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Section	A
Subject	Mechanics of solid II
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Question = 1

Given data

The state of stresses which are given as

$\sigma x = -40 Mpa$

$\sigma y = 70 Mpa$

$\tau xy = -50 Mpa$

Required data:-

Determine the state of stress at point rotated at 15° in clockwise direction

Solution:-

The given stresses are

$\sigma x = -40 Mpa$

$\sigma y = 70 Mpa$

$\tau xy = -50 Mpa$

When we rotate the element of beam at 15° in clockwise direction then we find σx_1, σy_1 and τxy_1 frist we find σx_1, As

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\Phi + \tau_{xy} \sin 2\Phi$$

So

$$\sigma_{x_1} = \frac{-40-70}{2} + \left(\frac{-40-70}{2}\right) \cos 2(-15^\circ) + (-50) \sin 2(-15^\circ)$$

$$\boxed{\sigma_{x_1} = -7.63 \text{ MPa}}$$

Now we have to find σ_{y_1}

$$\sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\Phi - \tau_{xy} \sin\Phi$$

$$\sigma_{y_1} = \frac{-40+70}{2} - \left(\frac{-40-70}{2}\right) \cos 2(-15) - (-50) \sin 2(-15)$$

$$\boxed{\sigma_{y_1} = 37.63 \text{ MPa}}$$

Also we have to find $\tau_{x_1y_1}$

$$\tau_{x_1y_1} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\Phi + \tau_{xy} \cos 2\Phi$$

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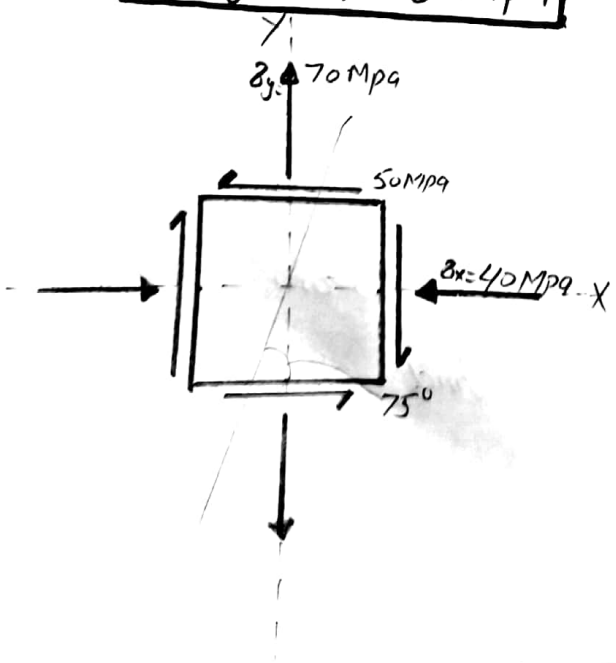
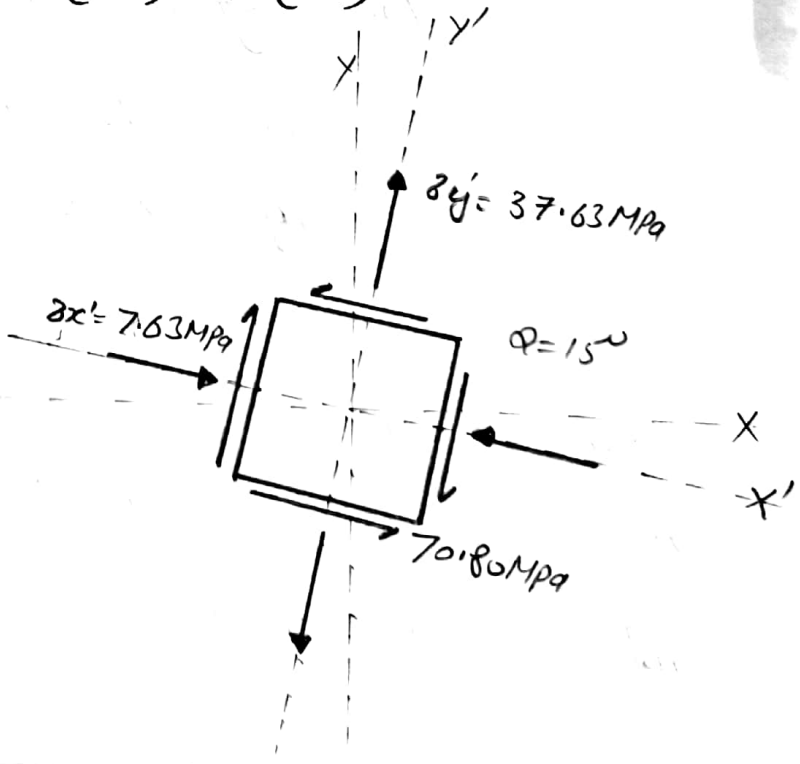
$$\tau_{x_1y_1} = -\frac{(-40-70)}{2} \sin 2(-15^\circ) + (-50) \cos 2(-15)$$

$$\boxed{\tau_{x_1y_1} = -70.80 \text{ MPa}}$$





Question No 2

Part A

Given Data

The state of stresses which are given as;

$\sigma_x = -40 \text{ Mpa}$

$\sigma_y = 70 \text{ Mpa}$

$\tau_{xy} = -50 \text{ Mpa}$

Required Data:-

principal plane = ?

principal stresses = ?

Solution:-

frist we find principal plane.

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_p = \frac{2(-50)}{-40-70}$$

$$\tan 2\theta_p = 0.9091$$

$$2\theta_p = \tan^{-1}(0.9091) = 42.27$$

$$\theta_{p_2} = \frac{42.27}{2}$$

$$\boxed{\theta_{p_2} = 21.14^\circ} \quad \text{for X-Axis}$$

Also

$$2\theta_{p_1} = 42.27 + 180$$

$$2\theta_{p_1} = 222.27$$

$$\theta_{p_1} = \frac{222.27}{2}$$

$$\boxed{\theta_{p_1} = 111.135^\circ} \quad \text{for Y-Axis}$$

Now we have to find principal stresses as

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{-40 + 70}{2} + \sqrt{\left(\frac{-40 - 70}{2}\right)^2 + (-50)^2}$$

$$\boxed{\sigma_1 = 89.33 \text{ MPa}}$$

Also

$$\sigma_2 = \frac{\sigma x + \sigma y}{2} - \sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau xy^2}$$

$$\sigma_2 = \frac{-40+70}{2} - \sqrt{\left(\frac{-40-70}{2}\right)^2 + (-50)^2}$$

$$\boxed{\sigma_2 = -59.33 \text{ Mpa}}$$

We must check which angle goes with which principal stresses

$$\sigma x_1 = \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2} \cos 2\Phi + \tau xy \sin 2\Phi$$

$$= \frac{-40+70}{2} + \left(\frac{-40-70}{2}\right) \cos 2(21.14) + (-50)(\sin(21.14))$$

$$\boxed{\sigma x_1 = -59.33 \text{ MPa}}$$

Hence

$\sigma_1 = 89.33$ Mpa with $\Theta p_1 = \underline{111.135}^\circ$

$\sigma_2 = -59.33$ Mpa with $\Theta p_2 = 21.14^\circ$

Part b

Given data

The state of stresses which are given as

$\sigma_x = -40\ Mpa$

$\sigma_y = 70\ Mpa$

$\tau_{xy} = -50\ Mpa$

Required data

Maximum In-plane shear stress = ?

Maximum shear plane ?

Solution :-

first we find Max. in-plane shear stress.

$$|\tau_{max\ in\ plan}| = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}}$$

$$= \sqrt{\left(\frac{-40-70}{2}\right)^2 + (-50)^2}$$

$$\boxed{|\tau_{max\ in\ plan}| = 74.3\ MPa}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-40+70}{2}$$

$$\boxed{\sigma_{avg} = 15}$$

Now we have to find Max. Shear plan

$$\tan 2\theta_s = \frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$= \frac{(-40-70)/2}{-50}$$

$$\tan 2\theta_s = 1.1$$

$$\theta_s = \frac{1}{2}\tan^{-1}(1.1)$$

$$\boxed{\theta_s = 23.90}$$