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Subject:- Advance Fluid Mechanics

Q # 1 (Part-A)

DRAG :-

In fluid dynamics drag is known as air Resistance, a type of friction or fluid ~~friction~~ Resistance or type of friction or fluid friction. is a force acting opposite to the relative motion of any object moving with Respect to a surrounding fluid. This can exist between two fluid layers (or surfaces) or a fluid and a solid surface.

It has two components

1. Pressure Drag
2. Friction Drag.

1. (PRESSURE DRAG) (Fp):-

It is equal to the Integration of component in direction of motion of all Pressure forces exerted on surface of body.

$$F_p = C_p \frac{\rho V^2}{2} A$$

where "Cp" depends on shape.

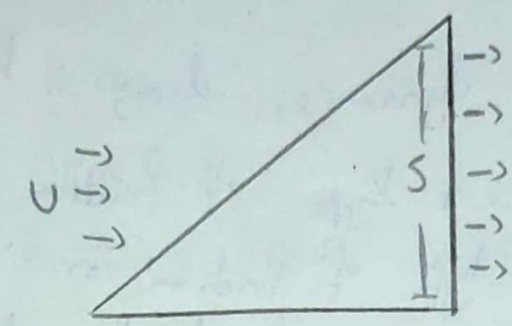
② FRICTION DRAG:-

It is Equal to Integration of Components of Shear stress along the Surface of the body in direction of motion

$$F_f = C_f \rho \frac{V^2}{2} BC$$

Cf -> Depends on viscosity.

FRICTION DRAG OF BOUNDARY



$$\bar{\tau}_0 = \mu \frac{U df(n)}{\delta d(n)}$$

$$\bar{\tau}_0 = \frac{\mu U B}{\delta}$$

As we ~~know~~ have $\tau_0 = \frac{\rho U^2 \alpha d\delta}{dx}$

Compare both

$$\frac{\rho U^2 \alpha d\delta}{dx} = \frac{\mu U B}{\delta}$$

$$\delta d\delta = \frac{\mu B}{\rho U \alpha} dx$$

Integrating on both sides.

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} x + C \qquad \because C = 0$$

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}} \qquad \because R_x = \frac{U x \rho}{\mu}$$

$B = 1.63$, $d = 0.135$

$$\delta = \frac{4.91}{\sqrt{Rx}} x \rightarrow (6)$$

where (Rx) is is local Reynold Number

we have

$$\tau_0 = \frac{\mu U B}{\delta}$$

$$F_x = \rho B U^2 \delta d$$

where d is a function of Boundary layer velocity distribution

Now to find Shear Stress

$$\tau = \frac{F_x}{A} = \frac{dF_x}{B\,dx} = \frac{dF_x}{\beta\,dx}$$

$$\tau_0 = \rho B U^2 d \frac{d\delta}{B\,dx} = \rho U^2 \alpha \frac{d\delta}{dx}$$

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$ – Generate Equation

LAMINAR BOUNDARY LAYER

$$\frac{u}{U} = f\left(\frac{y}{\delta}\right) \quad -(1)$$

$$\frac{y}{\delta} = \eta \quad \Rightarrow y = \delta\eta$$

$$dy = \delta\, d\eta \quad -(2)$$

$$\frac{u}{U} = f(\eta)$$

$$du = U\, df(\eta) \quad -(3)$$

For laminar flow

$$\tau_0 = \mu \frac{du}{dy} \quad -(4)$$

$$\tau_0 = \mu U \frac{df(\eta)}{\delta\, d(\eta)}$$

$$\tau_0 = \frac{\mu U \beta}{\delta} \quad -(5)$$

As we have $\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$

Compare both

$$\rho U^2 \alpha \frac{d\delta}{dx} = \frac{\mu U \beta}{\delta}$$

$$\delta\, d\delta = \frac{\mu \beta}{\rho U \alpha} dx$$

Integrating on both sides

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} x + C$$

$\because C = 0$

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}}$$

$$\because R_x = \frac{U x \rho}{\mu}$$

$$\delta = \frac{4.91 x}{\sqrt{R_x}} \quad \text{--- } (6)$$

where (R_x) is Local Reynold Number

As we have

$$\tau_0 = \frac{\mu U B}{\delta}$$

Put 6 in Equation 5

$$\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

Now

$$F_f = B \int_0^L \tau_0 \, dx$$

where $\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}$

$$\because R_x = \frac{x U \rho}{\mu}$$

then putting values we have

$$F_f = 0.664 \sqrt{\rho \mu L U^3}$$

we have

$$F_f = C_F \, \rho \frac{U^2}{2} BL$$

thus Equation both side.

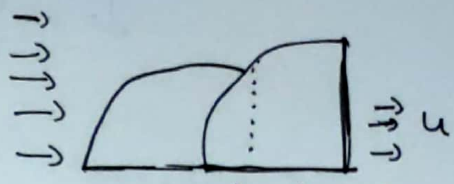
6

$$c_f = 1.328\sqrt{\frac{\mu}{\rho C U}} = \frac{1.328}{\sqrt{R_n}}$$

for Laminar $R \leq 500,000$

TURBULANT BOUNDARY LAYER:-

This fig shows the velocity distribution of boundary layer which is steeper near walls & flatter throughout reminder of layer.



The shear stress is greater in turbulant than in laminar thus

$$\tau_0 = \rho f \frac{V^2}{8}$$

where V is the average velocity to obtain relation between average and max we have.

$$\frac{V}{U_{max}} = \frac{1}{1.33\sqrt{f}} \quad \because f = 0.028$$

$$\frac{V}{U_{max}} = \frac{1}{1.33\sqrt{0.028}}$$

$$U = 1.235\,V$$

$$V = \frac{U}{1.235} \qquad \because R_n = \left(\frac{DU}{\nu}\right)$$

$$f = \frac{0.316}{(R_n)^{1/4}} \qquad D = 2\delta$$

$$\tau_0 = f \rho \frac{V^2}{8}$$

$$\tau_0 = \frac{0.316}{\left(\left(\frac{D}{\nu}\right)\left(\frac{D}{1.235}\right)\right)^{1/4}} \cdot \frac{f}{8}\left(\frac{U}{1.238}\right)^2$$

$$\tau_0 = \frac{0.023 \rho U^2}{\left(\frac{2\delta}{V}\right)^{1/4}} \quad -①$$

As we have general

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx} \quad -②$$

Equation 1 & ②

$u = 0 \quad , \delta = 0$

$$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\frac{V}{Ux}\right)^{1/5} . x$$

$d = 0.0972$

$$\delta = \frac{0.377}{(Rx)^{1/5}} . x \quad -③$$

$$\tau_0 = 0.0587 \int \frac{U^2}{2} \left(\frac{V}{Ux}\right)^{1/5}$$

Now

$$F_f = B \int_0^L \tau_0 \, dx$$

$$F_f = 0.0735 \int \frac{U^2}{2} \left(\frac{V}{UL}\right)^{1/5} . BL$$

$$F_f = C_f \int \frac{V^2}{2} . BL$$

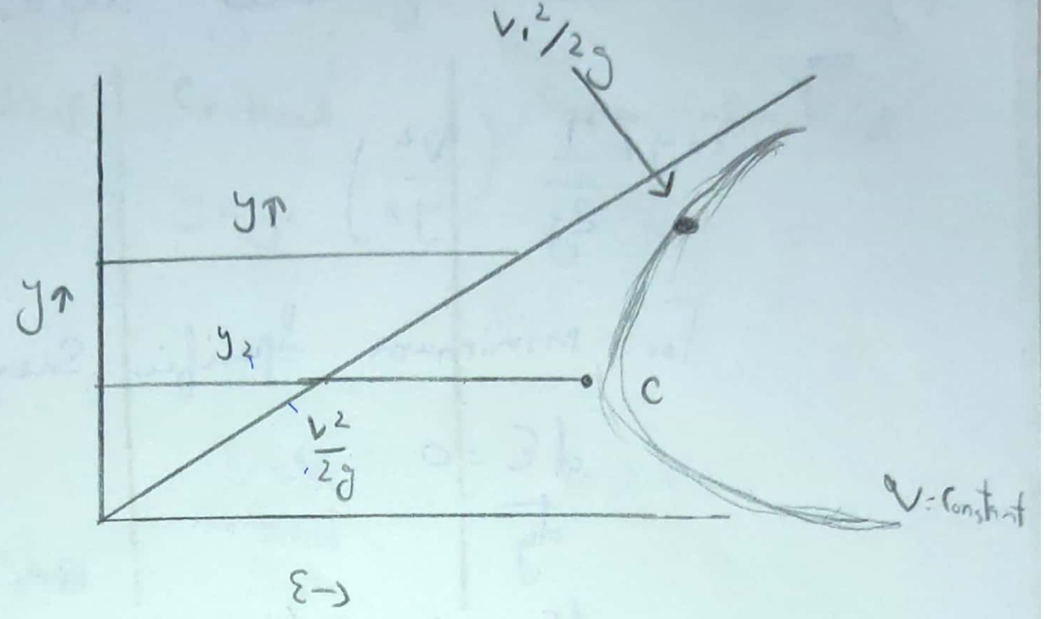
Equating on both Sides

$$\boxed{C_f = \frac{0.0735}{(R)^{\cancel{\log}\,1/5}}} \quad (500{,}000 < R < 10^7)$$

For e $R > 10^7$

$$\boxed{C_f = \frac{0.455}{(\log R)^{2.52}}}$$

Q2 Part (b)



This is Specific Energy equation:-

For particular q, there will be two kind of possible values of y ~~—~~ for given E. The equation is cubic with three roots with third being negative giving no values. Thus two Alternative Depths represents two totally different flow regimes - slow and deep an upper position & Fast & shallow & deep and upper position.

Point represent dividing point between two regimes of flow.

Thus for given "q" value of E is minimum & flow at this point is critical flow ~~Dater~~ Depth of flow at at this Point is ~~Bai~~ critical depth L/C

And Velocity at this point is Critical Velocity.

Thus Reaction of Critical depth Can be found as

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

For minimum Specific Energy

$$\frac{dE}{dy} = 0$$

$$\frac{dE}{dy} = 1 - \frac{2}{2g}\left(\frac{q^2}{y^3}\right)$$

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3}$$

$$1 = \frac{q^2}{gy^3} = q^2 = gy^3$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

As $q = vy$, $v_c^2 = gy^3$

or

$$v_c = \sqrt{gy_c}$$

$$y_c = \frac{v_c^2}{g}$$

Now

$$\frac{y_c}{2} = \frac{v_c^2}{2g}$$

$$E_{min} = y_c + \frac{v_c^2}{2g} = y_c + \frac{y_c}{2}$$

$\frac{3}{2} y_c$ or $y_{cr} = \frac{2}{3}$ Constant

	Sub-Critical	Critical	Super-Critical
Depth of flow	$y > y_c$	$y = y_c$	$y < y_c$
velocity	$V < V_c$	$V = V_c$	$V > V_c$
Slope	mild Slope $S_o < S_c$	Critical Slope	

Q#2

Given:-

Depth of Rectangular Channel (d) =?

Flow rate (Q) = 3.5 m^3/Sec

Slope of Bed (S_o) = 0.0008

$n = 0.0219$

width of Bed = 7386 mm

= 7.386

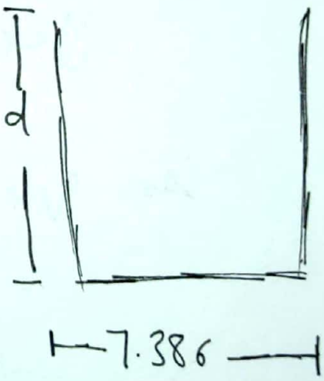
Critical Depth =?

Flow Sub Critical or Super Critical

Sol:-

Area = 7.386 × d

= 7.386d



Perimeter = d + 7.386 + d

= 7.386 + 2d

Hydraulic Radius (Rh) = A/P

$$= \frac{7.386d}{7.386 + 2d}$$

By using Manning Equation.

$$Q = \frac{1}{n} A R_h^{2/3} \cdot S_o^{1/2}$$

Putting values

$$3.5 = \frac{1}{0.0219} \times 7.386\, d \times \left(\frac{7.386\, d}{2d + 7.386}\right)^{2/3} \times (0.0008)^{1/2}$$

$$\boxed{d = 0.58\ m}$$

Area = $7.386(0.58)$

$\boxed{= 4.28\ m^2}$

Perimeter = $7.386 + 2(0.58)$

$\boxed{= 8.54\ m}$

Hydraulic Radius $(R_h) = \frac{4.28}{8.54}$

$\boxed{= 0.511\ m}$

Finding Critical Depth:

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

As $q = Q/B$

$= \frac{3.5}{7.386}$

$= 0.473\ m^2/sec$

$$\Rightarrow y_{cr} = \left(\frac{(0.473)^2}{9.81}\right)^{1/3}$$

$= 0.13$

As $y > y_{cr}$

$0.58 > 0.13$

So flow is sub-critical

Q#3

Given Data:-

Friction Drag (FD) = ?

width (B) = 200mm = 0.2m

Length (L) = 800mm : 0.8m

Specific Gravity (S) : 0.89

Undisturbed Velocity (U) = 5m/sec

Kinematic Velocity (ν) = 0.93×10^{-4} m²/sec

Sol:-

Checking weather flow is laminar or not By Reynold Number.

$$R = \frac{DU}{\nu}$$

For Smooth flat Plate

$D = L$, $V = U$

So $R = \frac{LU}{\nu}$

$$= \frac{0.8\times5}{0.93\times10^{-4}} = 43010$$

$43010 < 500,000 \rightarrow$ Laminar

By Using formula:-

$$F_f = C_f \cdot \rho \cdot \frac{U^2}{2} \cdot BL$$

where

$$C_f = \frac{1.328}{\sqrt{R}} = \frac{1.328}{\sqrt{43010}} = 0.0064$$

$$S = \frac{\rho_{oil}}{\rho_{water}} \Rightarrow 0.89 = \frac{\rho_{oil}}{1000}$$

$$\rho_{oil} = 0.89 \times 1000$$

$$\rho_{oil} = 890 \text{ kg/m}^3$$

$$F_f = C_f \cdot \rho \cdot \frac{U^2}{2} \cdot BL$$

$$= 0.0064 \times 890 \times \frac{(5)^2}{2} \times 0.2 \times 0.8$$

$$\boxed{F_f = 11.39 N}$$