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SUBJECT: DIFFERENTIAL EQUATION:

Question: 1:-

Use any of the methods for solving the ordinary differential Equations.

=) Solve and graph the solutions.

$$x^2y'' - 4xy' + 6y = 0 \quad , \quad y(1) = 0.4$$
$$y'(1) = 0.$$

Solution:-

putting $y = x^m$.

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

putting y'' into the give ODE:

$$x^2 m(m-1)x^{m-2} - 4xmx^{m-1} + 6x^m = 0.$$

$$x^2 m(m-1) x^m . x^{-2} - 4xmx^m . x^{-1} + 6x^m = 0.$$

since x^m is a common factor, dropping it gives:

$$m(m-1) - 4m + 6 = 0.$$

$$m_{1/2} = \frac{-2 \pm \sqrt{2^2 - (4)(0.75)}}{2}.$$

$$m_{1/2} = \frac{-2 \pm 1}{2}.$$

so, it has the distinct real roots:

$$m_1 = \frac{-1}{2} \wedge m_2 = -\frac{3}{2}$$

Real different roots provide two real solution.

$$y_1 = x^{m_1} = x^{-1/2} = x^{-0.5} \wedge y_2 = x^{m_2} = x^{-3/2}$$

So, the general solution is:

$$y = c_1 y_1 + c_2 y_2.$$

$$= c_1 x^{-0.5} + c_2 x^{-1.5}.$$

$$y' = -0.5 c_1 x^{-1.5} - 1.5 c_2 x^{-2.5}.$$

Now we need to determine c_1 & c_2 from IVP.

$$\begin{cases} 1 = y(1) = c_1 \cdot 1^{-0.5} + c_2 \cdot 1^{-1.5}. \\ 1.5 = y'(1) = -0.5 c_1 \cdot 1^{-1.5} - 1.5 c_2 \cdot 1^{-2.5} \end{cases}$$

Now we need to determine c_1 & c_2 from IVP.

$$\begin{cases} 0.4 = y(1) = c_1 \cdot 1^3 + c_2 \cdot 1^2 \\ 0 = y(1) = 3c_1 \cdot 1^2 + 2c_2 \cdot 1^1 \end{cases}$$

$$\begin{cases} 0.4 = C_1 + C_2 \\ 0 = 3C_1 + 2C_2 \end{cases}$$

$$\begin{cases} 0.4 - C_2 = C_1 \\ 0 = 3(0.4 - C_2) + 2C_2 \end{cases}$$

$$\begin{cases} 0.4 - C_2 = C_1 \\ 1.2 = C_2 \end{cases}$$

$$\begin{cases} 0.4 - 1.2 = C_1 \\ 1.2 = C_2 \end{cases}$$

$$\begin{cases} -0.8 = C_1 \\ 1.2 = C_2 \end{cases}$$

The particular solution ~~of~~ IVP is:

$$y = (-0.8x^3) + 1.2x^2$$

Answer.

Question:

$$x^2y'' + 3xy' + 0.75y = 0.$$

$$y(1) = 1,\ y'(1) = -1.5.$$

Solution :-

let,

$$y = x^m.$$

$$y' = m x^{m-1}$$

$$y'' = m(m-1)x^{m-2}.$$

put in the given ODE :

$$x^2 m(m-1)x^{m-2} + 3xmx^{m-1} + 0.75x^m = 0.$$

$$x^2 m(m-1)x^m \cancel{x^{-2}} + 3xmx^m \cancel{x^{-1}} + 0.75m = 0$$

Since x^m is common factor, by dropping it gives:

$$m(m-1) + 3m + 0.75 = 0.$$

$$m^2 + 2m + 0.75 = 0.$$

lets find the roots of equation.

$$m^2 + 2m + 0.75 = 0.$$

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14/ $x^2y'' + xy' + 9y = 0, \quad y(1) = 0, \quad y'(1) = 2.5$

Solution://

Let $y = x^m$, $y'' = m(m-1)x^{m-2}$

into The given ODE. This gives.

$$x^2 m(m-1)x^{m-2} + mx^m + 9x^m = 0$$

$$\cancel{x^2} m(m-1)x^m \cdot \cancel{x^{-2}} + mx^m + 9x^m = 0$$

We can see That x^m is a common factor, dropping it gives.

$$m(m-1) + m + 9 = 0 \Rightarrow m^2 - \cancel{m} + \cancel{m} + 9 = 0$$

$$\Rightarrow m^2 + 9 = 0 \; (*)$$

So $y = x^m$ is a solution of The given ODE if m is The root of equation.

Let find the root of equation.

$$m^2 + 9 = 0 \Leftrightarrow m^2 - (3i)^2 = 0 \Leftrightarrow (m - 3i)(m + 3i) = 0$$

So, it has the complex conjugate roots

$m_1 = 3i \quad \wedge \quad m_2 = 3i.$

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Neat subtract the second formula from the first & divide it by $2i$ after that:

$$x^{m_1} - x^{m_2} = \cancel{\cos(3\ln x)} + i\sin(3\ln x)$$
$$- \cancel{\cos(3\ln x)} + i\sin(3\ln x)$$
$$= 2i\sin(3\ln x)$$

Divide it by $2i$.

$$\frac{x^{m_1} - x^{m_2}}{2i} = \frac{\cancel{2i}\sin(3\ln x)}{\cancel{2i}} = \sin(3\ln x)$$

By the superposition principle, $\cos(3\ln x)$ & $\sin(3\ln x)$ are the solution of Euler-Cauchy Equation

Quotien is not constant, solution is

$y_1 = \cos(3\ln x)$ ∧ $y_2 = \sin(3\ln x)$.

are linearly independent & form a basis of solution.

So, general solution is :-

$y = c_1 y_1 + c_2 y_2$

And constitute a basis of solutions for the given ODE.

$y_1 = y \in R$.

So, the solution is.

$y = c_1 y_1 + c_2 y_2$

$= c_1 . \frac{1}{x} + c_2 . \frac{1}{x} . \ln x$

$= \boxed{\frac{1}{x} . (c_1 + c_2 \ln x)}$

$\Rightarrow y' = (x^{-1})^1 (c_1 + c_2 \ln x) + x^{-1} (c_1 + c_2 \ln x)'$

$= -x^{-2} (c_1 + c_2 \ln x + \frac{1}{x} \quad c_2 . \frac{1}{x}$

$= \frac{1}{x^2} (-c_1 - c_2 \ln x + c_2)$.

$\begin{cases} 3.6 = y(1) = \frac{1}{1} (c_1 + c_2 \ln 1) \\ 0.4 = y'(1) = \frac{1}{1^2} (-c_1 - c_2 \ln 1 + c_2) \end{cases} \Rightarrow \begin{cases} 3.6 = c_1 \\ 0.4 = -c_1 + c_2 \end{cases}$

$\Rightarrow \begin{cases} 3.6 = c_1 \\ 0.4 = -3.6 + c_2 \end{cases}$

$\Rightarrow \begin{cases} 3.6 = c_1 \\ 4.0 = c_2 \end{cases}$

The particular solution of IVP is

$y = (3.6 + 4.0 \ln x) \cdot 1/x$.

We can find a second linearly independent sol y_0 using the method of reduction of orders.

First we need to write the given ODE in standard form.

$$y'' + 3/x \cdot y' + \frac{1}{x^2} \cdot y = 0.$$

Now we can see that

$$p(x) = 3 \cdot \frac{1}{x} \Rightarrow \int p\,dx = 3\ln|x|$$

Put

$$y_0 = u y_1$$

where

$$u = \int U\,dx \;\&\; U = \frac{1}{y_1^2} e^{-\int p\,dx}.$$

Let find U

$$e^{-\int p\,dx} = e^{-3\ln|x|} = (e^{\ln|x|})^{-3} = x^{-3}$$

$$\Rightarrow U = x^{-3} \cdot \frac{1}{x^2} = x^{-3+2} = x^{-1} = \frac{1}{x}.$$

By integrating we have,

$$u = \int \frac{dx}{x} = \ln|x|.$$

So, $y_0 = u y_1 = y_1 \ln x = \frac{1}{x} \ln x$

Since quotient is not constant, y_1 & y_2 are linearly independant.

$(x^2D^2 - 3xD + 4I)y = 0$, $y(1) = -\pi$, $y'(1) = -2\pi$

First we need to apply the given operator to the given function.

$x^2D^2y - 3xDy + 4Iy = x^2D(Dy) - 3xDy + 4y.$

$= x^2y'' - 3xy' + 4y.$

Lets solve the equation.

Let subtitute

$y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$.

$x^2m(m-1)x^{m-2} - 3xmx^{m-1} + 4x^m = 0.$

$x^2 m(m-1)x^m \cdot \cancel{x^{-2}} - 3xmx^m \cdot \cancel{x^{-1}} + 4x^m = 0$

x^m is common factor, drop it.

$m(m-1) - 3m + 4 = 0 \Leftrightarrow m^2 - 4m + 4 = 0.$

So $y = x^m$ is a solution of the given ODE.

Let find the root of the equation.

$m^2 - 4m + 4 = 0 \Leftrightarrow (m-2)^2 = 0$.

So, it has the double root

$m = 2$.

Real double root m provide a real solution.

$y_1 = x^m = x^2$.

the given ODE, for all x for which $y_1 = y_2 \in R$

So solution is:

$y = C_1 y_1 + C_2 y_2$

$= C_1 x^2 + x^2 \ln x$.

$= \boxed{x^2 (C_1 + C_2 \ln x)}$.

$\Rightarrow y' = (x^2)' (C_1 + C_2 \ln x) + x^2 (C_1 + C_2 \ln x)'$.

$= 2x (C_1 + C_2 \ln x) + C_2 x^2 \cdot 1/x$.

$= 2C_1 x + 2C_2 x \ln x + C_2 x$.

$= 2C_1 x + C_2 x (2 \ln x + 1)$.

Now all we need to determine C_1 & C_2 from IVP.

$\begin{cases} -\pi = y(1) = 1^2 (C_1 + C_2 \ln 1) \\ 2\pi = y'(1) = 2C_1 + C_2 (2 \ln 1 + 1) \end{cases} \rightarrow \begin{cases} -\pi = C_2 \\ 2\pi = 2C_1 + C_2 \end{cases}$

The solution is

$$\boxed{y = x^2 (-\pi + 4\pi \ln x)}$$

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We can find a second linearly independent sol y_2 using the method of reduction of order.

First we need to write the given ODE in Standard form:

$$y'' - \frac{3}{x} \cdot y' + \frac{4}{x^2} \cdot y = 0.$$

Now we find that

$$p(x) = -3 \cdot \frac{1}{x} \Rightarrow \int p\,dx = -3 \ln|x|.$$

Put

$$y_2 = u y_1.$$

where

$$u = \int U\,dx \quad \wedge \quad U = \frac{1}{y_1^2} e^{-\int p\,dx}.$$

lets find U

$$e^{-\int p\,dx} = e^{3 \ln|x|} = (e^{\ln|x|})^3 = x^3.$$

$$\Rightarrow U = x^3 \cdot \frac{1}{(x^2)^2} = x^{3-4} = x^{-1} = \frac{1}{x}.$$

By integrating, we have.

$$u = \int \frac{dx}{x} = \ln|x|.$$

So

$$y_2 = u y_1 = y_1 \ln x = x^2 \ln x.$$

Since the quotient is not constant y_1 & y_2 are linearly independent.

Now we use the fact that $x = e^{\ln x}$.

$$x^{m_1} = x^{3i} = \left(e^{\ln x}\right)^{3i} = e^{3i\ln x}.$$

$$x^{m_2} = x^{-3i} = \left(e^{\ln x}\right)^{-3i} = e^{-3i\ln x}$$

Recall That.

$$e^{z} = e^{a+ib} = e^{a}(\cos b + i\sin b), \quad z \in i.$$

So we have.

$$e^{3i\ln x} = e^{0}\left(\cos(3\ln x) + i\sin(3\ln x)\right) = \cos(3\ln x) + i\sin(3\ln x)$$

This Gives.

$$x^{m_2} = \cos(3\ln x) + i\sin(3\ln x)$$

$$x^{m_2} = \cos(3\ln x) - i\sin(3\ln x).$$

Adding the formula gives.

$$x^{m_1} + x^{m_2} = \cos(3\ln x) + \cancel{i\sin(3\ln x)} + \cos(3\ln x) - \cancel{i\sin(3\ln x)} - i\sin(3\ln x)$$

divide it by 2.

$$\frac{x^{m_1} + x^{m_2}}{2} = \frac{2\cos(3\ln x)}{2} = \cos(3\ln x)$$

$m^2 - 5m + 6 = 0.$

Now, lets find root of equation.

$m^2 - 5m + 6 = 0.$

$$m_{1/2} = \frac{5 \pm \sqrt{(-5)^2 + 4 \cdot 6}}{2}.$$

$$m_{1/2} = \frac{5 \pm 1}{2}.$$

So, it has the distinct real root.

$m_1 = 3 \wedge m_2 = 2.$

Real different roots $m_1 < m_2$ provide two real solutions:

$y_1 = x^{m_1} = x^3 \wedge y^2 = x^{m_2} = x^2$

So, the solution is

$y = C_1 y_1 + C_2 y_2$

$= C_1 x^3 + C_2 x^2.$

$y' = 3C_1 x^2 + 2C_2 x.$

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17) $(x^2D^2 + xD + I)y = 0$, $y(1) = 1$, $y'(1) = 1$.

First Applying given operation to the function.

$$x^2 D^2 y + xDy + Iy = x^2 D(Dy) + xDy + y$$

$$= x^2 y'' + xy' + y.$$

Now

$$y = x^m,\ y' = mx^m,\ y'' = m(m-1)$$

$$= x^2 m(m-1) \cancel{x^m} \cdot \cancel{x^{-2}} \cancel{x}\, m x^m x^{-1} + x^m = 0$$

Dropping the common factor x^m

$$m(m-1) + m + 1 = 0 \Rightarrow m^2 - \cancel{m} + \cancel{m} + 1 = 0$$

$$= m^2 + 1 = 0.$$

Now finding the roots.

$$m^2 + 1 = 0,\ m^2 + 1^2 = 0,\ (m-i)(m+i) = 0.$$

$$m_1 = i \quad \wedge \quad m_2 = -i$$

$$x^m = x^i = (e^{\ln x})^i = e^{i\ln x}.$$

$$e = e^{a+b} = e^a(\cos b + i\sin b),\ z \in C.$$

$$= \cos(\ln x) + i\sin(\ln x).$$

$$x^{m_1} = \cos(\ln x) + i\sin(\ln x).$$

$$x^{m_2} = \cos(\ln x) - i\sin(\ln x).$$

$$\frac{x^{m_1} + x^{m_2}}{2} = \cos(\ln x) + i\sin(\ln x) + \cos(\ln x) - i\sin(\ln x)$$

$$= 2\cos(\ln x)$$

$$\frac{u^{m_1} + u^{m_2}}{2} = \cos(\ln(x))$$

$$u^{m_1} - u^{m_2} = \cancel{\cos(\ln x)} + i\sin(\ln x) - \cancel{\cos(\ln x)} + i\sin(\ln x)$$

$$\frac{u^{m_1} - u^{m_2}}{2} = \sin(\ln x)$$

$$y' = \cos(\ln x) \wedge y_2 = \sin(\ln x).$$

$$y' = -C_1 \sin(\ln x) \cdot (\ln x)^1 + C_2 \cos(\ln x)(\ln x)$$

$$= \frac{-C_1}{x} \sin(\ln x) + \frac{C_2}{x} \cos(\ln x)$$

To determine C_1 & C_2

$$\begin{cases} 1 = C_1 \cos(0) + C_2 \sin(0) \\ 1 = -C_1 \sin(0) + C_2 \cos(0) \end{cases}$$

$$\begin{cases} 1 = C_1 \\ 1 = C_2 \end{cases}$$

$$y = \sin(\ln x) + \cos(\ln x).$$

18) $(9x^2D^2 + 3xD + I)y = 0 \quad y(1) = 1, y'(1) = 0.$

Apply given operation to the equation.

$$9x^2D^2y + 3xDy + Iy = 9x^2D(Dy) + 3xDx$$
$$= 9x^2y'' + 3xy' + y.$$
$$9x^2y'' + 3xy' + y = 0.$$

let $y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$.

$$9x^2 m(m-1)x^{m-2} + 3xmx^{m-1} + x^m = 0.$$
$$9x^2(m-1) + 3m + 1 = 0, \quad 9m^2 - 9m + 3m + 1 = 0$$
$$= 9m^2 - 6x + 1 = 0.$$

find the roots of equation's.

$$m^2 - 4m + 4 = 0, \quad (m-2)^2 = 0.$$
$$9m^2 - 6m + 1 = 0 \quad m/2 = \frac{6 \pm \sqrt{6^2 - 4 \cdot 9}}{18}$$
$$m/2 = 6/18$$
$$m/2 = 1/3$$
$$m = 1/3$$

$$y_1 = x^m = x^{1/3}$$

$$y'' + \frac{1}{3x} \cdot y' + \frac{1}{9x^2} \cdot y^0 = 0$$

$$p(x) = \frac{1}{3} \cdot \frac{1}{x} \Rightarrow \int P\,dx = \frac{1}{3} \ln(x)$$

$$y_2 = u \cdot y_1$$

Finding u.

$$u\, e^{-\int p\,dx} = e^{-1/3 \ln(x)} = \left(e^{\ln|x|}\right)^{-1/3} = x^{-1/3}$$

$$u = x^{-1/3} \cdot \frac{1}{(x^{1/3})^2} = x^{-1/3 - 2/3} = u^{-1} = \frac{1}{x}.$$

$$u = \int \frac{dx}{x} = \ln|x|.$$

$$y_1 = u y_1 = y_1 \ln(x) = x^{1/3} \ln x.$$

here

$$y = c_1 y_1 + c_2 y_2.$$

$$x^{1/3} (c_1 + c_2 \ln x)$$

$$y' = (x^{1/3})'(c_1 + c_2 \ln x) + x^{1/3}(c_1 + c_2 \ln x)'.$$

$$= \frac{1}{3} \cdot x^{-2/3}(c_1 + c_2 \ln x) + x^{1/3}\, c_2 .$$

$$\begin{cases} 1 = y(1) = 1^{1/3}(c_1 + c_2 \ln 1) \\ 0 = y'(1) = \frac{1}{3} \cdot 1^{-2/3}(c_1 + c_2 \ln 1) + 1^{1/3} c_2 \end{cases}$$

$$\begin{cases} 1 = c_1 \\ 0 = \frac{c_1}{3} + c_2 \end{cases}$$

$$\begin{cases} 1 = c_1 \\ -1/3 = c_2 \end{cases}$$

$$y = x^{1/3}\left(1 - \frac{1}{3}\ln x\right)$$ Answer.

19) $(x^2D^2 - xD - 151)y = 0$. $y(1) = 1$

$y'(1) = 4.5$

Sol

$x^2D^2y - xDy - 151y = x^2D(Dy) - xDy - 15y$.

$\Rightarrow x^2y'' - xy' - 15y$.

$x^2 m(m-1)x^{m-2} - x\,mx^{m-1} - 15x^m = 0$.

$x^2 m(m-1)x^m - x^{-2} - xmx^m x^{-1} - 15x^m = 0$

Dropping x^m.

$= m(m-1) - m - 15 = 0$

$= m^2 - 2m - 15 = 0$.

Finding roots.

$m^2 - 2m - 15 = 0$.

$$m_{1/2} = \frac{2 \pm 8}{2}.$$

$m_1 = 5 \quad \wedge \quad m_2 = -3$.

The two real solutions are

$y = C_1y_1 + C_2y_2$

$= C_1x^5 + C_2x^{-3}$

$y' = 5C_1x^4 - 3C_2x^{-4}$.

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Now to determine the C_1 & C_2.

$$\begin{cases} 0.1 = y(1) = C_1 \cdot 1^5 + C_2 \cdot 1^{-3} \\ -4.5 = (y')(1) = 5C_1 \cdot 1^4 - 3C_2 \cdot 1^{-4} \end{cases}$$

$$\begin{cases} 0.1 - C_2 = C_1 \\ -4.5 = 5(0.1 - C_2) \cdot 3C_2 \end{cases}$$

$$\begin{cases} 0.1 - 0.625 = C_1 \\ 0.625 = C_2 \end{cases}$$

$$\begin{cases} -0.525 = C_1 \\ 0.625 = C_2 \end{cases}$$

Dertication solution:

$$y = -0.525x^5 + 0.625x^{-3}$$

Ans.

Q2-1-a $x' = \sqrt{x}$.

Sol:-

$$dx/dt = x.$$

$$\frac{dx}{\sqrt{x}} = 1 \cdot dt.$$

$$\int \frac{1}{\sqrt{x}} \cdot dx = \int dt.$$

$$2\sqrt{x} = t + c.$$

$$4(x) = (t+c)^2.$$

$$x = \frac{(t+c)^2}{4} \quad \text{Ans.}$$

Q2-1-b $x' = e^{-2x}$.

$$= \frac{dx}{dt} = e^{-2x} \Rightarrow \frac{dx}{e^{-2x}} = dt$$

$$= \int \frac{dx}{e^{-2x}} = \int dt.$$

$$= e^{2x}/2 = t + c.$$

$$= e^{2x} = 2(t+c)$$

$$= x = \frac{\ln 2(t+c)}{2} \quad \text{Ans.}$$

Q 1 - 1

(C) $y' = 1+y^2$.

Sol:-

$$\frac{dy}{dt} = 1+y^2 \Rightarrow \frac{dy}{1+y^2} = dt$$

$$\int dy \cdot \frac{1}{1+y^2} = \int dt$$

$$y(1) = \tan(t+c) \quad \text{Ans.}$$

Q 2 - 1

(d) $u' = 1/5-2u$

Sol:-

$$\frac{dy}{dt} = \frac{1}{5 \cdot 2u} \Rightarrow \frac{dy}{5 \cdot 2u} = dt(1)$$

$$= \int dy \frac{1}{5-2u} = \int dt$$

$$= -(u-5)u + C_1 = t + C_2$$

$$= -(u-5)u = t+C$$

$$u = \frac{t+C}{(-u-5)} \quad \text{Ans.}$$

Q2 - 1

f) $\theta' = \frac{\theta}{4+\theta^2}$

$\frac{d\theta}{dt} = \frac{\theta}{4+\theta} \therefore \frac{\theta^2 d\theta}{\theta} = dt$

$\int \frac{\theta^2}{\theta} \cdot d\theta = \int dt$

$3 \ln|\theta) + \frac{\theta^2}{2} + C$ Ans.

Q2 - 1

g) $x' = e^{x^2}$

$\frac{dx}{dt} = e^{x^2}$

$\frac{dx}{e^{x^2}} = dt$

$\int \frac{1}{e^{x^2}} \cdot dx = \int dt$

$\frac{\sqrt{\pi}\, erf(a)}{2} = t + C$

$$u = \frac{2t + c}{\sqrt{\pi}}$$

$$x(1) = \frac{2t + c}{\sqrt{\pi}} \quad \text{Ans.}$$

Q2 - 1

(h) $y' = r(a-y)$.

$$\frac{dy}{dt} = r(a-y).$$

$$\frac{dy}{r(a-y)} = dt.$$

$$\int \frac{1}{r(a-y)} \cdot dy = \int dt.$$

$$\frac{(a-y)u}{r} = t + c.$$

$$x(t) = \frac{r(t+c)}{a-y} \quad \text{Ans.}$$

Q 2 - 2

Solve $y' = r(a-y)$, where r & a are constants.

Sol:-

$$\frac{dy}{dt} = r(a-y)$$

$$\frac{dy}{r(a-y)} = dt$$

$$\int \frac{1}{r(a-y)} \, dy = \int dt$$

$$a - y = k(r)^{1/2}$$

Q 2 - 3, 1(a)

$(1a)$ 7, $x(0) = 1$

$$u(t) = \frac{(t+c)^2}{4}$$

$u(0) = 1$, $u(t) = u(0) = 1$

$$1 = \frac{(0+c)^2}{4}$$

$\boxed{4 = c^2}$, $\boxed{c = 2}$ Ans

1b

$$u(t) = \frac{\ln 2t}{2} + c$$

$x(0) = 1$

$$1 = \frac{\ln 2(0) + e}{2}.$$

$$1 = \frac{\ln (0) + c}{2}$$

$2 = \ln e^2$ Ans.

2-4)

(a) $u' = \frac{\cancel{2y}\ 2u}{t+1}.$

$$\frac{du}{dt} = \frac{2u}{t+1}.$$

$$\frac{dx}{2x} = \frac{1}{t+1} \cdot dt.$$

$$\int \frac{du}{2u} = \int \frac{1}{t+1} \cdot dt.$$

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$$\frac{x^2}{4} = \ln(t+1) + C$$

$$\frac{x^2}{4\ln(t+1)} = C \Rightarrow C = \frac{x^2}{4\ln(t+1)}$$ Ans.

Q 2-4)

(b)

$$\theta' = t\sqrt{t^2+1}\ \sec\theta.$$

$$\frac{d\theta}{dt} = t\sqrt{t^2+1}\ \sec\theta.$$

$$\frac{d\theta}{\sec\theta} = t\sqrt{t^2+1}\ dt.$$

$$\int \frac{d\theta}{\sec\theta} = \int t\sqrt{t^2+1}\cdot dt.$$

$$= \sin\theta = \frac{(t^2+1)^{3/2}}{3} + C.$$

Answer.

Q 2 - 4

(c) $(2u+1)u' - (t+1) = 0$

$$(2u+1)\frac{du}{dt} = (t+1)$$

$$(2u+1)\,du = (t+1)\cdot dt$$

$$\int (2u+1)\cdot du = \int (t+1)\cdot dt$$

$$u^2+u = \frac{t^2}{2} + t + c$$

$$\frac{2(u^2+u)}{t^2} - t = c$$

$$c = \frac{2(u^2+u)}{t^2} - t$$

Ans.

Q2-4

Q: (D)

$$R' = (t+1)(R^2+1)$$

$$\frac{dR}{dt} = (t+1)(R^2+1)$$

$$\frac{dR}{R^2+1} = (t+1) \cdot dt$$

$$\int \frac{dR}{R^2+1} = \int (t+1) \, dt$$

$$\cot(R) = \frac{t^2}{2} + t + C$$

$$C = \frac{2\cot(R)}{t^2} - t \quad \text{Ans}$$

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