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I.D = 7759

Section = "C"

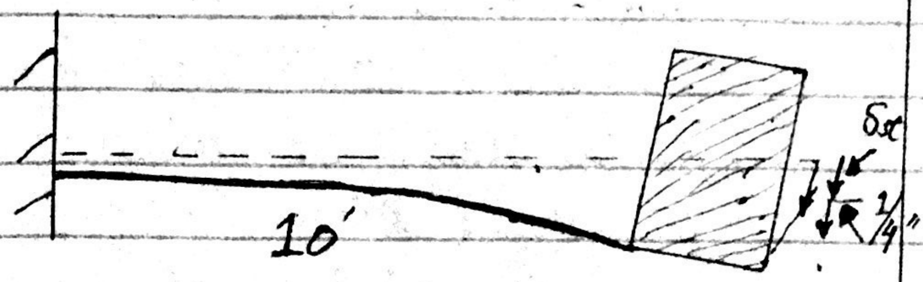
Assignment = Introduction to earth quake engineering.

Date : 29/06/2020

Question No. (01)

Question No. (01) Answer

Given Data:



$E = 29,000$ KSI
$I = 150$ inch^4
Load = 7759
Δ (Deflection) = $1/2"$

The general E.O.M for SDOF system is.

$$Ku + c\dot{u} + m\ddot{u} = P(t)$$

In our case system is undamped $(C=0)$ Undegoing free vibration $(P(t)=0)$.

Hence general EOM become

$$Ku + m\ddot{u} = 0 \quad \dots (1)$$

Question No. (02)

$K = 3EI/L^3$

$= \frac{3 \times 29000 \frac{K}{in^2} \times 150 in^4}{(10 \times 12 in)^3}$

$= 7.55 K/in$

In order to eliminate the chances of mistake during calculation, it is more appropriate to use fundamental units like lb, ft sec or Kg, m, sec.

$K = 7.55 K/in = 90625 lb/ft$

$m = \frac{7759 sec^2}{32.2 ft} = 241$ slug.

$\omega_n = \sqrt{K/m} = \sqrt{96025/241} =$

$W_n = 19.96$ rad/sec

$T_n = 2\pi/\omega_n = 2\pi/19.96$

$T_n = \frac{2 \times 3.14}{19.96} = 0.314$ sec

Subsituting the corresponding values in eq - (1)

$90625u + 241\ddot{u} = 0$

where "K" is in lb/ft and 'm' is in lb sec/ft²

General Solution to the EOM for Undamped Free Vibration is,

$$u(t) = u(0)\cos(\omega_n t) + \dot{u}(0)/\omega_n \sin(\omega_n t)$$

$$u(0) = 1/2'' = 1/24\ ft \text{ and } \dot{u}(0) = 0$$

$$u(t) = (1/24) \times \cos(19.96t) + 0 .$$

$$= (1/24) \times \cos(19.96t)$$

Equivalent static force at any time "t" is

$$F_s(t) = K.U(t) = \frac{90625 \times \cos(19.96t)}{24}$$

$$F_s(t) = ~~3549~~\ 3776(\cos 19.96t)$$

Amplitude of dynamic displacement, u_o for undamped free vibration is

$$u_o = \sqrt{\left[(u(0))^2 + \left(\dot{u}(0)/\omega_n \right)^2 \right]}$$

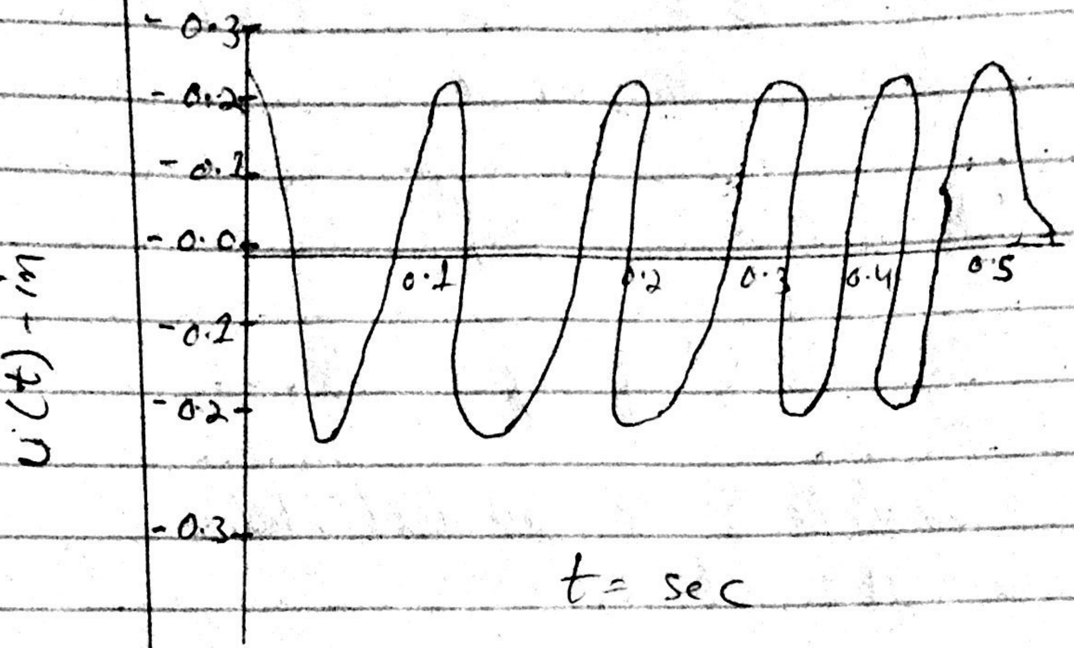
$$= \sqrt{\left((1/24)^2 + 0 \right)} = 1/24 \text{ ft}$$

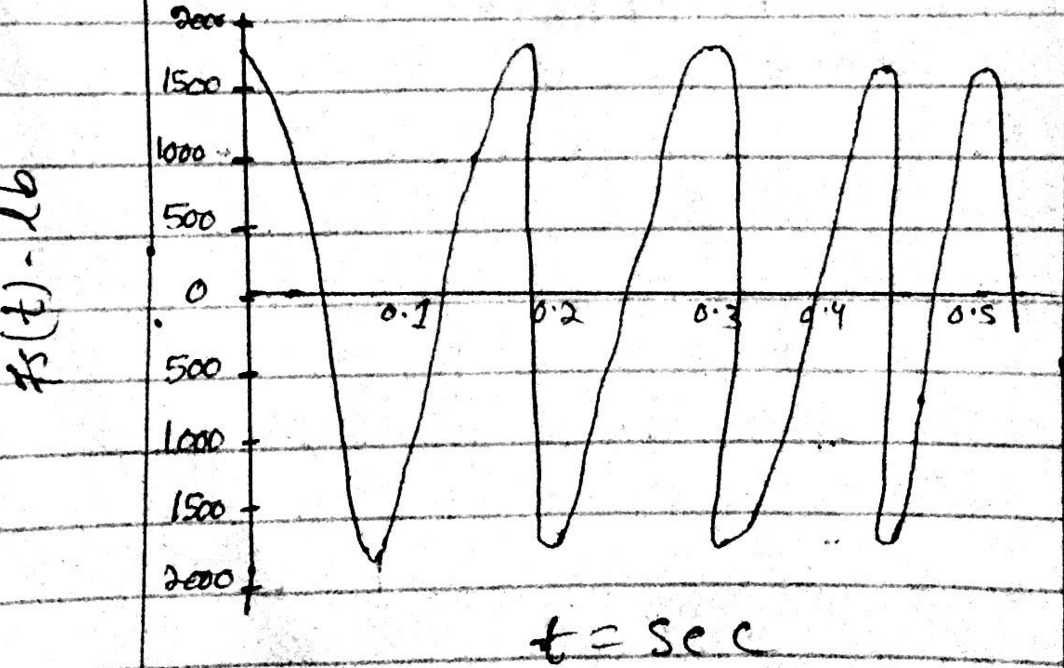
Amplitude of equivalent static force f_{so},

$$K u_o = 90625 \times 1/24$$
$$= 3776 \text{ lb}$$

Question No. (01)

Undamped free vibration:





Variation of equivalent static forces with time:

Question No.(02)

Answer Question No.(02)

Solution:

Given data:

$\Delta = 1/2''$

Load $= 7759\,lb$

$E = 29{,}000\,ksi$

$I = 150\,inch^4$

$\zeta = 2.5\%$

E.O.M for damped free vibration is:

$$Ku + C\dot{u} + m\ddot{u} = 0 \quad \ldots (1)$$

It is known from problem (1) that:

$K = 90625\,lb/ft$ and $m = 241\,lb.sec^2/ft$

$C = \zeta \times 2m\omega_n = 2 \times 241 \times 19.96 \times 0.025$

$C = 240.5\,lb.sec/ft$

By substituting values of K, C and m in (1) we get

$$90625u + 240.5\dot{u} + 241\ddot{u} = 0$$

Question No. (02)

Solution to the E.O.M for damped free vibration is:

$$u(t) = e^{-\zeta w_n t}\left(u(0)\cos(w_D t) + \frac{1}{w_D}[\dot{u}(0) + u(0)\zeta w_n].\sin(w_D t)\right)$$

$$w_D = 19.96 \text{ rad/sec}$$

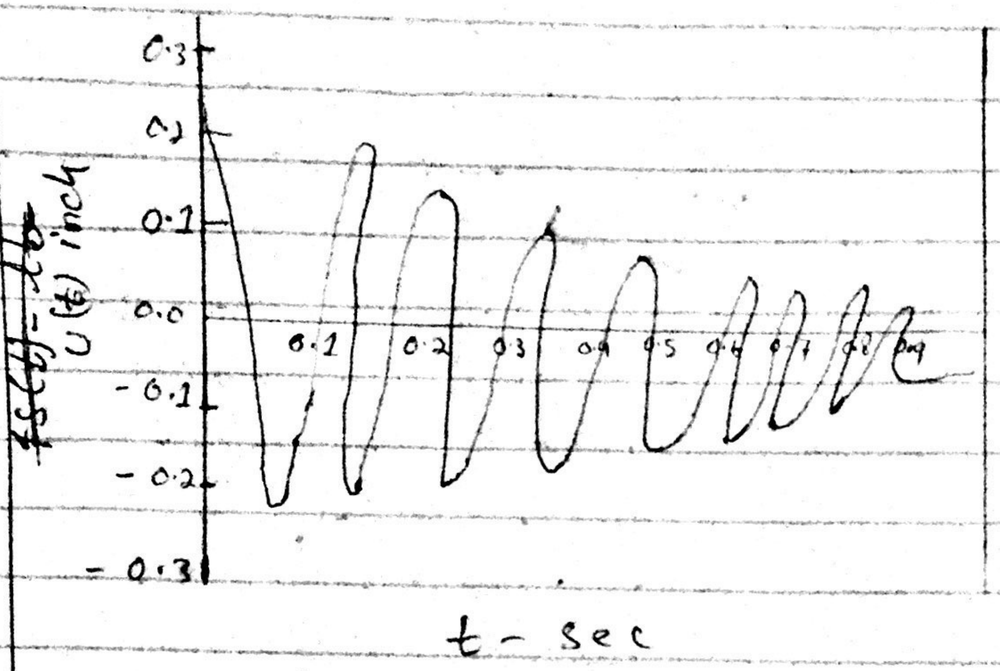
$$u(t) = e^{-0.025\times19.96}\left[\frac{1}{24}\times\cos(19.96t) + \frac{1}{19.96}\times\left[0 + \frac{1}{24}\times0.025\times19.96\times\sin(19.96t)\right)\right.$$

$$u(t) = e^{-0.499t}\left(0.041\times\cos(19.96t) + 0.00104\times\sin(19.96t)\right)$$

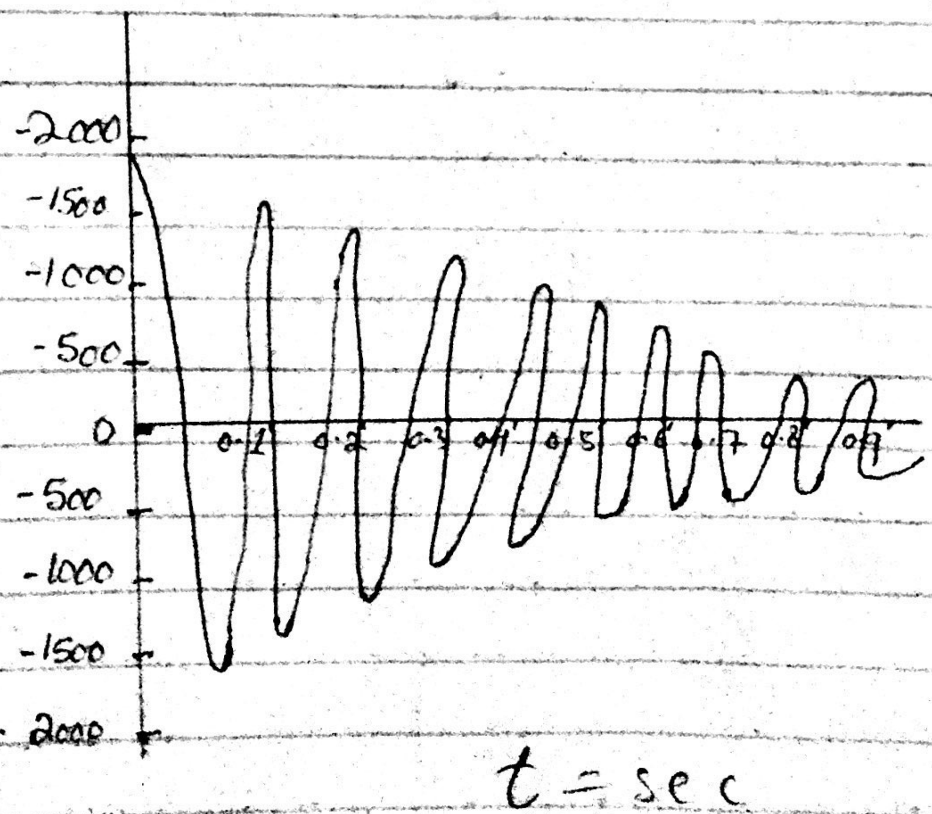
$$fs(t) = k.u(t) = 90625\times u(t)$$

$$fs(t) = e^{-0.499t}\left[3715\cos(19.96t) + 94.25\sin(19.96t)\right]$$

Question No. (02)



variation of displacement with time.

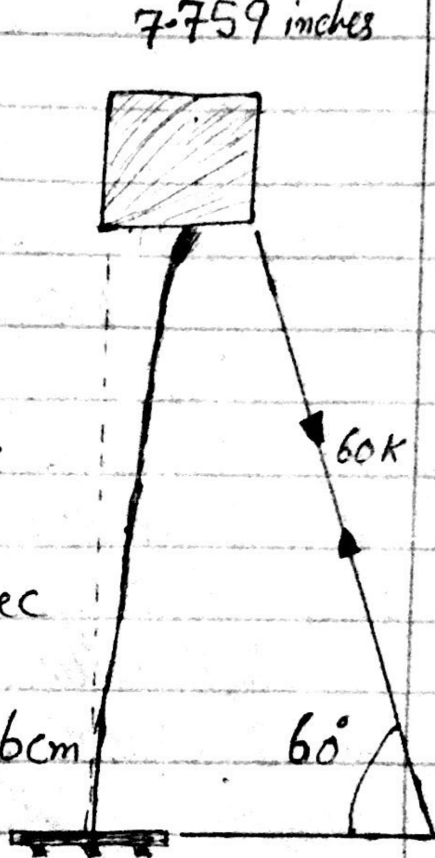


variation of equivalent static force with time.

Question No. (03)

Answer Question No. (03)

Given Data.



Figure

Force = 60 kips

Displacement the tank = (ID/1000)"
= (7759/1000)"
= 7.759 inches

cycles = 7

Time = 3.57 sec

Amplitude of displacement = 2.286 cm

Solution:

$u_1 = 7.759\,in$

After $j = 7$, $u_{j+1} = u_8 \frac{2.286}{2.54} = 0.9$

1 inch = 2.54 cm

(a) Damping ratio = ?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

Question No. (03)

$$7 = \frac{1}{(2 \times 3.14)(\delta)} \ln\left(\frac{7.759}{0.9}\right)$$

$$7 = \frac{1}{6.28(\delta)} \ln(8.62)$$

$$7 = \frac{1}{6.28(\delta)} \times 2.154$$

$$7 \times 6.28(\delta) = 2.154$$

$$44\delta = 2.154$$

$$\delta = \frac{2.154}{44}$$

$$\delta = 0.048 = 4.8\%$$

(b) Natural period of Un-damped vibration:

$T_n = ?$

7 cycles of vibration are completed in 3.57 sec

⇒ Time required to complete one cycle = 7/3.57 = 1.96 sec

Now $\boxed{T_D = 1.96 sec}$

$w_D = w_n \sqrt{1-\zeta^2}$

$\Rightarrow 2\pi / w_D = 2\pi / w_n \left(\sqrt{1-\zeta^2} \right)$

$T_D = T_n / \sqrt{1-\zeta^2}$

$\Rightarrow T_n = T_D \left(\sqrt{1-\zeta^2} \right)$

$= 1.96 \sqrt{1-(0.048)^2}$

$\Rightarrow \boxed{T_n = 1.86 \text{ sec}}$ natural period of undamped vibration

(c)

$K = ?$

$$K = \frac{60 \cos 60^\circ}{2} = 15 \text{ k/in}$$

$$= 18000 \text{ lb/ft}$$

(d) weight of the tank, $W = ?$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{(W/g)}} = \sqrt{\frac{K \times g}{W}}$$

$$\Rightarrow \omega_n^2 = K \times g / W$$

$$\Rightarrow W = K \times g / \omega_n^2$$

Also $\omega_n = 2\pi / T_n$

$$\Rightarrow W = Kg \Big/ \left(\frac{4\pi^2}{T_n^2}\right) = Kg \times \frac{T_n^2}{4\pi^2}$$

$$W = \frac{18000 \text{ lb}}{\text{ft}} \cdot \frac{32.2 \text{ ft}}{\text{sec}} \left(\frac{(1.96)^2}{4(3.14)^2}\right)$$

$$\boxed{W = 55.284 \text{ K lb}}$$

(e) Damping co-efficient.

$c = ?$

It is know that $\zeta = \frac{c}{2 m \omega_n}$

$c = \zeta (2 m \omega_n) = \zeta (2m(2\pi/T_n))$

But putting values

$c = \frac{0.0462 \left(2\left(\frac{56284}{32.2}\right)\left(2(3.14)\right)\right)}{1.96}$

$c = 519.286 \text{ lb sec/ft}$

(f) No of cycles to reduce displacement altitude friction from 6.872 to 0.59 inch.

$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$

$= \frac{1}{2(3.14)(0.0462)} \ln\left(\frac{6.872}{0.59}\right)$

$= 7$ cycles.

$j = 7$ cycles