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Program : BS.(C.S)

Assignment : Differential Equation

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Q.1 Solve the following 2nd order
b: linear homogenous / non-homogenous differential equations

$$(i) \quad 4y'' - 6y' + 7y = 0$$

Sol: finding the root of the characteristics equation

$$4\lambda^2 - 6\lambda + 7 = 0 \Rightarrow \lambda = \frac{6 \pm \sqrt{(-6)^2 - 4(4)(7)}}{2(4)}$$

$$\lambda = \frac{6 + \sqrt{36 - 112}}{8}$$

$$\lambda = \frac{6 \pm \sqrt{-76}}{8}$$

$$\lambda = \frac{6}{8} + \frac{2\sqrt{19}i}{8}$$

$$\lambda = \frac{3}{4} \pm \frac{\sqrt{19}i}{4}$$

$$\lambda_1 = \frac{3}{4} + \frac{\sqrt{19}i}{4}$$

$$\lambda_2 = \frac{3}{4} - \frac{\sqrt{19}i}{4}$$

As it has complex conjugate roots.

$$C_1(x) = e^{\lambda_1 x} \cos^{\lambda_1(x)}$$

$$C_2(x) = e^{\lambda_2 x} \sin^{\lambda_2(x)}$$

$$y = C_1 e^{\frac{3}{4}x} \cos^{\frac{i\sqrt{19}}{4}x} + e^{\frac{3}{4}x} \sin^{\frac{i\sqrt{19}}{4}x} C_2$$

Q. 1

$$b(ii) \quad y'' - 4y' - 12y = 3e^{5x}$$

Solⁿ

Characteristic equation and its roots.

$$y^2 - 4y - 12 = (y-6)(y+2) = 0$$

$$y_1 = -2, \quad y_2 = 6$$

p.T.O

The Complementary solution is

$$y(t) = C_1 e^{-at} + C_2 e^{at}$$

x \sim x \sim x

Q.2 Solve the following IVP for the 2nd order linear equations.

(i) $16y'' - 40y' + 25y = 0$ $y(0) = 3$ $y'(0) = -9/4$

Soln

Below are the characteristics and its roots.

$$16y^2 - 40y + 25 = (4y - 5)^2 = 0 \quad y_1 = 5/4 = y_2 = 5/4$$

general solution and its derivative are.

$$y(t) = C_1 e^{5t/4} + C_2 e^{5t/4} + \frac{5t}{4} C_2 e^{5t/4}$$

$$y'(t) = 5/4 C_1 e^{5t/4} + C_2 e^{5t/4} + 5/4 C_2 t e^{5t/4}$$

By putting in the initial condition

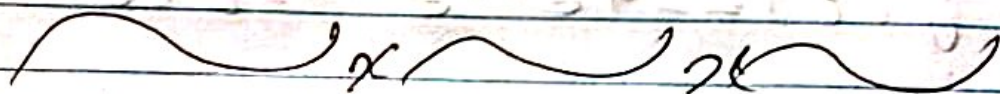
P.T.O

$$3 = y(0) = c_1$$

$$-9/4 = y'(0) = 5/4 c_1 + c_2$$

Solution for ivp is

$$y^t = 3e^{5t/4} - 6te^{5t/4}$$



Q.2

(ii) $y'' + 14y' + 49y = 0$ $y(-4) = -2$ $y'(-4) = 5$

Solⁿ

Characteristic equation & its roots are

$$y^2 + 14y + 49 = (y+7)^2 = 0 \quad y_1 = -7, \quad y_2 = -7$$

General sol & its derivative are

$$y(t) = C_1 e^{-7t} + C_2 t e^{-7t}$$

$$y'(t) = -7C_1 e^{-7t} + C_2 e^{-7t} - 7C_2 t e^{-7t}$$

Put the initial condition

$$-2 = y(-4) = -7C_1 e^{28} + C_2 e^{28} + 28C_2 e^{28}$$

$$= -7C_1 e^{28} + 29C_2 e^{28}$$

P.T.O

$$C_1 = -9e^{-28}$$

$$C_2 = -2e^{-28}$$

the sol for IVP is,

$$y(t) = -9e^{28}e^{-7t} - 2te^{-28}e^{-7t}$$

$$y(t) = -9e^{-7(t-4)} - 2te^{-2(t+4)}$$



Q. 2

(iii) $y'' - 4y' + 9 = 0$ $y(0) = 0$, $y'(0) = 8$

Sol:

characteristic equation is

$$y^2 - 4y + 9 = 0$$

The roots of the equation are

$$y_1 = 2 + \sqrt{5}i$$

$$y_2 = 2 - \sqrt{5}i$$

General solution to DE is

$$y(t) = C_1 e^{2t} \cos(\sqrt{5}t) + C_2 e^{2t} \sin(\sqrt{5}t)$$

By applying initial condition along with derivation

$$y(t) = C_2 e^{2t} \sin(\sqrt{5}t)$$

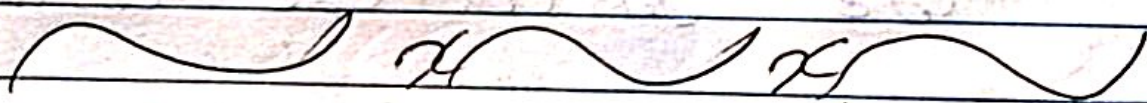
$$y'(t) = 2C_2 e^{2t} \sin(\sqrt{5}t) +$$

$$\sqrt{5} C_2 e^{at} \cos(\sqrt{5}t)$$

$$-8 = y'(0) = \sqrt{5} C_2 = C_2 = -8/\sqrt{5}$$

Solution is

$$y(t) = -8/\sqrt{5} e^{at} \sin$$



Q.2

iv

$$y'' - 8y' + 17y = 0 \quad y(0) = -4, \quad y'(0) = 1$$

Soln

Characteristic equation and its roots are

$$y^2 - 8y + 17 = 0$$

$$y_1 = 4 + i$$

$$y_2 = 4 - i$$

P.T.O

General sol and derivation
are

$$y(t) = C_1 e^{4t} \cos(t) + C_2 e^{4t} \sin(t)$$

$$y'(t) = 4C_1 e^{4t} \cos(t) - C_1 e^{4t} \sin(t) + 4C_2 e^{4t} \sin(t) + C_2 e^{4t} \cos(t)$$

Applying the initial conditions
we get the following

$$-y = y(0) = C_1$$

$$-1 = y'(0) = 4C_1 + C_2$$

Solution is

$$y(t) = -4e^{4t} \cos(t) + 5e^{4t} \sin(t)$$



Q.3

$$(i) f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$$

Soln

$$F(t) = \frac{6}{s-(-5)} + \frac{1}{s-3} + \frac{5 \cdot 3}{s^{3+1}} - \frac{9}{s}$$

$$= \frac{6}{3+s} + \frac{1}{s-3} + \frac{30}{s^2} - \frac{9}{s}$$

Q.3

(ii) $g(t) = 4 \cos(4t) - 9 \sin(4t) + 2 \cos(6t)$

Soln

$$g(t) = \frac{4s}{s^2 + (4)^2} - \frac{9 \cdot 4}{s^2 + (4)^2} + \frac{2s}{s^2 + (6)^2}$$

$$= \frac{4s}{s^2 + 16} - \frac{36}{s^2 + 16} + \frac{2s}{s^2 + 100}$$

Q.3

(iii) $H(t) = e^{3t} + \cos(6t) - e^{3t} \cos(6t)$

Soln

$$H(s) = \frac{1}{s-3} + \frac{s}{s^2 + (6)^2} - \frac{s-3}{(s-3)^2 + (6)^2}$$

$$\frac{1}{s-3} + \frac{3}{s^2 + 36} - \frac{s-3}{(s-3)^2 + 36}$$

⑨

Q.4
(i) $y'' - 10y' + 9y = 5t$, $y(0) = -1$, $y'(0) = 2$

Sol: taking transform of every term

$$\mathcal{L}\{y''\} - 10\mathcal{L}\{y'\} + 9\mathcal{L}\{y\} = \mathcal{L}\{5t\}$$

By formula

$$s^2 Y(s) - sy(0) - y'(0) - 10(sY(s) - y(0)) + 9Y(s) = \frac{5}{s^2}$$

putting in initial conditions

$$(s^2 - 10s + 9)Y(s) + s + 12 = \frac{5}{s^2}$$

Solving for $Y(s)$

$$Y(s) = \frac{s + 12}{s^2(s^2 - 10s + 9)} + \frac{5}{s^2(s^2 - 10s + 9)}$$

$$Y(s) = \frac{s + 12s^2 - 5s^2}{s^2(s-9)(s-1)}$$

the partial fraction be

$$Y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{D}{s-9}$$

$$S + 12S^2 - S^3 = A S(S-9)(S-1) + \frac{3(3-9)}{S-1} + \frac{S^2(S-1)}{S-9} + D S^2(S-9)$$

Solving for Constants

$$\begin{aligned} S=0 & \Rightarrow 12 = -9A \Rightarrow A = -\frac{4}{3} \\ S=1 & \Rightarrow 16 = -9D \Rightarrow D = -\frac{16}{9} \\ S=9 & \Rightarrow 248 = 648C \Rightarrow C = \frac{31}{81} \\ S=2 & \Rightarrow 45 = -14A + \frac{4345}{81} \Rightarrow A = \frac{50}{81} \end{aligned}$$

By plugging in the constants gives

$$Y(S) = \frac{50}{81} + \frac{3}{9} \frac{1}{S} + \frac{31}{81} \frac{1}{S-9} - \frac{2}{S-1}$$

taking the inverse transform the Laplace

$$y(t) = \frac{50}{81} + \frac{5}{9} t + \frac{31}{81} e^{3t} - \frac{2}{9} e^{-t}$$



Q.4

ii)

$$y'' - 6y' + 10y = 2 \sin(3t) \quad y(0) = 1$$

$$y'(0) = -4$$

Soln

P.T.O

Solⁿ

taking Laplace transform of every and plug it in initial condition

$$s^2 Y(s) - s y(0) - y'(0) - 6(s) Y(s) = y$$

$$(0) + 15 Y(s) = 2 \frac{3}{s^2 + 9}$$

$$(s^2 - 6s + 15) Y(s) + s - 2 = \frac{6}{s^2 + 9}$$

$$Y(s) = \frac{-s^3 + 2s^2 - 9s + 24}{(s^2 + 9)(s^2 - 6s + 15)}$$

$$(s^2 + 9)(s^2 - 6s + 15)$$

getting the partial fraction decomposition

$$Y(s) = \frac{As + B}{s^2 + 9} + \frac{(s + D)}{s^2 - 6s + 15}$$

Letting numerator equal gets

$$\begin{aligned} -s^3 + 2s^2 - 9s + 24 &= (As + B)(s^2 - 6s + 15) \\ &+ (s + D)(s + 9) = (A + C)s^3 + (-6A + B + D)s^2 \\ &+ (15A + 6B + 9C + 15B + 9D) \end{aligned}$$

Sol has constants

P.T.O

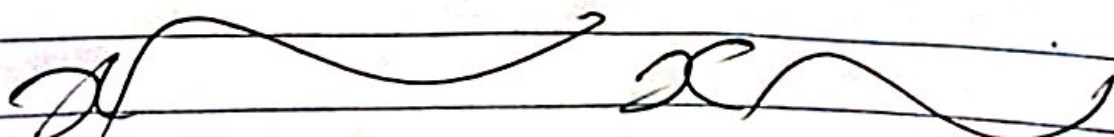
$$\begin{array}{l}
 S^3: A+C=-1 \\
 S^2: -6A+B+D=2 \\
 S^1: 18A-6B+9C=-9 \\
 S^0: 18B+9D=24
 \end{array}
 \left. \vphantom{\begin{array}{l} S^3 \\ S^2 \\ S^1 \\ S^0 \end{array}} \right\}
 \begin{array}{l}
 A=\frac{1}{10} \quad B=\frac{1}{10} \\
 C=\frac{-11}{10} \quad D=\frac{5}{2}
 \end{array}$$

Plugging the constant gives

$$\begin{aligned}
 y(s) &= \frac{1}{10} \left(\frac{s+1}{s^2+9} + \frac{-11s+28}{s^2-6s+18} \right) \\
 &= \frac{1}{10} \left(\frac{s+1\frac{3}{3}}{s^2+9} - \frac{11(s-3)}{(s-3)^2+6} \right)
 \end{aligned}$$

taking inverse function and sol will be as

$$\begin{aligned}
 y(t) &= \frac{1}{10} \left(\cos(3t) + \frac{1}{3} \sin(3t) - 11 \cos \sqrt{6}t \right. \\
 &\quad \left. - \frac{8}{\sqrt{6}} e^{3t} \sin \sqrt{6}t \right)
 \end{aligned}$$



Q3:

(a) Laplace:

Ans

Laplace transform is integral transform that converts a function of real variable (t) to the function of complex var(s).

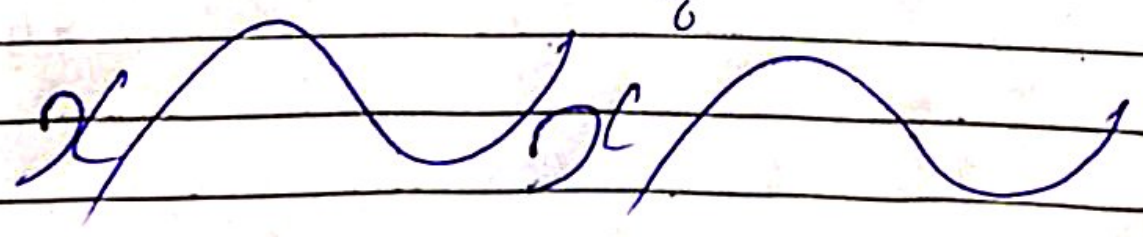
i.e.:

The Laplace transform y of a function f(t) for t > 0 is defined by the following

$$y \{ f(t) \} = \int_0^{\infty} e^{-st} f(t) dt.$$

general eq.:

$$f(s) = \int_0^{\infty} f(t) \cdot e^{-st} dt.$$



Q1:-

part a):-

Ans:- Homogeneous differential equation
 It involves only derivatives of y & terms involving y , & they are set 0, as in the example.

$$\frac{d^4 y}{dx^4} + x \frac{d^2 y}{dx^2} + y^2 = 0.$$

Non homogeneous differential eq:

Same as homogeneous differential equations, except they can have terms involving only x & y on right side as given.

$$\frac{d^4 y}{dx^4} + x \frac{d^2 y}{dx^2} + y^2 = 6x + 3.$$

Ans