Exam: MID SEMESTER ASSIGNMENT

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Q.1:

Ans.1: $\begin{bmatrix} 1 & 4 & 3 & 0 & 5 \\ 0 & 1 & 9 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$

As you know for solving the augmented matrix we first need to form triangular form as a circles above but here no need to do raw operation just we need to solve it and find the solution set.

We have four rows our solution set must have form value or elements.

$x1\ , x2\ , x3\ , x4$

$= \ 1(x) + 4(x2) + 36(3) + (0)(x4) = 5$

$= \ (x1) + 1(x2) + 9(x3) + 0(x4) = 7$

$= 0(x1) + 0(x2) + 1(x3) + 0(x4) = -6$

$= 0(x1) + 0(x2) + 0(x3) + 1(x4) = 4$

$= x_1 + 4x_2 + 3x_3 = 5$

$x_2 + 9x_3 = 7$

$x_3 = -6$

$x_4 = 4$

$x_3 = -6$, $x_4 = 4$ lets find $x_1 \& x_2$

$= x_2 + 9(-6) = 7$

$= x_2 - 54 = 7$

$= x_2 = 54 + 7$ ➔ $x_2 = 61$

➔ $x_1 + 4(61) + 3(-6) = 5$

$= x_1 + 244 - 18 - 5$

$= x_1 + 226 = 5$

$= x_1 = -226 + 5$ ➔ $x_1 = -221$

$= x_1 = -221$, $x_2 = 61$, $x_3 = -6$, $x_4 = 4$ ➔ Solution set $\{-221, 61, -6, 4\}$

Q.2(a):

Ans.2: (a) Put

i. Transforming the first matrix in to 2nd

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

R3 – 2R2

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

ii. Transforming the 2nd in to 1st matrix

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$R3 + 2R2$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

Q.2: (b)

Ans.2: (b)

a. It is an echelon form because its satisfies the three condition of the echelon form.
   As it has the leading entry in each raw which is non zero
   As it has all zero elements.
   Below the each leading entry.
   The whole column elements are zero.
b. It is also an echelon form its satisfied the three conditions.
   i. It has the all zero rows at the end.
   ii. It has the leading entry in each row.
   iii. It has zero element from the leading entry of the above row. Whole column must be zero.
c. It is not reduced echelon form because the leading entry is row 1 is 5. So it must be 1 that why it is not reduced echelon form.
d. It is not reduced echelon form because it has zero row in the middle that must be at the last row.

Q.3: (a)

Ans.3: (a) Difference between Echelon and Reduced Echelon form:

a. <u>Echelon Form of Matrix:</u>

A Matrix is said to be in echelon form if its satisfy the following properties.

i. In each successive non-zero row, the number of zero before the first non-zero entry of a row increase row by row.
ii. Every non-zero row in a precedes every zero row (if there is any)

For Example, the matrices $\begin{bmatrix} 2 & 3 & -4 & 1 \\ 0 & 1 & 5 & 3 \\ 0 & 0 & 0 & 6 \end{bmatrix}$ and $\begin{bmatrix} 0 & 2 & 3 \\ 0 & 0 & -5 \\ 0 & 0 & 0 \end{bmatrix}$ are in echelon form.

b. <u>Reduce Echelon form of Matrix:</u>

A Matrix is said to be in reduce echelon form if its satisfy the following properties.

i. It is an (row) echelon form.

ii. The first non-zero entry in R_1 lies in C_2 is 1 and all other entries of C_2 are zero.

Example:

The matrix $\begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

<u>Uses:</u>

it is used for solving the linear equation

For the resolution of linear systems, Gaussian elimination is preferred over Gauss-Jordan (these parallel the echelon and reduced echelon forms) as the former involves less operations (roughly n3/3n3/3 vs n3/2n3/2), for a similar numerical stability.

Anyway, Gauss-Jordan does not require a back substitution step, which makes the code a little more compact. On the other hand, the reduced form is not compatible with LU decomposition.

From a theoretical point of view, the reduced form has the advantage of being uniquely defined for a given matrix.

Anyway, it is worth to note that for an invertible matrix, the reduced form is simply a unit matrix, totally uninteresting. The concept is only useful for degenerate cases

Q.3: (b)

Ans.3: (b) Solution:

$$\begin{bmatrix} 1 & 5 & 8 \\ 2 & 8 & -1 \\ -4 & 0 & 0 \\ 1 & -4 & 19 \end{bmatrix}$$

R4 – R3 $$\begin{bmatrix} 1 & 5 & 8 \\ 2 & 8 & -1 \\ -4 & 0 & 0 \\ 5 & -4 & 19 \end{bmatrix}$$

R3/-4 $$\begin{bmatrix} 1 & 5 & 8 \\ 2 & 8 & -1 \\ 0 & 0 & 0 \\ 5 & -4 & 19 \end{bmatrix}$$

Swap R3 to R4 $$\begin{bmatrix} 1 & 5 & 8 \\ 2 & 8 & -1 \\ 5 & -4 & 10 \\ 0 & 0 & 0 \end{bmatrix}$$

R3 – 5R1 $$\begin{bmatrix} 1 & 5 & 8 \\ 2 & 8 & -1 \\ 0 & -29 & 21 \\ 0 & 0 & 0 \end{bmatrix}$$

R2 –2R1 and R3 X 1/-29 $$\begin{bmatrix} 1 & 5 & 8 \\ 0 & -2 & -17 \\ 0 & 0 & 21/-29 \\ 0 & 0 & 0 \end{bmatrix}$$ Echelon form.