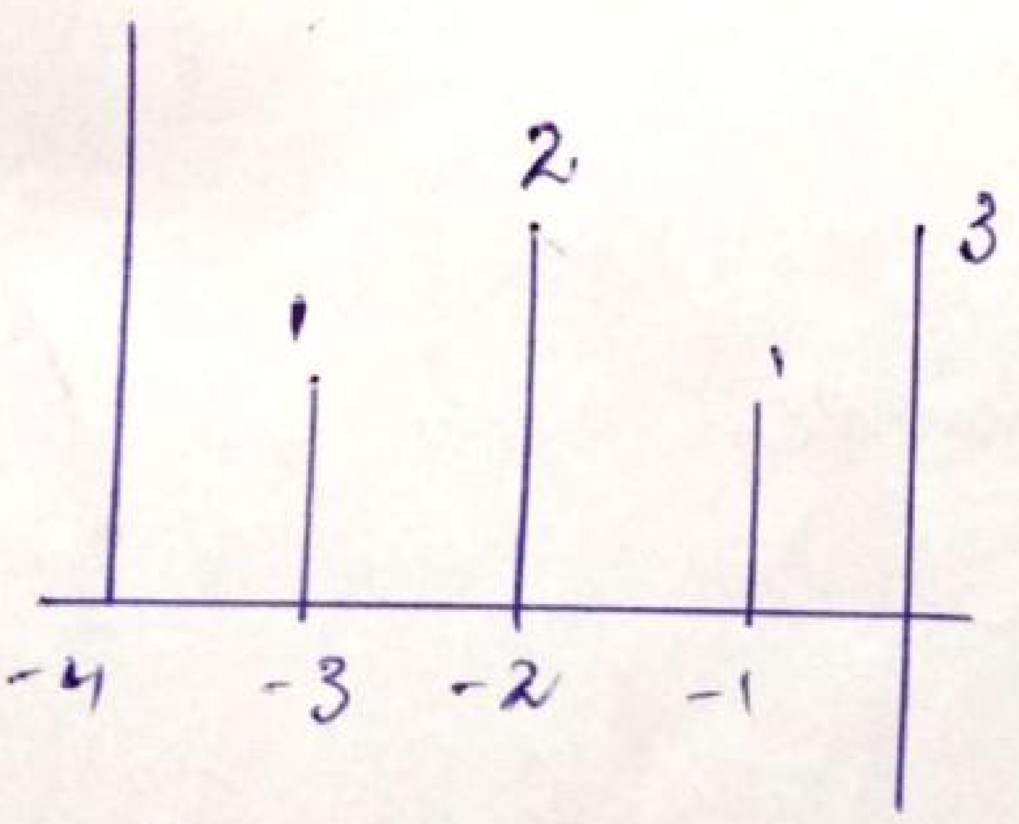
Solution:- $x[n] = \sum_{100 \infty}^{\infty} x[k]\, h(n-k)$



$h(k)$



(-k) folded signal



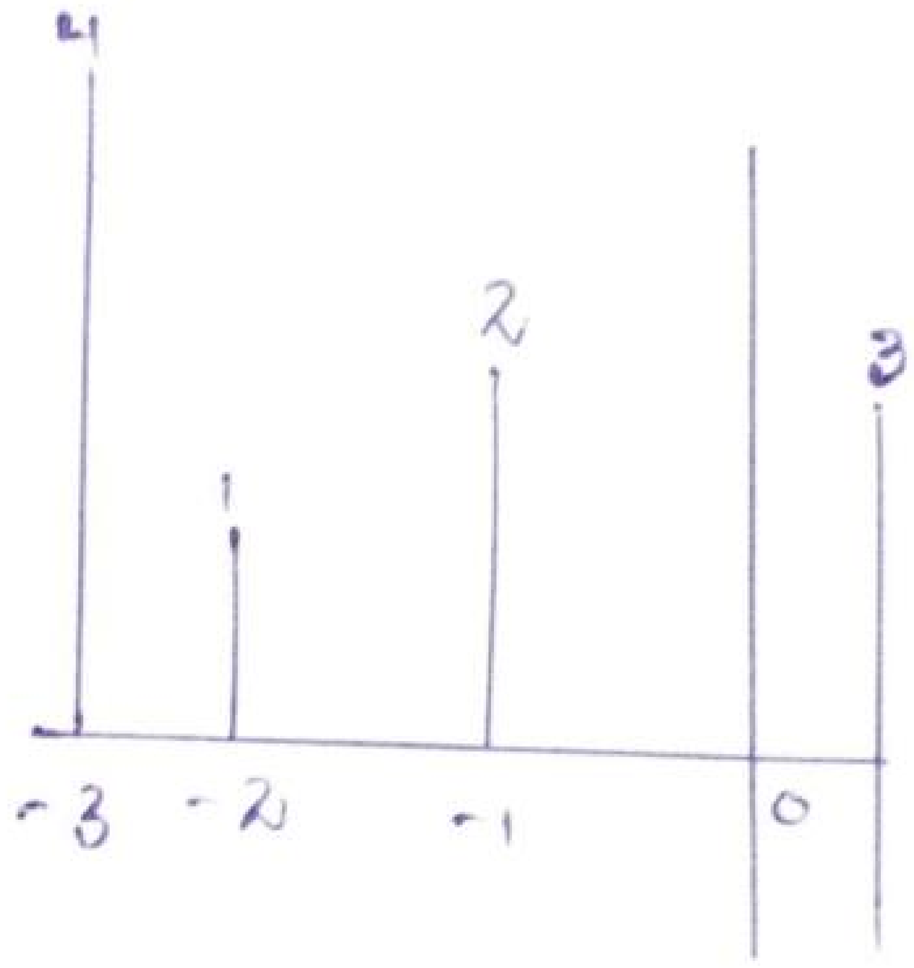
$$y[0] = \sum_{k=1}^{0} x(-1)\, h(-1) + x(0)\, h(0)$$

$$y(0) = 2(1) + (1)(3)$$

$$= 2 + 3 = 5$$

for $n=1$

$h(1-K)$

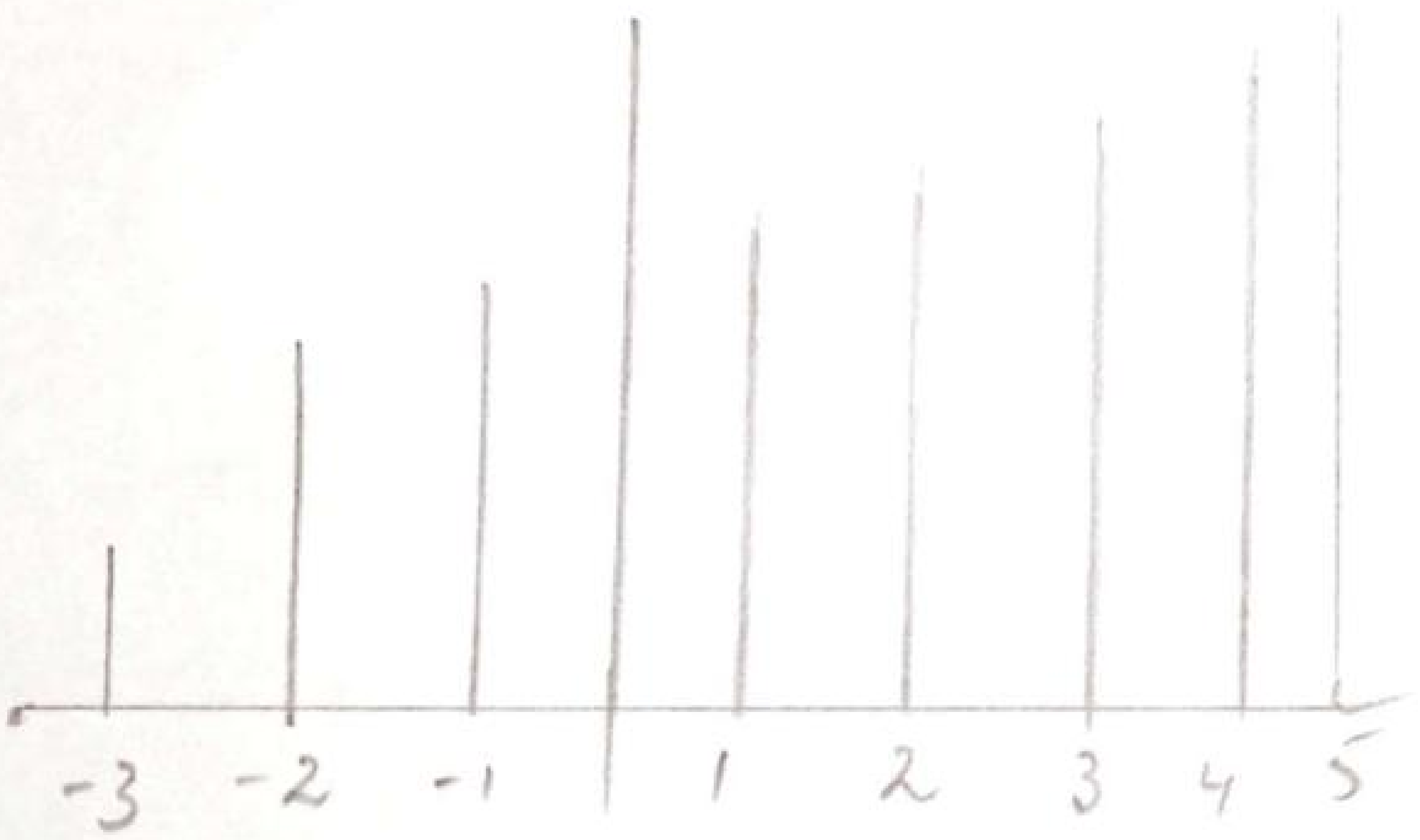


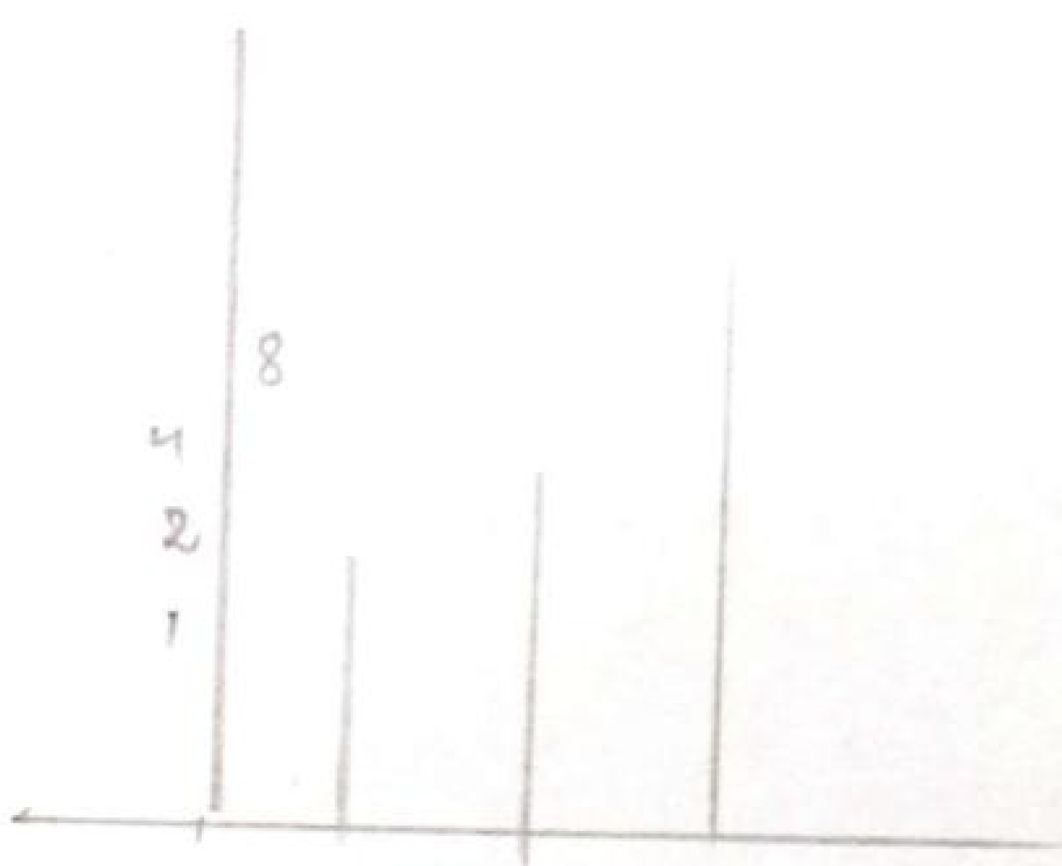
(5)

Compute the convolution $y(n)$ of the following signal

$$x(n)=\begin{cases}\alpha^{n+1}, & -3 \le n \le 5 \\ 0, & \text{elsewhere}\end{cases}$$

$$h(n)=\begin{cases}2^{n}, & 0 \le n \le 4 \\ 0, & \text{elsewhere}\end{cases}$$

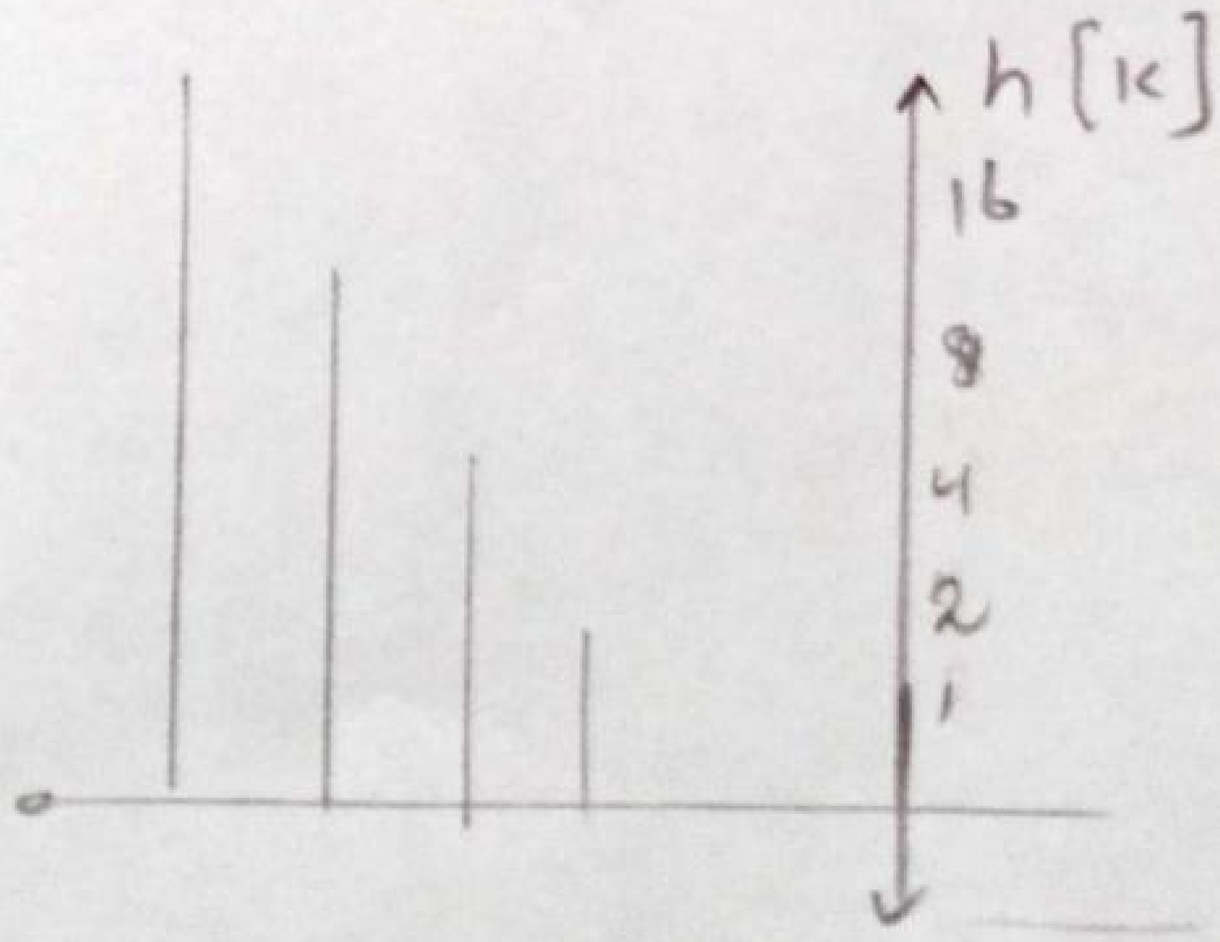




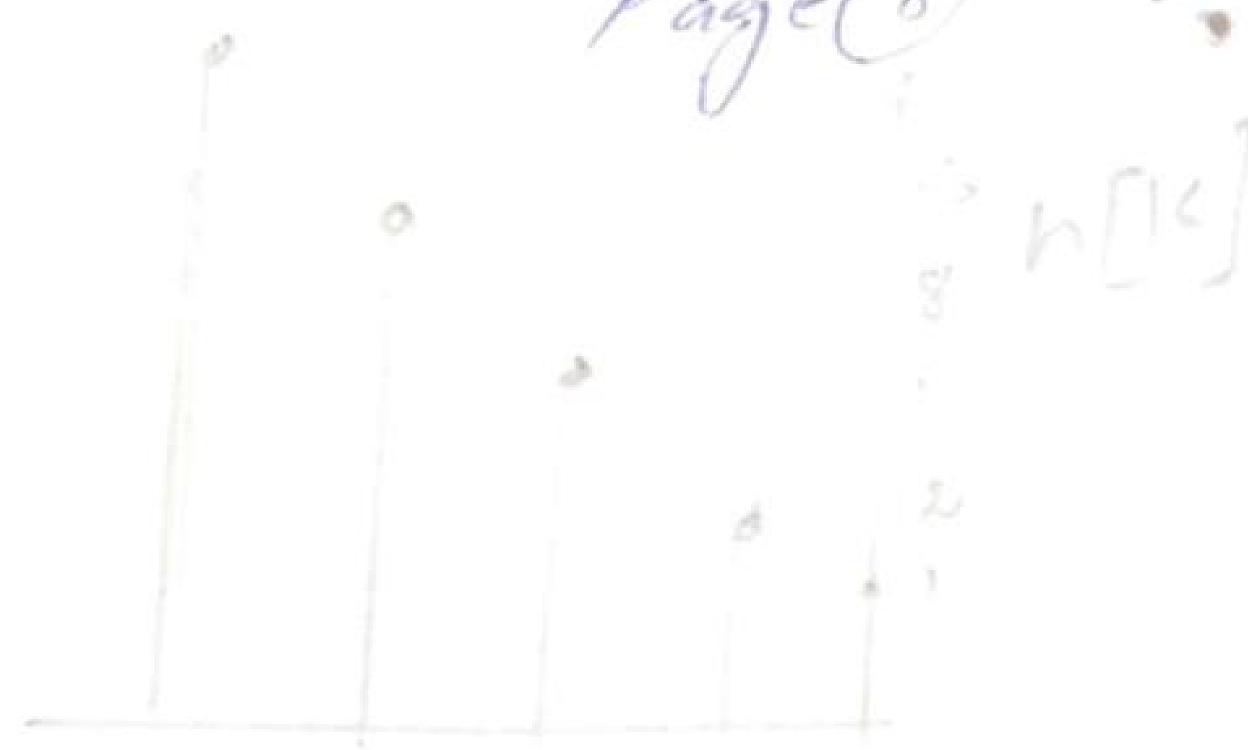
$$x[n]=\{a^{-2}, a^{-1}, 1, a, a^{2}, a^{3}, a^{4}, a^{5}, a^{6}\}$$

$$h[n]=\{1, 2, 4, 8, 16\}$$

$$y[n_0]=\sum_{K=-\infty}^{\infty} x[K]\,h[n_0-K]$$



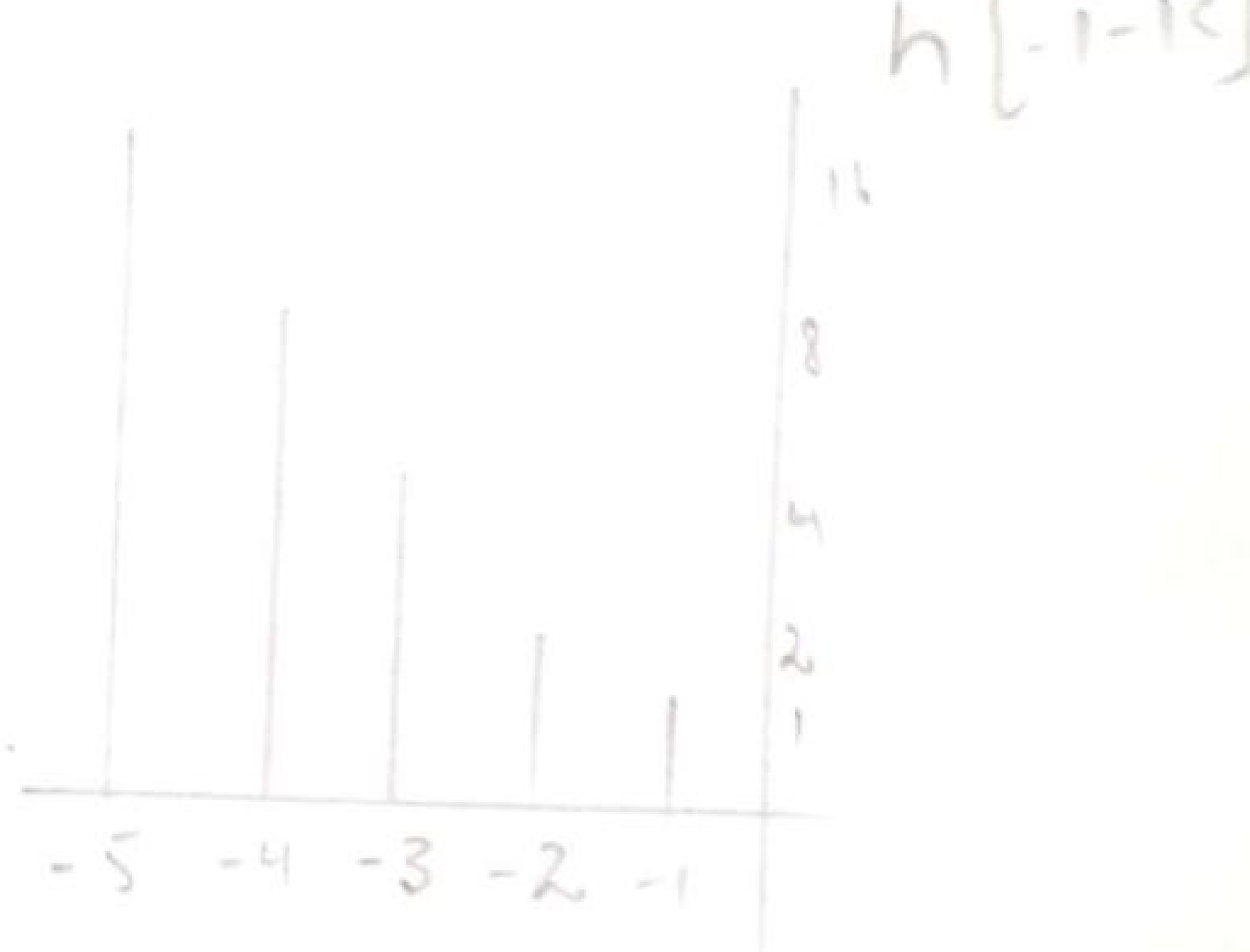
$y[0] = a - 2 + 4a^{-1} + 8a^{-2}$



$y[-1] = 1 + 2a^{-1} + 4a^{-2}$

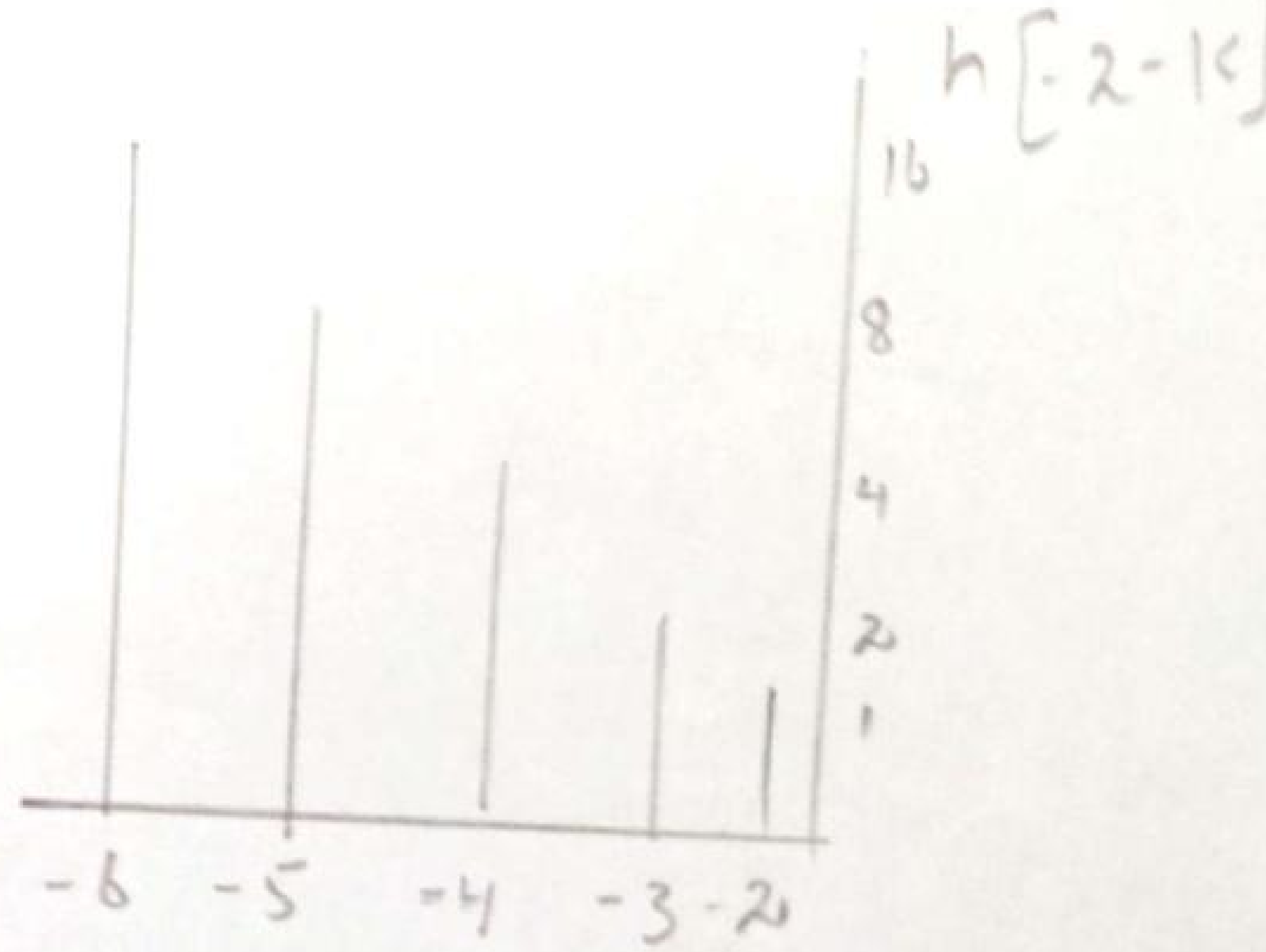
$y[-2] = 2a^{-2} + a^{-1}$

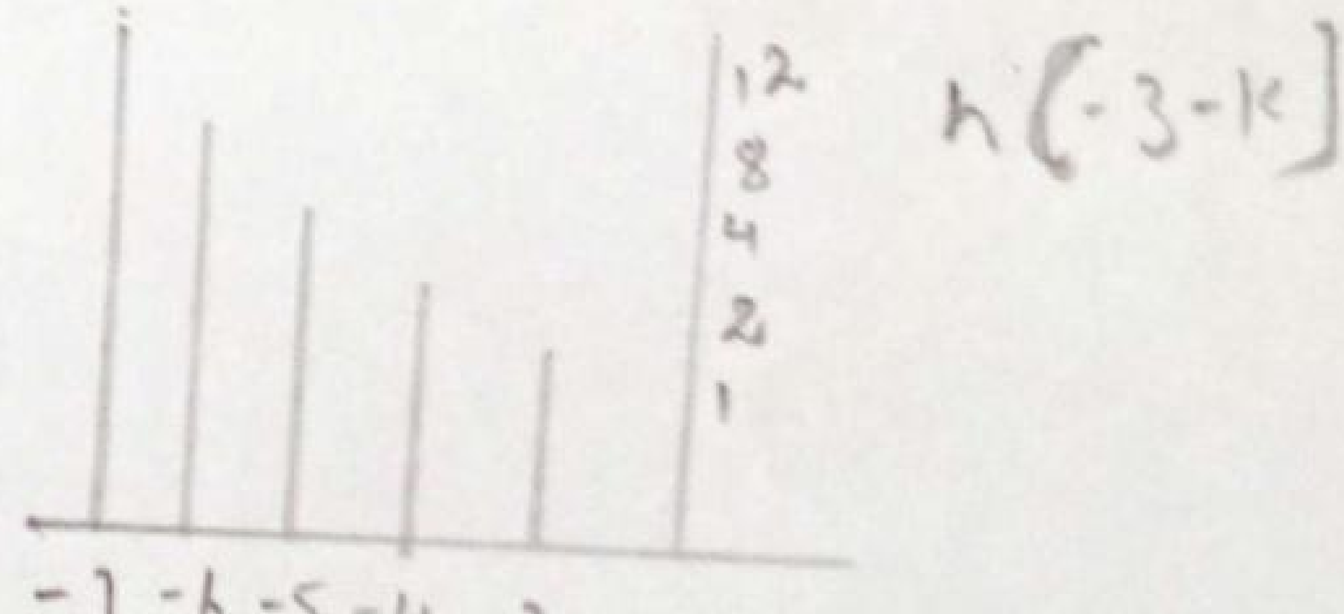
$y[-3] = a^{-2}$

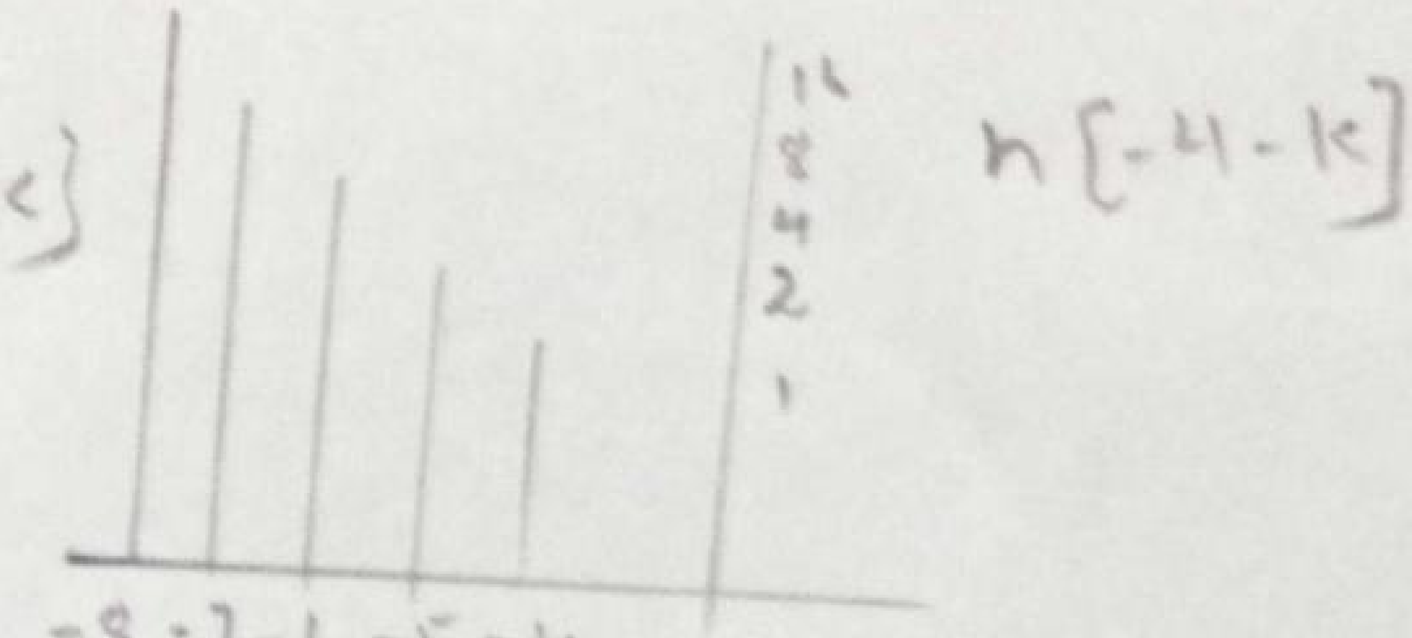


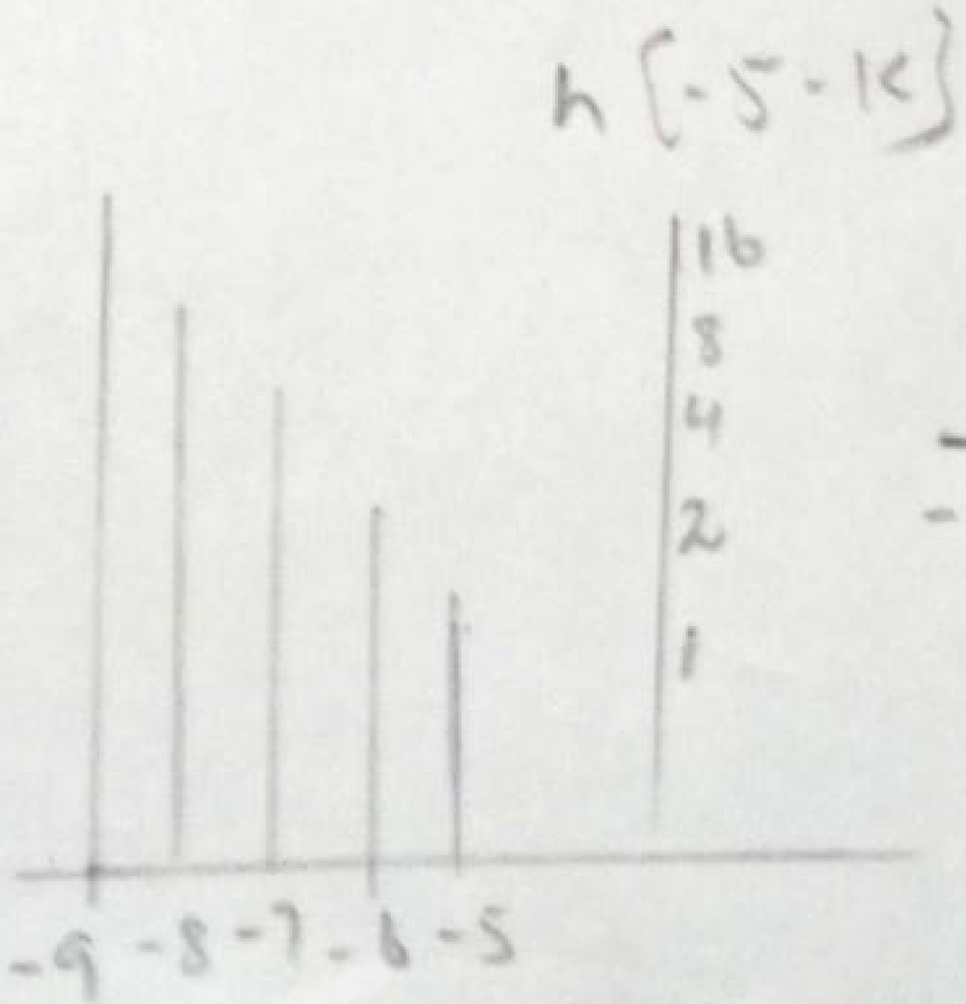
$y[1] = a^2 + 2a + 4 + 8a^{-1} + 16a^{-2}$

$y[2] = a^3 + 2a^2 + 4a + 8 + 16a^{-1}$

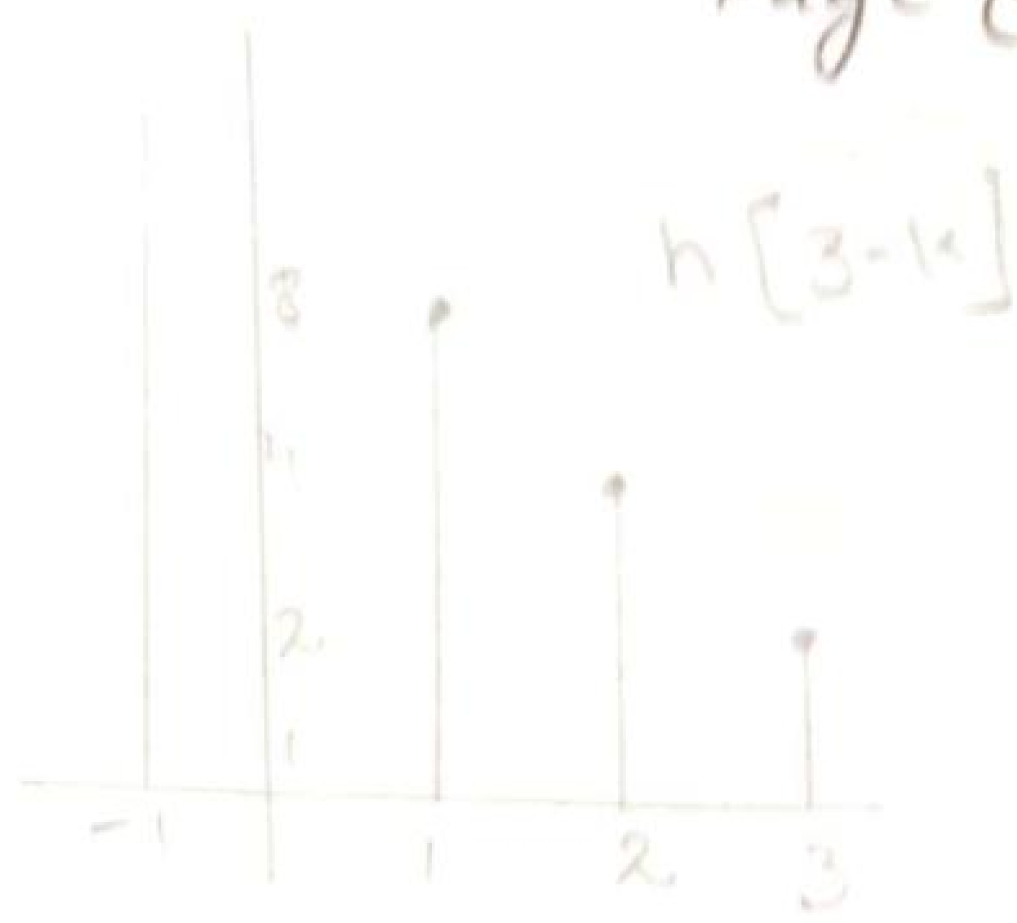






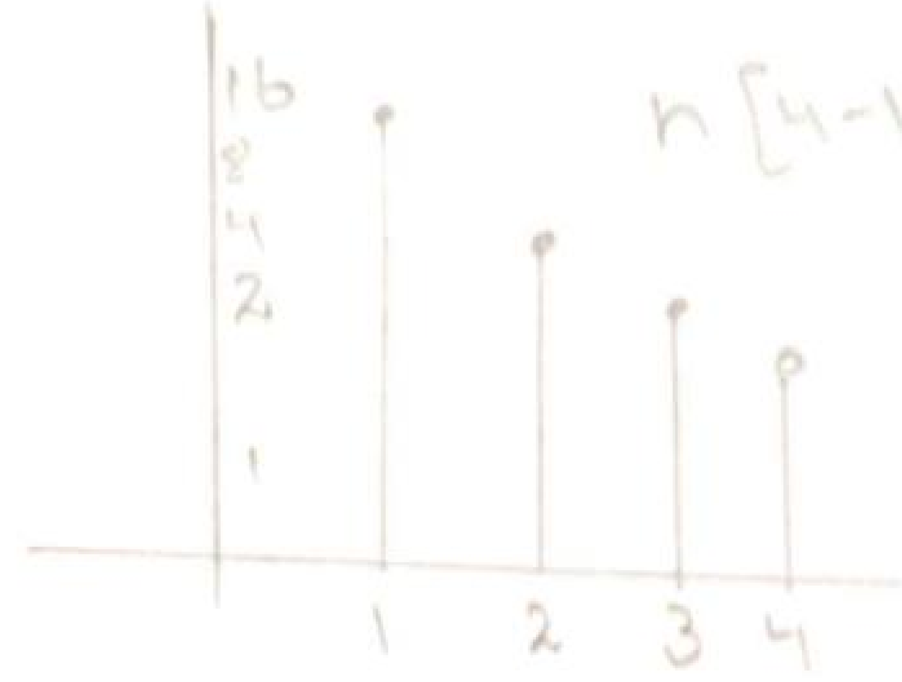


$y[3] = a^4 + 2a^3 + 4a^2 + 8a + 16$

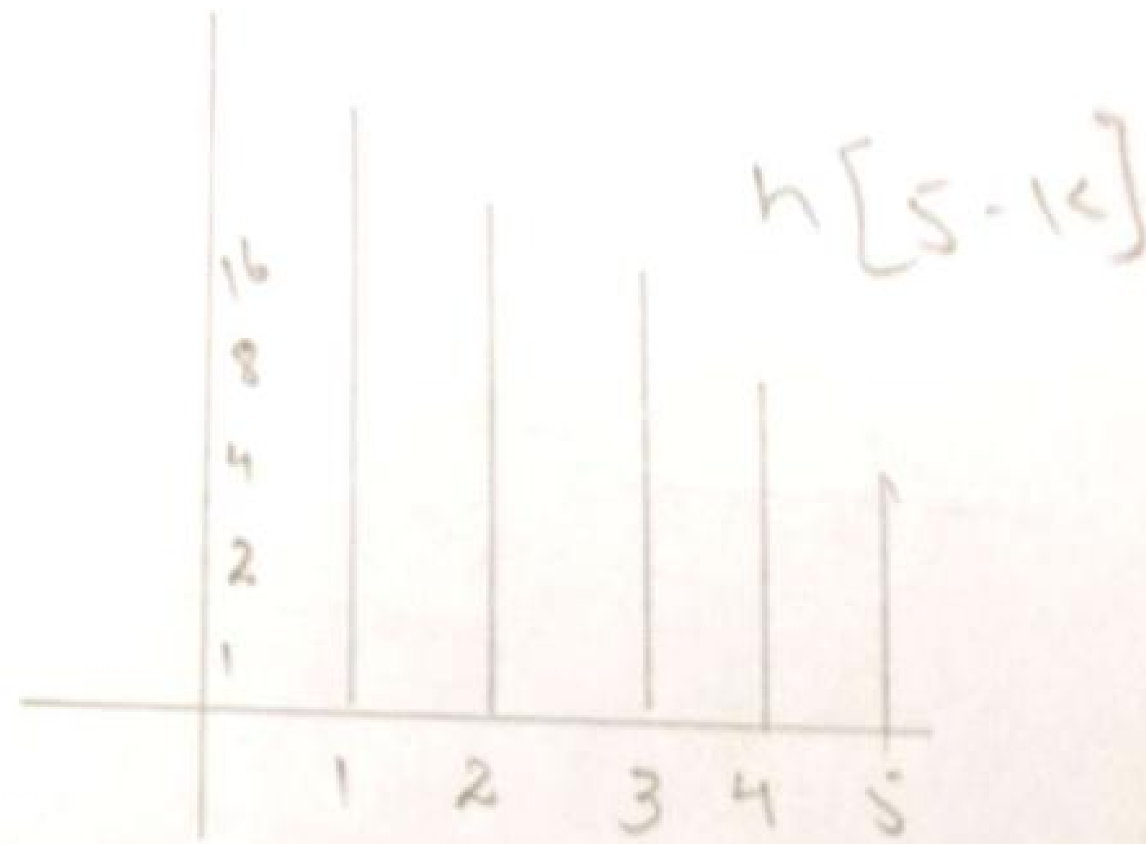


$y[4] = a^5 + 2a^4 + 4a^3 + 8a^2 + 16a$

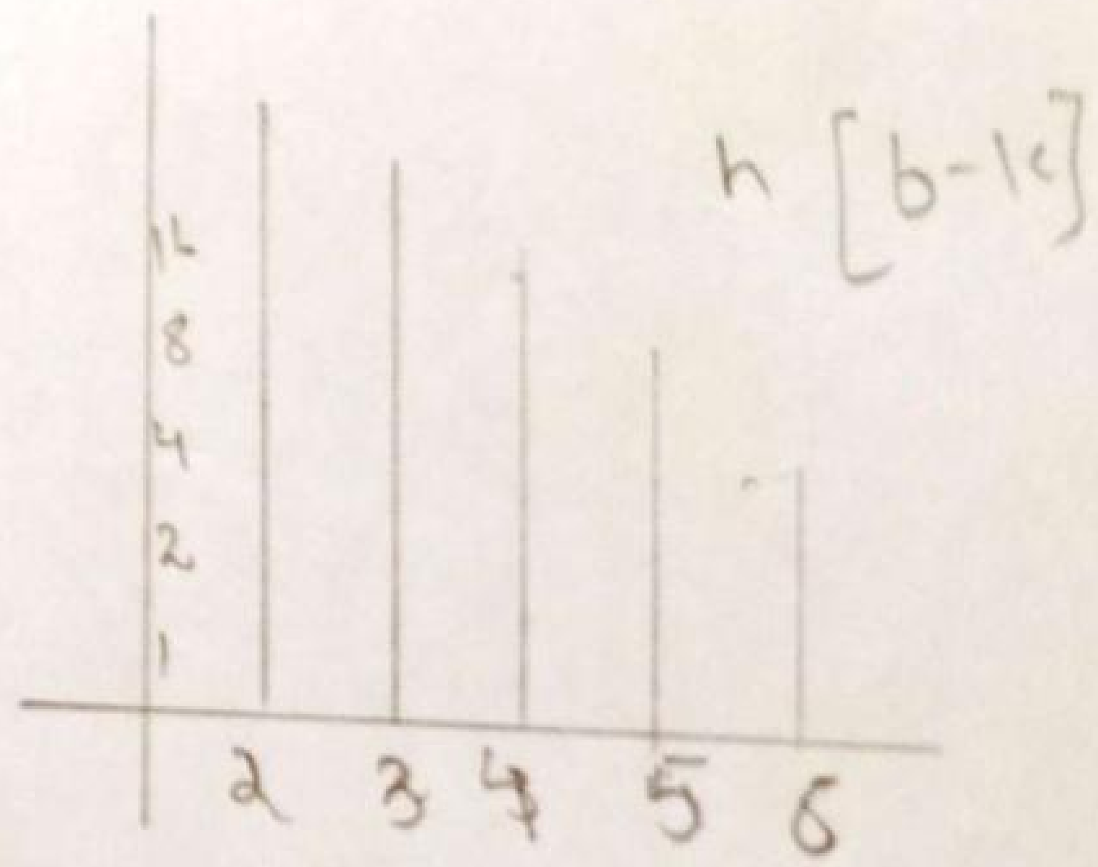
$y[$



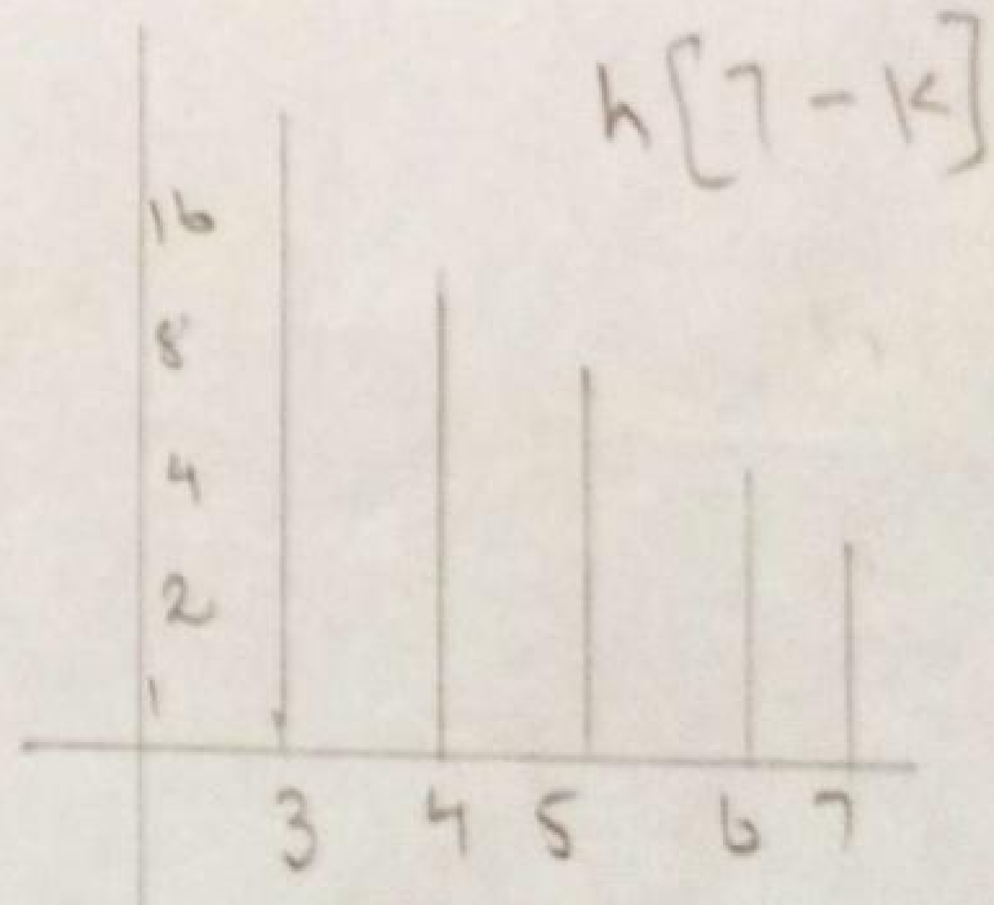
$y[5] = 16a^2 + 8a^3 + 4a^4 + 2a^5 + a^6$



$y[6] = 16a^3 + 8a^4 + 4a^5 + 2a^6$

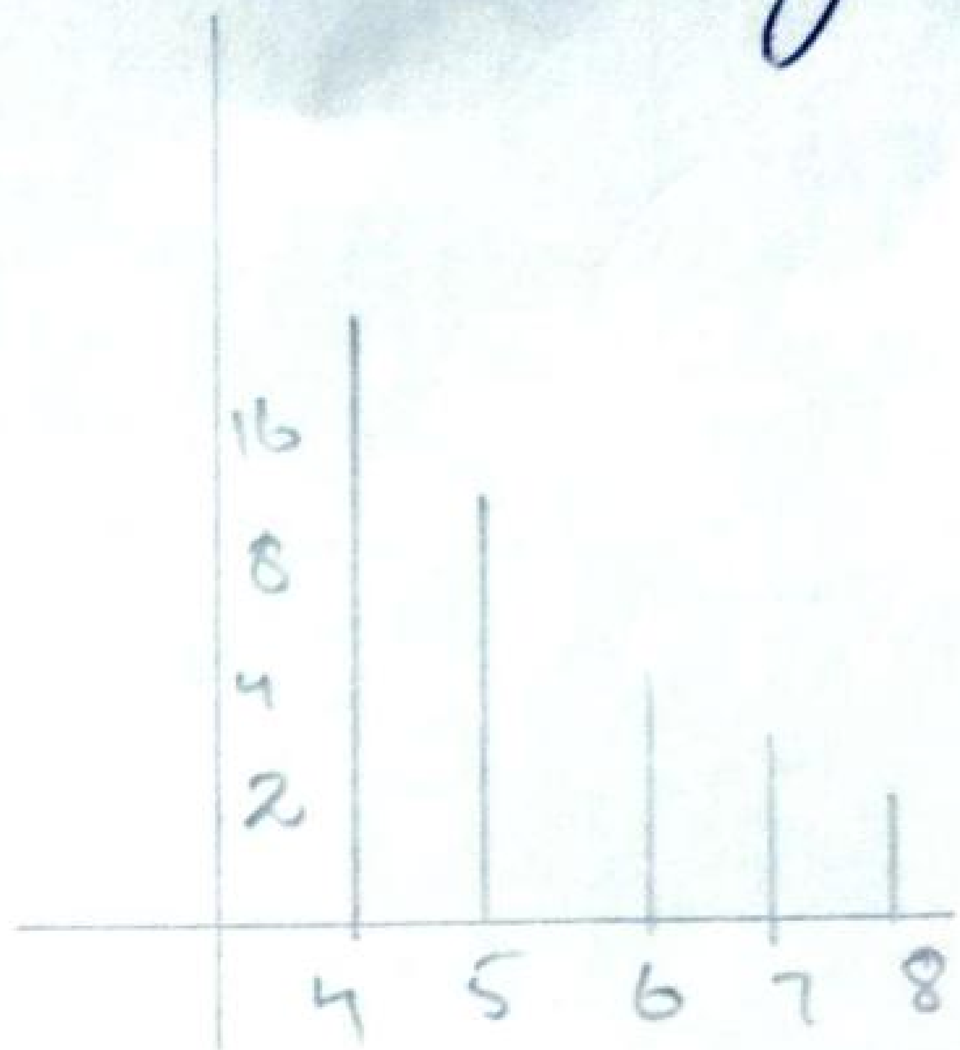


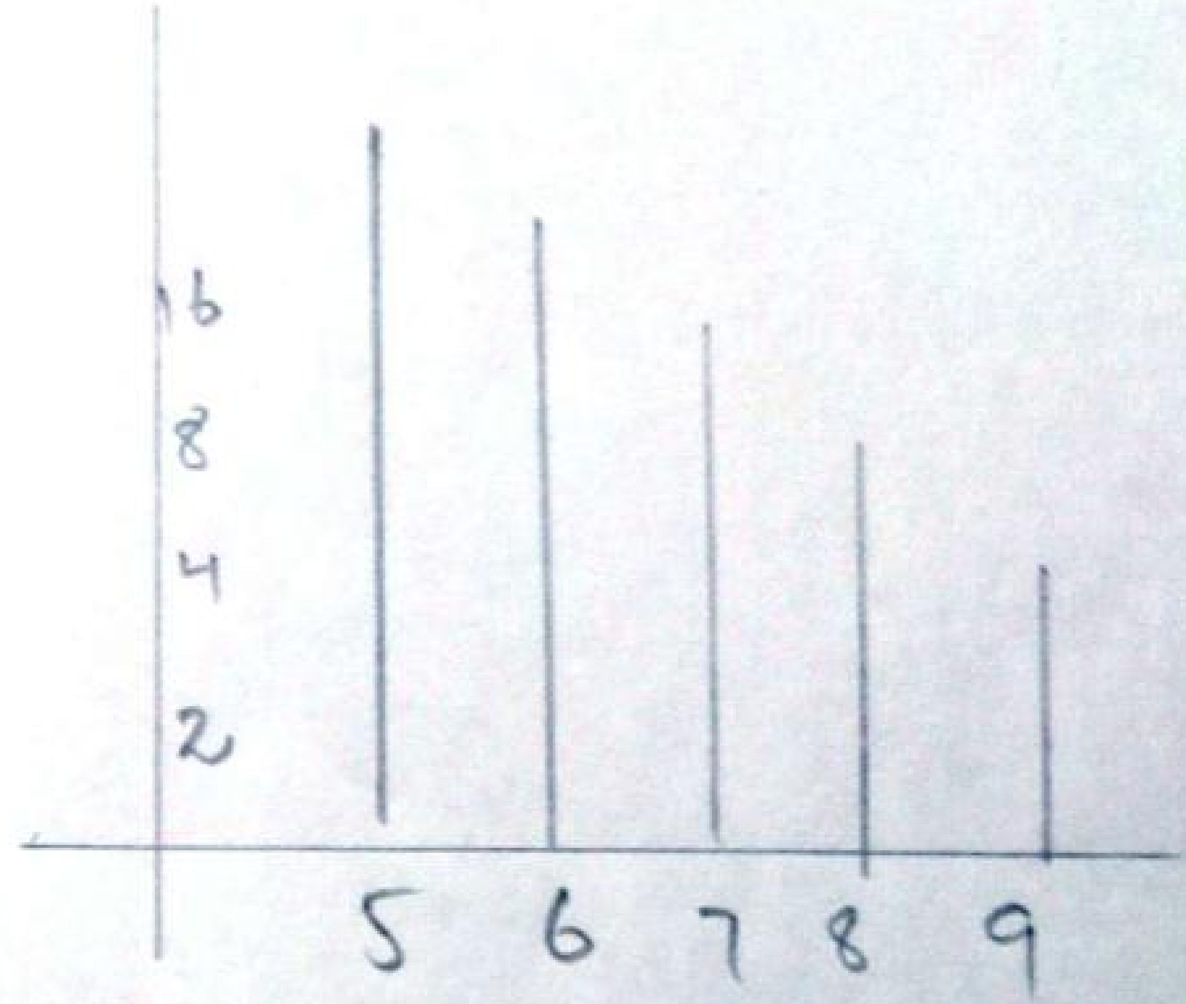
$y[7] = 16a^4 + 8a^5 + 4a^6$



$y[8] = 16a^5 + 8a^6$

$y[9] = 16a^6$





Q3:- Determine the z-transform of the following signals and also sketch its Region of Convergence (ROC)

i. $x(n) = \begin{cases} (\frac{1}{4})^n, & n \geq 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{cases}$

Sol:- The Z-transform pair is

$$x(n) = a^n u(n) \Rightarrow x(z) \frac{1}{1-\alpha z^{-1}} \quad ROC|z|$$

put the values of the above equation we get

$$x(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{n} - 1$$

using geometric series

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{3}z} - 1$$

taking LCM

$$= 1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1} - \left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)$$

$$= \frac{1-\frac{1}{3}z + 1-\frac{1}{4}z^{-1} - \left(1-\frac{1}{3}z - \frac{1}{4}z^{-1} + \frac{1}{12}\cancel{z^{-1}}\cancel{z}\right)}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$= \frac{1+\frac{1}{12}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

So ROC is $\frac{1}{4} < |z| < 3$

ii. $x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$

Sol:-

$x(n) = \begin{cases} \left(\frac{1}{2}\right) - 3, & n \geq 0 \\ 0 & \text{elsewhere} \end{cases}$

So in z-transform form

$$x(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} - \sum_{n=0}^{\infty} 3^n z^{-n}$$

$$\frac{1}{1 - \frac{1}{2} z^{-1}} - \frac{1}{1 - 3z^{-1}}$$

taking LCM

$$= \frac{1 - 3z^{-1} - 1 - 1/2\, z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - 3z^{-1}\right)}$$

taking LCM

$$= \frac{1 - 3z^{-1} - 1 - 1/2\, z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - 3z^{-1}\right)}$$

$$= \frac{-\frac{5}{2} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - 3z^{-1}\right)}$$

Hence the ROC is $|z| > 3$