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Program : BS.(C.S)

Assignment : Differential Equation

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Q.1 b: Solve the following 2nd order Linear homogenous / non-homogenous differential equation?

(i) $4y'' - 6y' + 7y = 0$

Sol: Finding the root of the characteristics equation

$4\lambda^2 - 6\lambda + 7 = 0 \Rightarrow \lambda = \frac{6 \pm \sqrt{(-6)^2 - 4(4)(7)}}{2(4)}$

$\lambda = \frac{6 \pm \sqrt{36 - 112}}{8}$

$\lambda = \frac{6 \pm \sqrt{-76}}{8}$

$\lambda = 6/8 + \frac{2\sqrt{19}i}{8}$

P.T.O

$$\lambda = \frac{3}{4} \pm \frac{\sqrt{19}\,i}{4}$$

$$\lambda_1 = \frac{3}{4} \pm \frac{\sqrt{19}\,i}{4}$$

$$\lambda_2 = 3/4 - \frac{\sqrt{19}\,i}{4}$$

As it has Complex Conjugate roots.

$\phi_1(x) = e^{\lambda_1(x)} \cos^{\lambda_1(x)}$

$\phi_2(x) = e^{\lambda_2 x} \sin^{\lambda_2(x)}$

$$y = C_1 e^{3/4 x} \cos^{i\sqrt{19}/4(x)} + e^{3x/4} \sin^{i\sqrt{19}/4 x} C_2$$

~ x ~

Q. 1

b(ii) $y'' - 4y' - 12y = 3e^{5x}$

Sol:

characteristic equation and its roots.

$$y^2 - 4y - 12 = (y-6)(y+2) = 0$$

$$y' = -2, \; y_2 = 6$$

P.T.O

The Complementary solution is

$$y(t) = C_1 e^{-2t} + C_2 e^{8t}$$

x ~~~ x ~~~ x ~~~

Q.2 Solve the following IVP for the 2nd order linear equations.

(i) $16y'' - 40y' + 25y = 0 \quad y(0) = 3 \quad y'(0) = -9/4$

Sol:
Below are the characteristics and its roots.

$16y^2 - 40y + 25 = (4y^{(2)} - 5)^2 = 0 \quad y_1 = 5/4 = y_2 = 5/4$

general solution and its derivative are.

$$y(t) = C_1 e^{5t/4} + C_2 e^{5t/4} ~~+ 5/4 C_2 t e^{5t/4}~~$$

$$y'(t) = 5/4\, C_1 e^{5t/4} + C_2 e^{5t/4} + 5/4\, C_2 t e^{5t/4}$$

By putting in the initial condition

P.T.O

$3 = y(0) = c_1$

$-9/4 = y'(0) = 5/4\ c_1 + c_2$

Solution for IVP is

$y^t = 3e^{5t/4} - 6te^{5t/4}$

Q.2 (ii) $y'' + 14y' + 49y = 0 \quad y(-4) = -1\ y'(-4) = 5$

Sol:
Characteristic equation & its root are

$y^2 + 14y + 49 = (y+7)^2 = 0 \quad y_1 = 7,\ y_2 = -7$

General sol & its derivative are

$y(t) = c_1 e^{-7t} + c_2 te^{-7t}$

$y't = -7c_1 e^{-7t} + c_2 e^{-7t} - 7c_2 te^{-7t}$

Putt the initial Condition

$-1 = y(-4) = -7c_1 e^{28} + c_2 e^{28} + 28c_2 e^{28}$

$= -7c_1 e^{28} + 29c_2 e^{28}$

P.T.O

$C_1 = -9e^{-28}$

$C_2 = -2e^{-28}$

the sol for IVP is,

$y(t) = -9e^{28}e^{-7t} - 2te^{-28}e^{-7t}$

$y(t) = -9e^{-7(t-4)} - 2te^{-7(t+4)}$

~~ x ~~ x ~~

Q.2 (iii) $y'' - 4y' + 9 = 0 \quad y(0) = 0, \; y'(0) = 8$

Sol:

Characteristic equation is

$y^2 - 4y + 9 = 0$

The roots of the equation are

$y_1 = 2 + \sqrt{5}i$

$y_2 = 2 \pm \sqrt{5}i$

General solution to DE is

$y(t) = C_1 e^{2t} \cos(\sqrt{5}t) + C_2 e^{2t} \sin(\sqrt{5}t)$

By applying initial Condition along with derivation

$y(t) = C_2 e^{2t} \sin(\sqrt{5}t)$

$y'(t) = 2C_2 e^{2t} \sin(\sqrt{5}t) +$

P.T.O

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$\sqrt{5}\, C_2 e^{2t} \cos(\sqrt{5}t)$

$-8 = y'(0) = \sqrt{5}(2) = C_2 = -8/\sqrt{5}$

Solution is

$$y(t) = -8/\sqrt{5}\, e^{2t} \sin$$

~ x ~ x ~

Q.2

iv $y'' - 8y' + 17y = 0 \quad y(0) = -4,\ y'(0) = -1$

Sol:

Characteristic equation and its roots are

$$y^2 - 8y + 17 = 0$$

$$y_1 = 4 + i$$

$$y_2 = 4 - i$$

P.T.O

General Sol and derivation are

$y(t) = C_1 e^{4t} \cos(t) + C_2 e^{4t} \sin(t)$

$y'(t) = 4C_1 e^{4t} \cos(t) - C_1 e^{4t} \cos(t) +$
$4C_2 e^{4t} \sin(t) + C_2 e^{4t} \cos(t)$

Applying the initial condition we get the following

$-y = y(0) = C_1$

$-1 = y'(0) = 4C_1 + C_2$

Solution is

$y(t) = -4e^{4t} \cos(t) + 15 e^{4t} \sin(t)$

x ~ x ~ x ~

Q.3
(i) $f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$

Sol

$F(t) = 6 \frac{1}{s-(-5)} + \frac{1}{s-3} + 5 \frac{3}{s^{3+1}} - 9 \frac{1}{s}$

$$= \frac{6}{3+s} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}$$

Q. 3
(ii) $g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

Sol:

$$g(t) = 4\frac{s}{s^2+(4)^2} - 9\frac{4}{s^2+(4)^2} + 2\frac{s}{s^2+(10)^2}$$

$$= \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}$$

Q. 3
(iii) $H(t) = e^{3t} + \cos(6t) - e^{3t}\cos(6t)$

Sol:

$$H(s) = \frac{1}{s-3} + \frac{s}{s^2+(6)^2} - \frac{s-3}{(s-3)^2+(6)^2}$$

$$\frac{1}{s-3} + \frac{3}{s^2+36} - \frac{s-3}{(s-3)^2+36}$$

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Q. 4
(i) $y'' - 10y' + 9y = 5t, \; y(0) = -1, \; y'(0) = 2$

Sol: taking transform of every term

$L\{y''\} - 10L\{y'\} + 9L\{y\} = L\{5t\}$

By formula

$s^2 Y(s) - s y(0) - y'(0) - 10(sY(s) - y(0)) + 9Y(s) = \frac{5}{s^2}$

putting in initial condition

$(s^2 - 10s + 9)Y(s) + s + 12 = 5/s^2$

Solving for $Y(s)$

$$Y(s) = \frac{5}{s^2(s+9)(s+1)} + \frac{-19-s}{(s-9)(s-1)}$$

$$Y(s) = \frac{5 + 12s^2 - s^3}{s^2(s-9)(s-1)}$$

the partial fraction be

$$Y(s) = \frac{A}{s} + \frac{B}{s^2} + C + \frac{D}{s-1}$$

P.T.O

$$S+12S^2-S^3 = AS(S-9)(S-1)+B(S-9)(S-1)$$
$$+CS^2(S-1)+DS^2(S-9).$$

Solving for Constants

$S=0$ $5=9B$ $\Rightarrow B=5/9$

$S=1$ $16=-8D$ $\Rightarrow D=-2$

$S=9$ $248=648C$ $\Rightarrow C=31/81$

$S=2$ $45=-14A+\frac{4345}{81}$ $\Rightarrow A=\frac{50}{81}$

By plugging in the Constant gives

$$Y(s)=\frac{50/81}{s}+\frac{5/9}{S^2}+\frac{31/81}{S-9}-\frac{2}{S-1}$$

taking the inverse transform the Sol is

$$y(t)=\frac{50}{81}+\frac{5}{9}t+\frac{31}{81}e^{9t}-2e^{t}$$

Q.4

ii) $y''-6y'+15y=2\sin(3t)$ $y(0)=-1$
$y'(0)=-4$

Sol:

P.T.O

Solo

taking laplace transform of every and plug it in initial condition

$$s^2 Y(s) - s y(0) - y'(0) - 6(s)Y(s) - y(0) + 15Y(s) = 2\frac{3}{s^2+9}$$

$$(s^2 - 6s + 15)Y(s) + s - 2 = \frac{6}{s^2+9}$$

$$Y(s) = \frac{-s^3 + 2s^2 - 9s + 24}{(s^2+9)(s^2-6s+15)}$$

getting the partial fraction decomposition

$$Y(s) = \frac{As+B}{s^2+9} + \frac{(s+D)}{s^2-6s+15}$$

Setting numerator equal gets

$$-s^3 + 2s^2 - 9s + 24 = (As+B)(s^2-6s+15) + (s+D)(s+9) = (A+C)s^3 + (-6A+B+D)s^2 + (15A) + 6s + 9c) + 15B + 9D$$

Sol has constants

P.T.O

$s^3: A+C=-1$

$s^2: -6A+B+D=2$

$s^1: 15A-6B+9C=-9$

$s^0: 15B+9D=24$

$A=\frac{1}{10}$ $B=\frac{1}{10}$ $C=\frac{-11}{10}$ $D=\frac{5}{2}$

Plugging the constant gives

$$Y(s)=\frac{1}{10}\left(\frac{s+1}{s^2+9}+\frac{-11s+25}{s^2-6s+15}\right)$$

$$=\frac{1}{10}\left(\frac{s}{s^2+9}+\frac{1}{3}\frac{3}{s^2+9}-\frac{11(s-3)}{(s-3)^2+6}\right)$$

taking inverse function and sol will be as

$$y(t)=\frac{1}{10}\left(\cos(3t)+\frac{1}{3}\sin(3t)-11\cos\sqrt{6}t\right.$$

$$\left.-\frac{8}{\sqrt{6}}\ e^{3t}\sin\sqrt{6}t\right)$$

Q3:-

(a):- **Laplace:**

Ans Laplace transform is integral transform that converts a function of real variable (t) to the function of complex var (s).

i.e:

The laplace transform $\mathcal{L}$ of a function f(t) for t>0 is defined by the following

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt.$$

general eu: $$f(s) = \int_0^\infty f(t) \cdot e^{-st}\,dt.$$

Q1:-
part a):-

Ans:- Homogeneous differential equation:
It involves only derivatives of y & terms involving y, & they are set 0, as in follow example.

$$\frac{d^4y}{dx^4} + x\frac{d^2y}{dx^2} + y^2 = 0.$$

Non homogeneous differential eq:

Same as homogeneous differential equations. except they can have terms involving only x on right side as given.

$$\frac{d^4y}{dx^4} + x\frac{d^2y}{dx^2} + y^2 = 6x + 3.$$

x ——— x ——— Ans