Graph Theory

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1. Graph

- A graph is defined as a set of vertices called “Nodes” and set of Arcs called “Edges”.
- A graph is denoted by G and is given as G(V, E).
- The graph G is a combination of vertices “v” and Edges “E”. E.g.

* In fig we have a graph with vertices “x” and “y” and an edge “e” between x and y.

Graph Terminologies

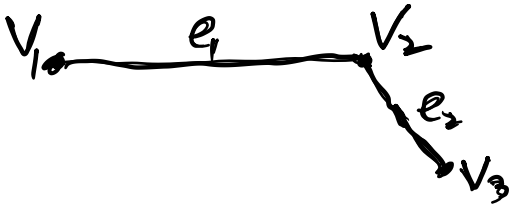
1. **Vertex**

* A set of elements is called a vertex Or
- It is a junction where something takes place
- It is also called a Node or Point usually represented by V1, V2, V3 ,……, Vn

2. **Edge**

* The line that joins two nodes or vertices, and some times it is attached to one node is called an Edge.

* The edges are usually represented by e1, e2 e3,…,en



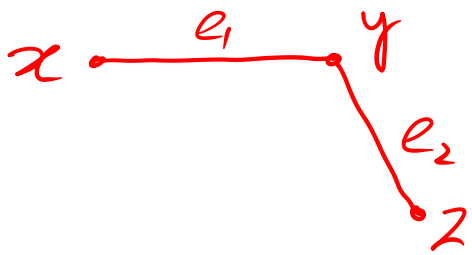
3. Adjacent Vertices / Nodes

- Two vertices are said to be Adjacent to each other if they are the end points of the same Edge.

- Here x and y are adjacent to each other as they are the end points of the same edge e

4. Adjacent Edges

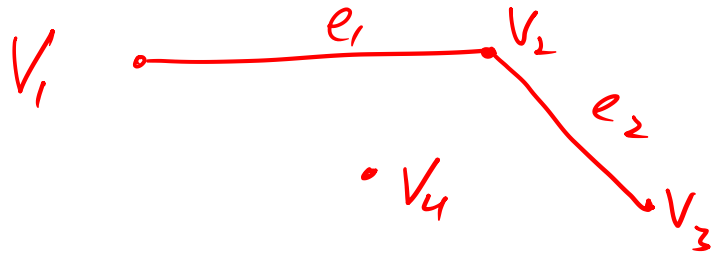
- Two edges are said to be adjacent to each other if they are incident on a single vertex.



- e1 and e2 are adjacent edges as they share the same vertex i.e. “y”

5. **Isolated Vertex**

- A vertex that is not connected to any other vertex in the given graph.



- V4 is the Isolated Vertex

6. **Path**

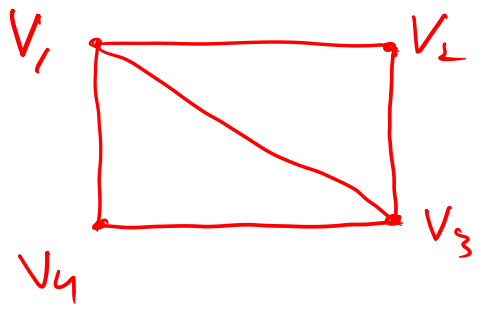
- A set of consecutive edges from one node to any node is a Path.

- A path of length “n” from node “u” to “v” is defined as a sequence of n+1 nodes i.e.

Here the length of path is 1 and it is defined between two nodes u and v

7. **Simple Path**

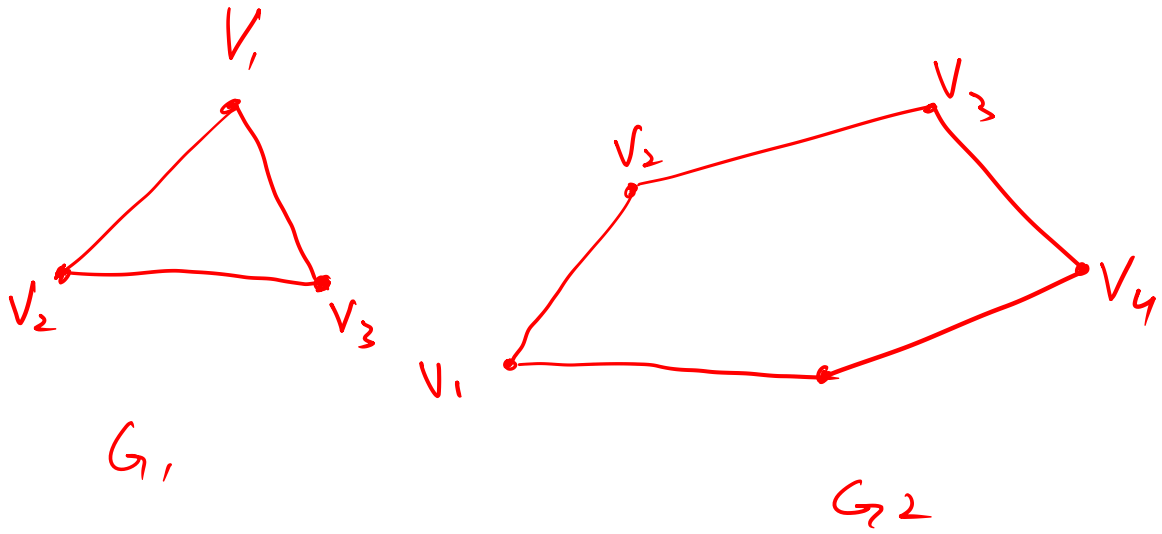
- A path from node “u” to “v”, covering minimum number of edges is a simple path.



- Here the Simple Path between V1 and V3 is the diagonal V1V3

8. Closed Path

- A path whose initial and final vertices are same is called a Closed Path

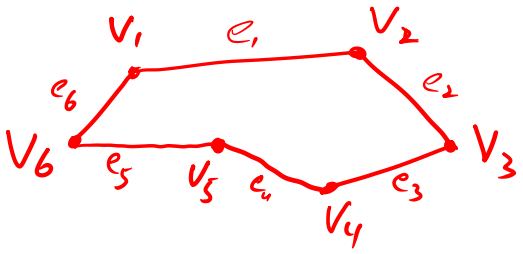


9. **Loop / Self Loop**

* It is a type of an edge that starts and end at the same vertex. E.g.

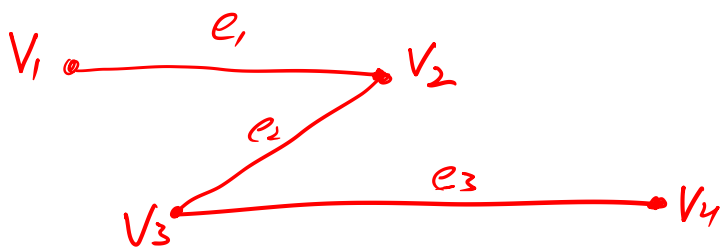
10. **Cycle**

* A closed path with numerically more than three edges e.g.



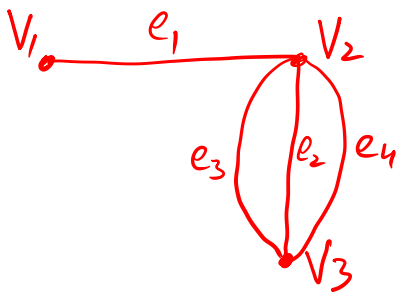
11. **Open Path**

- A path whose starting and ending vertices are different e.g.



12. **Multiple / Parallel Edges**

- When a graph has two or more edges joining the same pair of vertices, then the edges are called Multiple / Parallel edges.



- Here e2, e3 and e4 are parallel edges.

13. **Degree of a Node**

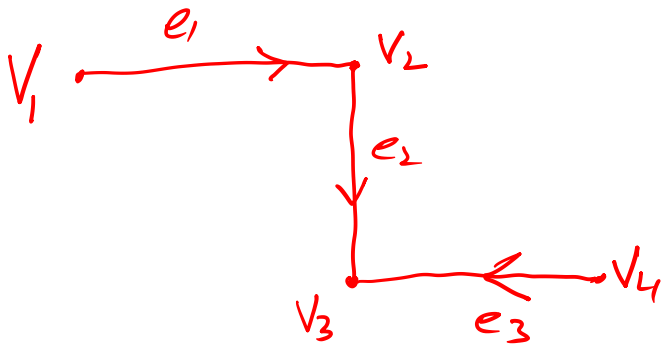
- It is the number of edges belonging to the node
- For a node “v”, its degree is given by d (v) e.g.

d (v1) = 1

d (v2) = 2

d (v3) = 2

d (v4) = 1



14. **In-Degree of a Node / Vertex**

- It is the number of edges ending on a node
- It is denoted by d (-v)
- Here in the graph given above

  d (-v1) = 0

  d (-v2) = 1

  d (-v2) = 2

  d (-v3) = 0

15. **Out-Degree of a Node**

- It is the number of edges starting from a node “v”

OR

- The number of edges leaving a node “v”
- It is defined as d (+v)
- In the graph given above

d (+v1) = 1

d (+v2) = 1

d (+v3) = 0

d (+v4) = 1

16. **Total Degree of a Node**

- Sum of In-Degree and Out-Degree of a node i.e.

$$d(v) = d(-v) + d(+v)$$

Therefore, from above given graph

$$d(v1) = 0 + 1 = 1$$

$$d(v2) = 1 + 1 = 2$$

$$d(v3) = 2 + 0 = 2$$

$$d(v4) = 0 + 1 = 1$$

17. **Source Node**

- A node “v” is called source if it has +ve Out-Degree and zero In-Degree, e.g. in above graph

d (-v1) = 0 and d (+v1) = 1

So v1 is a source node

18. **Sink**

* A node “v” is called Sink if it has +ve In-Degree and Zero Out-Degree, e.g. in above graph

d (-v3) = 2 and d (+v3) = 0

So v3 is a Sink