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Final-Term: Assignment

Subject : Differential Equation

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Program : BS (CS)

Q1 a:
Define 2^{nd} order linear homogenous/ non-homogenous equation along with examples?

Ans:- The general 2^{nd} order differential equation is of the form

$$P_0 \frac{d^n y}{dx^n} + P_1 \frac{d^{n-1}y}{dx^{n-1}} + \cdots + P_{n-1}\frac{dy}{dx} + P_n y = X \quad \text{—(1)}$$

where $(P_0 \neq 0)$
where X and the co-efficients $P_0, P_1, P_2, \ldots P_n$ are the constant or the function of x.
Now if X is zero (identically) then the linear equation (1) is said to homogenous
And if the $X \neq 0$, then the linear equation (1) is said to be non-homogenous differential equation
for example

$$4x^3y''' - 3x^2y'' + 6xy' + 9y = \sin x$$

is the non-homogeneous differential equation of order three

Now the equation

$$4y'' + 6y' - 8y = 4x^2$$

Now the equation

$$3y'' - 2y' + 8y = 0$$

is said to homogeneous differential equation of order two.

Qb

Solve the following 2^{nd} order Linear homogenous/ non-homogenous differential equation ~~[illegible]~~?

(i) $4y'' - 6y' - 7y = 0$

Sol: First finding $\sqrt{\ }$

So, $4\lambda^2 - 6\lambda + 7 = 0$

$$\lambda = 6 \pm \sqrt{\frac{(-6)^2 - 4(4)(7)}{2(4)}}$$

$$\lambda = \frac{6 \pm \sqrt{36 - 112}}{8} \Rightarrow \lambda = \frac{6 \pm \sqrt{-76}}{8}$$

$$\lambda = \frac{6}{8} + \frac{2\sqrt{19}\,i}{8} \Rightarrow \lambda\ \frac{3}{4} \pm \frac{\sqrt{19}i}{4}$$

$$\lambda_1 = \frac{3}{4} + \frac{\sqrt{19}i}{8}, \quad \lambda_2 = \frac{3}{4} - \frac{\sqrt{19}}{4}$$

So it has the Complex Conjugate roots

$$Q_1(x) = e^{\lambda_1 x} \cos \lambda'(x)$$

$$Q_2(x) = e^{\lambda_2 x} \sin \lambda_2(x)$$

$$y = c_1 e^{\frac{3}{4}x} \cos \frac{2\sqrt{19}}{4}(x) + e^{\frac{3x}{4}} \sin \frac{\sqrt{19}}{4} c_2$$

Ans.

Now for y_p

$$y_p = \frac{1}{F(D)} f(x)$$

$$\Rightarrow y_p = \frac{1}{D^2 - 4D - 12}$$

Replace D by $a = 5$ we have

$$y_p = \frac{1}{(5)^2 - 4(5) - 12} 3e^{5x}$$

$$\Rightarrow y_p = \frac{1}{25 - 20 - 12} 3e^{5x}$$

$$\Rightarrow \boxed{y_p = \frac{-3}{7} e^{5x}} \longrightarrow b$$

Now Putting a and b in *
we get

$$\boxed{y = c_1 e^{-2x} + c_2 e^{6x} - \frac{3}{7} e^{5x}}$$

is the required General Solution

Q.

(ii) $y'' - 4y' - 12y - 3e^{1}(5x)$

Sol:

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 12y = 3e^{5x}$$

$$\Rightarrow D^2y - 4Dy - 12y = 3e^{5x}$$

$$\Rightarrow (D^2 - 4D - 12)y = 3e^{5x}$$

$$\Rightarrow F(D)y = f(x)$$

Let $y = y_c + y_p \longrightarrow *$

is the required general solution. Now for y_c, the auxiliary equation will be

$$F(m) = 0$$

$$\Rightarrow m^2 - 4m - 12 = 0$$

$$\Rightarrow m^2 + 2m - 6m - 12 = 0$$

$$\Rightarrow m(m+2) - 6(m+2) = 0$$

$$\Rightarrow (m+2)(m-6) = 0$$

So

$m + 2 = 0$ or $m - 6 = 0$

$\Rightarrow m = -2$ and $m = 6$

So

$m_1 = -2$ and $m_2 = 6$

So the solution is

$$y_c = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

$$\Rightarrow \boxed{y_c = C_1 e^{-2x} + C_2 e^{6x}} \longrightarrow (a)$$

Q2:-

Solve the following IVP for the 2nd order linear equations.

(i)

$$16y'' - 40y' + 25y = 0 \quad y(0) = 3 \quad y'(0) = -9/4$$

Sol:

$$16r^2 - 40r + 25 = (4r-5)^2 = 0 \quad r_{1,2} = 5/4$$

$$y(t) = c_1 e^{\frac{5t}{4}} + c_2 t e^{\frac{5t}{4}}$$

$$y'(t) = \frac{5}{4} c_1 e^{\frac{5t}{4}} + c_2 e^{\frac{5t}{4}} + \frac{5}{4} c_2 t e^{\frac{5t}{4}}$$

$$3 = y(0) = c_1$$

$$-\frac{9}{4} = y'(0) = \frac{5}{4} c_1 + c_2$$

$$y(t) = 3e^{\frac{5t}{4}} - 6te^{\frac{5t}{4}}$$

Ans

(ii) $y'' + 14y' + 49y = 0 \quad y(-4) = -1 \quad y'(-4) = 5$

Sol

$$r^2 + 14r + 49 = (r+7)^2 = 0 \quad r_{1,2} = -7$$

$$y(t) = c_1 e^{-7t} + c_2 t e^{-7t}$$

$$y'(t) = -7c_1 e^{-7t} + c_2 e^{-7t} - 7c_2 t e^{-7t}$$

$$-1 = y(-4) = c_1 e^{28} - 4c_2 e^{28}$$

$$5 = y'(-4) = -7c_1 e^{28} + c_2 e^{28} + 28 c_2 e^{28}$$

$$= -7c_1 e^{28} + 29 c_2 e^{28}$$

$$c_1 = -9e^{-28} \quad c_2 = -2e^{-28}$$

$$y(t) = -9e^{-28} e^{-7t} - 2te^{-28} e^{-7t}$$

$$y(t) = -9e^{-7(t+4)} - 2te^{-7(t+4)}$$

Ans

(iii) $y'' - 4y' + 9y = 0 \quad y(0) = 0 \; y'(0) = -8$

Sol:

$r^2 - 4r + 9 = 0$

$y(t) = c_1 e^{2t} \cos(\sqrt{5}t) + c_2 e^{2t} \sin(\sqrt{5}t)$

$0 = y(0) = c_1$

$y(t) = c_2 e^{2t} \sin(\sqrt{5}t)$

$y'(t) = 2c_2 e^{2t} \sin(\sqrt{5}t) + \sqrt{5} c_2 e^{2t} \cos(\sqrt{5}t)$

$-8 = y'(0) = \sqrt{5} c_2 \Rightarrow c_2 = -\frac{8}{\sqrt{5}}$

$y(t) = \frac{8}{\sqrt{5}} e^{2t} \sin(\sqrt{5}t)$ Ans

(iv) $y'' - 8y' + 17y = 0 \quad y(0) = -4 \; y'(0) = -1$

Sol:

$r^2 - 8r + 17 = 0$

$y(t) = c_1 e^{4t} \cos(t) + c_2 e^{4t} \sin(t)$

$y'(t) = 4c_1 e^{4t} \cos(t) - c_1 e^{4t} \sin(t) + 4c_2 e^{4t}$

$\Rightarrow \sin(t) + c_2 e^{4t} \cos(t)$

$-4 = y(0) = c_1$

$-1 = y'(0) = 4c_1 + c_2$

$y(t) = -4e^{4t} \cos(t) + 15e^{4t} \sin(t)$

Q4:- Solve the following IVP using Laplace Transform.

(i) $y'' - 10y' + 9y = 5t, \quad y(0) = -1 \quad y'(0) = 2$

Sol:

$$\mathcal{L}\{y''\} - 10\mathcal{L}\{y'\} + 9\mathcal{L}\{y\} = \mathcal{L}\{5t\}$$

$$s^2Y(s) - sy(0) - y'(0) - 10(sY(s) - y(0))$$

$$\Rightarrow + 9Y(s) = \frac{5}{s^2}$$

$$(s^2 - 10s + 9)Y(s) + s - 12 = \frac{5}{s^2}$$

$$Y(s) = \frac{5}{s^2(s-9)(s-1)} + \frac{12 - s}{(s-9)(s-1)}$$

$$Y(s) = \frac{5 + 12s^2 - s^3}{s^2(s-9)(s-1)}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-9} + \frac{D}{s-1}$$

$$5 + 12s^2 - s^3 = As(s-9)(s-1)$$
$$\Rightarrow B(s-9)(s-1) + Cs^2(s-1) + Ds^2(s-9)$$

$s = 0 \qquad 5 = 9B \qquad \Rightarrow B = \frac{5}{9}$

$s = 1 \qquad 16 = -8D \qquad \Rightarrow D = -2$

$s = 9 \qquad 248 = 648C \qquad \Rightarrow C = \frac{31}{81}$

QA^3:-

Find the laplace transforms of the given Functions.

(i)

$$F(t) = 6(e^{\wedge}-5t)+e^{\wedge}3t+5(t^{\wedge}3)-9$$

Sol

$$F(s) = 6\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\frac{3!}{s^{3+1}} - 9\frac{1}{s}$$

$$= \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s} \quad \text{An}$$

(ii)

$$g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$$

Sol

$$G(s) = 4\frac{s}{s^2+(4)^2} - 9\frac{4}{s^2+(4)^2} + 2\frac{s}{s^2+(10)^2}$$

$$= \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100} \quad \text{An}$$

(iii) $h(t) = e^{\wedge}3t + \cos(6t) - e^{\wedge}(3t)\cos(6t)$

Sol:-

$$H(s) = \frac{1}{s-3} + \frac{s}{s^2+(6)^2} - \frac{s-3}{(s-3)^2+(6)^2}$$

$$= \frac{1}{s-3} + \frac{s}{s^2+36} - \frac{s-3}{(s-3)^2+36}$$

An

Q4:- Solve the following IVP using Laplace Transform.

(i) $y'' - 10y' + 9y = 5t, \quad y(0) = -1 \quad y'(0) = 2$

Sol: $\mathcal{L}\{y''\} - 10\mathcal{L}\{y'\} + 9\mathcal{L}\{y\} = \mathcal{L}\{5t\}$

$s^2 Y(s) - sy(0) - y'(0) - 10(sY(s) - y(0))$

$\Rightarrow + 9Y(s) = \frac{5}{s^2}$

$(s^2 - 10s + 9)Y(s) + s - 12 = \frac{5}{s^2}$

$Y(s) = \frac{5}{s^2(s-9)(s-1)} + \frac{12 - s}{(s-9)(s-1)}$

$Y(s) = \frac{5 + 12s^2 - s^3}{s^2(s-9)(s-1)}$

$Y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-9} + \frac{D}{s-1}$

$5 + 12s^2 - s^3 = As(s-9)(s-1)$
$\Rightarrow B(s-9)(s-1) + Cs^2(s-1) + Ds^2(s-9)$

$s = 0 \quad 5 = 9B \quad \Rightarrow B = \frac{5}{9}$

$s = 1 \quad 16 = -8D \quad \Rightarrow D = -2$

$s = 9 \quad 248 = 648C \quad \Rightarrow C = \frac{31}{81}$

$s = 2 \quad 45 = -14A + \frac{4345}{81} \Rightarrow A = \frac{50}{81}$

$$Y(s) = \frac{\frac{50}{80}}{s} + \frac{\frac{5}{9}}{s^2} + \frac{\frac{31}{81}}{s-9} - \frac{2}{s-1}$$

$$y(t) = \frac{50}{81} + \frac{5}{9}t + \frac{31}{81}e^{9t} - 2e^{t}$$

Ans

(ii) $y'' - 6y' + 15y = 2\sin(3t)$, $y(0) = -1$ $y'(0) = -4$

Sol:

$$s^2Y(s) - sy(0) - 6(sY(s) - y(0))$$

$$\Rightarrow + 15Y(s) = 2\frac{3}{s^2+9}$$

$$(s^2 - 6s + 15)Y(s) + s - 2 = \frac{6}{s^2+9}$$

$$Y(s) = \frac{-s^3 + 2s^2 - 9s + 24}{(s^2+9)(s^2-6s+15)}$$

$$Y(s) = \frac{As+B}{s^2+9} + \frac{Cs+D}{s^2-6s+15}$$

$$-s^3 + 2s^2 - 9s + 24 = (As+B)(s^2-6s+15)$$

$$\Rightarrow + (Cs+D)(s^2+9)$$

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$$S^3 = \quad A + C = -1 \quad \Rightarrow A = \frac{1}{10}$$

$$S^2 = \quad -6A + B + D = 2 \quad \Rightarrow B = \frac{1}{10}$$

$$S^1 = \quad 15A - 6B + 9C = -9 \quad \Rightarrow C = -\frac{11}{10}$$

$$S^0 = \quad 15B + 9D = 24 \quad \Rightarrow D = \frac{5}{2}$$

$$Y(s) = \frac{1}{10}\left(\frac{s+1}{s^2+9} + \frac{-11s+25}{s^2-6s+15}\right)$$

$$= \frac{1}{10}\left(\frac{s+1}{s^2+9} + \frac{-11(s-3+3)+25}{(s-3)^2+6}\right)$$

$$= \frac{1}{10}\left(\frac{s}{s^2+9} + \frac{1\,\frac{3}{3}}{s^2+9} - \frac{11(s-3)}{(s-3)^2+6} - \frac{8\,\frac{\sqrt{6}}{\sqrt{6}}}{(s-3)^2+6}\right)$$

$$y(t) = \frac{1}{10}\left(\cos(3t) + \frac{1}{3}\sin(3t) - 11e^{3t}\cos(\sqrt{6}t) - \frac{8}{\sqrt{6}}e^{3t}\sin(\sqrt{6}t)\right)$$

Ans