**Maxima and Minima Questions**

Q1 what positive number added to its reciprocal gives the minimum sum?

Solution 2

Let

x = the required positive number and

1/x = the reciprocal of the number

y = sum of x and 1/x

y=x+1/x

y=x+x−1

y′=1−x−2=0

x=1 answer

What number exceeds its square by the maximum amount?

**Solution 1**

Let  
x = the number and  
x2 = the square of the number  
y = the difference between x and x2

y=x−x2

y′=1−2x=0

x=1/2*answer*

**Problem 3**  
Q3The sum of two numbers is k. Find the minimum value of the sum of their squares.

**Solution 3**

Let  
x and y = the numbers  
z = sum of their squares

k=x+y

y=k−x  
z=x2+y2

z=x2+ (k−x) 2

dz/ dx =2x+2(k−x) (−1)=0

2x−k=0

x=1/2 k

y=k−1/2 k

y=1/2k

z= (1/2k)2+(1/2k)2

z=1/2 k2

Q4:The sum of two numbers is k. Find the minimum value of the sum of their cubes.

**Solution 4**

Let  
x and y = the numbers  
z = sum of their cubes

k=x+y

y=k−x

z=x3+y3

z=x3+(k−x)3

dz/dx=3x2+3(k−x)2(−1)=0

x2−(k2−2kx+x2)=0

x=1/2k

y=k−1/2k

y=1/2k

z= (1/2k)3+(1/2k)3

z=1/4k3

Q5: The sum of two positive numbers is 2. Find the smallest value possible for the sum of the cube of one number and the square of the other.

Solution

Let x and y = the numbers

x+y=2 → Equation (1)

1+y′=0 y′=−1

z=x3+y2 → Equation (2)

dz/ dx=3x2+2yy′=0

3x2+2y (−1) =0

y=3/2 x2

From Equation (1)

x+3/2x2=2

2x+3x2=4

3x2+2x−4=0

x=0.8685&−1.5352

Use

x=0.8685

y=32(0.86852)

y=1.1315

z=0.86853+1.13152

z=1.9354 answer

Q7: Find two numbers whose sum is *a*, if the product of one to the square of the other is to be a minimum.

Let x and y = the numbers  
x+y=a

x=a−y

z=xy2

z=(a−y)y2

z=ay2−y3

dz/dy=2ay−3y2=0

y=2/3a

x=a−2/3a

x=1/3a

The numbers are 1/3 a, and 2/3 a.          *Answer*