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# Section I

## Fundamentals

# 1

# Conceptual Bridge Design



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## 1.1 Introduction

Planning and designing of bridges is part art and part compromise, the most significant aspect of structural engineering. It is the manifestation of the creative capability of designers and demonstrates their imagination, innovation, and exploration [1,2]. The first question designers have to answer is what kind of structural marvel bridge design are they going to create?

The importance of conceptual analysis in bridge-designing problems cannot be emphasized strongly enough. The designer must first visualize and imagine the bridge in order to determine its fundamental function and performance.

Without question, the factors of safety and economy shape the bridge designer's thought in a very significant way. The values of technical and economic analysis are indisputable, but they do not cover the whole design process.

Bridge design is a complex engineering problem. The design process includes consideration of other important factors, such as choice of bridge system, materials, dimensions, foundations, aesthetics, and local landscape and environment. To investigate these issues and arrive at the best solution, the method of preliminary design is the subject of the discussion in this chapter.

## 1.2 Preliminary Design

### 1.2.1 Introduction

What is preliminary design? Basically, the design process of bridges consists of two major parts: (1) the preliminary design phase and (2) the final design phase. The first design phase is discussed in this section and the final design phase is discussed in Section 1.3 in more detail.

The preliminary design stage (see Tables 1.1 and 1.2)consists of a comprehensive search of current practical and analytical applications of old and new methods in structural bridge engineering. The final design stage consists of a complete treatment of a new project in all its aspects. This includes any material, steel, or concrete problems. The important argument is that with this approach a significant savings in design effort can be easily achieved, particularly in the final stage.

In order to plan and design a bridge, it is necessary first to visualize it. The fundamental creativity lies in the imagination. This is largely reflected by the designer's creativity and the designer's past experience and knowledge. Also, the designer's concept may be based on knowledge gained from comparisons of different bridge schemes.

Generally, the designer approaches the problem successively, in two steps. In preliminary design, the first and the most important part is the creation of bridge schemes. The second step is to check schemes and sketch them in a drawing. It will then be possible to determine other design needs. An examination process is then carried out for other design requirements (e.g., local conditions, span systems, construction height, profile, etc.). From an economics point of view, choice of span structure, configuration, etc. is very essential. From the cost and aesthetics prospective, the view against the local environment is important. Completing these two steps yields the desired bridge scheme that satisfies the project proposal [3].

In the preliminary design stage it is also required to find a rational scientific analysis scheme for the conceived design. Thus, an essential part of preliminary design is to select and refine various schemes in order to select the most appropriate one. This is not an easy task since there are no existing formulas and solution. It is based mainly on the designer's experience and the requirements dictated by the project.

The final stage requires a detailed study and analysis of structural behavior and stability. Economy and safety are also important aspects in bridge design, but considerable attention must be given to detailed study for the analysis, which involves the final choices of the structural system, dimensions, material, system of spans, location of foundations, wind factor, and many others.

However, the difference in preliminary schemes if all analysis is done accurately should not be substantial. Therefore, it is very important to have, from the first step, the design calculation exact and complete. The designer workload can be dramatically reduced through use of auxiliary coefficients. These coefficients can be used if the chosen scheme needs to be modified.

Design calculation is done on the basis of structural mechanics. Usually the analysis starts with the deck, stringers, and transverse beams which determine the weight of the deck. Final analysis includes a check of the main load-carrying members, determination of various loads and their effects, total weight, and analysis of bearings. Parallel to the analysis, correction of the initial construction scheme is normally carried out.

However, at the preliminary design stage it is only necessary to explain the characteristics of the alternatives. The comparison is normally based upon the weight and cost of the structure. It should also be highlighted that at this stage the weight of the structure cannot be determined with absolute precision. It is normally estimated on the basis of experimental coefficients.

As mentioned earlier, the aim of preliminary design is to compare various design schemes. This can be achieved efficiently by using computers. The designer can create a number of rational schemes and alternatives in a short period of time. A critical comparison between the various schemes should then be made. However, this is not an easy process and it is necessary to go to the next step. Various components of each scheme, such as the deck, the spans, supports, etc. should be compared with each other. It is important at this stage that the designer be able to visualize each component in the scheme, sketch it, and check its rationality, applicability, and economy. Following this, the analysis and drawings can be adjusted and corrected.

Finally, the chosen scheme should undergo a detailed design in order to establish the structure of the bridge. The analysis is applied to each component of the bridge and to the whole structure. Each part should be visualized first by the designer, sketched, analyzed, and checked for feasibility. Then it should be modified if necessary. In each case, the most beneficial alternative should be chosen. It is a very sensitive task because it is not easy to find immediate answers and the required solutions. The problem of making final choices could only be solved on the basis of general considerations and designer's particular point of view, which is undoubtedly based on personal experience and knowledge as well as professional intuition.

The sequence of analysis in detailed final design remains the same as for preliminary design except that it is more complete. The bridge structure at this stage has a physical meaning since each part has been formed and detailed on paper. Finally, the weight is estimated considering the actual volume of the bridge elements and is documented in a special form referred to as "specifications" or a list of weights. The specifications generally should be drafted at the end of the project. This sequence leads to the final stage of the project, but the process is still incomplete. The project will reach its final form only at the construction stage. For this reason, it is worth mentioning that the designer should from the beginning give serious consideration to construction problems and provide, in certain cases, complete instructions as well as methods for construction.

### 1.2.2 General Considerations for the Design of Bridge Schemes

Factually, the structural design scheme of the bridge presents a complex problem for the structural designer despite the presence of modern technology and advanced computer facilities. The scope of such a problem encompasses the determination of general dimensions of the structure, the span system (i.e., number and length of spans), the choice of a rational type of substructure. Also, within this scope, there is a demand to find the most advantageous solution to the problem in order to determine the maximum safety with minimum cost that is compatible with structural engineering principles. Fulfilling these demands will provide the proper solution to the technical and economic parameters, such as structure behavior, cost, safety, convenience, and external view.

Also, during the design of a bridge, crossing the river should take into consideration the cross section under the bridge that provides the required discharge of water. The opening of the bridge is measured from the level of high water as obtained at cross sections between piers, considering the configuration of the river channel, the coefficient of stream compression, and the permissible erosion of the riverbed. By changing the erosion coefficient and the cross-sectional area within the limits permitted by the standards, it is possible to obtain different acceptable dimensions of openings for the same bridge crossing. During the choice of the most expedient alternative, it is necessary also to consider that reducing the bridge opening is connected with increased cost of foundation as a result of the large depth of erosion and the need to apply more-complicated and expensive structures for stream flow. During the design of such structures as viaducts and overpasses, their total length is usually given, which may be determined by the general plan or by the landscape of the location and the relation of the cost of an embankment of great height and the bridge structure.

The design of the bridge usually starts with the development of a series of possible alternatives. By comparing alternatives, considering technical and economic parameters, we try to find the most expedient solution for the local site conditions. At the present time, the development and comparison of alternatives is the only way to find the most expedient solution. Factors influencing the choice of bridge scheme are various and their number is so great that obtaining a direct answer to what bridge scheme is most rational at a given local condition is a challenge. It is necessary to develop a few alternatives based on local conditions (geologic, hydrologic, shipping, construction, etc.) and apply the creative initiative of the designer to the choice of a structural solution. Providing structural schemes of bridge alternatives is a creative act., computers can be used to determine the

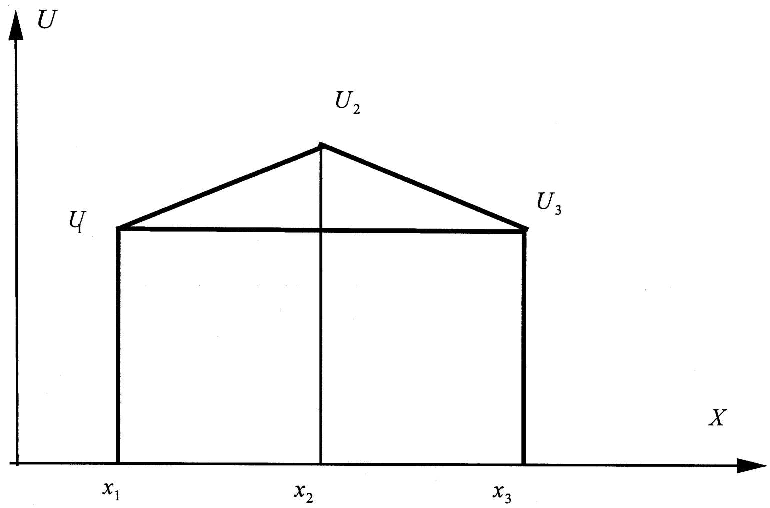


**FIGURE 1.1** Quality index of the structure.

most advantageous span length and span system, to find the number of girders on the bridge having a top deck or the number of panels in the truss, and to choose the substructure. However, using computers to make a choice of rational alternatives, considering a comparison of all technical and economic parameters, is impossible. Finding an optimum alternative using different points of view often leads to different conclusions. For example, the alternative may be the most advantageous by cost, but may require great expenditure on metal or require special erection equipment, which cannot be obtained. Some alternatives may not satisfy an architectural requirement, when considering city bridges. When using computers it is still impossible to refute the conventional design method, considering all problems of specific local condition, which are practically impossible to write into a computer program.

### 1.2.3 Theoretical Basic Method of Preliminary Design

Methods of design cannot be invented on the basis of certain arbitrary principles. They are developed from practice. In a given theoretical study, there are enough proofs that methods of design are changing depending upon the bridge-building practice and its basic problems. Therefore, today's applied method of preliminary design is mainly determined by empirical methods.

To achieve improvement, this method is based on consistency in its exact application and explanation of its logical basis. This advanced method of preliminary design makes it possible to develop a perfect final solution for the project. It is worth mentioning at this point the importance of calculation parameters in the considered design approach.

Using mathematical models, it is possible to express (see Figure 1.1) the quality indexes $U$ of the structure as a function of its parameters; $x$, $y$, $z$, i.e.,

$$U = u\,(x,\ y,\ z,\ \ldots) \tag{1.1}$$

Preliminary design provides means to determine the exact values of parameters and their quality indexes. The problem is similar to finding the limit of a function, as in calculus of variation. This analogy may be used to determine a logical basis for the method of preliminary design. It is clear that the problem of preliminary design cannot be solved in pure mathematics. The quality indexes cannot be expressed by algebraic functions. Note that the majority of parameters from one alternative to another change their size rapidly. Alternatives are shown only for consideration and to show the investigation process in order to prove the correctness of the accepted alternative.

Only in particular cases can a mathematical method be applied to find the limit. For instance, it is known that by this method it is possible to find exact dimensions of span lengths of simple-span trusses or exact heights of steel trusses those with parallel chords because of their behavior of minimal total weight of the structure.

To find the limit of the function *U*, it is possible to find the corresponding values of parameters *x*, *y*, *z* from the following equation:

$$\frac{\partial U}{\partial x} = 0, \quad \frac{\partial U}{\partial y} = 0, \quad \frac{\partial U}{\partial z} = 0 \tag{1.2}$$

These equations provide the tool to investigate the influence of each parameter as it changes the quality indexes of the structure. Leaving all other parameters constant, $\pm\Delta x$ is imposed to study the change of the value *U*. We can then find the value of the parameter for which $\Delta U$ changes its sign. This corresponds to the minimum of the function *U*.

Note that the separate parameters are interrelated. If one parameter is changed, it is necessary to modify the others. By exceeding certain limits, the span of the reinforced concrete bridge must change from a beam system to an arched system. Applying this method of preliminary design to bridges, the comparison of Eq. (1.2) leads to composition of alternatives. For each equation in Eq. (1.2), it is necessary to use a minimum of three alternatives. The first equation is formed from certain values of parameters $x_1$, $y_1$, $z_1$, etc. Leaving parameters $y_1$ and $z_1$ constant gives a new value of *x*, which is $x_2$ to compose a second alternative. Comparing this with the first, we establish the change of quality indexes of the bridge. If they have improved, it is necessary to change again the parameter *x* in the same direction, raising it to the new value $x_3$ to form a third alternative. Then, we compare this with the first two alternatives to determine the change of the quality indexes for the designed bridge. If, for example, they become worse, then their maximum value corresponds to $x_2$ (see Figure 1.1). If they improve, it is necessary to repeat the investigation for the second equation $\partial U/\partial y = 0$, and so on. All these equations must be solved simultaneously. In preliminary design, this means it is necessary to prepare many alternatives and compare them simultaneously. This process is difficult and tedious. The difficulty is increased because, unlike the purely mathematical method where the function U is given, in preliminary design the type of function is not known and should be determined.

Because of the above-mentioned difficulties, there is enough ground to assume that the first stage of the design process is based on creativity and invention.

How to build a bridge over a certain river? There are number of different answers to this question. The type of bridge can be steel or reinforced concrete, and for each case there are a number of applicable alternatives. If, for the given problem, there are several *known* solutions, there could be just as many or more undeveloped. This shows that building a bridge and creating a design are not easy tasks. Many undetermined problems face the designer. However, engineering science has proved that these difficulties could be solved in a systematic sequence, as illustrated below.

1. Equation (1.2) should be applied to the problem of structural design and solved by a method of successive approximations which follow preliminary design. This method is considered to be technically reliable and has been used in engineering successfully. In order to improve and accelerate this method (of successive approximation), it is of major importance to choose absolute precision. Experience, in a significant way, helps in making such a decision.
2. Preliminary design is generally the first approximation in the creation of a bridge project. It solves the equation for the most important parameters which have great influence on the quality indexes of the structure. Details of the structure may be investigated at a later stage.

Many solutions have been developed in practice for detailed structure. It is worth mentioning that when working with parameters, there are not many basic ones.

3. Some parameters are given and remain constant during design. Others take a limited number of values and this shortens the number of alternatives. Relations among the parameters, their correction, and the importance for the quality indexes of the bridge make it easier to carry out the methods of investigation of alternatives.

If it were possible to solve the problem by pure mathematics, then the solution would be simply to solve the equations. But we should remember that these equations (except those of the first degree) have several roots and arbitrary constants. In application to bridge design, this means, if a few equally valid alternatives are obtained, the investigation should be refined further.

The method of successive approximations should be accepted as the methodological principle because, as the process results in several alternatives for one project, we should consider only the best scientific solution. We may stop at an approximate solution, but only after we have been convinced that, in comparison with other solutions, it is the best scientific approach. This is the best way to generate designer success. The same method is applied for choosing a bridge system, as well as making a final choice for the material of bridge design, and so on.

### 1.2.4 Choice of Final Alternative for Reinforced-Concrete Bridges

The designer may, for instance, decide that, for a given material (say, reinforced concrete), a third alternative is chosen. Using this reasoning, the following imperfection arises: in the mathematical analogy, it was necessary to solve Eq. (1.2) simultaneously, but here each is solved separately. After determining a certain parameter, it will be kept constant and the choice for the others will follow.

There is an element of sensitivity within this method. The values of parameters are not chosen arbitrarily. The initial values are determined empirically so that their values are as exact as the real ones. The order of invention of individual parameters is also important. First, the parameters that affect the quality indexes of the structure most significantly are investigated. Then, investigation for the less important parameters follows.

Remember, the span structure implies length of span but also requires determination of the type of span structure, its form, its shape, its system, the varied types which could be uniform or unequal span structures, or the number of spans.

Within this method, the chosen alternative determines the system of span structure with minimum weight and maximum economy and safety. Now, a legitimate question may arise. Does it mean that the chosen alternative with varied type of span design (including span length, shape, form, and type) can be considered the *best* choice for the bridge project with maximum economy and safety? Although the answer may sound controversial and theocratically inconsistent, it is not. The answer is factually yes! There is a reason for that. Certain types of bridge projects require structures of various type of spans which represent economically and safely the least choice for bridge design. For refining, continue investigation by the method of successive approximation.

Note, when laying out spans for frame-beam bridges, an equal span system is often used because it provides maximum standardization of elements. However, the application of unequal span construction is also possible and in certain cases more favorable.

In following the above discussion, if an economically feasible span alternative is not satisfactory, say, does not meet the requirements of shipping regulations, or if the bridge length does not permit equal spans, then it is necessary by the method of successive approximation to find more sensitive alternative schemes with a system of spans unequal in shape, form, length, etc.

In conclusion, by changing the span system, the number of spans, or their form or shape or their combination or dimensions, it is possible to obtain a number of alternatives that will satisfy the best given local conditions at minimum cost.

For instance, changing the span system, say, by reducing the number of spans, results into a reexamination of the whole bridge design, and consequently a new bridge scheme should be drafted.

**TABLE 1.1** Preliminary Design — Example 1

| Design Stages | Beam System |
|---|---|
| First alternative | System of span structure, deck-type beam; bridge having three spans; construction of span structure from reinforced concrete having four main beams; supports are massive |
| Second alternative | Same, only two spans |
| Third alternative | Same, only four spans |
| Comparison of alternatives | The best alternative is four spans (third alternative); the first and second alternatives are canceled |
| Subalternatives of third alternative | 1. Four-span alternative with two main beams and supports from third alternative two columns |
| | 2. Same, with prestressed concrete |
| | 3. With application of welded reinforcing frame |
| Comparison of alternatives | The third alternative is chosen; four-span bridges with two main reinforced concrete beams |
| Fourth alternative | Arch-type, three spans with four separate arches and columns above arches; supports are massive |
| Fifth alternative | Same, two spans |
| Sixth alternative | Same, four spans |
| Comparison of alternatives | Fourth alternative is chosen: three-span bridge with four separate arches |
| Subalternatives of fourth alternative | 1. With two narrow arches and walls above arches |
| | 2. With box-type arches |
| Comparison of alternatives | Fourth alternative is chosen: three-span bridges with four separate arches |

If the weight of the span structure is relieved, the span system should be modified either by decreasing the span length or span shape or form or other aspects of span structure.

It worth mentioning at this point that the significance of choosing the right alternative for the span system should not be underestimated. This is because the choice of material type for the bridge structure (e.g., monolith or prefabricated, conventional or prestressed, or reinforced concrete) is of lesser importance in cost value than the total span system, which consists of length, shape, form, number of spans, etc. Due to the relative simplicity of reinforced concrete shapes of spans and supports, calculating their volume is not a difficult process if their dimensions are given or determined from preliminary calculations.

The greatest advantage of applying theoretical methods is that the process of design is not abstract and is based on scientific analysis and quantifiable information. Therefore, during the process of choosing the best alternatives for solution, there are opportunities for eliminating imperfections for each scheme. Typical for this method is searching for the best solution through detailed investigation for each material, superstructure, bridge system, etc.

The number of alternatives obtained could be large. In the scheme of variation given in Table 1.2 it was decided to compose these alternatives with subalternatives. It takes extensive investigation, which is not always necessary. In some cases, shorter methods may be applied. In the example shown in Table 1.2, it is possible to choose the bridge material first, thus composing one alternative for steel and one for reinforced concrete systems. It is advisable to consider information from experience using an empirical approach for bridge schemes.

## Example 1.1: Preliminary Design of Highway Bridge

Given

Clearance, design loads, location, bridge span, type of foundation (wells), and the material is reinforced concrete and steel.

Solution

The detailed design process is shown in Table 1.2. It can be observed that the shortened method is achieved to reduce investigation and the number of alternative projects and to increase the use of

**TABLE 1.2** Preliminary Design — Example 2

| Design Stages | Beam System |
|---|---|
| First alternative | Reinforced-concrete beam, three-span bridge of deck system, with four main beams; supports are massive |
| Second alternative | Steel beams two spans |
| Comparison of alternatives | The first alternative is chosen: reinforced-concrete bridge |
| Third alternative | Reinforced-concrete arch, two spans having four separate arches |
| Comparison of alternatives | After comparison of the first and third alternatives, the first alternative is chosen: beam bridge |
| Fourth alternative | Two spans, reinforced-concrete beam bridge |
| Fifth alternative | Four spans, reinforced-concrete beam bridge |
| Comparison of alternatives | After comparison of the first, fourth, and fifth alternatives, the fifth alternative is chosen: four-span bridge |
| Subalternatives of fifth alternative | 1. With two main beams, monolith |
| | 2. Same, with four beams, prestressed concrete prefabricated |
| | 3. Same, with welded reinforcing frame |
| Comparison of alternatives | By comparison of the fifth basic and additional alternatives, fifth subalternative 2 is chosen |

existing available data in practice. Thus, new investigations are unnecessary and this results in significant savings in design analysis and endeavor.

## 1.3 Final Design

### 1.3.1 Basic Trends in the Design of Bridges

In many aspects, the design of bridges is based on exact analysis and for this reason it is analogous to the solution of mathematical problems, where the results are obtained by examining the problem data and utilizing mathematical methods to arrive at a solution. This approach works well for technical and economic analyses which present very important aspects of bridge design, but it leaves out a significant part of the project.

This is because, first of all, many problems cannot be solved numerically. Second, the analysis may not correspond exactly to the actual situation. Technical analysis is valid for providing information for construction, but not significant for the solution of basic problems: choice of bridge system, choice of material, general dimensions, foundation problems, etc. These problems are solved on the basis of general considerations and the designer's judgment.

For the same problems in technical analysis or basic problems, for a bridge project there could be as many proposals as the number of the participating designers involved in engineering disputes [6]. The final choice of alternative depends to some extent on the attending participants who defend their view and support their arguments technically. It is necessary to analyze the different reasoning and determine which proposal is the most consistent with prevailing and accepted standards in the present circumstances.

The assistance of different methodological trends in bridge design is inevitable considering centuries of steady improvement and progress in bridge engineering. Progress in techniques of bridge construction depends on scientific and technological developments at each historical moment in the creation of a bridge; traditions are preserved and present views are formed.

An investigation of the history of bridges demonstrates that bridge construction has passed through several industrial stages [7]. We can separate these stages into primitive, industrial, architectural, and engineering phases. These can be subdivided still further into simpler forms and characteristics.

The influence of previous centuries on bridge design indicates that, to best understand the present trends, one must study the evolution of bridge engineering. Note that it reflects involvement of materials, the spiritual culture of the society, and the transfer of heritage. Concerning technological advances universities have had a large influence. The future engineers take from their professors the basic knowledge and new trends in design.

### 1.3.2 Creative Trends

In the 20th century, bridge design has undergone considerable change. With increasing demand for reinforced-concrete bridges, the need for and the creation of a new system was inevitable. The old methods had many limitations and will not be discussed here. They actually presented many obstacles for further developments in bridge engineering. It was necessary to create new specifications for reinforced-concrete structures.

The construction of highway bridges and the application of reinforced concrete presented designers with a basic problem regarding the choice of the bridge system. This created strong demand for preliminary design. This new concept required developing new methods and has put pressure on designers to look at the bridge not as a condensation of essential parts but rather as a monolithic compound unit with interrelated parts.

Because of the growing demand for reinforced-concrete and suspension bridges, the designer had large choice of materials and means to develop new bridge systems and the idea of cable-stayed bridges followed. The new century created strong demand for an analytical approach and necessitated a growing need for preliminary design with more schemes.

The acceptance that for each case there is no *one* solution but, rather, that there are several from which it is possible to choose the one most consistent with prevailing, accepted standards and most effective for the actual project leads to the basic characteristic of the second significant trend in bridge design, which will be called "creative."

Therefore, the design of each bridge is a process of finding a solution to a new problem. If there is no solution available, it must be sought. Considering the role of personal creation, this second trend may provide original new projects. Supporters of such a trend believe that creation of a bridge depends upon personal predisposition, capability, and vision. Design is considered to be a creative process that consists of a combination of structural expressions based on required knowledge and professional intuition.

### 1.3.3 Practical Trends

Practicability is the main consideration in this trend. The word *practical* goes hand in hand with scientific investigation using modern technology. Designers use both scientific principles and creativity for their designs only in order to solve the actual problem. In this trend, the bridge is considered as part of the highway or railway and its basic purpose is to satisfy the requirements of transportation.

The bridge should satisfy the basic requirements of safety and economic factors. The construction of the bridge should also follow the pattern of successful industrial methods.

Supporters of the creative trend considered highways and railways as areas to apply their creative capabilities and for testing their new inventions. Followers of scientific analysis investigation considered highways as large laboratories for their investigations. Adherents of practical design have borrowed their concepts from both trends, insisting that bridges must be first safe and permitted experimental structures only on secondary highways. Practical designers suggested that the structure should be standardized for industrial preparation because it could lead to faster ways of reconstruction or rehabilitation. Also, practical designers insist on use of construction techniques that require minimum maintenance and do not affect the traffic flow.

### 1.3.4 Basic Assumptions of Design

Methodological rules compatible with technical and applicable requirements in bridge engineering play a major role in modern progressive methods for designing bridges.

Nowadays, time is an important factor, especially in bridge construction. Progressive methods must satisfy technical swift performance as well as requirements of astute engineering economy. Such majestic structures must function effectively and, in addition, be aesthetically appealing. Bridges play the major role in the transportation system crossing rivers or other obstructions.

At different times, bridges were built for more than one purpose. The following are examples:

1. Roman bridges and those built in the Middle Ages served not only for transportation or for chariots, but also for joyful, exuberant activities for the population. These traditions were continued at later times.
2. Another trend that appeared in the Middle Ages is the construction of bridges for fortresses, castles, and towers as a protective measure against attacks by enemies. An example is the bridge at Avignon, France; also "London Tower Bridge," which was built with towers for aesthetic purposes only.
3. Another trend in the same era was to build chapels on bridges and to collect tolls to maintain them, the same old problem of upkeep (e.g., Italy, Spain, Germany).
4. During the Middle Ages and later, bridges were built to serve as dams for water mills, which were important parts of the economy in those days (e.g., Holland).
5. During the 16th and 17th centuries bridges were built as wide structures for shops and convenience in general. Good examples are London Bridge, England and Ponte Vecchio, Florence, Italy. Construction of these types of bridges was terminated toward the beginning of the 19th century.
6. In Western civilizations, bridges are sometimes built as majestic monuments to commemorate outstanding events or achievements of national importance for an important person or national hero. Examples are the monument to George Washington, the George Washington Bridge, New York City; the monument to Princess Margaret of Great Britain, The Princess Margaret Bridge, Fredericton, New Brunswick, Canada (this bridge was designed by M. S. Troitsky); the monument to the victory at the Battle of Waterloo, The Waterloo Bridge, London, England; the monument to Russian Tzar Alexander the Third, The Alexander IIIrd Bridge, Paris, France (one of the most beautiful cast-iron bridges of imperial style); the monument of the Sarajevo Association, The Gavrilo Princip Bridge, Sarajevo, Yugoslavia; the monument to Napoleon Bonaparte's victory at the battle of Austerlitz, Austerlitz Bridge, Austria.

The 19th century was characterized by industrial growth, and the use of bridges was confined to transportation as a result of the boom in building railways. Later, with Ford promoting "auto-vehicles," the building of bridges for highways became in great demand. This new trend in transport requirements put on pressure to improve safety factors as well. As a result, it is very important in modern bridge engineering to determine the carrying capacity of the bridge or the maximum value of the temporary vertical load that the bridge can bear.

Also, to avoid interruption in traffic flow, the calculations should consider the maximum number of vehicles passing in a given time. For bridges crossing navigable rivers, passing clearance must be considered. Also, similar consideration should be given to underpasses. The carrying capacity of a bridge is defined by the number of lanes, their width, and the accepted lateral clear distances of shoulders and medians required for safety considerations.

To avoid interrupted traffic flow, it is necessary for the width of the bridge to be greater than that required by the calculated carrying capacity. For example, in long bridges, it is necessary to provide an extra parking space for possible emergency cases in order to prevent a traffic jam. As a rule, the width of the roadway on the bridge is equal to the width of the highway. However, there may be deviations from this rule. For instance, although the highway may accommodate three lanes for traffic, the number of lanes on the bridge could be reduced. Also, there are examples of the reversed situation.

The condition of maximum traffic suitability and convenience is not a requirement but is preferred and attention should be paid to this issue during planning the project. Also, this issue could be considered as one of the criteria for the appraisal of the project, provided that the cost is not prohibively excessive.

The most efficient functional bridge structure is considered to be the one that embodies the most requirements of transport, with top safety factors, carrying capacity, that contains extra convenience facilities, that is most effective in labor and material, and that can be completed in a reasonable time. Since Henry Ford's time, extra pressure has been put on the transportation system, primarily on highways and railways, which has directly affected innovation in bridges. Modern-day transport is increasing in number and weight. This means bridges must be designed so that their carrying and passing capacities can accommodate heavier vehicles and larger numbers of vehicles. Designers must be resourceful and have means to overcome difficult situations effectively and to cope with the growing demands of faster and larger moving transport with the greater reserves for future growth, the longer the bridge stands without needing repair or reinforcement.

Note that by increasing the reserves for passing and carrying capacities, the cost of the bridge will increase. Determining the necessary reserve is a problem that needs to be resolved by engineering economy. The Romans did not visualize the fast development of transport and means for transportation, but concentrated their conceptual design on timelessness of the bridge structure and, for this purpose, provided great reserves for passing and carrying capacities.

The property of material is not necessarily the basic factor that defines the service time and safety of the bridge. More often, bridges are reconstructed for other reasons: too small passing and carrying capacity, insufficient clearance under the bridge, straightening of lanes or reduction of the grade.

### 1.3.5 Basic Requirement of the Bridge under Design

Choosing the right location is crucial for designing and planning a bridge. But above all, safety considerations that govern the technical, functional, economic, efficiencies, expeditiousness, and aesthetic requirements are very important. It is necessary for the bridge and each of its components to be safe, durable, reliable, and stable. This is usually checked by analysis using current specifications. But not all questions of durability, reliability, and stability may be answered by analysis. Therefore, in some cases it is necessary to provide special measures such as testing the performance of the structure and examining its behavior under maximum loading on the construction site.

Specifications and technical requirements should be satisfied because they guarantee the carrying capacity of the structure. From the safety point of view, all bridges designed according to the technical requirements are equal. But practically speaking, different aspects of technical requirements may be satisfied with different margins of safety.

Regarding the various bridge components, it is necessary to know that for engineering structures, the best solution should provide the appropriate material and carrying capacity.

During comparison of projects, the technical requirements should be considered. Because technical requirements may be accomplished using alternatives, consideration should always be given to additional guarantees for safety. Never compromise the safety of the passengers. Essential requirements naturally should have great importance, but they are basically satisfied by accepted clearance. Also, additional consideration must be given to issues other than elementary demands in order to make traffic flow efficiently. Note that the height of the bridge and the elevation of the roadway must be determined at an early stage, because they have influence on the traffic flow. Also, greater or smaller grades of the approaches should be designed earlier in the project. Maximum grades are defined by specifications, but for practical purposes minimum grades are the most convenient. Further, it is important to define the number of joints in the roadway that correlate to the division of the structure in separate sections.

Conditions of minimum wear of the parts of carrying construction under the influence of moving vehicles are also important to consider. Regarding the maintenance of the roadway and the bridge, it is possible to consider this as a general expense and therefore relate it to economic considerations.

Essential requirements indicate that the total cost of the bridge at all conditions should be economically rational. The overall cost of construction and bridge erection is determined in significant part by the quantity of material and the unit price. Yet, the tendency to reduce the quantity of material in order to achieve lower cost does not always lead to minimum overall cost. There are other factors that should be taken into consideration. Take, for example, steel structures: consideration should be given to quantity of steel and on top of that special attention must be given to modern industrial practices in production which in its turn may lead to conveniences in erection resulting from heavy construction with lower cost.

During comparisons of various projects, analysis of their economic criteria may reveal principles of expedience that can be applied to the project under consideration. Construction requirements are connected to economic constraints because, when the amount of material is small, the work is simple and the time required is shorter. Also, the unit price is considered as part of the economic criteria, which implies the cost of preparation and erection. All these factors affect the overall cost.

For conventional bridges to be built from a certain material, construction is carried out by established methods. Therefore, during comparison of alternatives, construction criteria are not so important. In special cases of complicated erection of bridges having large spans, or for urgent work, construction requirements are very important and may influence the choice of the bridge system and material. In these cases, it may be necessary to use a great quantity of materials, thus increasing the cost of construction and ignoring other requirements. For example, during the initial period of application, assembled reinforced-concrete constructions were more expensive than monolithic ones. However, with increased use of these constructions, the application of assembled structures is more rational and economical.

### 1.3.6 Aesthetic Requirements

Apart from the basic requirements of the bridge design, there are often additional demands. The first is the problem of aesthetics. Beauty should be achieved as a result of good proportions of the whole bridge and its separate parts. In spite of the tendency to build economical structures, we should not forget beauty. The importance of the architecture of the bridge should not be ignored because of economic and technical requirements. In fact, the most famous bridges are remembered by their architectural standards and magnificent structures (examples, Brooklyn Bridge, Verrazano Narrows Bridge, Golden Gate Bridge, Tower Bridge, Alexander IIIrd Bridge, Ponte Vecchio Bridge, Revelstoke Bridge, British Columbia (designed by M.S. Troitsky), Skyway Bridge, Ontario (designed by M.S. Troitsky), etc.).

There are different views regarding aesthetic practices in bridge engineering. Supporters of the rational analytical trend feel that aesthetic demands are not important and not necessary for bridges outside cities. On the other hand, designers of the creative trend consider these aesthetic values to be more important than the economic ones and equivalent to the requirements of strength and longevity.

Because of the conflicting views, this problem requires special consideration. All designers inevitably want their structure to be the most beautiful. This wish is natural and shows love and interest of the work and is necessary in order to make the designed structure head toward perfection.

During the process of design, the engineer is occupied with detailed calculations. The engineer also may be occupied with particularities and may lose sight of the complete structure. By checking the creation from an aesthetic point of view, the engineer gives attention to the wide scope and shape of the structure and has the opportunity to design details and correct if necessary. If the designer is aesthetically unsatisfied with the creation, the designer will improve it and try to find workable solutions. But the designer should always be aware of technical, economic, and safety values of the structure. Note, the architecture of bridges should not contradict either as a whole or in details the purpose of the structure. The designer's ideas should be compatible with the technical concept, surrounding conditions and environment (for example, London Tower Bridge).

It is necessary to be technically literate. Moreover, it is not enough just to design the external view of the bridge. Bridges satisfying demands and requirements of modern engineering requests and properly designed will achieve recognition and will deserve worldwide acknowledgment and credit. If designers are guided by fanciful tastes of their own, regardless of the technical concepts, they will not achieve this goal. A beautiful shape alone cannot be invested and applied to the bridge. The design should consider both the technical concepts and the structural shape.

The critical rules of proportion and the use of purely geometric shapes had, in their time, not so much an aesthetic but a technical basis. Designers based their theories on the principle of initiations and relations that they observed in nature. Historical investigation indicates that many aesthetic rules were preserved from previous centuries when they had a different basis. Even today, a bridge is considered beautiful when it has an even number of supports because it is classic and not easy to achieve with tough natural conditions. According to Palladio [7] it is clear that this rule is accepted because all birds and animals have an even number of extremities which give them better stability. Freeing themselves from prejudice and carrying out independent investigations to find the shape corresponding to the contents should lead designers toward the development of the theory of true aesthetics in bridge engineering. History has shown how the shapes of bridges were changed depending upon the general development of the cultural and economic life of a nation. For this reason, the problem of aesthetics in bridge engineering should be viewed in a historical perspective. A designer should be able to judge the bridge by considering its external view and scheme of construction.

Followers of the historic direction renounced such investigations and by this changed their principles and were more attracted to the design of bridges. However, a joint venture by engineers and architects is not always useful for solving a problem of bridge design. Nowadays, architects specialize in the construction of buildings which is reflected in their aesthetic taste. Although architectural rules and views may be correct for buildings, they may not be applicable to bridges. For example, when designing a building, architects usually use steel construction as a frame for the building which requires certain covering. For the bridge designer, steel construction is a force polygon that clearly demonstrates the transfer of forces. For an attractive external view for the bridge, detailed design and proper accomplishment of the construction are important. The external view may be spoiled by careless work. The technical concepts of structure and the architectural shape should not be separate, but should satisfy the local conditions and cover a wide scope of requirements. By understanding the validity of recognizing special aesthetic criteria a proper alternative can be selected. The final choice of alternative is the solution of some technical problem in correspondence with the basic purpose of the bridge as part of the roadway.

If the bridge is not considered a monument commemorating an outstanding event or an outstanding historic figure or a significant happening in the world, but serves only for traffic for a certain period of time, then it is not necessary to design this bridge as a highly aesthetic creation. We may be satisfied by more modest wishes with regard to its external view. Practice indicates that designers may create, and actually have created, attractive bridges even when they were governed only by the technical and economic requirements during the design process.

A bridge that is properly designed from the technical and economic point of view cannot contradict the basic rules of architecture. The general basis of architecture consists of the idea that masses of material should be distributed expediently. The properties of the material should be used correspondingly, and the whole structure should correspond to its purpose.

Generally, economic considerations of bridge design are the same as those stated above. An economic design is achieved by (1) the expedient distribution of material, choice of the most economical system, cross sections of the members, and considering working conditions; and (2) the use of proper material (members in tension use steel, members in compression use concrete).

Therefore, economic expediency and architectural conception are determined by the same criteria. From this, it is impossible to contrast aesthetic criteria with technical and economic aspects.

For example, it is advisable to reject a beam bridge for an arch in the case when the first by all other properties is better, or to prefer a single-span bridge to the more expedient two spans. Also, it is possible to say that the choice of alternative, considering technical and economic criteria, should not deviate from the proper way to achieve the aesthetic aims.

Finally, the bridge will only be perfect in an aesthetic sense, when its system as a whole and its separate members are chosen not on the basis of personal taste of the designer, but considering technical and economic expedience.

All other proofs that are often applied by the authors of separate projects to defend unsuccessful technical and economic alternatives should be rejected. All these proofs are based on the unstable and changeable bases of personal opinion. Such proofs are only declarations of personal impressions and tend not to prove anything but only to convince people by the use of feeble verbal arguments.

Many definitions are expressed using varied terminology synonymous in meaning, but with drastically different shades in the positive and negative sense. For example, regarding the structure of the bridge, when the deck is at the bottom chord the defender may say that this structure is "expressive," "easily seen," or "stands out with a beautiful shape on the sky." The opponent, however, may object and say that this structure "obstructs view," "hangs on the observer," etc. By the skillful use of such terminology, it is possible to convince the inexperienced that a beautifully presented perspective is not as worthy of praise as a less successful project.

### 1.3.7 Requirement for Scientific Research

The second additional requirement sometimes asked of bridges under design is called the scientific research or "innovation." This requires that the bridge contain a new achievement due to scientific research or a new invention.

The design of a bridge always contains something new. Even if the project is worked using old examples and applying typical projects, the designer uses new contributions along with the known. Therefore, there is always a certain degree of novelty. A good designer or engineer should not only be familiar with previous designs but should also be updated with modern scientific research and benefit from that by using advanced technical sciences in the design as the project changes.

It is natural for the designer to search for novelty; yet new solutions should be born only from the tendency to reach the best solution by starting from the existing conditions at the project. Therefore, the "novelty" requirement cannot run contrary; they should complement each other.

The history of the evolution of bridge engineering is the progression from simple to more complex, and it was achieved gradually and unevenly. Some periods were distinguished by invention and the appearance of new shapes, systems, and types of bridges; other periods were characterized by mastering and perfecting existing systems and the development of scientific research work. For example, at the end of the 19th century, a great step forward was made in the area of stone bridges. Perhaps the most significant achievement in the modern era was the appearance of the cast-iron arch and iron-suspension systems with different members of trusses and large spans. All these novelties resulted from the impact of growing industry and transportation.

The first 40 or 50 years of the 19th century were spent creating the iron beam bridges, and the second part of the century was devoted to developing expedient systems and improving the construction. Significant periods in later history were devoted to the development of reinforced-concrete bridges. The initial period of trials and creation of the construction was 1880 to 1890, and the period of mastery was 1900 to 1910. However, it is necessary to note that with the general development of science and technology, the role of scientific research is increasingly racing together with novelty, rationalization, and invention. It is obvious that the necessity for novelty results from the general economic conditions and sociocultural requirements.

The attitude toward novelty in bridge engineering has been modified. Adherents of the rational, analytical direction preferred to hold on to some classical models, considering that the search for new shapes should be related only to scientific research work of creative direction, however, tending toward the new and original by ignoring any old pattern. A realistic approach to a new idea should be based on understanding that novelty is not an aim in itself and that the new idea should be a solid ground for improvement.

It is necessary to consider the criterion of novelty because it sometimes appears as an independent factor during appraisal of projects and choice of alternatives. Because novelty is not an aim in itself, it should not be a special criterion, forcing a preference for new construction irrespective of its quality. When by basic conditions the new idea is better and there is no doubt regarding its quality, then it should be adopted and should replace the old. In the opposite case, it should be refused.

Not every novelty leads to progress in bridge engineering. If the novelty is sound, it may be developed to such a degree that it would lead to a new method, but if it is not better than the old method or not yet developed, its development at a later stage may be helped by abstaining from early application. Early application leads to lowering the quality of bridges and may compromise new ideas before they reach full appreciation and are fully evaluated.

The criterion of novelty may be considered independent only in separate cases when economy requires the introduction of a new type of construction. An example is the introduction of prefabricated reinforced-concrete construction. At the present time it is expedient to use prefabricated reinforced-concrete construction, but initially it was more expensive than conventional construction. The criterion of novelty then was contrary to other criteria. It had to be solved for each case, especially when the novelty was not an aim in itself, but was required for economic and commercial demands.

One reason for introducing new construction techniques is related to the necessity of experimental and practical checking of the scientific research work, which is certainly necessary.

Regarding bridges on main highways, however, it is not advisable to subject them to experiment, because their basic designation is to serve transportation. Only separate experimental structures and special controls are permitted. However, in each case, the problems of special scientific research and structural experimentation should be performed at a scientific institution.

### 1.3.8 Basic Parameters of the Bridge

The quality of the structure is evaluated considering different criteria: technical, functional, economic, construction, and, in addition, the material of the system and the geometric dimensions of the bridge. All these criteria are temporary parameters defining the quality of the structure.

The problem of design generally consists of the way to find the values of these parameters that will correspond to a better quality of the structure. It is necessary to consider first, in detail, basic factors influencing the quality of the structure. All the parameters interact, but their influence on the quality of the structure is different. Their influence on each other is different: one may depend little on another; another may greatly influence the other. For example, basic parameters for material may not influence basic parameters of foundation and so on.

During preliminary design, the determination of basic parameters interacts and has major influence in making decisions about the location of the bridge, the span, the material, the type of foundation, the system of the bridge, the length of separate spans, the type of superstructure, and the type of supports.

The location of the bridge usually does not much depend on other parameters, but does have an impact on them. For small bridges, the location is defined by the intersection of the highway with the river, ravine, etc. For medium to large bridges, it is possible to compare a number of alternatives, such as the basic value of the highway and the cost of approaches and highway installations. The cost of the bridge itself plays a deciding role because its span at all alternatives is usually an unchangeable constant. For this reason, during selection of bridge location it is possible to propose an often-used bridge type without detailed study. However, there are two exceptions to this general rule. First, if the river is not used for shipping and has sandbanks, then at the location of largest curvature the span of the bridge obtained is smaller, but the depth of the water here is greater. Therefore, foundations are complicated and the installation of pile supports may be impossible. On the sandbanks where the span is increased, but the water depth is shallower, it is possible to build a simple viaduct-type bridge supported by the piles. If the bridge is proposed to be built from timber, its location should be chosen over sandbank. Therefore, during choice of crossing, it is necessary to consider both alternative types of bridges.

The second exception is the design of viaducts across mountain ravines. In this case, the change of the crossing has substantial impact on the choice of the span of the bridge and it is reflected in its cost. It is true that the type of the bridge for the first comparison may be left unchanged (e.g., reinforced concrete arch type, etc.), but it may be designed for all alternatives because the cost of the viaduct will have impact on the choice of location of the crossing.

The above exceptions do not occur often and should be considered separately; for this reason the location of the crossing may be chosen before preliminary design and must be made by the investigators with designers' efforts only in order to check the correctness of the choice. The size of the bridge opening is defined by hydraulic and hydroanalytic investigations and is assumed for the design. It some cases, however, during the design process it is possible to change the span. The size of opening, as shown above, depends on the crossing location. It also depends on the type and depth of the foundation. At greater depths, greater washout is permitted, with corresponding diminishing of the opening. At shallow foundations the reverse could occur.

In principle, two opposite solutions may exist:

1. Build bridge supports as safe against washing, squeeze the river by flow-directed dikes, and obtain a minimum opening.
2. Not squeeze the river, cross the whole river during flood, and thus the concern that the supports will wash out will no longer be a problem.

The first solution is used as a rule for rivers on the plain and can be justified economically and technically. Only for a timber bridge is it expedient to cover the whole flood area by the approach viaducts. Here the size of the opening depends upon the bridge material. The second solution may often be expedient for mountain rivers in which the main channel is often changing and threatens to wash out the flood embankment.

Generally, the size of the opening may change a little depending on the type of foundation. If the type of foundation as a whole is determined by the local conditions (e.g., by using caissons or wells), then the size of the opening for all alternatives remains unchanged. Choice of material is the most substantial problem during preliminary design and depends not only on the designer's point of view but also on other conditions that must be considered before preparing the project. Each material has its own area of application and the problem of material choice arises when these areas intersect.

Timber bridges are usually used as temporary structures. Spans greater than 160 ft often present difficulties. For permanent bridges, the choice is usually between reinforced concrete and steel structures. The following are some recommendations concerning the material selection for the bridge:

1. For spans ranging between 65 to 100 ft reinforced-concrete beam-type bridges are mainly used and steel is considered for overpasses and underpasses.
2. For spans ranging between 330 and 500 ft, steel bridges are often preferred.
3. For spans ranging between 650 and 800 ft, it is expedient to use steel bridges.

Therefore, the choice between reinforced concrete and steel bridges is generally for spans ranging between 65 and 330 ft.

The type of foundation for the bridge is determined mainly by the geologic investigation of ground in the riverbanks and in the main channel, and also by the depth and behavior of the water.

Relatively, the type of foundation influences the superstructure, size of separate spans, and type of supports. Foundations built at the present time may be divided into two basic groups:

1. Piler foundation in which timber pilers are used for shallow foundations and reinforced concrete and steel piles are used for deep foundations.
2. Massive, shallow foundation (between others or piles) and deep foundations (caissons and wells). It is obvious that for large spans it is necessary to use a massive foundation.

Shallow pile foundations are possible for viaduct bridges having small spans. Regarding the bridge system, it should be emphasized that pile foundations almost define the beam system and arches. Suspension bridges require a massive foundation and supports, but there might be other alternatives. During design, the following parameters remain constant or are slightly modified for the bridge system:

1. Size of spans (unequal or uniform);
2. Span system;
3. Type of supports.

### 1.3.9 Bridge System

The bridge system (i.e., beam, arch, suspension) is integrally related to the chosen material. Beam systems are mostly used for small and medium spans. An arch system is mainly used for large spans and a suspension system is used for long spans.

When using reinforced-concrete bridges, the following should be taken into consideration:

1. For spans up to 130 ft, a beam system is recommended.
2. For spans ranging between 130 and 200 ft, either a beam or arch system can be used.
3. For longer spans, an arch system is recommended.

For steel bridges, beam systems are mainly used. The arch system is expedient to use for spans longer than 160 ft. All the above span lengths are approximate and can be used as preliminary guidance in the early stage of the investigation in order to determine the appropriate system to use. The bridge system depends also on other parameters. It is impossible to investigate all other parameters without assuming the material type for the structure and the bridge system in the early stage of the investigation.

### 1.3.10 Size of Separate System

The size of the separate system greatly influences the cost of bridges. Determination of the span system involves a number of basic problems that need to be solved during the preliminary design.

For beam bridges having steel trusses, a known rule exists. The cost of the main truss with bracing per span should equal the cost of one pier with foundation. For all other cases, the length of span depends upon the type of foundation and pier.

Similarly, the system of the span construction has influence on the system of the span. With arch bridges, the cost of support is generally greater than that of beam type. For this reason (all things being equal), the span of the arch bridge should be greater than a beam bridge. The exceptions are high viaducts having rising high arches which are more economical. The limits of changes to span length are governed by clearances for ships and typical uses of span structures. The clearances for ships regulate the minimum size of the span. Usually the span is greater than the most economical length. For this reason, during crossings of navigable rivers the size of the span at the main channel in most cases is predetermined. It is necessary to change only side and approach spans. When choosing approach spans, it is necessary to consider typical projects because the use of typical construction is more rational and useful.

From this it follows that the length of spans is not arbitrary. They are chosen from defined conditions. The span length is closely connected with the system of span structure. Therefore, it is necessary at the early stage of the project to assume the proper system of span structure, noting that the choice of the system significantly determines the bridge system.

### 1.3.11 Type of Span Construction

The type of span construction is closely related to the bridge system. After assuming a bridge system, the span structure should be determined. There might be some problems related to the type of structure (e.g., solid or truss type for steel, monolith or prefabricated for reinforced concrete), the number of main girders, the basic dimensions, etc. Detailed study for each case is needed. Many problems common to particular cases can be investigated earlier, during the preparation of typical projects.

The use of typical projects substantially helps the individual design. For example, in the majority of medium-span bridges typical projects may be used. The use of typical projects simplifies fabrication of the structure, reduces the time necessary for design and construction, and makes the structure more economical to execute. However, the immediate use of typical projects should not be considered as a rule. They should be considered as a first solution, which in many cases can be improved. Each project has different circumstances, and typical projects do not provide solutions to all possible design problems. In some projects there might be some local conditions that need to be dealt with and were not addressed in previous projects. This problem is especially recognized in the design and construction of long-span bridges. Examples of already built bridges may provide a rational starting point. Together with this experience in the design and building of bridges, it is possible to establish some useful relations such as the ratio of truss height to span to the number and length of panels, etc.

The design of bridge structures starts with the critical study and the use of existing bridges to prepare the first alternative of the structure and continues during the investigation to separate parameters to prepare the next alternatives.

### 1.3.12 Type of Support

Supports can be divided into two groups: columns and massive supports. The second group is used in the presence of large floating ice and arch-type span structures. Column-type supports are most expedient with small-beam structures.

## 1.4 Remarks and Conclusions

A proper design method should meet two basic criteria:

1. First, the design method should be based on scientific engineering research and analysis. From comprehensive research, design derives logical conclusions.
2. Design methods should be achieved by practice and previous experience in the design and construction of bridges. Also, modifications should always be performed to improve the design. This is largely reflected by the designer's creative capability, sense of invention, and innovation.

Therefore, the integrated part of preliminary design is a comprehensive search of scientific, practical findings and analysis.

## References

1. Waddell, J. A. L., *Bridge Engineering,* Vol. 1, John Wiley & Sons, New York, 1916, 267–280.
2. Mitropolskii, N. M., *Methodology of Bridges Design,* scientific-technical edition, Avtotransportni Literatury, Moscow, 1958, 215–242 [in Russian].

3. Polivanov, N. I., *Design and Calculation of the Reinforced Concrete and Metal Highway Bridges,* Transport, Moscow, 1970, 5–36 [in Russian].
4. Steinman, D. B. and Watson. S. R., *Bridges and Their Builders,* Dover, New York, 1957, 378–391.
5. DeMare, E., *Your Book of Bridges,* Faber and Faber, London, 1963, 11–28.
6. Vitruvius, *The Ten Books on Architecture,* translated by M. H. Morgan, Dover, New York, 1960, 13–16.
7. Palladio, A., *The Four Books on Architecture,* Stroiizdat, Moscow, 1952 [in Russian].
8. Holgate, A., *The Art in Structural Design,* Clarendon Press, Oxford, 1986, 1–6, 24–30, 187–195.
9. Francis, A. J., *Introducing Structures,* Pergamon Press, New York, 1980, 221–260.

Leonhardt, F. "Bridge Aesthetics - Basics."
*Bridge Engineering Handbook.*
Ed. Wai-Fah Chen and Lian Duan
Boca Raton: CRC Press, 2000

# 2

# Aesthetics — Basics*1 Introduction



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## 2.1 Introduction

Aesthetics falls within the scope of philosophy, physiology, and psychology. How then, you may ask, can I as an engineer presume to express an opinion on aesthetics, an opinion which will seem to experts to be that of a layman. Nevertheless, I am going to try.

For over 50 years I have been concerned with, and have read a great deal about, questions concerning the aesthetic design of building projects and judgment of the aesthetic qualities of works in areas of the performing arts. I have been disappointed by all but a few philosophical treatises on aesthetics. I find the mental acrobatics of many philosophers — whether, for example, existence is the existence of existing — difficult to follow. Philosophy is the love of Truth, but truth is elusive and hard to pin down. Books by great building masters are full of observations and considerations from which we can learn in the same way that we study modern natural scientists.

My ideas on aesthetics are based largely on my own observations, the results of years of questioning — why do we find this beautiful or that ugly? — and on innumerable discussions with architects who also were not content with the slogans and "isms" of the times, but tried to think critically and logically.

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The question of aesthetics cannot be understood purely by critical reasoning. It reaches to emotion, where logic and rationality lose their precision. Undaunted, I will personally address these questions, so pertinent to all of us, as rationally as possible. I will confine myself to the aesthetics of building works, of man-made objects, although from time to time a glance at the beauty of nature as created by God may help us reinforce our findings.

I would beg you to pardon the deficiencies that have arisen because of my outside position as a layman. This work is intended to encourage people to study questions of aesthetics using the methods of the natural scientist (observation, experiment, analysis, hypothesis, theory) and to restore the respect and value which it enjoyed in many cultures.

## 2.2 The Terms

The Greek word *aisthetike* means the science of sensory perception and very early on was attributed to the perception of the beautiful. Here we will define it as follows:

**Aesthetics:** The science or study of the quality of beauty an object possesses, and communicates to our perceptions through our senses (expression and impression according to Klages [1]).

**Aesthetic:** In relation to the qualities of beauty or its effects; aesthetic is not immediately beautiful but includes the possibility of nonbeauty or ugliness. Aesthetic is not limited to *forms*, but includes surroundings, light, shadows, and color.

## 2.3 Do Objects Have Aesthetic Qualities?

Two different opinions were expressed in old philosophical studies of aesthetics:

1. Beauty is not a quality of the objects themselves, but exists only in the imagination of the observer and is dependent on the observer's experience [2]. Smith said in his "Plea for Aesthetics" [3], "Aesthetic value is not an inborn quality of things, but something lent by the mind of the observer, an interpretation by understanding and feeling." But how can we interpret what does not exist? Some philosophers went so far as questioning the existence of objects at all, saying they are only vibrating atoms, and everything we perceive is subjective and only pictured by our sensory organs. This begs the question, then, is it possible to picture the forms and colors of objects on film using a camera? These machines definitely have no human sensory organs.
2. The second school of thought maintains that objects have qualities of beauty. Kant [4] in his *Critique of Pure Reason* said, "Beauty is what is generally and without definition, pleasing." It is not immediately clear what is meant by "without definition," perhaps without explaining and grasping the qualities of beauty consciously. What is "generally pleasing" must mean that the majority of observers "like" it. Paul [5] expressed similar thoughts in his *Vorschule der Aesthetik* and remarked that Kant's constraint "without definition" is unnecessary. Thomas Aquinas (1225–1274) simply said, "A thing is beautiful if it pleases when observed. Beauty consists of completeness, in suitable proportions, and in the luster of colors." At another time, Kant said that objects may arouse pleasure independent of their purpose or usefulness. He discussed "disinterested pleasure," a pleasure free from any interest in objects: "When perceiving beauty, I have no interest in the existence of the object." This emphasizes the subjective aspect of aesthetic perception, but nonetheless bases the origin of beauty in the object.

Is one right? Most would side with Kant and grant that all objects have aesthetic qualities, whether we perceive them or not. Aesthetic value is transmitted by the object as a message or simulation and its power to ourselves depends on how well we are tuned for reception. This example drawn from modern technology should be seen only as an aid to understanding. If a person is receptive to transmissions of beauty, it then depends very largely on how sensitive and developed are the person's senses for aesthetic messages, whether the person has any feeling for quality at all. We will look at this question more closely in Section 2.4.

On the other hand, Schmitz, in his *Neue Phänomenologie* [6], sees in this simple approach "one of the worst original sins in the theory of cognition." …This *physiologism* limits the information for human perception to messages that reach the sensory organs and the brain in the form of physical signals and are therefore metaphysically raised to consciousness in a strangely transformed shape." We must see the relationships between the object and circumstances, associations, and situations. More important is the situation and observer's background and experience. The observer is "affectively influenced," [6] i.e., the effect depends on the health of the observer's senses, on the observer's mood, on the observer's mental condition; the observer will have different perceptions when sad or happy. The observer's background experience arouses concepts and facts for which the observer is prepared subconsciously or which are suggested by the situation. Such "protensions" [6] influence the effects of the object perceived, and include prejudices which are held by most people and which are often a strong and permanent hindrance to objective cognition and judgment. However, none of this phenomenology denies the existence of the aesthetic qualities of objects.

Aesthetic quality is not limited to any particular fixed value by the characteristics of the object, but varies within a range of values dependent on a variety of characteristics of the observer. Judgment occurs in a process of communication. Bahrdt [7], the sociologist, said, "As a rule aesthetic judgment takes place in a context of social situations in which the observers are currently operating. The observers may be a group, a public audience, or individuals who may be part of a community or public. The situation can arise at work together, during leisure time, or during a secluded break from the rush of daily life. In each of these different situations the observer has a different perspective and interpretation, and thus a different aesthetic experience [impression]."

Aesthetic characteristics are expressed not only by form, color, light, and shadow of the object, but by the immediate surroundings of the object and thus are dependent on object environment. This fact is well known to photographers who can make an object appear much more beautiful by careful choice of light and backdrop. Often a photograph of a work of art radiates a stronger aesthetic message than the object itself (if badly exhibited) in a gallery. With buildings, the effect is very dependent on the weather, position of the sun, and on the foreground and background. It remains undisputed that there is an infinite number and variety of objects (which all normal healthy human beings find beautiful). Nature's beauty is a most powerful source of health for humans, giving credence to the suggestion that we have an inborn aesthetic sense.

The existence of aesthetic qualities in buildings is clearly demonstrated by the fact that there are many buildings, groups of buildings, or civic areas which are so beautifully designed that they have been admired by multitudes of people for centuries, and which today, despite our artless, materialistic attitudes to life, are still visited by thousands and still radiate vital power. We speak of classical beauty. All cultures have such works, and people go to great lengths to preserve and protect them; substantial assistance has come from all over the world to help preserve Venice, whose enchanting beauty is so varied and persuasive.

We can also give negative evidence for the existence of aesthetic qualities in objects in our manmade environment. Think of the ugliness of city slums, or depressing monotonous apartment blocks, or huge blocky concrete structures. These products of the "brutalist" school have provoked waves of protest. This affront to our senses prompted the Swiss architect Rolf Keller to write his widely read book *Bauen als Umweltzerstörung* [8].

All these observations and experiences point to the conclusion that objects have aesthetic qualities. We must now look at the question of how humans receive and process these aesthetic messages.

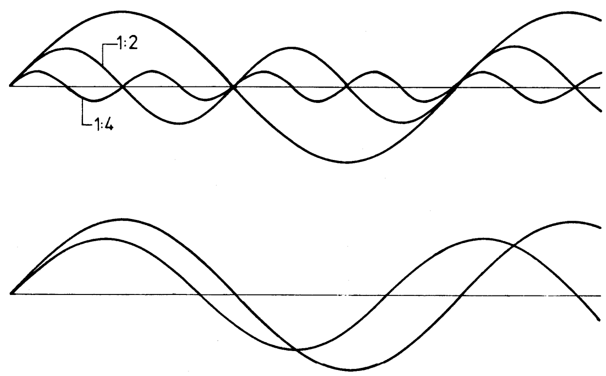


**FIGURE 2.1** Wave diagrams for consonant and dissonant tones.

## 2.4 How Do Humans Perceive Aesthetic Values?

Humans as the receivers of aesthetic messages use all of their senses: they see with their eyes, hear with their ears, feel by touch, and perceive temperature and radiation by sensors distributed in the body, sensors for which there is no one name. Our sensory organs receive different waveforms, wavelengths, and intensities. We read shapes by light rays, whose wavelengths give us information about the colors of objects at the same time. The wavelength of visible light ranges from 400 μm (violet) to 700 μm (red) (1 μm = 1 millionth of 1 mm). Our ears can hear frequencies from about 2 to 20,000 Hz.

The signals received are transmitted to the brain and there the aesthetic reaction occurs — satisfaction, pleasure, enjoyment, disapproval, or disgust. In modern Gestalt psychology, Arnheim [9] explained the processes of the brain as the creation of electrochemical charge fields which are topologically similar to the observed object. If such a field is in equilibrium, the observer feels aesthetic satisfaction, in other cases the observer may feel discomfort or even pain. Much research needs to be done to verify such explanations of brain functions, but they do seem plausible. However, for most of us we do not need to know brain functions exactly.

During the course of evolution, which we assume to have taken many millions of years, the eye and ear have developed into refined sensory organs with varied reactions to different kinds of waveforms. Special tone sequences can stimulate so much pleasure that we like to hear them — they are consonant or in harmony with one another. If, however, the waveforms have no common nodes (Figure 2.1) the result is dissonance or beats, which can be painful to our ear. Dissonances are often used in music to create excitement or tension.

The positive or negative effects are a result not only of the charge fields in the brain, but the anatomy of our ear, a complex structure of drum oscular bones, spiral cochlea, and basilar membrane. Whether we find tones pleasant or uncomfortable would seem to be physiological and thus genetically conditioned. There are naturally individual differences in the sense of hearing, differences which occur in all areas and in all forms of plant and animal life.

There are also pleasant and painful messages for the eye. The effects are partly dependent on the condition of the eye, as, for example, when we emerge from a dark room into light. Color effects of a physiological nature were described in much detail by Goethe in his color theory [10]. In the following, we will discuss the effects of physical colors on the rested, healthy eye, and will not address color effects caused by the refraction or reflection of light.

Some bright chemical colors cause painful reactions, but most colors occurring naturally seem pleasant or beautiful. Again, the cause lies in waves. The monotonous waves of pure spectral colors have a weak effect. The eye reacts more favorably to superimposed waves or to the interaction of two separate colors, especially complementary colors.

We feel that such combinations of complementary colors are harmonious, and speak of "color harmony." Great painters have given us many examples of color harmony, such as the blue and yellow in the coat of Leonardo da Vinci's *Madonna of the Grotto* .

We all know that colors can have different psychological effects: red spurs aggression; green and blue have a calming effect. There are whole books devoted to color psychology and its influence on human moods and attitudes.

We can assume that the eye's aesthetic judgment is also physiologically and genetically controlled, and that harmonic waveforms are perceived as more pleasant than dissonant ones. Our eyes sense not only color but can form images of the three-dimensional, spatial characteristics of objects, which is vital for judging the aesthetic effects of buildings. We react primarily to proportions of objects, to the relationships between width and length and between width and height, or between these dimensions and depth in space. The objects can have unbroken surfaces or be articulated. Illumination gives rise to an interplay of light and shadow, whose proportions are also important.

Here the question of whether there are genetic reasons for perceiving certain proportions as beautiful or whether upbringing, education, or habit play a role cannot be answered as easily as for those of acoustic tone and color. Let us first look at the role proportions play.

## 2.5 The Cultural Role of Proportions

Proportions exist not only between geometric lengths, but between the frequencies of musical tones and colors. An interplay between harmonic proportions in music, color, and geometric dimensions was discovered very early, and has preoccupied the thinkers of many different cultural eras.

Pythagoras of Samos, a Greek philosopher (571–497 B.C.) noted that proportion between small whole numbers (1:2, 2:3, 3:4, or 4:3, and 3:2) has a pleasing effect for tones and lengths. He demonstrated this with the monochord, a stretched string whose length he divided into equal sections, comparing the tones generated by the portions of the string at either side of an intermediate support or with the open tone [11–13].

In music these harmonic or consonant tone intervals are well known, for example,

| String Length | Frequencies | |
|---|---|---|
| 1:2 | 2:1 | Octave |
| 2:3 | 3:2 | Fifth |
| 3:4 | 4:3 | Fourth |
| 4:5 | 5:4 | Major third |

The more the harmonies of two tones agree, the better their consonance; the nodes of the harmonies are congruent with the nodes of the basic tones. Later, different tone scales were developed to appeal to our feelings in a different way depending on the degree of consonance of the intervals; think of major and minor keys with their different emotional effects.

A correspondence between harmonic proportions in music and good geometric proportions in architecture was suggested and studied at an early stage. In Greek temples many proportions corresponding with Pythagoras's musical intervals can be identified. Kayser [14] has recorded these relationships for the Poseidon temple of Paestum.

H. Kayser (1891–1964) dedicated his working life to researching the "harmony of the World." For him, the heart of the Pythagorean approach is the coupling of the tone of the monochord string with the lengths of the string sections, which relates the qualitative (tone perception) to the quantitative (dimension). The monochord may be compared with a guitar. If you pull the string of a guitar, it gives a tone; the height of the tone (quality) depends on the length (dimension = quantity) and the tension of the string. Kayser considered the qualitative factor (tones) as judgment by emotional feeling. It is from this coupling of tone and dimension, of perception and logic, of feeling and knowledge, that the emotional sense for the proportions of buildings originates — the tones of buildings, if you will.

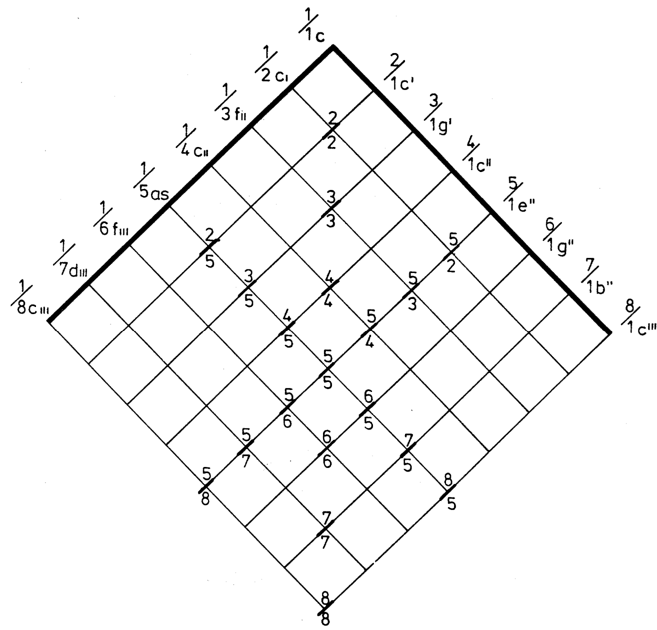


**FIGURE 2.2** Giorgio numerical analogy in Λ-shape.

Kayser also had shown that Pythagorean harmonies can be traced back to older cultures such as Egyptian, Babylonian, and Chinese, and that knowledge of harmonic proportions in music and building are about 3000 years old. Kayser's research has been continued by R. Haasse at the Kayser Institute for Harmonic Research at the Vienna College of Music and Performing Arts.

Let us return to our historical survey. In his famous 10 books *De Architectura,* Marcus Vitruvius Pollio (84–14 B.C.) noted the Grecian relationships between music and architecture and based his theories of proportion on them.

Wittkower [12] mentions an interesting text by the monk Francesco Giorgio of Venice. Writing in 1535 on the design of the Church of S. Francesco della Vigna in Venice (shortened extract):

> To build a church with correct, harmonic proportions, I would make the width of the nave nine double paces, which is the square of three, the most perfect and holy number. The length of the nave should be twenty-seven, three times nine, that is an octave and a fifth. … We have held it necessary to follow this order, whose master and author is God himself, the great master builder. … Whoever should dare to break these rules, he would create a deformity, he would blaspheme against the laws of Nature."

So strictly were the laws of harmony, God's harmony, obeyed.

In his book *Harmonia,* Francesco Giorgio represented his mystic number analogies in the form of the Greek letter Λ. Thimus [15] revised this "Lambdoma" for contemporary readers (Figure 2.2).

"Rediscovered" for curing the ills of today's architecture, Andea di Piero da Padova — known to us as *Palladio* [16], was a dedicated disciple of harmonic proportions. He wrote, "The pure proportions of tones are harmonious for the ear, the corresponding harmonies of spatial dimensions are harmonious for the eye. Such harmonies give us feelings of delight, but no-one knows why — except he who studies the causes of things."

Palladio's buildings and designs prove that beautiful structures can be created using these harmonic proportions when they are applied by a sensitive master. Palladio also studied proportions in spatial perspective, where the dimensions are continuously reduced along the line of vision. He confirmed the view already stated by Brunelleschi (1377–1446) that objective laws of harmony also apply to perspective space.

Even before Palladio, Leon Batista Alberti (1404–1472), had written about the proportions of buildings, Pythagoras had said:

> The numbers which thrill our ear with the harmony of tones are entirely the same as those which delight our eye and understanding. … [We] shall thus take all our rules for harmonic relationships from the musicians who know these numbers well, and from those particular things in which Nature shows herself so excellent and perfect.

We can see how completely classical architecture, particularly during the Renaissance, was ruled by harmonic proportions. In the Gothic age master builders kept their canon of numbers secret. Not until a few years ago did the book *Die Geheimnisse der Kathedrale von Chartres* (The Secrets of Chartres Cathedral) by the Frenchman L. Charpentier appear [13], in which he deciphered the proportions of this famous work. It reads like an exciting novel. The proportions correspond with the first Gregorian scale, based on *re* with the main tones of *re-fa-la*. Relationships to the course of the sun and the stars are demonstrated.

Ancient philosophers spent much of their time attempting to prove that God's sun, moon, stars, and planets obeyed these harmonic laws. In his work *Harmonice Mundi* Johannes Kepler (1571–1630) showed that there are a great number of musical harmonies. He discovered his third planetary law by means of harmonic deliberations, the so-called octavoperations. Some spoke of "the music of the spheres" (Boethius, Musica mundana).

Villard de Honnecourt, the 13th-century cathedral builder from Picardy, gave us an interesting illustration of harmonic canon for division based on the upper tone series 1–½–⅓–¼, etc. For Gothic cathedrals he started with a rectangle of 2:1. This Villard diagram (Figure 2.3) [13, 17] was probably used for the design of the Bern cathedral. Whole-number proportions of the fourth and third series can be seen in the articulation of the tower of Ulm Cathedral. A Villard diagram can be drawn for a square, and it then, for example, fits the cross section of the earlier basilica of St. Peter's Cathedral in Rome.

When speaking of proportion, many think of the golden mean, but this does not form a series of whole-number relationships and does not play the important role in architecture which is often ascribed to it. This proportion results from the division of a length $a + b$ where $b < a$ so that

$$\frac{b}{a} = \frac{a}{a+b} \tag{2.1}$$

This is the case if

$$a = \frac{\sqrt{5} + 1}{2} b = 1.618b \tag{2.2}$$

the reciprocal value is $b = 0.618a$, which is close to the value of the minor sixth at $5/8 = 0.625$ or $8/5 = 1.6$. The golden mean is a result of the convergence of the Fibonacci series, which is based on the proportion of $a{:}b$, $b{:}(a + b)$, etc.:

$$\begin{array}{lcrl}
a{:}b & = & 1{:}\,2 & = 0.500 = \text{octave} \\
b{:}(a+b) & = & 2{:}\,3 & = 0.667 = \text{fifth} \\
 & & 3{:}\,5 & = 0.600 = \text{major sixth} \\
 & & 5{:}\,8 & = 0.625 = \text{minor sixth} \\
 & & 8{:}\,13 & = 0.615 \\
 & & 13{:}\,21 & = 0.619 \\
 & & 21{:}\,34 & = 0.618 = \text{Golden Mean}
\end{array}$$

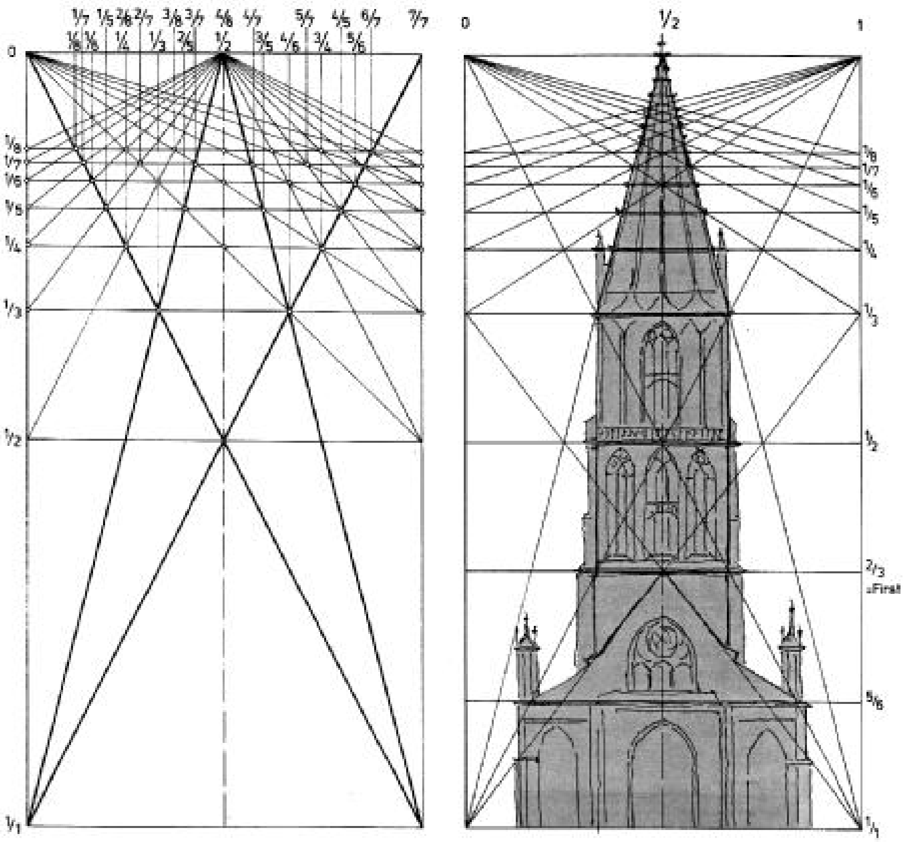


**FIGURE 2.3** The Villard diagram for rectangle 2:1.

This numerical value is interesting in that:

$$\frac{1.618}{1.618-1} = \frac{1.618}{0.618} = 2.618$$

and

$$2.168\ (6/5) = 3.1416 = \pi$$

The golden mean thus provided the key to squaring the circle, as can be found in Chartres Cathedral. It can be constructed by dividing the circle into five (Figure 2.4).

The Fibonacci series is also used to construct a logarithmic spiral, which occurs in nature in snail and ammonite shells, and which is considered particularly beautiful for ornaments. Le Corbusier (1887–1965) used the golden mean to construct his "Modulor" based on an assumed body height of 1.829 m but the Modulor is in itself not a guarantee of harmony.

An interesting proportion is $a$: $b$ = 1: $\sqrt{3}$ = 1: 1.73. It is close to the golden mean but for technical applications has the important characteristic that the angles to the diagonals are 30° or 60° (equilateral triangle) and the length of the diagonal is $2a$ or $2b$ (Figure 2.5). A grid with sides in the ratio of 1: $\sqrt{3}$ was patented on July 8, 1976 by Johann Klocker of Strasslach. He used this grid to design carpets, which were awarded prizes for their harmonious appearance.

During the last 50 years architects have largely discarded the use of harmonic proportions. The result has been a lack of aesthetic quality in many buildings where the architect did not choose

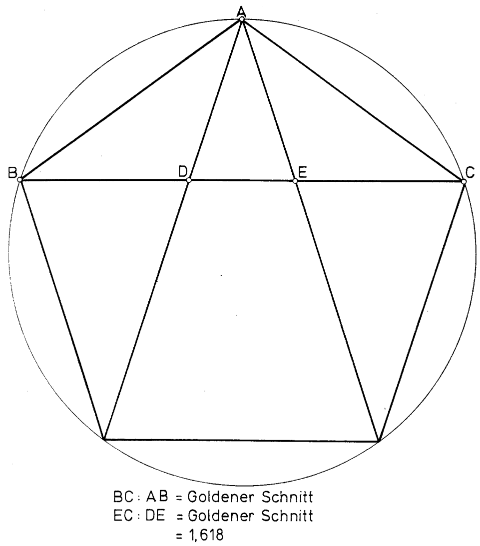


**FIGURE 2.4** The golden mean in a pentagon.

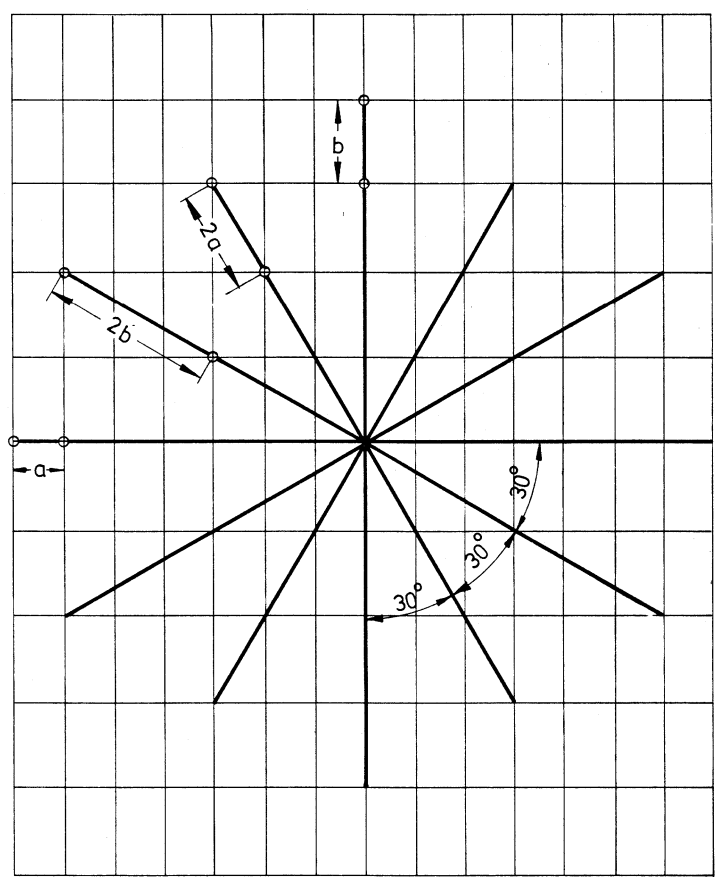


**FIGURE 2.5** The Kloecker grid with $a{:}b = 1{:}\ \sqrt{3}$ .

good proportions intuitively as a result of his artistic sensitivity. There were exceptions, as always. The Swiss architect Andre M. Studer [18] and the Finn Aulis Blomstdt consciously built "harmonically." One result of the wave of nostalgia of the 1970s is a return in many places to such aesthetics. Kayser in Reference [14] and P. Jesberg in the *Deutsche Bauzeitschrift* DBZ 9/1977 gave a full description of harmonic proportions.

## 2.6 How Do We Perceive Geometric Proportions?

In music we can assert plausibly that a feeling and sense for harmonic tone series is controlled genetically and physiologically through the inborn characteristics of the ear. What about the proportions of lengths, dimensions of objects, and volumes? Helmcke [19], of the Technical University of Berlin, wholeheartedly supported the idea of a genetic basis for the aesthetic perception of proportions and he argued as follows:

> During the evolution of animals and Man the choice of partner has undoubtedly always played an important role. Since ancient times men have chosen women as partners, who in their eyes were the most beautiful and well proportioned and equally women have chosen men as partners the strongest and most well-built in their eyes. Through natural selection [Darwin] during the evolution of a species this must have led to the evolution of aesthetic perception and feeling and resulted in the development in Man of a genetically coded aesthetic ideal for human partners, passed on from generation to generation. We fall in love more easily with a beautiful partner; love at first sight is directed mostly by an instinctive feeling for beauty, and not by logic. Nobody who knows Man and his history will doubt that there is an inherited human ideal of beauty. Every culture has demonstrated its ideal of human beauty, and if we study the famous sculptures of Greek artists we recognize that the European ideal of beauty in female and male bodies has not changed in the last 3000 years.

For the Greeks the erotic character of the beauty of the human body played a dominant role. At the Symposium of Xenophon (ca. 390 B.C.) Socrates made a speech in praise of Eros. According to Grassi [20], the term *beautiful* is used preferentially for the human body.

The Spanish engineer Eduardo Torroja (1899–1961), whose structures were widely recognized for their beauty, wrote in his book *The Logic of Form* [21] that "truly the most perfect and attractive work of Nature is woman." Helmcke said that "Man's aesthetic feeling, while perceiving certain proportions of a body, developed parallel to the evolution of Man himself and is programmed genetically in our cells as a hereditary trigger mechanism."

According to this the proportions of a beautiful human body would be the basis of our hereditary sense of beauty. This view is too narrow because thousands of other natural objects radiate beauty, but let us continue to study "Man" for the time being.

Fortunately, all humans differ in their hereditary, attributes, and appearance, although generally only slightly. This means that our canons of beauty cannot be tied to strictly specific geometric forms and their proportions. There must be a certain range of scatter. This range covers the differences in the ideals of beauty held by different races. It ensures that during the search for a partner each individual's ideal will differ, keeping the competition for available partners within reasonable bounds.

We can also explain this distribution physiologically. Our eyes have to work much harder than our ears. The messages received by the eye span a range about a thousand times wider than the scale of tones to be processed by the ear. This means that with colors and geometric proportions harmony and disharmony are not so sharply defined as with musical tones. The eye can be deceived more easily and is not as quickly offended or aggravated as the ear, which reacts sensitively to the smallest dissonance.

More evidence for a hereditary sense of beauty is provided by the fact that even during their first year, children express pleasure at beautiful things and are offended, even to the point of weeping, by ugly objects. How children's eyes sparkle when they see a pretty flower.

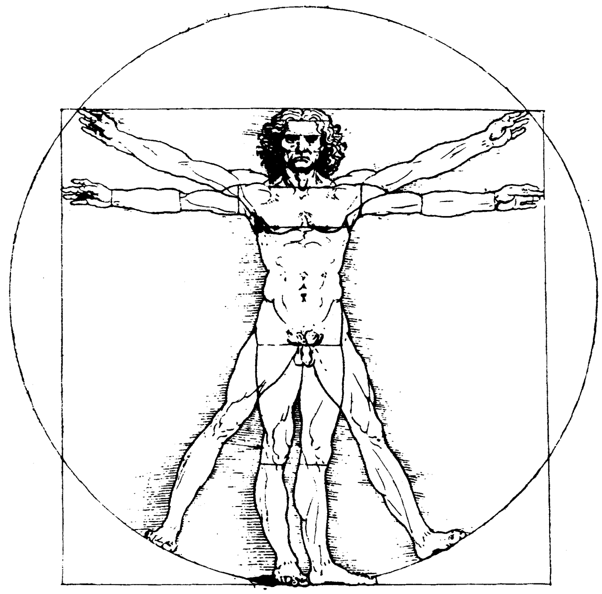

**FIGURE 2.6** Image of man in circle and square according to Leonardo da Vinci.

Evidence against the idea that we have a hereditary sense of beauty is suggested by the fact that people argue so much about what is beautiful or ugly, demonstrating a great deal of insecurity in the judgment of aesthetic qualities. We will give this further thought in Section 2.7.

Our ability to differentiate between good and bad using our senses of taste and smell has also developed genetically and with certain variations is the same for most people [22]. With this background of genetic development it is understandable that the proportions of those human bodies considered beautiful have been studied throughout the ages. A Greek sculptor Polyklet of Kikyon (465–420 B.C.) defined the following proportions:

| | | |
|---|---|---|
| two handbreadths | = | height of the face and height of the breast, distance breast to navel, navel to end of trunk |
| three handbreadths | = | height of skull, length of foot |
| four handbreadths | = | distance shoulder to elbow, elbow to fingertips |
| six handbreadths | = | ear to navel, navel to knee, length of trunk, length of thigh |

Plyklet based his "canon for the ideal figure" on these relationships. These studies had the greatest influence on art during the age of humanism, for example, through the *Vier Büecher von menschlicher Proportione* 1528 by Albrecht Dürer (1471–1528).

Vitruvius also dealt with the human body in his books *De Architectura* and used the handbreadth as a unit of measure. Leonardo da Vinci followed Vitruvius's theories when drawing his image of man inscribed in a square (Figure 2.6). Leonardo's friend, the mathematician Luca Pacioli (ca. 1445–1514) began his work *De Divina Proportione,* 1508, with the words:

> Let us first speak of the proportions of Man because all measures and their relationships are derived from the human body and here are to be found all numerical relationships, through which God reveals the innermost secrets of Nature. Once the ancients had studied the correct proportions of the human body they proportioned all their works, particularly the temples, accordingly. (Quoted by Wittkower [12])

The human body with outstretched arms and legs inscribed in a square and circle became a favorite emblem for humanistically oriented artists right up to Le Corbusier and Ernst Neufert. Let us close this section with a quotation from one of Helmcke's [19] works:

> The intellectual prowess of earlier cultures is revealed to us whenever their artists, architects, and patrons succeeded in incorporating, consciously or unconsciously, our hereditary, genetically programmed canon of proportions in their works; in achieving this they come close to our genetically controlled search for satisfaction of our sense of aesthetics. It reveals the spiritual pauperism of today's artists, architects and patrons when, despite good historical examples and despite advances in the natural sciences and the humanities they do not know of these simple biologically, anatomically based relationships or are too ungifted to perceive, understand, and realize them. Those who deprecate our search for the formal canons of our aesthetic feelings as a foolish and thus unnecessary pastime must expect to have their opinion ascribed to arrogant ignorance and to the lack of a sure instinctive sense of beauty, and already ethnologically known as a sign of decadence due to domestication.
>
> The only criticism which, in my [Helmcke's] opinion can be leveled at the thousands of years' old search for universally valid canons of form, lies in the assumption that these canons shall consist of fixed proportions and shall thus be valid for all mankind. …
>
> What is needed is experience of and insight into the range of scatter of proportional relations and insight into the limits within our hereditary aesthetic sense reacts positively, and beyond which it reacts negatively.

## 2.7 Perception of Beauty in the Subconscious

We are not generally aware of how strongly our world of feelings, our degree of well-being, comfort, disquiet, or rejection is dependent on impressions from our surroundings. Neurologists know that parts of our brain are capable of reacting to external stimuli without reference to the conscious mind and of processing extensive amounts of information. This takes place in the limbic system of the primitive structures of the midbrain and the brain stem. For all those activities of the subconscious which deal with the processing of aesthetic messages, Smith [3] used the phrase "limbic aesthetics" and dedicated a whole chapter of his very readable book to them.

Our subconscious sense of beauty is almost always active, whether we are at home, in the city marketplace, in a church, in a beautiful landscape, or in the desert. Our surroundings affect us through their aesthetic characteristics even if our conscious thoughts are occupied with entirely different matters and impressions.

Smith wrote of the sensory appetite of these primitive parts of the brain for pleasant surroundings, for the magic of the city, and for the beauty of nature. The limbic system reacts to an oversupply of stimuli with rejection or anxiety.

Symbolic values connected with certain parts of our environment also act on the subconscious. The home, the church, school, garden, etc. have always possessed symbolic values created by learning and experience. These are related mostly to basic human situations and cause emotional reactions, without ever reaching the conscious level.

This perception of beauty at the subconscious level plays a particularly strong role in city dwellers. Their basic feeling of well-being is doubtless influenced by the aesthetic qualities of their environment in this way. This has social consequences (see Section 2.10) and underlines our responsibility to care about the beauty of the environment.

## 2.8 Aesthetic Judgment and Taste

When two observers are not agreed in their judgment of a work of art, the discussion is all too often ended with the old proverb, "De gustibus non disputandum est." We like to use a little Latin to show our classical education, which, as we know, is supposed to include an understanding of art. This "there's no accounting for taste" is an idle avoidance tactic, serving only to show that the speaker has never really made a serious effort to study aesthetics and thus has educational deficiencies in the realm of assessing works of art.

Of course, taste is subject to continual change, which in turn depends on current ideals, fashions, and is dependent on historical and cultural background. The popular taste in any given period of time or even the taste of single individuals is never a reliable measure of aesthetic qualities.

On the other hand, genetic studies have shown that we have a certain basic hereditary sense of beauty. Smith [3] said that this aesthetic perception has developed into one of the highest capabilities of our central nervous system and is a source of deep satisfaction and joy.

The judgment of aesthetic characteristics is largely dependent on feelings which are derived from our sensory perceptions. Beauty, then, despite some theories (Bense, Maser) cannot be rationally measured. When looking at the nature of feelings we must admit the fact that despite all our research and science, we know very little about humanity or about ourselves. We can, however, call upon observations and experiences which are helpful.

We repeatedly experience that the majority of people agree that a certain landscape, great painting, or building is beautiful. When entering a room, for example, in an old church, or while wandering through a street, feelings are aroused which are pleasant, comfortable, even elevating, if we sense a radiation of beauty. If we enter a slum area, we feel revulsion or alarm, as we perceive the disorder and decay. We can be more or less aware of these feelings, depending on how strongly our thoughts are occupied elsewhere. Sensitivities and abilities to sense beauty naturally differ from person to person, as is true of our other talents. This sensitivity is influenced by impressions from our environment, by experience, by relationships with our companions at home, at school, and with our friends. Two people judging the qualities of beauty of an object are likely to give different opinions.

Beautiful surroundings arouse feelings of delight in almost all people, but an ugly, dirty environment causes discomfort. Only the degree of discomfort will differ. In our everyday life such feelings often occur only at a subconscious level and often their cause is only perceived after subsequent reflection.

We can develop a clear capacity for judging aesthetic qualities only when we study the message emanated by an object consciously and ask ourselves whether or not we like a building or a room. Next, we must ask ourselves why. Why do I like this and not that? Only by frequent analysis, evaluation, and consideration of consciously perceived aesthetic values can we develop that capacity of judgment which we commonly call taste — taste about which we must argue, so that we can strengthen and refine it. Taste, then, demands self-education, which can be cultivated by critical discussion with others or by guidance from those more experienced. Good judgment of aesthetic values requires a broad education. It can be compared to an art and requires skill, and like art it takes not only talent, but a lot of work.

We need not be afraid that such analysis will weaken our creative skills; in fact, the opposite is true: the goal of analysis is the discovery of the truth through creative thinking [23]. People have different talents and inclinations since they grow up in different circles with different cultural backgrounds and therefore their tastes will always differ. In any given culture, however, there is a certain polarity on the judgment of beauty. Psychologists call this agreement "normal behavior," "a normal reaction of the majority." This again corresponds with Kant's view that beauty is what is generally thought to be beautiful by the majority of people.

Beauty cannot be strictly proved, however; so we must be tolerant in questions of taste and must give freedom to what is generally felt to be beautiful and what ugly. That there is a generally recognized concept of beauty is proved by the consistent judgment of the classical works of art of all great cultures, visited year after year by thousands of people. Think of the popularity of exhibitions of great historic art today. It is history that has the last word on the judgment of aesthetic values, long after fashions have faded.

*Fashions*: Artistic creation will never be entirely free from fashion. The drive to create something new is the hallmark of creative beings. If the new becomes popular, it is soon copied, and so fashions are born. They are born of the ambition and vanity of humans and please both. The desire to impress often plays a role. Up to a certain point, fashions are necessary; in certain new directions true art may develop through the fashionable, acquiring stability through a maturing process and enduring beyond the original fashion. Often, such new developments are rejected, because we are strongly influenced by the familiar, by what we are used to seeing, and only later realize the value of the new. Again, history pronounces a balanced judgment.

Confusion is often caused in our sense of judgment by modern artists who deliberately represent ugliness in order to mirror the warped mental state of our industrial society. Some of this work has no real quality, but is nonetheless acclaimed as modern art. The majority dares not question this for fear of rejection, slander, and peer pressure.

Although some works that consciously display ugliness or repulsiveness may well be art, we must seriously question the sanity and honesty of the patrons of primitive smearings, tangles of scrap iron, or old baby baths covered in Elastoplast strips (J. Beuys) when such efforts are exhibited as works of art. Happily, the courage to reject clearly such affronts and to put them in their place is on the increase. We only need to read Claus Borgeest's book, *Das Kunsturteil,* [24], in which he wrote, "the belief in such 'art' is a modern form of self-inflicted immaturity, whose price is the self-deprivation of reason, man's supreme attribute."

In any case, it would be wrong to describe as beautiful works, those haunted by ugliness, even if they have the quality of art. The artist intends to provoke and to encourage deliberation. However, the educational effects of such artistic creations are questionable, because we usually avoid their repeated study. Painters and sculptors, however, should be free to paint and sculpt as hatefully and repulsively as they wish — we do not have to look at their works. It is an entirely different case with buildings; they are not a private affair, but a public one. It follows that the designer has responsibility to the rest of humankind and a duty to produce beautiful buildings so that the designer does not give offence. Rightly, the ancient Greeks forbade public showings of ugliness, because their effects are largely negative.

We seldom find anyone who will hang ugly works of art in his or her home. It is beyond a doubt that in the long term we feel comfortable only in beautiful surroundings and that beauty is a significant requirement for the well-being of our soul; this is much more important for people's happiness than we today care to admit.

## 2.9 Characteristics of Aesthetic Qualities Lead to Guidelines for Designing

The search for explanations, the analysis of aesthetic values, are bound to lead to useful results, at least for man-made buildings and structures. We will now try to subject matters of feelings, emotions, to the clear light of recognition and understanding.

If we do this, we can certainly find answers to the question, "Why is this beautiful and this ugly?" For recognized masterpieces of architecture generally considered beautiful, there have been answers since ancient times, many of which are given in the quoted literature on proportions. Such buildings reveal certain characteristics of quality and from these we can deduce guidelines for design, such as certain proportions, symmetry, rhythm, repeats, contrasts, and similar factors, The master schools of old had such rules or guidelines, such as those of Vitruvius and Palladio. Today, these rules are surely valid and must be rediscovered for the sake of future architecture. They can prove a valuable aid in the design of building structures and at the very least contribute toward avoiding gross design errors.

Many architects and engineers reject rules, but in their statements about buildings we still find references to harmony, proportion, rhythm, dominance, function, etc. Torroja [21] rejected rules, but he said "the enjoyment and conscious understanding of aesthetic pleasure will without doubt be much greater if, through a knowledge of the rules of harmony, we can enjoy all the refinements and perfections of the building in question." Rules of harmony are based on rules of proportion, and somehow the striving for individual artistic freedom prevents us from recognizing relationships often imposed upon us by ethics.

Let us then attempt to formulate such characteristics, rules, or guidelines as they apply to building structures, particularly bridges.

### 2.9.1 Fulfillment of Purpose–Function

Buildings or bridge structures are erected for a purpose. The first requirement is that the buildings and bridges be designed to optimally suit this purpose. To meet the specific purpose, a bridge may have different structural types: arches, beams, or suspensions. The structure should reveal itself in a pure, clear form and impart a feeling of stability. We must seek simplicity here. The form of the basic structure must also correspond to the materials used. Brick and wood dictate different forms from those for steel or concrete. We speak of form justified by the material, or of "logic of form" [21]. This reminds us of the architect Sullivan's rule "form follows function" which became an often misunderstood maxim for building design. The function of a building is not only that it stand up. One must fulfill all the various requirements of the people that inhabit the building. These include hygiene, comfort, shelter from weather, beauty, even cosiness. The fulfillment of the functional requirements of buildings includes favorable thermal, climatic, acoustic, and aesthetic qualities. Sullivan undoubtedly intends us to interpret his rule in this sense. For buildings the functional requirements are very complex, but in engineering structures, functions besides load-carrying capacity must be fulfilled, such as adequate protection against weather, limitation of deformation and oscillation, among others, and all these factors affect design. Quality and beauty must be united, and quality takes first priority!

### 2.9.2 Proportion

An important characteristic necessary to achieve beauty of a building is good, harmonious proportions, in three-dimensional space. Good proportions must exist between the relative sizes of the various parts of a building, between its height, width, and breadth, between masses and voids, closed surfaces and openings, between the light and dark caused by sunlight and shadow. These proportions should convey an impression of balance. Tassios [25] preferred "expressive proportions" which emphasize the desired character of a building (see Section 2.9.8).

For structures it is not sufficient that their design is "statically correct." A ponderous beam can be as structurally correct as a slender beam, but it expresses something totally different. Not only are the proportions of the geometric dimensions of individual parts of the building important, but also those of the masses of the structure. In a bridge, for instance, these relationships may be between the suspended superstructure and the supporting columns, between the depth and the span of the beam, or between the height, length, and width of the openings. Harmony is also achieved by the repetition of the same proportions in the entire structure or in its various parts. This is particularly true in buildings.

Sometimes contrasting proportion can be a suitable element. The detailed discussion is referred to Chapter 4 of my book [26], which shows what good proportions can mean for bridges.

### 2.9.3 Order

A third important rule is the principle of order in the lines and edges of a building, an order achieved by limiting the directions of these lines and edges to only a few in space.

Too many directions of edges, struts, and the like create disquiet, confuse the observer, and arouse disagreeable emotions. Nature offers us many examples of how order can lead to beauty; just think of the enchanting shapes of snow crystals and of many flowers [27, 28]. Good order must be observed between the proportions occurring in a building; for instance, rectangles of 0.8:1 should not be placed next to slim rectangles of 1:3. Symmetry is a well-tried element of order whenever the functional requirements allow symmetry without constraint.

We can include the repetition of equal elements under the rule of order. Repetition provides rhythm, which creates satisfaction. Too many repetitions, on the other hand, lead to monotony, which we encounter in the modular architecture of many high-rise buildings. Where too many repetitions occur, they should be interrupted by other design elements.

The selection of one girder system throughout the structure provides an element of good order. Interrupting a series of arches with a beam gives rise to aesthetic design problems. Under the principle of order for bridges we may include the desire to avoid unnecessary accessories. The design should be so refined that we can neither remove nor add any element without disturbing the harmony of the whole.

### 2.9.4 Refining the Form

In many cases, bodies formed by parallel straight lines appear stiff and static, producing uncomfortable optical illusions. Tall bridge piers or towers with parallel sides appear from below to be wider at the top than at the bottom, which would be unnatural. Nor does this uniform thickness conform to our concept of functionality, because the forces decrease with increasing height. For this reason, the Egyptians and Greeks gave the columns of their temples a very slight taper, which in many cases is actually curved. Towers are built tapered or stepped. On high towers and bridge piers, a parabolic taper looks better than a straight taper.

The spans of a viaduct crossing a valley should become smaller on the slopes, and even the depth of the girders or edge fascia can be adjusted to the varying spans. Long beams of which the bottom edge is exactly horizontal look as if they are sagging, and so we give them a slight camber.

We must also check the appearance of the design from all possible vantage points of the future observer. Often the pure elevation on the drawing board is entirely satisfactory, but in skew angle views of unpleasant overlapping are found. We must also consider the effects of light and shadow. A wide cantilever deck slab can throw bridge girders into shadow and make them appear light, whereas similar shadows break the expressive character of an arch. Models are strongly recommended for checking a design from all possible viewpoints.

These refinements of form are based on long experience and must be studied with models from case to case.

### 2.9.5 Integration into the Environment

As the next rule, we recognize the need to integrate a structure or a building into its environment, landscape, or cityscape, particularly where its dimensional relationships and scale are concerned. In this respect many mistakes have been made during the past decades by placing massive concrete blocks in the heart of old city areas. Many factories and supermarkets also show this lack of sensitive integration. Sometimes long-span bridges with deep, heavy beams spoil lovely valley landscapes or towns with old houses lining the riverbank.

The dimensions of buildings must also be related to the human scale. We feel uneasy and uncomfortable moving between gigantic high-rise buildings. Heavy, brutal forms are often deliberately chosen by architects working with prefabricated concrete elements, but they are simply offensive. It is precisely their lack of scale and proportion that has led to the revolt against the brutality of this kind of architecture.

### 2.9.6 Surface Texture

When integrating a building with its surroundings, a major role is played by the *choice of materials*, the *texture of the surfaces*, and particularly by *color*. How beautiful and vital a natural stone wall can appear if we choose the right stone. By contrast, how repulsive are many concrete facades; not only do they have a dull gray color from the beginning, but they weather badly, producing an ugly patina and appear dirty after only a few years. Rough surfaces are suitable for piers and abutments; smooth surfaces work well on fascia-beams, girders, and slender columns. As a rule, surfaces should be matte and not glossy.

### 2.9.7 Color

*Color* plays a significant role in the overall aesthetic effect. Many researchers have studied the psychological effects of color. Here, too, ancient rules of harmonious color composition apply, but today successful harmonious color schemes are rare. Often, we find the fatal urge for sensation, for startling aggressive effects, which can be satisfied all too easily with the use of dissonant colors, especially with modern synthetic pop — or shocking — colors. We can find, however, many examples of harmonious coloring, generally in town renovation programs. Bavaria has provided several examples where good taste has prevailed.

### 2.9.8 Character

A building and bridge should have character; it should have a certain deliberate effect on people. The nature of this desired effect depends on the purpose, the situation, the type of society, and on sociological relationships and intentions. Monarchies and dictatorships try to intimidate by creating monumental buildings, which make people feel small and weak. We can hope this belongs to the past. Only large banks and companies still make attempts to impress their customers with monumentalism. Churches should lead inward to peace of mind or convey a sense of release and joy of life as in the Baroque or Rococo. Simple dwellings should radiate safety, shelter, comfort, and warmth. Beautiful houses can stimulate happiness.

Buildings of the last few decades express an air of austere objectivity, monotony, coldness, confinement, and, in cities, confusion, restlessness, and lack of composition; there is too much individuality and egoism. All this dulls people's senses and saddens them.

We seem to have forgotten that people also want to meet with joy in their man-made environment. Modern buildings seem to lack entirely the qualities of cheerfulness, buoyancy, charm, and relaxation. We should once again become familiar with design features that radiate cheerfulness without lapsing into Baroque profusion.

### 2.9.9 Complexity — Stimulation by Variety

Smith [3] postulated a "second aesthetic order," suggested by findings made by biologists and psychologists [29]. According to this, beauty can be enhanced by the tension between variety and similarity, between complexity and order. Baumgarten expressed this as early as 1750, "Abundance and variety should be combined with clarity. Beauty offers a twofold reward: a feeling of well being both from the perception of newness, originality and variation as well as from coherence, simplicity, and clarity." Leibniz in 1714 demanded for the achievement of perfection as much variety as possible, but with the greatest possible order.

Berlyne [30] considered the sequence of tension and relaxation to be a significant characteristic of aesthetic experience. Venturi [31], a rebel against the "rasteritis" (modular disease) architecture of Mies van der Rohe, said, "A departure from order — but with artistic sensitivity — can create pleasant poetic tension."

A certain amount of excitement caused by a surprising object is experienced as pleasant if neighboring objects within the order ease the release of tension. If variety dominates our orientation, reflex is overtaxed and feelings ranging from distaste to rejection are aroused. Disorder is not beautiful.

This complexity doubtless requires artistic skill to be successful. It can be used well in bridge design if, for instance, in a long, multispan bridge the main span is accented by a variation in the girder form. The interplay of complexity and order is important in architecture, particularly in city planning. Palladio was one of the first to extend the classical understanding of harmony by means of the complexity of architectural elements and ornamentation.

### 2.9.10 Incorporating Nature

We will always find the highest degree of beauty in nature, in plants, flowers, animals, crystals, and throughout the universe in such a variety of forms and colors that awe and admiration make it extremely difficult to begin an analysis. As we explore deeper into the realm of beauty we also find in nature rules and order, but there are always exceptions. It must also remain possible to incorporate such exceptions in the masterpieces of art made by creative humans [28].

The beauty of nature is a rich source for the needs of the soul, and for humans' psychic well-being. All of us know how nature can heal the effects of sorrow and grief. Walk through beautiful countryside — it often works wonders. As human beings we need a direct relationship with nature, because we are a part of her and for thousands of years have been formed by her.

This understanding of the beneficial effects of natural beauty should lead us to insist that nature again be given more room in our man-made environment. This is already happening in many of our cities, but we must introduce many more green areas and groups of trees. Here we must mention the valuable work of Seifert [32] during the building of the first autobahns in Germany.

### 2.9.11 Closing Remarks on the Rules

We must not assume that the simple application of these rules will in itself lead to beautiful buildings or bridges. The designer must still possess imagination, intuition, and a sense for both form and beauty. Some are born with these gifts, but they must be practiced and perfected. The act of designing must always begin with individual freedom, which in any case will be restricted by all the functional requirements, by the limits of the site, and not least by building regulations that are usually too strict.

The rules, however, provide us with a better point of departure and help us with the critical appraisal of our design, particularly at the model stage, thus making us aware of design errors.

The artistically gifted may be able to produce masterpieces of beauty intuitively without reference to any rules and without rational procedures. However, the many functional requirements imposed on today's buildings and structures demand that our work must include a significant degree of conscious, rational, and methodical reasoning.

## 2.10 Aesthetics and Ethics

Aesthetics and ethics are in a sense related; by ethics we mean our moral responsibility to humanity and nature. Ethics also infers humility and modesty, virtues which we find lacking in many designers of the last few decades and which have been replaced by a tendency toward the spectacular, the sensational, and the gigantic in design. Due to exaggerated ambition and vanity and spurred by the desire to impress, unnecessary superlatives of fashions were created, lacking true qualities of beauty. Most of these works lack the characteristics needed to satisfy the requirements of the users of these buildings.

As a responsibility, ethics requires a full consideration of all functional requirements. In our man-made environment we must emphasize the categories of quality and beauty. In his *Acht Todsünden der Menschheit,* Loreanz [33] once said that "the senses of aesthetics and ethics are apparently very closely related, so that the aesthetic quality of the environment must directly affect Man's ethical behavior." He said further, "The beauty of Nature and the beauty of the man-made cultural environment are apparently both necessary to maintain Man's mental and psychic health. Total blindness of the soul for all that is beautiful is a mental disease that is rapidly spreading today and which we must take seriously because it makes us insensitive to the ethically obnoxious."

In one of his last important works, in *To Have or to Be* [34] Erich Fromm also said that the category of "goodness" must be an important prerequisite for the category "beauty," if beauty is to be an enduring value. Fromm goes so far as to say that "the physical survival of mankind is dependent on a radical spiritual change in Man." The demand for aesthetics is only a part of the general demand for changes in the development of "Man." These changes have been called for at least in part and at intervals by humanism, but their full realization in turn demands a new kind of humanism, as well expressed in the appeal by Peccei [35].

## 2.11 Summary

In order to reach a good capacity of judging aesthetic qualities of buildings or bridge structures, it is necessary to go deep into our human capacities of perception and feelings. The views of many authors who treated aesthetics may help to come to some understanding, which shall help us to design with good aesthetic quality.

### References


1. Klages, L., *Grundlagen der Wissenschaft vom Ansdruck*, 9, Auflage, Bonn, Germany, 1970.
2. Hume, H., *On the Standard of Taste*, London, U.K., 1882.
3. Smith, P. F, *Architecktur und Aesthetik*, Stuttgart, Germany, 1981. Original: *Architecture and the Human Dimension*, London, 1979.
4. Kant, I., *Kritik der Urteilskraft, Reklam*, Stuttgart, Germany, 1995.
5. Paul, J., *Vorschule der Aesthetik*, München, Germany, 1974.
6. Schmitz, H., *Neue Phänomenologie*, Bonn, Germany, 1980.
7. Bahrdt, H. P., *Vortrag Hannover*, Stiftung FVS, Hamburg, Germany, 1979.
8. Keller, R., *Bauen als Umweltzerstörung*, Zürich, Switzerland, 1973.
9. Arnheim, R., *Art and Visual Perception*, Berkeley, CA, 1954. German: *Kunst und Sehen*, Berlin, Germany, 1978.
10. Goethe, J. W., *Farbenlehre*, Stuttgart, Germany, 1979.
11. Szabo, I., *Anfänge der griechischen Mathematik*, Berlin, Germany, 1969.
12. Wittlkower, R., *Architectural Principles in the Age of Humanism*, London, 1952. German: *Grundlagen der Architecktur im Zeitalter des Humanismus*, München, Germany, 1969.
13. Charpentier, L., *Die Geheimnisse der Kathedrale von Chartres*, Köln, Germany, 1974.
14. Kayser, H., *Paestum*, Heidelberg, Germany, 1958.
15. Thimus, A. von, *Die Harmonische Symbolik des Altertums*, Köln, Germany, 1968–1976.
16. Puppi, L., *Andrea Palladio*, Stuttgart, Germany, 1977.
17. Strübin, M., Das Villard Diagramm, *Schw. Bauz.*, 1947, 527.
18. Studer, A. M., Architektur, Zahlen und Werte, *Dtsch Bauz.*, 9, Stuttgart, Germany, 1965.
19. Helmcke, J. G., *Ist das Empfinden von aesthetisch schoenen Formen angeboren oder anerzogen*, Heft 3 des SFB 64 der Universität Stuttgart, Germany, 1976, 59; see also *Grenzen menschlicher Anpassung*, IL 14, Universität Stuttgart, Germany, 1975.
20. Grassi, E., *Die Theorie des Schönen in der Antike*, Köln, Germnay, 1980.
21. Torroja, E., *Logik der Form*, München, Germany, 1961.
22. Tellenbach, H., *Geschmack und Atmosphaere*, Salzburg, Austria, 1968.
23. Grimm, C. T., Rationalized esthetics in civil engineering, *J. Struct. Div., ASCE*, 1975.
24. Borgeest, C., *Das Kunsturteil*, Frankfurt, Germany, 1979. Also: *Das sogenannte Schöne*, Frankfurt, Germany, 1977.
25. Tassios, T. P., Relativity and optimization of aesthetic rules for structures, *IABSE Congr. Rep.*, Zürich, Switzerland, 1980.
26. Leonhartdt, F., *Bridges — Aesthetics and Design*, DVA, Stuttgart, Germany, 1984.

27. Heydemann, B., Auswirkungen des angeborenen Schönheitssinnes bei Mensch und Tier, *Nat., Horst Sterns Umweltmag.*, 0, 1980.
28. Kayser, H., *Harmonia Plantarum*, Basel, Switzerland. Also: H. Akroasis, *Die Lehre von der Harmonik der Welt*, Stuttgart, Germany, 1976.
29. Humphrey, N., The illusion of beauty, *Perception Bd.*, 2, 1973.
30. Berlyne, D. E., *Aesthetics and Psycho-Biology*, New York, 1971.
31. Venturi, R., *Complexity and Contradiction in Architecture*, New York, 1966.
32. Seifert, A., *Ein Leben für die Landschaft*, Düsseldorf-Köln, Germany, 1962.
33. Lorenz, K., *Acht Todsünden der zivilisierten Menschheit*, München, Germany, 1973.
34. Fromm, E., *Haben oder Sein*, Stuttgart, Germany, 1976.
35. Peccei, A., *Die Zukunft in unserer Hand*, München, Germany, 1981.

Billington, D., Gottemoeller, F. "Bridge Aesthetics - Structural Art."
*Bridge Engineering Handbook.*
Ed. Wai-Fah Chen and Lian Duan
Boca Raton: CRC Press, 2000

# 3

# Bridge Aesthetics — Structural Art




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## 3.1 Introduction

In recent years it has become apparent that the real problems of bridge design include more than the structural or construction issues relating to the spanning of a gap. The public often expresses concern over the appearance of bridges, having recognized that a bridge's visual impact on its community is lasting and must receive serious consideration.

The public knows that civilization forms around civil works: for water, transportation, and shelter. The quality of public life depends, therefore, on the quality of such civil works as aqueducts, bridges, towers, terminals, and meeting halls: their efficiency of design, their economy of construction, and the visual appearance of their completed forms. At their best, these civil works function reliably, cost the public as little as possible, and, when sensitively designed, become works of art.

Thus, engineers all over the world are being forced to address the issues of aesthetics. Engineers cannot avoid aesthetic issues by taking care of the structural elements and leaving the visual quality to someone else. It is the shapes and sizes of the structural components themselves that dominate the appearance of the bridge, not the details, color, or surfaces. Since they control the shapes and sizes of the structural components, engineers must acknowledge the fact that they are ultimately responsible for the appearance of their structures. Engineers are used to dealing with issues of performance, efficiency, and cost. Now, they must also be prepared to deal with issues of appearance.

**FIGURE 3.1** Thomas Telford's Craigellachie Bridge.

## 3.2 The Engineer's Aesthetic and Structural Art

"Aesthetics" is a mysterious subject to most engineers, not lending itself to the engineer's usual tools of analysis. It is a topic rarely taught in engineering schools. Many contemporary engineers are not aware that a long line of engineers have made aesthetics an explicit element in their work, beginning with the British engineer Thomas Telford. In 1812, Telford defined structural art as the personal expression of structure within the disciplines of efficiency and economy. Efficiency here meant reliable performance with minimum materials, and economy implied the construction with competitive costs and restricted maintenance expenses. Within these bounds, structural artists find the means to choose forms and details that express their own vision, as Telford did in his Craigellachie Bridge (Figure 3.1). The arch is shaped to be an efficient structural form in cast iron, while his diamond pattern of spandrel bars, at a location in the bridge where structural considerations permit many options, is clearly chosen with an eye to its appearance.

Those engineers who were most conscious of the centrality of aesthetics for structure have also been regarded as the best in a purely technical sense. Starting with Thomas Telford (1757–1834), we can identify Gustave Eiffel (1832–1923) and John Roebling (1806–1869) as the undisputed leaders in their fields during the 19th century. They designed the largest and most technically challenging structures, and they were leaders of their professions. Telford was the first president of the first formal engineering society, the Institution of Civil Engineers, and remained president for 14 years until his death. Eiffel directed his own design–construction–fabrication company and created the longest spanning arches and the highest tower; Roebling founded his large scale wire rope manufacturing organization while building the world's longest spanning bridges ( Figure 3.2).

In reinforced concrete, Robert Maillart (1872–1940) was the major structural artist of the early 20th century. First in his 1905 Tavanasa Bridge, and later with the 1930 Salginatobel (Figure 3.3) and 1936 Vessy designs, he imagined a new form for three-hinged arches that included his own invention of the hollow box in reinforced concrete. The Swiss engineer Christian Menn (1927– ) has demonstrated how a deep understanding of arches, prestressing, and cable-stayed forms can lead to structures worthy of exhibition in art museums. Especially noteworthy are the 1964 Reichenau Arch, the 1974 Felsenau prestressed cantilever, and the 1980 concrete cable-stayed Ganter Bridge. Meanwhile, German engineer

**FIGURE 3.2** John Roebling's Brooklyn Bridge.

Jorg Schlaich has developed new ideas for light structures often using cables, characterized by a series of elegant footbridges in and around Stuttgart (Figure 3.4).

The engineers' aesthetic results from the conscious choice of form by engineers who seek the expression of structure. It is neither the unconscious result of the search for economy nor the product of supposedly optimizing calculations. Many of the best structural engineers have recognized the possibility for structural engineering to be an art form parallel to but independent of architecture. These people have, over the past two centuries, defined a new tradition, structural art, which we take here to be the ideal for an engineer's aesthetic.

Although structural art is emphatically modern, it cannot be labeled as just another movement in modern art. For one thing, its forms and its ideals have changed little since they were first expressed by Thomas Telford. It is not accidental that these ideals emerged in societies that were struggling with the consequences not only of industrial revolutions but also of democratic ones. The tradition of structural art is a democratic one.

In our own age the works of structural art provide evidence that the common life flourishes best when the goals of freedom and discipline are held in balance. The disciplines of structural art are efficiency and economy, and its freedom lies in the potential it offers the individual designer for the expression of a personal style motivated by the conscious aesthetic search for engineering elegance. These are the three leading ideals of structural art — efficiency, economy, and elegance.

**FIGURE 3.3** Robert Maillert's Salginotobel Bridge.

**FIGURE 3.4** One of Jorg Schlaich's footbridges.

## 3.3 The Three Dimensions of Structure

Its first dimension is a scientific one. Each working structure or machine must perform in accordance with the laws of nature. In this sense, then, technology becomes part of the natural world. Methods of analysis useful to scientists for explaining natural phenomena are often useful to engineers for describing the behavior of their artificial creations. It is this similarity of method that helps to feed the fallacy that engineering is applied science. But scientists seek to discover preexisting form and explain its behavior by inventing formulas, whereas engineers want to invent forms, using preexisting formulas to check their designs. Because the forms studied by scientists are so different from those of engineers, the methods of analysis will differ; yet, because both sets of forms exist in the natural world, both must obey the same natural laws. This scientific dimension is measured by efficiency.

Technological forms live also in the social world. Their forms are shaped by the patterns of politics and economics as well as by the laws of nature. The second dimension of structure is a social one. In the past or in primitive places of the present, completed structures and machines might, in their most elementary forms, be merely the products of a single person; in the civilized modern world, however, these technological forms, although at their best designed by one person, are the products of a society. The public must support them, either through public taxation or through private commerce. Economy measures the social dimension of structure.

Technological objects visually dominate our industrial, urban landscape. They are among the most powerful symbols of the modern age. Structures and machines define our environment. The locomotive of the 19th century has given way to the automobile and airplane of the 20th. Large-scale complexes that include structures and machines become major public issues. Power plants, weapons systems, refineries, river works — all have come to symbolize the promises and problems of industrial civilization.

The Golden Gate, the George Washington, and the Verrazano Bridges carry on the traditions set by the Brooklyn Bridge. The Chicago Hancock and Sears Towers, and the New York Woolworth, Empire State, and World Trade Center Towers all bring the promise of the Eiffel Tower into the utility of city office and apartment buildings. The Astrodome, the Kingdome, and the Superdome carry into the late 20th century the vision of huge permanently covered meeting spaces first dramatized by the 1851 Crystal Palace in London and the 1889 Gallery of Machines in Paris.

Nearly every American knows something about these immense 20th-century structures, and modern cities repeatedly publicize themselves by visual reference to these works. As Montgomery Schuyler, the first American critic of structures, wrote in the 19th century for the opening of the Brooklyn Bridge, "It so happens that the work which is likely to be our most durable monument, and to convey some knowledge of us to the most remote posterity, is a work of bare utility; not a shrine, not a fortress, not a palace but a bridge. This is in itself characteristic of our time." [1].

So it is that the third dimension of technology is symbolic, and it is, of course, this dimension that opens up the possibility for the new engineering to be structural art. Although there can be no measure for a symbolic dimension, we recognize a symbol by its elegance and its expressive power. Thus, the Sunshine Skyway (Figure 3.5) has become a symbol of both Florida's Tampa Bay area and the best of late-20th-century technology.

There are three types of designers who work with forms in space: the engineer, the architect, and the sculptor. In making a form, each designer must consider the three dimensions or criteria we have discussed. The first, or scientific criterion, essentially comes down to making structures with a minimum of materials and yet with enough resistance to loads and environment so that they will last. This efficiency–endurance analysis is arbitrated by the concern for safety. The second, or social criterion, comprises mainly analyses of costs as compared with the usefulness of the forms by society. Such cost–benefit analyses are set in the context of politics. Finally, the third criterion, the symbolic, consists of studies in appearance, along with a consideration of how elegance can be achieved within the constraints set by the scientific and social criteria. This is the aesthetic/ethical basis upon which the individual designer builds his or her work.

For the structural designer the scientific criterion is primary (as is the social criterion for the architect and the symbolic criterion for the sculptor). Yet the structural designer must balance the primary criterion with the other two. It is true that all structural art springs from the central ideal of artificial forms controlling natural forces. Structural forms will, however, never get built if they do not gain some social acceptance. The will of the designer is never enough. Finally, the designer must think aesthetically for structural form to become structural art. All of the leading artists of

**FIGURE 3.5** The Sunshine Skyway.

structure thought about the appearance of their designs. These engineers consciously made aesthetic choices to arrive at their final designs. Their writings about aesthetics show that they did not base design only on the scientific and social criteria of efficiency and economy. Within those two constraints, they found the freedom to invent form. It was precisely the austere discipline of minimizing materials and costs that gave them the license to create new images that could be built and endure.

## 3.4 Structure and Architecture

The modern world tends to classify towers, stadiums, and even bridges as architecture, creating an important, but subtle, fallacy. Even the word is a problem, because *architect* comes from the Greek word meaning chief technician. But, beginning with the Industrial Revolution, structure has become an art form separate from architecture. The visible forms of the Eiffel Tower, Seattle's Kingdome, and the Brooklyn Bridge result directly from technological ideas and from the experience and imagination of individual structural engineers. Sometimes, the engineers have worked with architects just as with mechanical or electrical engineers, but the forms have come from structural engineering ideas.

Structural designers give form to objects that are of relatively large scale and of single use, and these designers see forms as the means of controlling the forces of nature to be resisted. Architectural designers, on the other hand, give form to objects that are of relatively small scale and of complex human use, and these designers see forms as the means of controlling the spaces to be used by people. The prototypical engineering form — the public bridge — requires no architect. The prototypical architectural form — the private house — requires no engineer. Structural engineers and architects learn from each other and sometimes collaborate fruitfully, especially when, as with tall buildings, large scale goes together with complex use. But the two types of designers act predominately in different spheres.

The works of structural art have sprung from the imagination of engineers who have, for the most part, come from a new type of school — the polytechnical school, unheard of prior to the late 18th century. Engineers organized new professional societies, worked with new materials, and stimulated political thinkers to devise new images of future society. Their schools developed curricula that decidedly cut whatever bond had previously existed between those who made architectural forms and those who began to make — out of industrialized metal and later from reinforced concrete — the new engineering forms by which we everywhere recognize the modern world. For these forms the ideas inherited from the masonry world of antiquity no longer applied; new ideas were essential in order to build with the new materials. But as these new ideas broke so radically with conventional taste, they were rejected by the cultural establishment.

This is, of course, a classic problem in the history of art: new forms often offend the academics. In this case, it was beaux arts against structural arts. The skeletal metal of the 19th century offended most architects and cultural leaders. New buildings and city bridges suffered from valiant attempts to cover up or contort their structure into some reflection of stone form. In the 20th century, the use of reinforced concrete led to similar attempts. Although some people were able to see the potential for lightness and energy, most architects tried gamely to make concrete look like stone or, later on, like the emerging abstractions of modern art. There was a deep sense that engineering alone was insufficient.

The conservative, plodding, hip-booted technicians might be, as the architect Le Corbusier said, "healthy and virile, active and useful, balanced and happy in their work, but only the architect, by his arrangement of forms, realizes an order which is pure creation of his spirit … it is then that we experience the sense of beauty." The belief that the happy engineer, like the noble savage, gives us useful things but only the architect can make them into art is one that ignores the centrality of aesthetics to the structural artist. In towers, bridges, free-spanning roofs, and many types of industrial buildings, aesthetic considerations provide important criteria for the engineer's design. The best of such engineering works are examples of structural art, made by engineers, and they have appeared with enough frequency to justify the identification of structural art as a mature tradition with a unique character. One of the most recent manifestations is Christian Menn's Sunniberg Bridge (Figure 3.6).

## 3.5 Application to Everyday Design

Many of today's engineers see themselves as a type of applied scientist, analyzing preexisting structural forms that have been established by others. Seeing oneself as an applied scientist is an unfortunate state of mind for a design engineer. It eliminates the imaginative half of the design process and forfeits the opportunity for the integration of form and structural requirements that can result in structural art. Design must start with the selection of a structural form. It is a decision that can be made well only by the engineer because it must be based on a knowledge of structural forms and how they control forces and movements.

In the case of most everyday bridges the selection of form is based largely on precedents and standards established by the bridge-building agency. For example, the form of a highway overpass may be predetermined by the client agency to be a welded plate girder bridge because that is what the agency prefers or what local steel fabricators are used to or even because the steel industry is a dominant political force in the state. In other cases, the form may be established by an architect or urban designer for reasons outside structural requirements. Thus the form is set without any serious consideration of whether or not that is in fact the best form for that particular site.

Creative form determination consists not of applying free visual imagination alone nor in applying rigorous scientific analysis alone, but of applying both together, at the same time. The art starts with a vision of what might be. The development of that vision is the key. Many engineers call the development of the vision conceptual engineering. It is the most important part of design. It is the stage at which all plausible forms are examined. The examination must include, to a rough level of precision, the whole range of considerations; performance, cost, and appearance. All that follows, including the aesthetic impression the bridge makes, will depend on the quality of the form selected. This stage is often ignored or foreclosed, based on precedents, standards, preconceived ideas or prior experience that may or may not apply.

**FIGURE 3.6** Christian Menn's Sunniberg Bridge.

The reasons often given for shortchanging this stage include, "Everybody knows that [steel plate girders, precast concrete girders, cast-in-place concrete] are the most economical structure for this location," or "We always build [steel plate girders, precast concrete girders, cast-in-place concrete] in this state," or "Let's use the same design as we did for [any bridge] last year."

At this point someone will protest that other considerations (costs, the preferences of the local contracting industry, etc.) will indeed differentiate and determine the form. Too often these reasons are based on unexamined assumptions, such as, "The local contracting industry will not adjust to a different form," or "Cost differentials from [a past project] still apply," or "The client will never consider a different idea." Or the belief is based on a misleading analysis of costs which relies too much on assumed unit costs. Or that belief may be simply habit — either the engineer's or the client's — often expressed in the phrase, "We've always done it that way." Accepting these assumptions and beliefs places an unfortunate and unnecessary limitation on the quality of the resulting bridge for, by definition, improvements must come from the realm of ideas not tried before.

As Captain James B. Eads put it in the preliminary report on his great bridge over the Mississippi River at St. Louis:

> Must we admit that because a thing has never been done, it never can be, when our knowledge and judgment assure us that it is entirely practicable?[2].

**FIGURE 3.7** MD 18 over U.S. 50.

**FIGURE 3.8** Another possibility for MD 18 over U.S. 50.

The engineer's first job is to question all such determinations, assumptions and beliefs. From that questioning will come the open mind that is necessary to develop a vision of what each structure can be at its best.

Unless such questioning is the starting point it is unlikely that the most promising ideas will ever appear. No design will occur. Instead, there will be a premature assumption of the bridge form, and the engineer will move immediately into the analysis of the assumed form. That is why so many engineers mistake analysis for design. Design is more correctly the selection of the form in the first place, which most engineers have not been permitted to do. Design is by far the more important of the two activities.

Engineers also focus on analysis in the belief that the form (shape and dimensions) will be determined by the forces as calculated in the analysis. But, in fact, there are a large number of forms that can be shown by the analysis to work equally well. It is the engineer's option to choose among them, and in so doing to determine the forces by means of the form, not the other way around.

Take the simple example of a two-span continuous girder bridge, using an existing structure, MD 18 over U.S. 50 (Figure 3.7). Here the engineer has a wide range of possibilities such as a girder with parallel flanges, or with various haunches having a wide range of proportions (Figure 3.8). The moments will depend on the stiffness at each point, which in turn will depend on the presence or absence of a haunch and its shape (Figure 3.9). The engineer's choice of shape and dimensions will determine the moments at each point along the girder. The forces will follow the choice of form. Within limits, the engineer can direct the forces.

Let's examine which form the engineer should choose. Allcan support the required load. Depending on the specifics of the local contracting industry, many of them will be essentially equal in cost. All would perform equally well and all are comparable in cost, leaving the engineer a decision that can only be made on aesthetic grounds. Why not pick the one the engineer believes looks best?

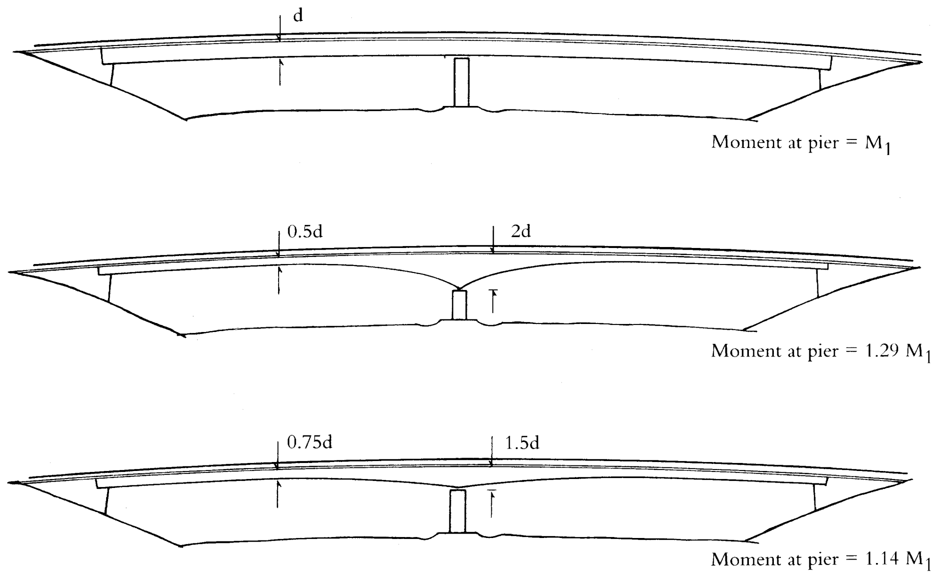


**FIGURE 3.9** Forces determined by the engineer's choice of form.

That, in a nutshell, is the process that all of the great engineers have followed. Maillart's work, as one example, shows that the engineer cannot choose form as freely as a sculptor, but the engineer is not restricted to the discovery of preexisting forms as the scientist is. The engineer invents form, and Maillart's career shows that such invention has both a visual and a scientific basis. When either is denied, engineering design ceases. For Maillart, the dimensions were not to be determined by the calculations, and even the results of the calculations could be changed (by adjusting the form) because a designer rather than an analyst is at work. Analysis and calculation are the servants of design. Design, analysis, and must work together. In the words of Spanish engineer Eduardo Torroja,

> The imagination alone cannot reach such [elegant] designs unaided by reason, nor can a process of deduction, advancing by successive cycles of refinement, be so logical and determinate as to lead inevitably to them.[3]

The engineering challenge is not just to find the least costly solution. The engineering challenge is to bring forth elegance from utility: We should not be content with bridges that only move vehicles and people. They should move our spirits as well.

## 3.6 The Role of Case Studies in Bridge Design

Bridge design, even of highway overpasses, often involves standard problems but always in different situations. Case studies can help in the design of these standard problems by showing models and points of comparison for a large number of bridges without implying that each such bridge be mere imitation.

The primary goals of a case study are to look carefully at all major aspects of the completed bridge, to understand the reasons for each design decision, and to discuss alternatives, all to the end of improving future designs. Such cases help to define more general ideas or principles. Case studies are well recognized by engineers when designing for acceptable performance and low cost; they can be useful when considering appearance as well.

A common organization of these studies will help identify standard problems and make comparisons easier. First comes an overall evaluation of the bridge as a justification for studying it. Is it a good example that can be better? Is it a model of near perfection? Is it a bad example to be avoided?, Second comes a description of the complete bridge, which is divided into parts roughly coinciding with easily identifiable costs and including modifications to each part as suggested improvements. In this major description section there is an order to the parts that implies a priority for the structural engineer: concept and form of the entire structure, superstructure, supports, deck, color, and landscaping.

1. The *concept and form* of the completed bridge goes together with a summary of the bridge performance history (including maintenance) and of its construction cost, usually given per square meter of bridge. Required clearances, foundation conditions, hydraulic requirements, traffic issues, and other general requirements would be covered here.
2. The *superstructure* here includes primarily the main horizontal spanning members such as continuous girders, arches, trusses, etc. In continuous steel girder bridges, the cost is primarily identified with the fabricated steel cost. Modification in design by haunching, changing span lengths, or making girders continuous with columns would be discussed including their influence on cost.
3. The *pier supports* are most frequently columns or frames either in the median or outside the shoulders, or at both places in highway overpasses. These are normally highly visible elements and can have many possible forms. Different designs for the relationship among steel girder, bearings, and columns can make major improvements in appearance without detriment to cost or performance.
4. The *abutment supports* are also highly visible parts of the bridge, which include bearings, cantilever walls, cheek walls, and wing walls.
5. The *deck* includes the concrete slab or orthotropic steel deck, overhangs, railings, parapets, and provisions for drainage, all of which have an influence on performance as well as on the appearance either when seen in profile or from beneath the bridge.
6. The *color* is especially significant for steel structures that are painted, and *texture* can be important for concrete surfaces of piers, abutments, and deck.
7. The *landscaping/guardrail* includes plantings and other features that can have important visual consequences to the design.

The order of these parts is significant because it focuses attention on the engineering design. The performance of a weak structural concept cannot be saved by good deck details. An ugly form cannot be salvaged by color or landscaping. The first four parts are structural, the fifth is in part structural, whereas the last two, while essential for the bridge engineer to consider, involve primarily nonstructural ideas.

Third, the case study can give a critique of the concept and form by comparison with other similar bridges or bridge designs for similar conditions, including those with very different forms, as a stimulus to design imagination.

Fourth and finally, the case should conclude with some discussion of the relationship of this study to a theory of bridge design. Clearly, any such study must be based upon a set of ideas about design which often implicitly bias the writer who should make these ideas explicit. This conclusion should show how the present study illustrates a theory and even at times forces a modification of it. General ideas form only out of specific examples.

## 3.7 Case Study in Colorado: Buckley Road over I-76

Colorado's Buckley Road over I-76 (Figure 3.10) offers the application of an innovative form to prestressed concrete girders in order to achieve longer than normal spans, with a visually unique result. It is therefore a worthy subject for a case study.

**FIGURE 3.10** Buckley Road over I-76.

### 3.7.1 Description of the Bridge

In *concept* this is a three-span continuous beam bridge with a 47° skew made of precast prestressed girders set onto cast-in-place concrete piers and abutments.

The *superstructure* consists of seven girders spaced approximately 3 m apart and each made up of five precast prestressed concrete segments (Figure 3.11). The main span is approximately 56 m and each side span is approximately 50 m. Segments one and five are 37.8 m long and behave essentially as simply supported beams between the abutments and the cantilever segments two and four. These latter cantilever 12 m into the side spans and 15 m into the main span. Segment three is 25.6 m long and behaves approximately as a simply supported beam within the main span. There are 0.15 m spacings between segments for closure pours. The cantilever segments have a linear haunch of 0.6 m from the girder depth of 1.8 m for the other three segments, which are Colorado BT72 girders.

The two *piers* each consist of a pier cap beam 1.2 m wide by 1.8 m deep and 28 m long supported by three walls each 1.2 m wide, about 7.6 m high, and 6 m long at their tops tapering to 3 m long at the footings. The cap beam extends about 1.5 m beyond the centerline of the exterior girder or about 1.1 m beyond the edge of that girder's bottom flange. The pier next to the railroad has a crash wall built into the three tapered walls.

The *abutments* are shallow concrete beams 0.9 m wide supported on piles and carrying the precast girders. The *deck* is a series of precast pretensioned concrete panels made composite with the precast girders. Bounded by Jersey barriers, the deck is 20.4 m in width and overhangs the exterior girders by 0.96 m or slightly over half the depth of the unhaunched girder segments.

### 3.7.2 Critique of the Bridge

The *concept* of a fully precast superstructure, a three-span continuous beam, and cast-in-place piers has led to an economical structure and fits well the site conditions of crossing both I-76 and the double-track railroad. Other reasonable concepts include a two-span bridge and a three-span bridge with the cantilever segments two and four cast in place with the piers (as illustrated by the Stewart

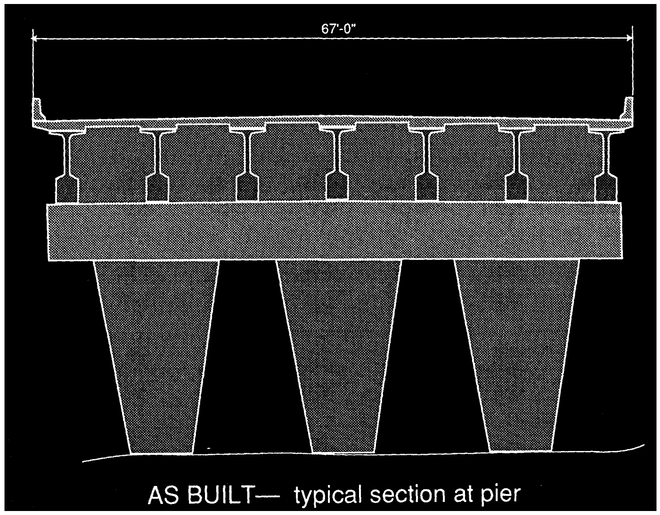


**FIGURE 3.11** Typical section of Buckley Road as built.

Park Bridge in Oregon). This critique will confine itself to the present concept, but a comparison of this bridge and the Oregon one will follow. In each case, the ideas of structural art will form the basis for a critique.

The *superstructure* represents an unusual use of a precast bulb T girder whose bulb is extended vertically to create a haunch at the two interior supports. The profile view expresses the increased forces at the interior supports and the construction photos (with temporary walkway) show the lightness achieved by an overhang that is about the same dimension as the girder depth.

The *haunches* would be more effective visually were they deeper and the segments one, three, and five correspondingly shallower. For example, with the Colorado C68 girder, the depth would decrease to 1.7 m and a haunch of 2.6 m would more strikingly express the flow of forces. At the same time, the girder spacing would be reduced to 2.9 m to permit an overhang of 1.4 m.

Another solution would be to retain the Colorado BT72 girders, increase the haunch to 3 m, and reduce the number of girders from seven to six, thus again increasing the overhang. If the six girders were spaced 3.35 m on centers, then the overhang would be 1.7 m or nearly the depth of the BT72 girders.

The *piers* are visually prominent and look heavy. They also have a formal shaping which does not clearly express the structure. Specifically, the horizontal lines of the hammerhead beam separate it from the supporting walls and the 1.8-m depth of that beam is far greater than needed to carry the girder loads over the 2.4-m span between the wide supporting walls below. Since these piers are relatively short compared with the long spans of the girders, their massive appearance is accentuated by the lack of structural expression. It is clear from beneath the bridge that the 6-m-wide walls can easily be made to support all the girders directly without any hammerhead beam (Figure 3.12). The walls will therefore be higher and, if carefully shaped, will form a striking integration with the deck girders. The cast-in-place diaphragm can then be structurally integrated with the walls and the girders to form a cross frame for live loads.

The *abutments* can be improved by eliminating the wall that hides the girder ends and bearings. Along with the lighter-looking girders, this structural expression at the abutments will increase the already striking appearance of the bridge profile.

The *deck* overhang, by being increased, lends lightness to the girders. Otherwise, the system used is good and avoids the staining that can arise when metal slab forms are left in place.

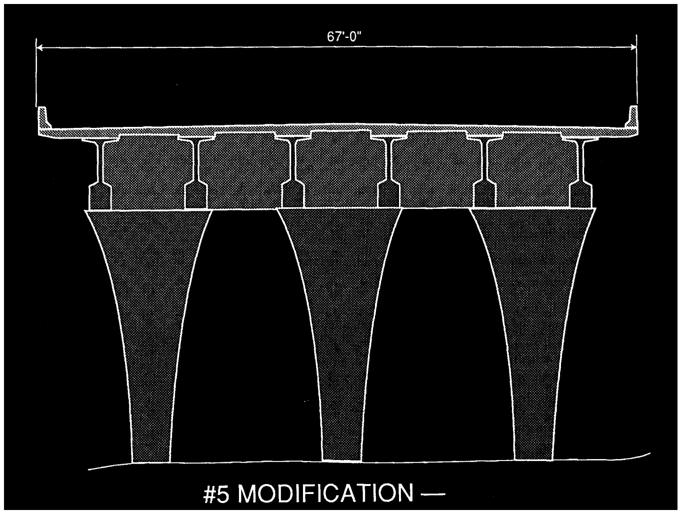


**FIGURE 3.12** Possible modification to Buckley Road.

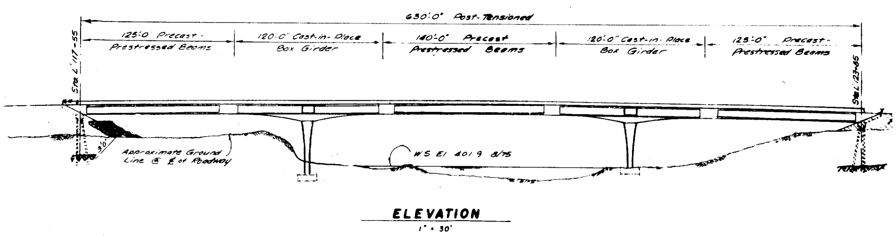


**FIGURE 3.13** Elevation of Louis Pierce's Stewart Park Bridge.

### 3.7.3 The Stewart Park Bridge

The *concept* for this 1978 bridge (Figure 3.13) designed by Louis Pierce is the same as for Buckley Road except for the cantilever segments two and four which are cast-in-place prestressed concrete hollow boxes. Because the spans (56.4, 79.2, 56.4 m) are longer than those for Buckley Road, segments one, three, and five are each made of two separate precast pieces.

The *superstructure* and the *piers* are thus integrated into one form rather than separated into two forms as at Buckley Road. The boxes are haunched from the 2.4 m of the constant section segments to 3.65 m at the two interior supports for a ratio of 1.55. But the boxes are 2.4 m deep along their exterior faces and haunch laterally to 3.65 m over a distance of 2.5 m. Just as at Buckley Road, the *deck* overhang is too short, about 0.8 m for a girder depth of 2.4 m.

The shape of the two piers are walls 7.6 m wide, 2.3 m thick at the top, tapering to 5.8 m wide and 1.4 m thick at the base. The total height is 12.7 m above the footing but only about 7.6 m above the ground line. This shaping of piers, having about the same height as those of Buckley Road, gives an impression of lightness missing from the latter structure (Figure 3.14).

### 3.7.4 Summary

The Buckley Road bridge represents a good design. A similar concept can be improved in future designs by relatively small changes in the superstructure through stronger haunching and wider deck overhangs and by major changes in the pier form. The use of cast-in-place cantilever sections offers increased possibilities for elegant forms and closer integration of superstructure with piers.

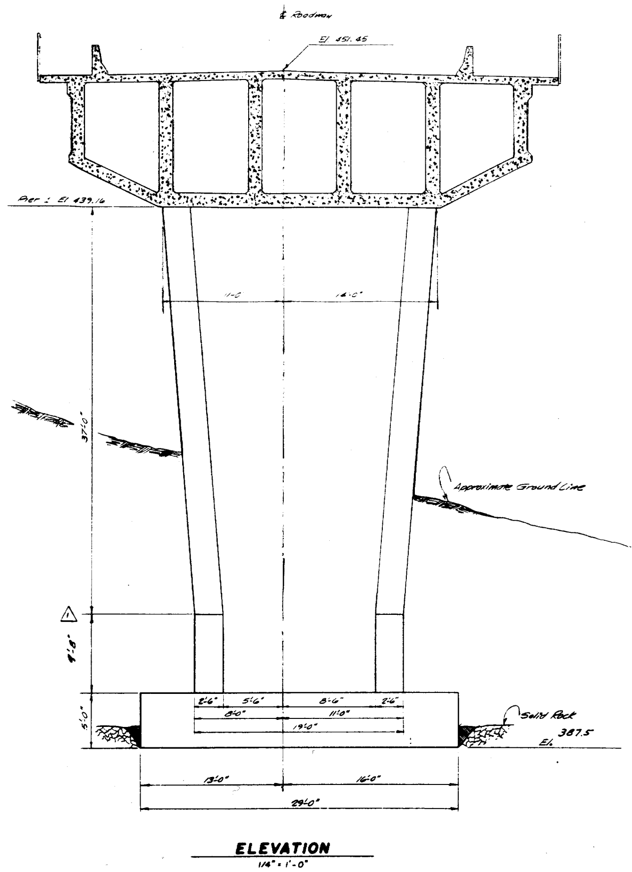


**FIGURE 3.14** Typical section of Stewart Park Bridge at pier.

This case study gives an example of how a good bridge can provide an excellent basis for further study and improvement.

## 3.8 Achieving Structural Art in Modern Society: Computer Analysis and Design Competitions

Most people would agree that the ideals of structural art coincide with those of an urban society: conservation of natural resources, minimization of public expenditures, and the creation of a more visually appealing environment. As the history of structural art shows, some engineers have already turned these ideals into realities. But these are isolated cases. How might they become the rule instead of the exception? We can address this question historically, by identifying the central ideas that have been associated with great structural art. These ideas reflect each of the three dimensions: the scientific, social, and symbolic.

The leading scientific idea might be stated as that of reducing analysis. In structural art, this idea has coexisted with the opposite tendency to overemphasize analysis, which today is typified by the heavy use of the computer for structural calculations. One striking example comes from the design of thin concrete vaults — thin shell roofs. Here, the major advances between 1955 and 1980 — a time of intense analytic developments — were achieved, not by performing complex analyses using computers, but rather by reducing analysis to very simple ideas based on observed physical behavior. Roof vaults characterize this advance and they carry forward the central scientific idea in structural art: the analyst of the form, being also the creator of the form, is free to change shapes so that analytic complexity disappears.

The form controls the forces and the more clearly that designers can visualize those forces the surer they are of their forms. The great early and mid-20th-century structural artists such as Robert Maillart and Pier Luigi Nervi have all written forcefully against the urge to complicate analysis. We see the same arguments put forth by the best designers in the late 20th century. When the form is well chosen, its analysis becomes astoundingly simple. The computer, of course, has become more and more useful as a time saver for routine calculations that come after the design is set. It is also increasingly valuable in aiding the designer through computer graphics. But like any machine, although it can reduce human labor, it cannot substitute for human creativity.

Turning to the social dimension, a leading idea that has come out of structural art is the effectiveness of public design competitions. Design quality arises from the stimulus of competing designs for the same project rather than from complex regulations imposed upon a single designer. The progress of modern bridge design illustrates the benefit and meaning of alternative designs. Many alternative designs have been prepared pursuant to design competitions, which bring the public into the process in a positive way. It is not enough for the public merely to protest the building of ugly, expensive designs. A positive activity is essential, and that can only come about when the public sees the alternative designs that are possible for a project. Thus, governments can ensure better designs by relinquishing some of their control over who designs and on what forms are chosen, and by giving some of this control to an informed jury which includes representatives of the lay public.

Although there is little tradition in the United States for design competitions in bridges, such a tradition is firmly rooted elsewhere, with results that are both politically and aesthetically spectacular. Switzerland has the longest and most intensive tradition of bridge design competitions, and it is no coincidence that, by nearly common consent, the two greatest bridge designers of the first half of the 20th century were Swiss: Robert Maillart (1872–1940), who designed in concrete, and Othmar Amman (1879–1966), designer of the George Washington and Verrazano Bridges, who designed in steel. That Switzerland, one sixth the size of Colorado, and with fewer people than New York City, could achieve such world prominence is due to the centrality of economics and aesthetics for both their engineering teachers and their practicing designers, a centrality which is encouraged by design competitions.

Maillart's concrete arches in Switzerland were often the least expensive proposals in design competitions, and they were later to provide the main focus for the first art museum exhibition ever devoted exclusively to the work of one engineer: the New York City Museum of Modern Art's 1947 exhibition on Maillart's structures. Amman has been similarly honored. His centennial was celebrated by symposia both in Boston and in New York and by an exhibition held in Switzerland. Both Maillart and Amman wrote articulately on the appearance as well as on the economy of bridges. They are prime examples of structural artists.

This Swiss bridge tradition continues today with a large number of striking new bridges in concrete that follow Maillart in principle if not in imitative detail. The most impressive post-World War II works are those of Christian Menn, whose long-span arches and cantilevers extend the new technique of prestressing to its limits, as Maillart's three-hinged and deck-stiffened arches did earlier with reinforced concrete.

Design competitions stimulated these engineers and also educated the general public. To be effective, uch competitions must be accepted by political authorities, judged by engineers and informed lay members whose opinions will be debated in the public press, and controlled by carefully drawn rules.

It is false images of engineering that keep us from insisting on following our normal instinct for open competition. The American politics of public works falsely compares the engineering designer either with a medical doctor or with a building contractor.

Supporters of the first comparison argue that you would never hold a competition to decide who will repair your heart; rather, you would choose professionals on the basis of reputation and then leave them alone to do the skilled work for which they are trained. However, there is a key difference between hearts and bridges. For most people, there is only one heart which will do the job. Picking a "best" heart is not a consideration. On the other hand, for a given bridge site, there are many bridge designs that will solve the problem. The more minds that are put to the problem, the more likely that an outstanding design will emerge. After all, the ultimate goal is to pick the best bridge, not the best bridge designer.

Furthermore, developing the engineer's imagination creates a valuable asset for society. That imagination needs more chances to exercise than there are chances to build, and it is stimulated by competition. However, frustrating it may be to lose a competition, the activity is healthy and maturing, especially when even the losers are compensated financially for their time, as they often are in Switzerland.

For proponents of the second false comparison, design competitions are to be run just as building competitions in which the lowest bid for design cost gets the design contract. In American public structures, design and construction are legally distinct activities. The cost of design is normally 5% or less of the cost of construction. Therefore, a brilliant engineer might spend more preparing a design which, as can often happen, will cost the owners substantially less overall. By the same token, an engineer who cuts the design fee to get the job may have to make a more conservative design which could easily cost the owner more in overall costs. Hence, large amounts of potential savings to the public are lost by a foolish policy of saving a little during the first stage of a project.

In one type of Swiss design competition, a small number of designers are invited to compete, some of their costs are covered, and they get additional prize funds in the order recommended by the jury. The winner usually gets the commission for the detailed design. Only several such competitions a year are needed to stimulate the entire profession and to show the general public the numerous possibilities available as good solutions to any one problem. This method of design award opens up the political process to local people far more than does the cumbersome and largely negative one of protest, legal action, and negation of building that so dominates public action in late-20th-century America.

The state of Maryland is leading the way in the United States. In 1988, Maryland held a design competition for a new structure over the Severn River adjoining the U.S. Naval Academy in Annapolis. The competition was patterned on the Swiss practice. The results of the competition resolved an acrimonious community controversy. The winning structure, by Thomas Jenkins (Figure 3.15), was recognized by the American Institute of Steel Construction as the outstanding medium-span structure constructed in 1995–96. In 1998, Maryland, together with the state of Virginia, the District of Columbia, and the Federal Highway Administration, conducted a competition to select the design of the new Woodrow Wilson Bridge over the Potomac River at Washington, D.C. The winning design (Figure 3.16) was prepared by a team led by the Parsons Transportation Group.

Properly defined design competitions reveal truths about society that are otherwise difficult to define. The resulting designs, therefore, became unique symbols of their time and place. This brings us to the third leading idea that has been associated with great structural art — the idea that its materials and forms possess a particular symbolic significance. Perceptive painters, poets, and writers have recognized in structural art a new type of symbol — first in metal and then in concrete — which fits mysteriously closely both to the engineering possibilities and to the possibilities inherent in democracy. The thinness and openness of the Eiffel Tower, Brooklyn Bridge, and Maillart's arches, as well as the stark contrast between their forms and their surroundings, have a deep affinity to both the political traditions and era in which they arose. They symbolize the artificial rather than the natural, the democratic rather than the autocratic, and the transparent rather than the impenetrable. Their forms reflect directly the inner springs of creativity emerging from contemporary industrial societies.

**FIGURE 3.15** Thomas Jenkins's U.S. Naval Academy Bridge over the Severn River.

These forms imply a democratic rather than an autocratic life. When structure and form are one, the result is a lightness, even fragility, which closely parallels the essence of a free and open society. The workings of a democratic government are transparent, conducted in full public view, and although a democracy may be far from perfect, its form and its actual workings (its structure) are inseparable. Furthermore, the public must continually inspect its handiwork: constant maintenance and periodic renewal are essential to its exposed structure. Politicians do not have life tenure; they must be inspected, chastised, and purified from time to time, and replaced when found corrupt or inept. So it is with the works of structural art. They, too, are subject to the weathering and fatigue of open use. They remind us that our institutions belong to us and not to some elite. If we let them deteriorate, as we flagrantly have in our older cities and transportation networks, then that outward sign betokens an inner corruption of the common life in a free democratic society.

## 3.9 The Engineer's Goal

The ideal bridge is structurally straightforward and elegant. It should provide safe passage and visual delight for drivers, pedestrians, and people living or working nearby. Society holds engineers responsible for the quality of their work, including its appearance. For the same reason engineers would not build a bridge that is unsafe, they should not build one that is ugly. Bridge designers must consider visual quality as fundamental a criterion in their work as performance, cost, and safety.

There are no fast rules or generic formulas conducive to outstanding visual quality in bridge design. Each bridge is unique and should be studied individually, always taking into consideration all the issues, constraints, and opportunities of its particular setting or environment. Nevertheless, by observing other bridges, using case studies and design guidelines, engineers can learn what makes bridges visually outstanding and develop their abilities to make their own bridges attractive. They can achieve outstanding visual quality in bridge design while maintaining structural integrity and meeting their budgets.

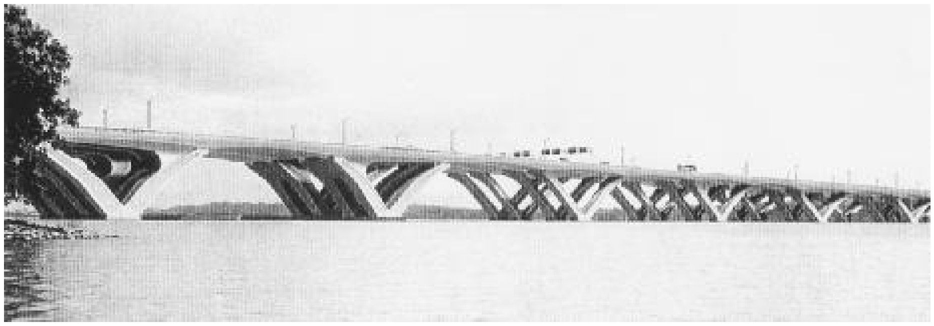

**FIGURE 3.16** The competition-winning design for the new Woodrow Wilson Bridge over the Potomac River at Washington, D.C.

## References


1. Schuyler, M., The bridge as a monument, *Harpers Weekly* 27, 326, 1883; reprinted in *American Architecture and Other Writings,* W. H. Jordy and R. Coe, Eds., Atheneum, New York, 1964, 164.
2. Eads, J.B., Report of the Engineer-in-Chief of the Illinois and St. Louis Bridge Company, St. Louis, Missouri Democrat Book and Job Printing House, 1868. Reprinted in *Engineers of Dreams,* H. Petroski, Alfred A. Knopf, New York, 1995, 54.
3. Torroja, E., *The Structures of Eduardo Torroja, an Autobiography of Engineering Accomplishment,* F.W. Dodge, New York, 1958, 7.


## Bibliography

Much of the discussion in this chapter is contained in a more complete form in the following four books by the authors:


David P. Billington, *Robert Maillart and the Art of Reinforced Concrete,* MIT Press, Cambridge, MA, 1990.

David P. Billington, *Robert Maillart's Bridges — The Art of Engineering,* Princeton University Press, Princeton, NJ, 1979.

David P. Billington, *The Tower and the Bridge — The New Art of Structural Engineering,* Basic Books, New York, 1983.

Frederick Gottemoeller, *Bridgescape, the Art of Designing Bridges,* John Wiley & Sons, New York, 1998.


The following should be part of the reference library of any engineer interested in bridge aesthetics (not in any particular order):


Fritz Leonhardt, *Brucken,* MIT Press, Cambridge, MA, 1982.

Max Bill, *Robert Maillart, Bridges & Construction,* 1st ed. 1949, Praeger, Westport, CT, 1969.

Steward C. Watson and M. K. Hurd, *Esthetics in Concrete Bridge Design,* American Concrete Institute, Detroit, 1990.

Hans Jochen Oster et al., *Fussgangerbrucken, Jorg Schlaich und Rudolph Bergermann Katalog zur Ausstellung an der ETH Zurich,* Institut für Baustatik und Konstruktion, Zürich, Switzerland, 1994.

Martin P. Burke, Jr. and the General Structures Committee, *Bridge Aesthetics around the World,* Transportation Research Board, Washington, D.C., 1991.

Ostenfeld, K.H., Hommel, D.L., Olsen, D., Hauge, L. "Planning of Major Fixed Links."
*Bridge Engineering Handbook.*
Ed. Wai-Fah Chen and Lian Duan
Boca Raton: CRC Press, 2000

# 4

# Planning of Major Fixed Links



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## 4.1 Introduction

### Characteristics of Fixed Links

Within the infrastructure of land transportation, fixed links are defined as permanent structures across large stretches of water allowing for uninterrupted passage of highway and/or railway traffic with adequate safety, efficiency, and comfort.

Traffic services are often provided by ferries before a fixed link is established. Normally, a fixed link offers shorter traveling times and higher traffic capacities than the ferry services. The establishment of a fixed link may therefore have a strong positive impact on the industrial and economic development of the areas to be served by the link. This together with an increased reliability in connection with climatic conditions are the major reasons for considering the implementation of a fixed link.

The waters to be passed by the links are often navigable; the link structures may present obstacles to the vessel traffic and are thus subject to the risk of impact from vessels. If the vessel traffic is important, the link traffic may be better separated from the crossing vessel traffic for general traffic safety. The water flow is often influenced by the link structures and this may affect the environment both near and far from the site. Furthermore, the water stretches and areas to be passed are part of beautiful territories forming important habitats for wildlife fauna and flora. The protection and preservation of the environment will therefore often be a major issue in the political discussions prior to the establishment of the links. These aspects have to be realized and considered in the very beginning of the planning process.

**TABLE 4.1** Major Fixed Links Opened Since 1988

| Name of Link | Total Length and Types of Structures | Status Early 1998 | Traffic Mode |
|---|---|---|---|
| Confederation Bridge, Canada | 12.9 km, high-level concrete box girder bridge | Open to traffic 1997 | Highway traffic |
| Vasco da Gama Bridge, Portugal | 12.3 km, viaducts and high-level cable-stayed bridge | Open to traffic 1998 | Highway traffic |
| Second Severn Bridge, Great Britain | 5.1 km, viaducts and high-level cable-stayed bridge | Open to traffic 1996 | Highway traffic |
| Honshu–Shikoku Connection, Japan | | | |
| • Kojima–Sakaide Route | 37.3 km, a o high-level suspension bridges | Open to traffic 1988 | Highway and railway |
| • Kobe–Naruto Route | 89.6 km, a o high-level suspension bridges | South part open 1998 | Highway traffic |
| • Onomichi–Imabari Route | 59.4 km, a o high-level suspension bridges | Under construction | Highway traffic |
| Lantau Fixed Crossing, Hong Kong | 3.4 km<br>High-level suspension bridge | Open to traffic 1997 | Highway and railway |
| • Tsing Ma Bridge<br>• Kap Shui Mun Bridge | High level cable stayed bridge | Open to traffic 1997 | Highway and railway |
| Boca Tigris Bridge, China | 4.6 km, high-level suspension bridge | Open to traffic 1997 | Highway traffic |
| Great Belt link, Denmark | 17.5 km | | |
| • West Bridge | 6.6 km, low-level concrete box girder bridge | Open to traffic 1997 | Highway and railway |
| • East Bridge | 6.8 km, high-level suspension bridge | Open to traffic 1998 | Highway traffic |
| • East Railway Tunnel | 8.0 km, bored tunnel, two tubes | Open to traffic 1997 | Railway traffic |
| Øresund link, Sweden–Denmark | 16 km, immersed tunnel, artificial island, high-level cable-stayed bridge, viaducts | Under construction | Highway and railway |
| Rion–Antirion Bridge, Golf of Corinthe, Greece | 2.9 km, high-level cable-stayed bridge, viaducts | Construction started in 1998 | Highway traffic |
| Channel Tunnel, Great Britain–France | 50.5 km, bored tunnel, three tubes | Open to traffic 1994 | Railway with car and lorry shuttle |
| Trans-Tokyo Bay Crossing, Japan | 15.1 km, bored tunnel, artificial islands, high- and low-level steel box girder bridges | Open to traffic 1997 | Highway traffic |

Generally, the term *fixed link* is associated with highway and/or railway sections of considerable length and a fixed link may comprise a combination of different civil engineering structures such as tunnels, artificial islands, causeways, and different types of bridges. Selected examples of major fixed links opened or are under construction since 1988 are listed in Table 4.1.

## Planning Activities for Major Fixed Links

Major fixed links represent important investments for the society and may have considerable influence on the development potential of the areas they serve.

The political discussions about the decision to design a fixed link may be extended over decades or even centuries. In this period planning activities on a society level are necessary to demonstrate the need for the fixed link and to determine positive and negative effects of the implementation. These early planning considerations are outside the scope of this chapter, but the outcome of the early planning activities may highly influence the tasks in the later planning phases after the final decision is made.

In the early planning phases, basic principles and criteria are dealt with, such as

- Ownership and financing
- Approximate location
- Expected service lifetime
- Necessary traffic capacity
- Considerations for other forms of traffic like vessel traffic and air traffic
- Principles for environmental evaluation
- Risk policy
- International conventions

Section 4.2 will explain the later planning phases by describing major steps in project development with emphasis on the consideration of all relevant aspects. The focus will be on the technical and civil engineering aspects of bridges as fixed links, but most of the methods and principles described can be applied to other types of link structures. In the case of complex fixed link arrangements (comprising more than one type of structure), some of these structures may be alternative solutions. Several combined solutions are therefore studied and for each combination it is normally necessary to perform the planning for the entire link as a whole.

The elements of the project basis for a major fixed link are further detailed in Section 4.3, and examples of major fixed links recently built, under construction, or in the planning stage in the Scandinavian area are described in Section 4.4.

The chapter does not treat aesthetic and environmental issues individually, but assumes that all alternatives are evaluated according to the same principles. Public approval processes are beyond the aim of this chapter; readers are referred to References [1–3].

Fixed links are unique in size and cost, and the political environment differs from project to project. It is thus not possible to provide a recipe for planning major fixed links. The present chapter describes some of the elements, which the authors believe are important in the complex, multidisciplinary planning process of all fixed links.

Many important fixed links still remain to be planned and built. One of the more spectacular ones is the Gibraltar link between Africa and Europe. Figure 4.1 shows an artist's impression of the bridge pylons for the planned Gibraltar link. Examples of other future links are the Messina Strait crossing in Italy, the Mallaca Strait crossing between Malaysia and Indonesia, and the Río de la Plata Bridge connecting Argentina and Uruguay.

## 4.2 Project Development

### 4.2.1 Initial Studies

The first step in project development consists of a review of all information relevant to the link and includes an investigation of the most likely and feasible technical solutions for the structures.

The transportation mode, highway and railway traffic, and the amount of traffic is determined based on a traffic estimate. The prognosis of traffic is often associated with considerable uncertainty since fixed links will not only satisfy the existing demands but may also create new demands due to the increased quality of the transport. For railway traffic, it has to be decided whether a railway line will accommodate one or two tracks. Similarly, the highway traffic can either be transported on shuttle trains or the bridge can be accommodated with a carriageway designed to a variety of standards, the main characteristics being the number of lanes.

The decision on the expected traffic demands and the associated traffic solution models is often based on a mix of technical, economic, socioeconomic, and political parameters. The decision may be confirmed at later stages of the planning when more information is available.

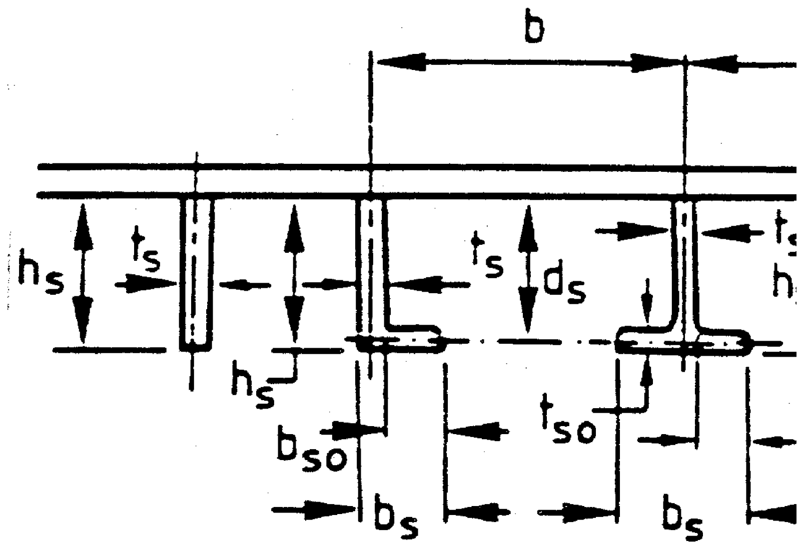


**FIGURE 4.1** Artist's impression of 465-m-high pylon on 300 m water for planned Gibraltar link with 3,500 m spans. (Courtesy of Dissing +Weitling, Architects, Denmark.)

A *fixed link concept study* will review alignment possibilities and define an appropriate corridor for further studies. It will consider the onshore interchanges for the anticipated traffic modes and identify potential conflict areas. It describes all feasible arrangements for the structures from coast to coast, and reviews the requirement for special onshore structures. Finally, the study defines the concepts to be investigated in greater depth in subsequent phases.

An *environmental condition study* aims to identify potential effects the structures may have on the environment and to review the legal environmental framework. It also identifies important conflict areas and describes the project study area. It will review the available information on the marine and onshore environment and define the need for additional investigations.

A *technical site condition study* will address the geological, the foundation, the navigation, the climatic, and the hydraulic conditions. It will review the topographic situation and define additional studies or investigations for the following project phase.

A *preliminary design basis study* will review the statutory requirements, codes, and standards and identify the need for relevant safety and durability requirements.

Finally, a *preliminary costing basis study* will define the cost estimation technique and provide first preliminary cost estimates.

Considering the results of these studies a comprehensive investigation program for the next project step — the conceptual study — will be defined.

### 4.2.2 Conceptual Study

The conceptual study is an iterative process, where all the aspects likely to influence the project should be considered, weighted, and clarified to achieve the most suitable solution for the intended purpose and location. These aspects are cost, construction, structural, navigational, environmental, aesthetic, risk, geological, vessel collision, wind, and earthquake.

After an interim selection of various alternatives, conceptual studies are undertaken for each selected alternative solution. Preliminary site investigations like subsoil investigations in the defined alignment corridor, wind, earthquake, and vessel traffic investigations will be carried out simultaneously with the conceptual studies.

The conceptual study comprises development of a project basis, including

- Defining functional requirements
- Reviewing and defining the navigational aspects
- Establishing risk policy and procedures for risk management
- Specifying design basis including structure-specific requirements
- Developing the costing basis

Each of the selected solutions will be developed in a conceptual design and described through drawings and descriptions. The conceptual design will comply with the project basis and further consider:

- Preliminary site investigations
- Structural aspects
- Architectural aspects
- Environmental aspects
- Mechanical and electrical installations and utilities
- Definitions and constraints for operation and maintenance
- Cost aspects
- Major construction stages

Practically, it is not possible to satisfy all the above requirements, but effort should be made to achieve a balanced solution. The conceptual study phase is concluded by a comparison analysis with predeterminant weighting of parameters, which provides the technical ranking of all alternative solutions.

### 4.2.3 Project Selection and Procurement Strategy

#### Project Selection

By using the results from the technical ranking of the solutions, the basis for a project selection has to be established by the owner organization. This requires information from other investigations carried out in parallel with the technical studies that cover:

- Environmental impact assessment including hydraulic studies;
- Traffic demand studies including possible tariff structures;
- Layout, cost, and requirements for connections to the existing network outside the study areas;
- Definition of the project implementation and the tendering procedure.

The information obtained from the above studies may be used as input into a cost–benefit model of the anticipated solutions. These final results usually provide the basis to make a decision that will best consider local and global political viewpoints. Public hearings may be necessary in addition to the investigations. The result of this process is the selection of the solution of choice.

#### Procurement Strategy

The optimum procurement method should ensure that the work and activities are distributed on and executed by the most qualified party (owner, consultant, contractor) at all phases, to meet the required quality level, at the lowest overall cost. The procurement strategy should clarify tendering procedures with commercial and legal regulations for the region. In the following, three main contracting concepts in the definition of the procurement strategy for a fixed link project are presented.

### Contracting Concepts

The three main concepts are as follows:

- *Separate Design and Construction* (*SDC*) is a concept in which the construction contract documents are prepared by the owner, often assisted by an engineering consulting team, and the construction is performed by a contractor.
- *Design-Build* (*DB*) is a concept in which both engineering design and construction responsibilities are assigned to a single entity, most often the contractor.
- *Design-Built-Operate and Transfer* (*BOT*) is a concept in which the financing, design, construction, and operation are assigned to a concessionaire. After an agreed number of years of operation, the link is transferred to the owner. The BOT is not described further because it uses the same design and construction procedure as the DB.
- The *SDC* concept requires that the owner and the consultant participate actively during all phases to influence and control the quality and performance capability of the completed facility. The main differences between the various forms of the SDC are the degree of detailing at the tender stage and whether alternatives will be permitted. Completing the detailed design prior to inviting tenders is good if the strategy is to obtain lump-sum bids in full compliance with the owner's conditions.

Tendering based on a partial design — often 60 to 70% — represents a compromise between initial design costs and definitions of the owner's requirements to serve as a reference for alternative tenders. This procurement strategy has been applied for large construction works from the 1970s. Advantages are that the early start of the construction work can be achieved while completing the design work and that innovative ideas may be developed between the owner, contractor and the consultant, and incorporated in the design. The procedure usually allows contractors to submit alternatives in which case the tender design serves the important purpose of outlining the required quality standards. A disadvantage, however, is the risk for later claims due to the fact that the final design is made after awarding the construction contract. The more aggressive contracting environment and the development of international tender rules have made it desirable to procure on a completely fixed basis.

The *DB* concept assigns a high degree of autonomy to the contractor, and, as a consequence, the owner's's direct influence on the quality and performance of the completed facility is reduced. To ensure that the contractor delivers a project that meets the expectations of the owner, it is necessary to specify these in the tender documents. Aesthetic, functional, maintenance, durability, and other technical standards and requirements should be defined. Also legal, environmental, financial, time, interface, and other more or less transparent constraints to the contractor's freedom of performance should be described in the tender documents in order to ensure comparable solutions and prices. Substantial requirements to the contractor's quality assurance system are essential in combination with close follow-up by the owner.

Tenders for major bridges may be difficult to evaluate if they are based on substandard and marginal designs or on radical and unusual designs. The owner then has the dilemma of either rejecting a low tender or accepting it and paying high additional costs for subsequent upgrading.

### Contract Packaging

The total bridge project can be divided into reasonable contract parts:

- Vertical separation (e.g., main bridge, approach bridges, viaducts, and interchanges);
- Horizontal separation (e.g., substructure and superstructure);
- Disciplinary separation (e.g., concrete and steel works).

The application of these general principles depends on the specific situation of each project. Furthermore, the achievement of the intended quality level, together with contract sizes allowing for competitive bidding, should be considered in the final choice. Definition and control of interfaces between the different contractors is an important task for the owner's organization.

### 4.2.4 Tender Design

The main purpose of a tender design or a bid design is to describe the complexity of the structure and to determine the construction quantities, allowing the contractors to prepare a bid for the construction work. The goal for a tender design is as low cost as possible within the given framework. This is normally identical to the lowest quantities and/or the most suitable method. It is essential that the project basis be updated and completed prior to the commencement of the tender design. This will minimize the risk of contract disputes.

It is vital that a common understanding between consultant and owner is achieved. Assumptions regarding the physical conditions of the site are important, especially subsoil, wind, and earthquake conditions. Awareness that these factors might have a significant impact on the design and thereby on the quantities and complexity is important. The subsoil conditions for the most important structures should always be determined prior to the tender design to minimize the uncertainty.

Determination of the quantities is also necessary. For instance, if splice lengths in the reinforcement are included, if holes or cutouts in the structure are included, what material strengths are assumed. There must be stipulated an estimate of the expected variation of quantities (global or local quantities).

The structures in the tender design shall be constructible. In an SDC contract, the tender design should be based on safe and well-established production and erection procedures. In the case of DB, the tender design is carried out in close cooperation between the, contractor and the consultant. This assures that the design accommodates the contractor's methods and the available equipment.

The tender design is often carried out within a short period of time. It should focus on elements with large cost impact and on elements with large uncertainties in order to arrive as closely as possible at the actual quantities and to describe the complexity of the structure efficiently from a costing point of view. A tender design comprises layout drawings of the main structural elements, detailed drawings of typical details with a high degree of repetition, typical reinforcement arrangement, and material distribution.

Aesthetics are normally treated during the tender design. It is important that extreme event loads such as vessel collision, train derailment, cable rupture, earthquake, and ice impact should be considered in the tender design phase as they often govern the design. Durability, operation, and maintenance aspects should be considered in the tender design. Experience from operation and maintenance of similar bridges allows a proper service life design to be carried out. It is at the early design stages that the construction methods should be chosen which have a significant effect on further operation and maintenance costs.

It is not unusual that a tender design is prepared for more than one solution to obtain the optimal solution. It could, for instance, be two solutions with different materials (concrete and steel) as for the Storebælt East Bridge. It could also be two solutions with traffic arranged differently (one level or two levels) as for the Øresund link. Different structural layouts as cable stayed and suspension bridge could be relevant to investigate under certain conditions. After the designs are prepared to a certain level, a selection can be carried out based on a preliminary pricing, and one or more solutions are brought all the way to tender.

Tender documents to follow the drawings should be prepared. The tender documents comprise bill of quantities, special specifications, and the like.

### 4.2.5 Tender Evaluation

The objective of tender evaluation is to select the overall most advantageous tender including capitalized owner's risk and cost for operation and maintenance. A basis should be established via a rating system where all tenders become directly comparable. The rating system is predefined by the owner, and should be part of the tender documents.

The tender evaluation activities can be split up in phases:

1. Preparation
2. Compilation and checking of tenders
3. Evaluation of tenders
4. Preparation for contract negotiations
5. Negotiation and award of contract

The *preparation* phase covers activities up to the receipt of tenders. The main activities are as follows:

- Establish the owner's risk for each of the tendered projects, using the owner's cost estimate;
- Define tender opening procedures and tender opening committee;
- Quantify the differences in present value due to function, operation, maintenance, and owner's risk for each of the tendered projects, using the owner's cost estimate.

After receipt of tenders, a summary report, which collects the information supplied in the different tenders into a single summarizing document and presents a recommendation of tenders for detailed review, as a result of *compilation and checking of tenders,* should be prepared. Typical activities are as follows:

- Check completeness of compliance of all tenders, including arithmetical correctness and errors or omissions;
- Identify possible qualifications and reservations;
- Identify parts of tenders where clarification is needed, or more detailed examination required;
- Prepare a preliminary list of questions for clarification by the tenderers;
- Review compliance with requirements for alternative designs;
- Upgrade alternative tender design and pricing to the design basis requirements for tender design.

The *evaluation of tenders* comprises the following:

- Provide initial questionnaires for tender clarification to tenderers, arrange clarification meetings, and request tenderers' written clarification answers;
- Adjust tender prices to a comparable basis taking account of revised quantities due to modified tender design effects of combined tenders, alternatives, options, reservations, and differences in present value;
- Appraise the financial components of the tenders;
- Assess owner's risk;
- Review technical issues of alternatives and their effect on interfaces;
- Review the proposed tender time schedule;
- Evaluate proposed subcontractors, suppliers, consultants, testing institutes, etc.
- Review method statements and similar information;
- Establish list of total project cost.

The assessment of owner's risk concerns exceeding budgets and time limits. An evaluation of the split of financial consequences between contractor and owner should be carried out.

*Preparation for contract negotiations* should be performed, allowing all aspects for the actual project type to be taken into account. Typical activities are as follows:

- Modify tender design to take current status of the project development into account to establish an accurate contract basis;
- Modify tender design to accommodate alternatives;
- Coordinate with the third parties regarding contractual interfaces;
- Coordinate with interfacing authorities;
- Establish strategies and recommendations for contract negotiations.

The probable extent and nature of the negotiations will become apparent from the tender evaluation. Typical activities during *negotiations and award of contract* are as follows:

- Prepare draft contract documents;
- Clarify technical, financial, and legal matters;
- Finalize contract documents.

### 4.2.6 Detailed Design

The detailed design is either carried out before (SDC contracts) or after signing of the construction contact (DB contracts). In the case the detailed design is carried out in parallel with the construction work, the completion of the detailed design should be planned and coordinated with the execution. A detailed planning of the design work is required when the parts of the structure, typically the foundation structures, need to be designed and constructed before the completion of the design of the entire structure. Design of temporary works is normally conducted in-house by the contractor, whereas the design of the permanent works is carried out by the consultant.

The purpose of the detailed design is to prepare drawings for construction in accordance with various requirements and specifications. Detailed design drawings define all measures and material qualities for the structure. Shop drawings for steel works are generally prepared by the steel fabricator. Detailed reinforcement arrangements and bar schedules are either prepared by the contractor or the consultant. It is important that the consultant prescribes the tolerance requirements of the design.

The detailed design should consider the serviceability limit state (deflection and comfort), the ultimate limit state (strength and stability), and the extreme event limit state (collapse of the structure). To ensure the adequacy of the design, substantial analyses, including three-dimensional global finite-element analyses, local finite-element analyses, and nonlinear analyses both in geometry and materials, should be carried out. Dynamic calculations, typically response spectrum analyses, are usually performed to determine the response from wind. The dynamic amplifications of traffic loads and cable rupture are determined by a time-history analysis, which is also frequently used for vessel collision and earthquake analyses.

For large cable-supported bridges, wind tunnel testing is conducted as part of the detailed design. Preliminary wind tunnel testing is often carried out in the tender design phase to investigate the aerodynamic stability of the structure. Other tests, such as scour protection and fatigue tests can be carried out to ensure design satisfactions. Detailed subsoil investigations for all foundation locations are carried out prior to, or in parallel with, the detailed design.

The operation and maintenance (O&M) objectives should be implemented in the detailed design in a way which:

- Gives an overall cost-effective operation and maintenance;
- Causes a minimum of traffic restrictions due to O&M works;
- Provides optimal personnel safety;
- Protects the environment;
- Allows for an easy documentation of maintenance needs and results.

In addition, the contractor should provide a forecast schedule for the replacement of major equipment during the lifetime of the bridge.

### 4.2.7 Follow-Up during Construction

During the construction period the consultant monitors the construction work to verify that it is performed in accordance with the intentions of the design. This design follow-up, or general supervision, is an activity which is carried out in cooperation with (and within the framework of) the owner's supervision organization.

The general supervision activities include review of the contractor's quality assurance manuals, method statements, work procedures, work instructions, and design of temporary structures, as well as proper inspections on the construction site during important construction activities. The quality of workmanship and materials is verified by spot-checking the contractor's quality control documentation.

When the work results in mistakes or nonconformances, the general supervision team evaluates the contractor's proposals for rectification or evaluates whether or not the structural element in question can be used as built, without any modifications. The general supervision team also evaluates proposals for changes to the design submitted by the contractor and issues recommendations on approval of such proposals.

The duties of the general supervision team also include preparation of technical supervision plans, which are manuals used by the supervision organization as a basis for the technical supervision of the construction work. These manuals should be based on inputs from the consultants and experienced engineers to avoid mistakes during the construction work.

The general supervision team monitors the performance of the supervision organization and receives feedback on experience gained by the supervision organization, as in some cases it may be found necessary and advantageous to adjust the design of the project to suit the contractor's actual performance.

The general supervision team provides advice on the necessity for expert assistance, special testing of materials, and special investigations. The general supervision team evaluates the results of such activities and issues recommendations to the owner. Special testing institutes are often involved in the third-party controls which normally are performed as spot checks only. Examples are nondestructive testing of welds, mechanical and chemical analyses of steel materials, and testing of concrete constituents such as cement, aggregates, and admixtures at official laboratories.

The general supervision team assists the supervision organization with the final inspection of the works prior to the contractor's handing over of the works. The general supervision team assists the consultant with the preparation of operation and maintenance manuals and procedures for inspections and maintenance during the operation phase. Some of these instructions are based on detailed manuals prepared by the contractor's suppliers. This can apply to bearings, expansion joints, electrical installations, or special equipment such as dehumidification systems or buffers. Preparation of these manuals by the suppliers is part of their contractual obligations, and the manuals should be prepared in the required language of the country where the project is situated.

## 4.3 Project Basis

### 4.3.1 Introduction

The project basis is all the information and requirements that are decisive in the planning and design of a fixed link. The project basis is developed simultaneously with the early design activities, and it is important to have the owner's main requirements defined as early as possible, and to be precise about what types of link solutions are to be included.

## 4.3.2 Geometric Requirements

Most geometric requirements for the fixed links stem from the operational requirements of traffic and all the important installations. However, geometric requirements may also be necessary to mitigate accidents and to provide the needed space for safety and emergency situations. Geometric considerations should be addressed in the risk analyses.

## 4.3.3 Structural Requirements

### Design Basis

A main purpose of a design basis is to provide a set of requirements to ensure an adequate structural layout, safety, and performance of the load-bearing structures and installations for the intended use.

### Structural Design Codes

The structures must resist load effects from self-weight and a variety of external loads and environmental phenomena (climate and degradation effects). To obtain an adequately uniform level of structural safety, the statistical nature of the generating phenomena as well as the structural capacity should be considered. A rational approach is to adapt probabilistic methods, but these are generally inefficient for standard design situations, and consequently it is recommended that a format as used in codes of practice be applied. These codes are calibrated to achieve a uniform level of structural safety for ordinary loading situations, and probabilistic methods can subsequently be used to calibrate the safety factors for loads and/or design situations that are not covered by the codes of practice.

The safety level — expressed as formal probability of failure or exceeding of limit state — is of the order $10^{-6}$ to $10^{-7}$/year for ultimate limit states for important structures in major links.

## 4.3.4 Environmental Requirements

Fixed links crossing environmentally sensitive water stretches need to be developed with due attention to environmental requirements. Environmental strategies should be directed toward modification of the structural design to reduce any impact and to consider compensation or mitigation for unavoidable impacts. Guidelines for environmental considerations in the structural layout and detailing and in the construction planning are developed by a consultant, and these should typically address the following areas:

- Geometry of structures affecting the hydraulic situation;
- Space occupied by bridge structures, ramps and depot areas;
- Amount and character of excavated soils;
- Amount of external resources (raw materials winning);
- Methodology of earth works (dredging and related spill).

Consequences of the environmental requirements should be considered in the various project phases. Typical examples for possible improvements are selection of spans as large as possible or reasonable, shaping of the underwater part of foundations to reduce their blocking effect, orientation of structures parallel to the prevailing current direction, minimizing and streamlining of protection structures, reduction of embankment length, optimal layout of depot areas close to the shorelines, and reuse of excavated material.

The process should be started at the very early planning stages and continued until the link is completed and the impact on the environment should be monitored and assessed.

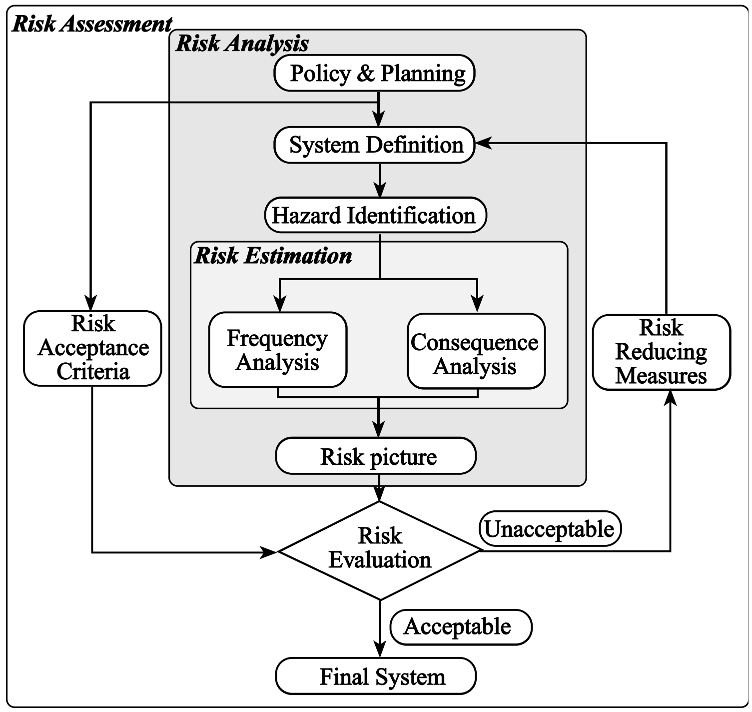


**FIGURE 4.2** Risk management components.

### 4.3.5 Risk Requirements

#### Types of Risk

Risk studies and risk management have gained a widespread application within the planning, design, and construction of fixed links. Risks are inherent in major transportation links, and therefore it is important for the owner and society that risks are identified and included in the project basis together with the technical and economic aspects. Risks are often studied separately according to the consequences of concern:

- Economic risk (rate of interest, inflation, exceeding of budget, changing traffic patterns);
- Operational risk (accidents, loss of lives, impact to environment, disruption of the traffic, loss of assets, loss of income);
- Construction risks (failure to meet time schedule or quality standards, unexpected ground conditions, accidents).

Economic risks in the project may be important for decisions on whether to initiate the project at all. The construction risk may have important implications on the selection of the structural concept and construction methods.

#### Risk Management Framework

The main risk management components are shown in Figure 4.2. The risk policy is formulated by the owner in few words: "The safety of the transportation link must be comparable with the safety for the same length of similar traffic on land."

The risk acceptance criteria are an engineering formulation of the risk policy in terms of upper limits of risk. The risk policy also specifies the types of risk selected to be considered, typically user fatalities and financial loss. In some cases, other risks are specifically studied, e.g., risk of traffic disruption and risk of environmental damage, but these risks may conveniently be converted into financial losses.

The risk analysis consists of a systematic hazard identification and an estimation of the two components of the risk, the likelihood and the consequence. Finally, the risk is evaluated against the acceptance criteria. If the risk is found unacceptable, risk-reducing measures are required. It is recommended to develop and maintain an accurate accounting system for the risks and to plan to update the risk assessment in pace with the project development.

In the following, three common risk evaluation methods are discussed: fixed limits, cost efficiency, and ALARP, i.e., as low as reasonably practicable.

*Fixed limits* is the classical form of acceptance criteria. Fixed limits are also known from legislation and it may easily be determined whether a determined risk is acceptable or not. On the other hand, the determination of limits, which can ensure an optimal risk level, may be difficult.

With a pure *cost efficiency* consideration, an upper limit is not defined, but all cost-efficient risk-reducing measures are introduced. For this cost–benefit consideration it is necessary to establish direct quantification of the consequences in units comparable to costs.

The *ALARP* method applies a cost–benefit consideration in which it, however, is stated that the risk shall be reduced until the cost of the reduction measures is in disproportion with the risk-reducing effect. This will result in a lower risk level than the pure cost efficiency. In ALARP a constraint of the acceptable risk is further introduced as an upper limit beyond which the risk is unconditionally unacceptable.

Often it is claimed that society regards one accident with 100 fatalities as worse than 100 accidents each with one fatality. Such an attitude toward risk aversion can be introduced in the risk policy and the risk acceptance criteria. The aversion against large accidents can also be modeled with aversion factors that are multiplied on the consequences of accidents with many fatalities; the more fatalities, the higher the factor. The sensitivity of the evaluations of risk should be considered by the representation of the uncertainty of the information in the models.

#### Risk Studies in Different Project Phases

The general result of the risk management is a documentation of the risk level, basis for decisions, and basis for risk communication. The specific aims and purposes for risk management depend on the phase of the project. Here some few examples of the purposes of risk management in conceptual study, tender design, detailed design, and operation, are given.

During initial studies, the risk should be crudely analyzed using more qualitative assessments of the risks. A risk management framework should be defined early in the design process. In the beginning of the project, some investigations should be initiated in order to establish a basis for the more-detailed work in later phases; for example, vessel traffic observations should be performed to provide the basis for the estimation of vessel impact probability. In later phases detailed special studies on single probabilities or consequences may be undertaken.

In the conceptual study the most important activities are to identify all relevant events, focus on events with significant risk contributions and risks with potential impact on geometry (safety, rescue, span width). Extreme event loads are established based on the risk studies. In the tender design phase, the main risks are examined in more detail, in particular risks with potential impact on the project basis. In the detailed design phase, the final documentation of the risk level should be established and modifications to the operational procedures should be made.

### 4.3.6 Aesthetic Requirements

The final structures and components of a fixed link are a result of a careful aesthetic appraisal and design of all the constituent elements. The purpose of aesthetic requirements is to obtain an optimal technical and sculptural form of individual elements and to obtain an overall aesthetic quality and visual consistency between the elements and the setting. Although difficult, it is recommended to establish guidelines for aesthetic questions.

### 4.3.7 Navigation Conditions

The shipping routes and the proposed arrangement for a major bridge across navigable waters may be such that both substructure and superstructure could be exposed to vessel collisions. General examples of consequences of vessel collisions are as follows:

- Fatalities and injuries to users of the bridge and to crew and vessel passengers;
- Pollution of the environment, in the case of an accidental release of the hazardous cargo;
- Damage or total loss of bridge;
- Damage or total loss of vessels;
- Economic loss in connection with prolonged traffic disruption of the bridge link.

A bridge design that is able to withstand worst-case vessel impact loads on other piers than the navigation piers is normally not cost-effective. Furthermore, such a deterministic approach does not reduce the risk to the environment and to the vessels. Therefore, a probabilistic approach addressing the main risks in a systematic and comprehensive way is recommended. This approach should include studies of safe navigation conditions, vessel collision risk analysis and vessel collision design criteria, as outlined below.

Navigation risks should be addressed as early as possible in the planning phases. The general approach outlined here is in accordance with the IABSE Green Ship Collision Book [4] and the AASTHO Guide Specification [5]. The approach has been applied in the development of the three major fixed links discussed in Section 4.4.

#### Safe Navigation Conditions

Good navigation conditions are a prerequisite for the safe passage of the bridge such that vessel collisions with the bridge will not occur under normal conditions, but only as a result of navigation error or technical failure on-board during approach.

The proposed bridge concept should be analyzed in relation to the characteristics of the vessel traffic. The main aspects to be considered are as follows:

- Preliminary design of bridge;
- Definition of navigation routes and navigation patterns;
- Data on weather conditions, currents, and visibility;
- Distribution of vessel movements with respect to type and size;
- Information on rules and practice for navigation, including use of pilots and tugs;
- Records of vessel accidents in the vicinity of the bridge;
- Analysis of local factors influencing the navigation conditions;
- Identification of special hazards from barges, long tows and other special vessels;
- Future navigation channel arrangements;
- Forecast of future vessel traffic and navigation conditions to the relevant study period;
- Identification of largest safe vessel and tow and of preventive measures for ensuring full control with larger passing vessels.

#### Vessel Collision Analysis

An analysis should be used to support the selection of design criteria for vessel impact. Frequencies of collisions and frequencies of bridge collapse should be estimated for each bridge element exposed to vessel collision. Relevant types of hazards to the bridge should be identified and modeled, hazards from ordinary vessel traffic which is laterally too far out of the ordinary route, hazards from vessels failing to turn properly at a bend near the bridge, and from vessels sailing on more or less random courses.

The frequencies of collapse depend on the design criteria for vessel impact. The overall design principle is that the design vessels are selected such that the estimated bridge collapse frequency fulfills an acceptance criterion.

### Vessel Collision Design Criteria

Design criteria for vessel impact should be developed. This includes selection of design vessels for the various bridge elements which can be hit. It also includes estimates of sizes of impact loads and rules for application of the loads. Both bow collisions and sideways collisions should be considered. Design capacities of the exposed girders against impact from a deck house shall be specified.

The vessel impact loads are preferably expressed as load indentation curves applicable for dynamic analysis of bridge response. Rules for application of the loads should be proposed. It is proposed that impact loads will be estimated on the basis of general formulas described in Ref. [4].

## 4.3.8 Wind Conditions

Bridges exposed to the actions of wind should be designed to be consistent with the type of bridge structure, the overall wind climate at the site, and the reliability of site-specific wind data. Wind effects on traffic could also be an important issue to be considered.

### Susceptibility of Bridge Structures to Wind

Winds generally introduce time-variant actions on all bridge structures. The susceptibility of a given bridge to the actions of wind depends on a number of structural properties such as overall stiffness, mass, and shape of deck structure and support conditions.

Cable-supported bridges and long-span beam structures are often relatively light and flexible structures in which case wind actions may yield significant contributions to structural loading as well as influence user comfort. Site-specific wind data are desirable for the design. Engineering codes and standards often provide useful information on mean wind properties, whereas codification of turbulence properties are rare. Guidelines for turbulence properties for generic types of terrain (sea, open farmland, moderately built-up areas) may be found in specialized literature. If the bridge is located in complex hilly/mountainous terrain or in the proximity of large structures (buildings, bridges, dams), it is advisable to carry out field investigations of the wind climate at the bridge site. Important wind effects from isolated obstacles located near the planned bridge may often be investigated by means of wind tunnel model testing.

In general, it is recommended that aerodynamic design studies be included in the designs process. Traditionally, aerodynamic design have relied extensively on wind tunnel testing for screening and evaluation of design alternatives. Today, computational fluid dynamics methods are becoming increasingly popular due to speed and efficiency as compared with experimental methods.

### Wind Climate Data

The properties of turbulence in the atmospheric boundary layer change with latitude, season and topography of the site, but must be known with a certain accuracy in order to design a bridge to a desired level of safety. The following wind climatic data should be available for a particular site for design of wind-sensitive bridge structures:

Mean wind:

- Maximum of the 10-min average wind speed corresponding to the design lifetime of the structure;
- Vertical wind speed profile;
- Maximum short-duration wind speed (3-s gust wind speed).

Turbulence properties (along-wind, cross-wind lateral, and cross-wind vertical):

- Intensity;
- Spectral distribution;
- Spatial coherence.

The magnitude of the mean wind governs the steady-state wind load to be carried by the bridge structure and is determinant for the development of aeroelastic instability phenomena. The turbulence properties govern the narrowband random oscillatory buffeting response of the structure, which is similar to the sway of trees and bushes in storm winds.

### 4.3.9 Earthquake Conditions

Structures should be able to resist regional seismic loads in a robust manner, avoiding loss of human lives and major damages, except for the very rare but large earthquake. The design methods should be consistent with the level of seismicity and the amount of available reliable information. Available codes and standards typically do not cover important lifeline structures such as a fixed link, but they may be used for inspiration for the development of a design basis.

### 4.3.10 Ice Conditions

The geographic location of a bridge site indicates whether or not ice loads are of concern for that structure. The ice loads may be defined as live loads or extreme event loads (exceptional environmental loads which are not included in live loads).

From recent studies carried out for the Great Belt link, the following main experience was obtained:

- Ice loads have a high dynamic component very likely to lock in the resonance frequencies of the bridge structure;
- Bearing capacity of the soil is dependent on number and type of load cycles, so dynamic soil testing is needed;
- Damping in soil and change of stiffness cause important reductions in the dynamic response;
- If possible, the piers should be given an inclined surface at the water level;
- High ductility of the structure should be achieved.

### 4.3.11 Costing Basis

The cost estimate is often decisive for the decision on undertaking the construction of the link, for selection of solution models, and for the selection of concepts for tendering. The estimate may also be important for decisions of detailed design items on the bridge.

#### Cost Uncertainty Estimation

To define the cost uncertainty it may be helpful to divide it into two conceptions which may overlap: (1) the uncertainties of the basis and input in the estimation (mainly on cost and time) and (2) risk of unwanted events. The uncertainty is in principle defined for each single item in the cost estimate. The risks can in principle be taken from a construction risk analysis.

#### Cost Estimates at Different Project Phases

Cost estimates are made in different phases of the planning of a project. In the first considerations of a project, the aim is to investigate whether the cost of the project is of a realistic magnitude and whether it is worthwhile to continue with conceptual studies. Later, the cost estimates are used to compare solution concepts, and to evaluate designs and design modification until the final cost estimate before the tender is used to evaluate the overall profitability of the project and to compare with the received bids. Different degrees of detailing of the estimates are needed in these stages. In the early phases an "overall unit cost" approach may be the only realistic method for estimating a price, whereas in the later phases it is necessary to have a detailed breakdown of the cost items and the associated risks and uncertainties.

### Life Cycle Costs

The life cycle cost is an integration of the entire cost for a bridge from the first planning to the final demolition. The life cycle cost is normally expressed as a present value figure. Hence, the interest rate used is very important as it is a weighting of future expenses against initial expenses.

It shall initially be defined how the lifetime costs are to be considered. For example, disturbance of the traffic resulting in waiting time for the users can be regarded as an operational cost to society whereas it is only a cost for the owner if it influences the users' behavior so that income will be less.

Important contributors to the life cycle cost for bridges are as follows:

- The total construction cost, including costs for the owner's organization;
- Future modifications or expansion of the bridge;
- Risks and major repairs;
- Income from the operation of the bridge;
- Demolition costs.

### Comparison Analysis

In the development of a project numerous situations are encountered in which comparisons and rankings must be made as bases for decisions. The decisions may be of different nature, conditions may be developing, and the decision maker may change. The comparisons should be based on a planning and management tool which can rationalize, support, and document the decision making.

A framework for the description of the solutions can be established and maintained. This framework may be modified to suit the purpose of the different situations. It is likely that factual information can be reused in a later phase.

The main components of the comparison can be as follows:

- Establishment of decision alternatives;
- Criteria for evaluation of the alternatives;
- Quantitative assessment of impacts of the various alternatives utilizing an evaluation grid;
- Preference patterns for one or more decision makers with associated importance of criteria;
- Assessment of uncertainties.

The decision maker must define the comparison method using a combination of technical, environmental and financial criteria. Quantitative assessment of all criteria is performed. Decision-making theories from economics and mathematical tools are used.

### Establishment of Alternatives and Their Characteristics

All decision alternatives should be identified. After a brief evaluation, the most obviously nonconforming alternatives may be excluded from the study. In complex cases a continued process of detailing of analysis and reduction of number of alternatives may be pursued. The selection of parameters for which it is most appropriate to make more-detailed analyses can be made on the basis of a sensitivity analysis of the parameters with respect to the utility value.

Risks may be regarded as uncertain events with adverse consequences. Of particular interest are the different risk pictures of the alternatives. These risk pictures should be quantified by the use of preferences so that they can be part of the comparison.

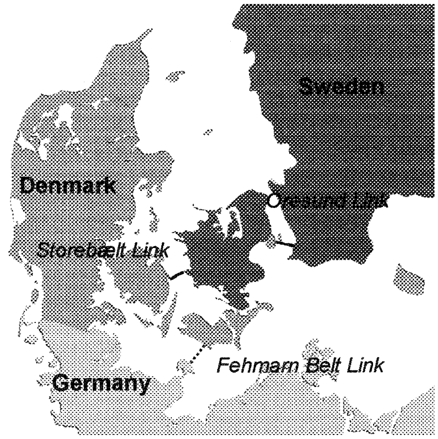


**FIGURE 4.3** Denmark and neighboring countries.

### Comparisons at Different Project Stages

After the initial identification of all possible solution models, the purpose of the first comparison may be to reduce the number of solution models to be investigated in the later phases. The solution models may here be alternative design concepts. In this first ranking the detail of the analysis should be adequate to determine the least attractive solutions with an appropriate certainty. This will in most cases imply that a relatively crude model can be used at this stage. A partly qualitative assessment of some of the parameters, based on an experienced professional's judgment, can be used.

At a later phase decisions should be made on which models to select for tender design, and later in the tender evaluation, which tenderer to award the contract. In these comparisons the basis and the input should be more well established, as the comparison here should be able to select the single best solution with sufficient certainty.

Weighting the criteria is necessary. Although a strictly rational weighting and conversion of these criteria directly into terms of financial units may not be possible, it is often sufficient if the weighting and selection process are shown to the tenderers before the tender.

An example of the comparison and selection process can be the following, which is performed in stages. Each stage consists of an evaluation and shortlisting of the tenders eliminating the low ranked tenders. At each stage the tenderers not on the shortlist are informed about the weak points and they are given the opportunity of changing their tender within a short deadline. At the last stage the remaining tenderers are requested to state their final offer improving on the technical quality and financial aspects raised by the owner during negotiations. Then the owner can select the financially most advantageous tender.

## 4.4 Recent Examples of Fixed Links

### 4.4.1 Introduction

Since the 1980s three major fixed links have been designed or planned in Denmark and neighboring countries, Figure 4.3. A combined tunnel and bridge link for railway and highway traffic has been constructed across the 18-km-wide Storebælt, a 16-km tunnel and bridge link for railway and highway traffic between Denmark and Sweden will be inaugurated in year 2000, and the conceptual study has been completed (1998) for a fixed link across the 19-km-wide Fehmarn Belt between Denmark and Germany.

### 4.4.2 The Storebælt Link

Over the years, more or less realistic projects for a fixed link across the Storebælt have been presented. At 18-km-wide, the belt is part of the inland sea area and divides Denmark's population and economy into nearly equal halves.

The belt is divided into two channels, east and west, by the small island, Sprogø, which has been as an obvious stepping-stone, an integral part of all plans for fixed link projects. The international vessel traffic between the Baltic Sea and the North Sea navigates the eastern channel, whereas the western channel is a national waterway. To bridge the eastern channel has therefore always been the main challenge of the project.

The first tender design for a combined railway and highway bridge across the eastern channel was prepared in 1977–78. However, only 1½ month short of issuing tender documents and call for bids, the progress of the project was temporarily stopped by the government. This was in August 1978. Several state-of-the-art investigations such as vessel impact, fatigue, and wind loads were carried out for two selected navigation spans: a 780-m main span cable-stayed bridge and a 1416-m main span suspension bridge, both designed for a heavy duty double-track railway and a six-lane highway.

The construction of the fixed link was again politically agreed upon on June 12, 1986, and the main principles for the link were set out. It should consist of a low-level bridge for combined railway and highway traffic, the West Bridge, across the western channel; whereas the eastern channel should be crossed by a bored or an immersed tunnel for the railway, the East Tunnel, and a high-level bridge for the highway traffic, the East Bridge.

A company, A/S Storebælt, was established January 23, 1987 and registered as a limited company with the Danish State as sole shareholder. The purpose of the company was to plan, design, implement, and operate the fixed link. The project is financed by government-guaranteed commercial loans to be paid back via user tolls. A/S Storebæltsforbindelsen has published a series of reports on the link structures, see Reference [6–8].

#### The East Bridge

Project Development

In 1987 conceptual design was carried out for the East Bridge. The main objectives were to develop a global optimization with regard to the following:

- Alignment, profile, and navigation clearance;
- Position of main navigation channel;
- Navigation span solutions, based on robust and proven design and construction technology;
- Constructable and cost-competitive solutions for the approach spans, focusing on repetitive industrialized production methods onshore;
- Master time schedule;
- Master budget.

In 1989–90 pretender studies, tender design, and tender documents were prepared. During the pretender phase, comparative studies of four alternative main bridge concepts were carried out to evaluate thoroughly the technical, financial, and environmental effects of the range of main spans:

| | |
|---|---|
| Cable-stayed bridge | 916 m main span |
| Cable-stayed bridge | 1204 m main span |
| Suspension bridge | 1448 m main span |
| Suspension bridge | 1688 m main span |

Navigation risk studies found only the 1688-m main span adequate to cross the existing navigation route without affecting the navigation conditions negatively. This during tender design was reduced to 1624 m, which together with a relocated navigation route, proved to be sufficient and was selected for tender and construction.

The pylons were tendered in both steel and concrete. For the approach span superstructure, 124-m-long concrete spans and 168-m-long steel spans as well as composite steel/concrete concepts were developed. Although an equally competitive economy was found, it was decided to limit the tender designs to concrete and steel spans. The East Bridge was tendered as SDC.

The tender documents were subdivided into four packages to be priced by the contractors: superstructure and substructure inclusive pylons for the suspension bridge (2) and superstructures and substructures for the approach spans. The tender documents were released to prequalified contractors and consortia in June 1990.

In December 1990 the tenders were received. Eight consortia submitted 32 tenders inclusive smaller alternatives and four major alternatives to the basic tender design.

In October 1991, construction contracts were signed with two international consortia; a German, Dutch, and Danish joint venture for the substructures, inclusive of concrete pylons, and an Italian contractor for an alternative superstructure tender where high-strength steel was applied to a more or less unchanged basic cross section, thereby increasing the span length for the approach spans from 168 m to 193 m.

The suspension span is designed with a main cable sag corresponding to $\frac{1}{9}$ of span length. The steel bridge girder is suspended from 800-mm-diameter main cables in hangers each 24 m.

The girder is continuous over the full cable-supported length of 2.7 km between the two anchor blocks. The traditional expansion joints at the tower positions are thus avoided. Expansions joints are arranged in four positions only, at the anchor blocks and at the abutments of the approach spans.

The concrete pylons rise 254 m above sea level. They are founded on caissons placed directly on crushed stone beds.

The anchor blocks must resist cable forces of 600,000 tonnes. They are founded on caissons placed on wedge-shaped foundation bases suitable for large horizontal loading. An anchor block caisson covers an area of 6100 $m^2$.

The caissons for the pylons, the anchor blocks, and the approach spans as well as for the approach span pier shafts have been constructed at a prefabrication site established by the contractor 30 nautical miles from the bridge site. The larger caissons were cast in two dry docks, and the smaller caissons and the pier shafts for the approach spans on a quay area, established for this purpose. A pylon caisson weighed 32,000 tonnes and an anchor block caisson 36,000 tonnes when they were towed from the dry dock by tug boats to their final position in the bridge alignment.

Both the suspension bridge girder and the approach span girders are designed as closed steel boxes and constructed of few basic elements: flat panels with trough stiffeners and transverse bulkhead trusses. The two approach bridges, 2530 and 1538 m, respectively, are continuous from the abutments to the anchor blocks. The suspension bridge girder is 31.0 m wide and 4.0 m deep; the girder for the approach spans is 6.7 m deep. They are fabricated in sections, starting in Italy. In Portugal, on their way by barge to Denmark, a major preassembly yard was established for girder sections to be assembled, before they were finally joined to full-span girders in Denmark. The East Bridge (Figure 4.4), was inaugurated by the Danish Queen on June 14, 1998 and the link was opened to highway traffic.

#### Project Basis

The project basis was throughout its development reviewed by international panels of experts.

#### Structural Requirements

Danish codes, standards, rules, and regulations were applied wherever applicable and supplemented with specific additional criteria and requirements, regarding various extreme event loads.

#### Environmental

The environmental design criteria required that the construction should be executed with no effect on the water flow through the belt. This was achieved by dredging, short ramps, long spans, and hydraulic shaped piers and pylons. The blocking effect to be compensated for was only about 0.5% of the total flow in the belt.

**FIGURE 4.4** Storebælt East Bridge.

Risk

Risk acceptance criteria were established early and a series of risk analyses regarding train accidents, fire and explosion, ice loads, and vessel collision were carried out to ensure adequate and consistent safety level for the entire link. The acceptance criteria required that the probability of disruption of a duration of more than 1 month should not exceed a specified level, and that the risk level for fatalities for crossing should be comparable to the risk for a similar length of traffic on land. The analyses were followed up by risk management through the subsequent phases to ensure that the objectives were met.

Navigation

With 18,000 vessel passages each year through the eastern channel, important considerations were given for navigation. Comprehensive vessel simulations and collision analysis studies were performed, leading to an improved knowledge about safe navigation conditions and also to a set of probabilistically based criteria for the required impact resistance of the bridge piers and girders. Vessel impact has been the governing load criterion for all the bridge piers. A vessel traffic service (VTS) system was established mainly for prevention of collision accidents to the low West Bridge.

Wind

The local wind climate at Storebælt was investigated by measurements from a 70-m-high tower on Sprogø. For the East Bridge, aerodynamic investigations were carried out on 16 different highway girder box section configurations in a wind tunnel. The testing determined the critical wind speed for flutter for the selected girder shape to be 74 m/s which was safely above the design critical wind speed of 60 m/s. For the detailed design an aeroelastic full bridge model of 1:200 scale was tested under simulated turbulent wind conditions.

### The West Bridge

The 6.6-km West Bridge (Figure 4.5) was tendered in three alternative types of superstructure; a double-deck composite girder, triple independent concrete girders side by side, and a single steel box girder. All three bridge alternatives shared a common gravity-founded sand-filled caisson substructure, topped by pier shafts of varying layout.

**FIGURE 4.5** Storebælt West Bridge.

Tender documents were issued to six prequalified consortia in April 1988, and 13 offers on the tender solutions as well as three major alternatives and nine smaller alternatives were received from five groups. Tender evaluation resulted in selecting an alternative design: two haunched concrete box girders with a typical span length of 110.4 m, reduced to 81.75 m at the abutments and the expansion joints. The total length was subdivided into six continuous girders, requiring seven expansion joints.

It was originally intended to tender the West Bridge as an SDC, but as an alternative design was selected, the contract ended up being similar to a DB contract.

Altogether, 324 elements, comprising 62 caissons, 124 pier shafts, and 138 girders, have been cast in five production lines at a reclaimed area close to the bridge site. All the elements were cast, moved by sliding, stored on piled production lines, and later discharged without use of heavy gantry cranes. The maximum weight of an element was 7400 tonnes. The further transportation and installation was carried out by *Svanen,* a large purpose-built catamaran crane vessel.

By this concept, which was originally presented in the tender design, but further developed in the contractor's design, the entire prefabrication system was optimized in regard to resources, quality, and time. The bridge was handed over on January 26, 1994.

## The East Tunnel

Two immersed tunnel solutions as well as a bored tunnel were considered for the 9 km wide eastern channel. After tender, the bored tunnel was selected for financial and environmental reasons.

The tunnel consists of two 7.7-m-internal-diameter tubes, each 7412 m long and 25 m apart. At the deepest point, the rails are 75 m below sea level.

Four purpose-built tunnel boring machines of the earth-balance pressure type have bored the tunnels, launched from each end of both tubes. The tunnel tubes are connected at about 250 m intervals by 4.5-m-diameter cross passages which provide safe evacuation of passengers and are the location for all electrical equipment. About 250 m of reinforced concrete cut-and-cover tunnels are built at each end of the bored tubes. The tunnel is lined with precast concrete segmental rings, bolted together with synthetic rubber gaskets. Altogether, 62,000 segments have been produced. A number of protective measures has been taken to ensure a 100-year service life design.

**FIGURE 4.6** Storebælt East Tunnel.

On April 7, 1995, the final tunnel lining segment was installed. Thus, the construction of the tunnel tubes was completed, almost 5 years after work commenced. Railway systems were installed and in June 1997 the railway connection (Figure 4.6) was opened to traffic and changes in the traffic pattern between East and West Denmark started.

### 4.4.3 The Øresund Link

The 16-km fixed link for combined railway and highway traffic between Denmark and Sweden consists of three major projects: a 3.7-km immersed tunnel, a 7.8-km bridge, and an artificial island which connects the tunnel and the bridge.

The tunnel contains a four-lane highway and two railway tracks. The different traffic routes are separated by walls, and a service tunnel will be placed between the highway's two directions. The tunnel will be about 40 m wide and 8 m high. The 20 reinforced concrete tunnel elements, 175 m long and weighing 50,000 tonnes, are being prefabricated at the Danish side, and towed to the alignment.

The owner organization of the Øresund link is Øresundskonsortiet, established as a consortium agreement between the Danish company A/S Øresundsforbindelsen and the Swedish company Svensk-Danska Broförbindelsen on January 27, 1992. The two parties own 50% each of the consortium. The purpose of the consortium is to own, plan, design, finance, construct and operate the fixed link across Øresund.

The project is financed by commercial loans, guaranteed jointly and severally between the Danish and the Swedish governments. The highway part will be paid by user tolls, whereas the railway companies of the two countries will pay fixed installments per year. The revenue also has to cover the construction work expenses for the Danish and Swedish land-based connections.

Prequalified consultants were asked in February 1993 to prepare a conceptual design, as part of a proposal to become the in-house consultant for the owner. Two consultants were selected to prepare tender documents for the tunnel, the artificial island and the bridge, respectively.

#### The Øresund Bridge

In July 1994, the Øresundkonsortiet prequalified a number of contractors to build the bridge on a design and construct basis.

**FIGURE 4.7** Øresund Main Bridge.

The bridge was tendered in three parts; the approach bridge from Sweden, the high-level bridge with a 490-m main span and a vertical clearance of 57 m, and the approach spans toward Denmark. Two solutions for the bridge were suggested: primarily, a two-level concept with the carriageway on the top deck and the two-track railway on the lower deck; secondarily a one-level bridge. Both concepts were based on cable-stayed main bridges.

Five consortia were prequalified to participate in the competition for the high-level bridge, and six consortia for the approach bridges. In June 1995, the bids for the Øresund Bridge were delivered. The two-level concept was selected as the financially most favorable solution. In November 1995, the contract for the entire bridge was awarded to a Swedish–German–Danish consortium.

The 7.8-km bridge includes a 1090-m cable-stayed bridge (Figure 4.7) with a main span of 490 m. The 3013 and 3739 m approach bridges have spans of 140 m. The entire superstructure is a composite structure with steel truss girders between the four-lane highway on the upper concrete deck and the dual-track railway on the lower deck.

Fabrication of the steel trusses and casting of the concrete deck of the approach bridges are carried out in Spain. The complete 140-m-long girder sections, weighing up to 7000 tonnes, are tugged on flat barges to the bridge site and lifted into position on the piers. Steel trusses for the cable-stayed bridge are fabricated in Sweden and transported to the casting yard close to the bridge site, where the concrete decks are cast.

On the cable-stayed bridge the girder will also be erected in 140-m sections on temporary supports before being suspended by the stays. This method is unusual for a cable-stayed bridge, but it is attractive because of the availability of the heavy-lift vessel *Svanen,* and it reduces the construction time and limits vessel traffic disturbance. (*Svanen* was, as mentioned earlier, purpose-built for the Storebælt West Bridge. After its service there *Svanen* crossed the Atlantic to be upgraded and used for the erection works at the Confederation Bridge in Canada. Back again in Europe *Svanen* performs an important job at Øresund). During the construction period two VTS systems have been in operation, the Drogden VTS on the Danish side, and the Flint VTS on the Swedish side. The main tasks for the VTS systems are to provide vessels with necessary information in order to ensure safe navigation and avoid dangerous situations in the vicinity of the working areas. The VTS systems have proved their usefulness on several occasions.

The cable system consists of two vertical cable planes with parallel stays, the so-called harp-shaped cable system. In combination with the flexural rigid truss girder and an efficient pier support in the side spans, a high stiffness is achieved.

The module of the truss remains 20 m both in the approach and in the main spans. This results in stay cable forces of up to 16,000 tonnes which is beyond the range of most suppliers of prefabricated cables. Four prefabricated strands in a square configuration have therefore been adopted for each stay cable.

The concrete pylons are 203.5 m high and founded on limestone. Caissons, prefabricated on the Swedish side of Øresund, are placed in 15 m water depth, and the cast-in-place pylon shafts are progressing. Artificial islands will be established around the pylons and nearby piers to protect against vessel impact. All caissons, piers, and pier shafts are being prefabricated onshore to be assembled offshore. The bridge is scheduled to be opened for traffic in year 2000.

## Project Basis

### General Requirements

The Eurocode system was selected to constitute the normative basis for the project. Project application documents (PADs) have been prepared as companion documents to each of the Eurocodes. The PADs perform the same function as the national application documents (NADs) developed by the member countries implementing the Eurocodes.

The partial safety and load combination factors are determined by reliability calibration. The target reliability index of $\beta = 4.7$, specified by the owner, corresponds to high safety class as commonly used for important structures in the Nordic countries.

In addition to the Eurocodes and the PADs, general design requirements were specified by the owner to cover special features of a large civil work. This is in line with what is normally done on similar projects. The general design requirements cover the following areas:

- Functional and aesthetic requirements as alignment, gradients, cross sections, and clearance profiles;
- Civil and structural loads, load combinations, and partial safety coefficients; methods of structural analysis and design;
- Soil mechanics requirements to foundation design and construction, including soil strength and deformation parameters;
- Mechanical and electrical requirements to tunnel and bridge installations, including systems for supervision, control and data acquisition (SCADA), power distribution, traffic control, communication.

### Risk

LHRisk acceptance criteria were developed such that the individual user risk for crossing the link would be equal to the average risk on a highway and railway on land of similar length and traffic intensity. In addition, the societal risk aspects concerning accidents with larger numbers of fatalities were controlled as well.

The ALARP-principle — as presented in Section 4.3 — was applied to reduce consequences from risks within a cost–benefit approach. Especially the disruption risks were controlled in this way. Risk-reducing measures were studied to reduce the frequency and consequences of hazardous events. The analyses carried out addressed main events due to fire, explosion, toxic releases, vessel collision and grounding, flooding, aircraft crash, and train derailment.

### Navigation

Øresund is being used by local vessel traffic and vessels in transit up to a certain limit set by the water depth in the channels Drogden and Flinterännan. The Drogden channel near the Danish coast will be crossed by the immersed tunnel and only requirements regarding accidental vessel impact to tunnel structures have been specified. The Flinterännan near the Swedish coast is being crossed by the bridge, and the navigation route will be improved for safety reasons. Design criteria against vessel impact have been specifically developed on a probabilistic basis, and main piers will be protected by artificial islands.

### 4.4.4 The Fehmarn Belt Crossing

In 1995, the Danish and German Ministry of Transport invited eight consulting consortia to tender for the preliminary investigations for a fixed link across the 19-km-wide Fehmarn Belt.

Two Danish/German consortia were selected; one to carry out the geological and the subsoil investigations, and the other to investigate technical solution models, the environmental impact, and to carry out the day-to-day coordination of all the investigations.

In the first phase, seven different technical solutions were investigated, and in the second phase five recommended solutions were the basis for a concept study:

- A bored railway tunnel with shuttle services;
- An immersed railway tunnel with shuttle services;
- A combined highway and railway bridge;
- A combined highway and railway bored tunnel;
- A combined highway and railway immersed tunnel.

With a set of more detailed and refined functional requirements, various concepts for each of the five solution models have been studied in more detail than in the first phase. This concept study was finalized in early July 1997 with the submission of an interim report. The conceptual design started in December 1997 and is planned to last 7 months. To provide an adequate basis for a vessel collision study and the associated part of the risk analysis, vessel traffic observations are carried out by the German Navy. In parallel, the environmental investigations are continued, whereas the geological and the subsoil investigations are concluded.

The results of the study will constitute the basis for public discussions and political decisions whether or not to establish a fixed link, and also which solution model should be preferred.

## References


1. The Danish Transport Council, Facts about Fehmarn Belt, Report 95-02, February 1995.
2. The Danish Transport Council, Fehmarn Belt. Issues of Accountability, Report 95-03, May 1995.
3. The Interaction between Major Engineering Structures and the Marine Environment, Report from IABSE Colloquium, Nyborg, Denmark, 1991.
4. Ole Damgaard Larsen: Ship Collision with Bridges, IABSE Structural Engineering Documents, 1993.
5. Guide Specification and Commentary for Vessel Collision Design of Highway Bridges, Vol. 1, Final Report, AASHTO, 1991.
6. The Storebælt Publications: East Tunnel, A/S Storebæltsforbindelsen, København, Denmark, 1997.
7. The Storebælt Publications: West Bridge, A/S Storebæltsforbindelsen, København, Denmark, 1998.
8. The Storebælt Publications: East Bridge, A/S Storebæltsforbindelsen, København, Denmark, 1998.

Kulicki, J.M. "Desgn Philosophies for Highway Bridges."
*Bridge Engineering Handbook.*
Ed. Wai-Fah Chen and Lian Duan
Boca Raton: CRC Press, 2000

# 5

# Design Philosophies for Highway Bridges




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## 5.1 Introduction

Several bridge design specifications will be referred to repeatedly herein. In order to simplify the references, the "Standard Specifications" means the *AASHTO Standard Specifications for Highway Bridges* [1], and the sixteenth edition will be referenced unless otherwise stated. The "LRFD Specifications" means the *AASHTO LRFD Bridge Design Specifications* [2], and the first edition will be referenced, unless otherwise stated. This latter document was developed in the period 1988 to 1993 when statistically based probability methods were available, and which became the basis of quantifying safety. Because this is a more modern philosophy than either the load factor design method or the allowable stress design method, both of which are available in the Standard Specifications, and neither of which have a mathematical basis for establishing safety, much of the chapter will deal primarily with the LRFD Specifications.

There are many issues that make up a design philosophy — for example, the expected service life of a structure, the degree to which future maintenance should be assumed to preserve the original resistance of the structure or should be assumed to be relatively nonexistent, the ways brittle behavior can be avoided, how much redundancy and ductility are needed, the degree to which analysis is expected to represent accurately the force effects actually experienced by the structure, the extent to which loads are thought to be understood and predictable, the degree to which the designers' intent will be upheld by vigorous material-testing requirements and thorough inspection during construction, the balance between the need for high precision during construction in terms of alignment and positioning compared with allowing for misalignment and compensating for it in the design, and, perhaps most fundamentally, the basis for establishing safety in the design specifications. It is this last issue, the way that specifications seek to establish safety, that is dealt with in this chapter.

## 5.2 Limit States

All comprehensive design specifications are written to establish an acceptable level of safety. There are many methods of attempting to provide safety and the method inherent in many modern bridge design specifications, including the LRFD Specifications, the Ontario Highway Bridge Design Code [3], and the Canadian Highway Bridge Design Code [4], is probability-based reliability analysis. The method for treating safety issues in modern specifications is the establishment of "limit states" to define groups of events or circumstances that could cause a structure to be unserviceable for its original intent.

The LRFD Specifications are written in a probability-based limit state format requiring examination of some, or all, of the four limit states defined below for each design component of a bridge.

- The *service limit state* deals with restrictions on stress, deformation, and crack width under regular service conditions. These provisions are intended to ensure the bridge performs acceptably during its design life.
- The *fatigue and fracture limit state* deals with restrictions on stress range under regular service conditions reflecting the number of expected stress range excursions. These provisions are intended to limit crack growth under repetitive loads to prevent fracture during the design life of the bridge.
- The *strength limit state* is intended to ensure that strength and stability, both local and global, are provided to resist the statistically significant load combinations that a bridge will experience in its design life. Extensive distress and structural damage may occur under strength limit state conditions, but overall structural integrity is expected to be maintained.
- The *extreme event limit state* is intended to ensure the structural survival of a bridge during a major earthquake, or when collided by a vessel, vehicle, or ice flow, or where the foundation is subject to the scour that would accompany a flood of extreme recurrence, usually considered to be 500 years. These provisions deal with circumstances considered to be unique occurrences whose return period is significantly greater than the design life of the bridge. The joint probability of these events is extremely low, and, therefore, they are specified to be applied separately. Under these extreme conditions, the structure is expected to undergo considerable inelastic deformation by which locked-in force effects due to temperature effects, creep, shrinkage, and settlement will be relieved.

## 5.3 Philosophy of Safety

### 5.3.1 Introduction

A review of the philosophy used in a variety of specifications resulted in three possibilities, allowable stress design (ASD), load factor design (LFD), and reliability-based design, a particular application of which is referred to as load and resistance factor design (LRFD). These philosophies are discussed below.

### 5.3.2 Allowable Stress Design

ASD is based on the premise that one or more factors of safety can be established based primarily on experience and judgment which will assure the safety of a bridge component over its design life; for example, this design philosophy for a member resisting moments is characterized by design criteria such as

$$\Sigma M/S \le F_y/1.82 \tag{5.1}$$

where

$\Sigma M$ = sum of applied moments

$F_y$ = specified yield stress

$S$ = elastic section modules

The constant 1.82 is the factor of safety.

The "allowable stress" is assumed to be an indicator of the resistance and is compared with the results of stress analysis of loads discussed below. Allowable stresses are determined by dividing the elastic stress at the onset of some assumed undesirable response, e.g., yielding of steel or aluminum, crushing of concrete, loss of stability, by a safety factor. In some circumstances, the allowable stresses were increased on the basis that more representative measures of resistance, usually based on inelastic methods, indicated that some behaviors are stronger than others. For example, the ratio of fully yielded cross-sectional resistance (no consideration of loss of stability) to elastic resistance based on first yield is about 1.12 to 1.15 for most rolled shapes bent about their major axis. For a rolled shape bent about its minor axis, this ratio is 1.5 for all practical purposes. This increased plastic strength inherent in weak axis bending was recognized by increasing the basic allowable stress for this illustration from 0.55 $F_y$ to 0.60 $F_y$ and retaining the elastic calculation of stress.

The specified loads are the working basis for stress analysis. Individual loads, particularly environmental loads, such as wind forces or earthquake forces, may be selected based on some committee-determined recurrence interval. Design events are specified through the use of load combinations discussed in Section 5.4.1.4. This philosophy treats each load in a given load combination on the structure as equal from the viewpoint of statistical variability. A "commonsense" approach may be taken to recognize that some combinations of loading are less likely to occur than others; e.g., a load combination involving a 160 km/h wind, dead load, full shrinkage, and temperature may be thought to be far less likely than a load combination involving the dead load and the full design live load. For example, in ASD the former load combination is permitted to produce a stress equal to four thirds of the latter. There is no consideration of the probability of both a higher-than-expected load and a lower-than-expected strength occurring at the same time and place. There is little or no direct relationship between the ASD procedure and the actual resistance of many components in bridges, or to the probability of events actually occurring.

These drawbacks notwithstanding, ASD has produced bridges which, for the most part, have served very well. Given that this is the historical basis for bridge design in the United States, it is important to proceed to other, more robust design philosophies of safety with a clear understanding of the type of safety currently inherent in the system.

### 5.3.3 Load Factor Design

In LFD a preliminary effort was made to recognize that the live load, in particular, was more highly variable than the dead load. This thought is embodied in the concept of using a different multiplier on dead and live load; e.g., a design criteria can be expressed as

$$1.30M_D + 2.17\left(M_{L+I}\right) \le \phi M_u \tag{5.2}$$

where

$M_D$ = moment from dead loads

$M_{L+I}$ = moment from live load and impact

$M_u$ = resistance

$\phi$ = a strength reduction factor

Resistance is usually based on attainment of either loss of stability of a component or the attainment of inelastic cross-sectional strength. Continuing the rolled beam example cited above, the distinction between weak axis and strong axis bending would not need to be identified because the cross-sectional resistance is the product of yield strength and plastic section modulus in both cases. In some cases, the resistance is reduced by a "strength reduction factor," which is based on the possibility that a component may be undersized, the material may be understrength, or the method of calculation may be more or less accurate than typical. In some cases, these factors have been based on statistical analysis of resistance itself. The joint probability of higher-than-expected loads and less-than-expected resistance occurring at the same time and place is not considered.

In the Standard Specifications, the same loads are used for ASD and LFD. In the case of LFD, the loads are multiplied by factors greater than unity and added to other factored loads to produce load combinations for design purposes. These combinations will be discussed further in Section 5.4.3.1.

The drawback to load factor design as seen from the viewpoint of probabilistic design is that the load factors and resistance factors were not calibrated on a basis that takes into account the statistical variability of design parameters in nature. In fact, the factors for steel girder bridges were established for one correlation at a simple span of 40 ft (12.2 m). At that span, both load factor design and service load design are intended to give the same basic structure. For shorter spans, load factor design is intended to result in slightly more capacity, whereas, for spans over 40 feet, it is intended to result in slightly less capacity with the difference increasing with span length. The development of this one point calibration for steel structures is given by Vincent in 1969 [5].

## 5.3.4 Probability- and Reliability-Based Design

Probability-based design seeks to take into account directly the statistical mean resistance, the statistical mean loads, the nominal or notional value of resistance, the nominal or notional value of the loads, and the dispersion of resistance and loads as measured by either the standard deviation or the coefficient of variation, i.e., the standard deviation divided by the mean. This process can be used directly to compute probability of failure for a given set of loads, statistical data, and the designer's estimate of the nominal resistance of the component being designed. Thus, it is possible to vary the designer's estimated resistance to achieve a criterion which might be expressed in terms, such as the component (or system) must have a probability of failure of less than 0.0001, or whatever variable is acceptable to society. Design based on probability of failure is used in numerous engineering disciplines, but its application to bridge engineering has been relatively small. The AASHTO "Guide Specification and Commentary for Vessel Collision Design of Highway Bridges" [6] is one of the few codifications of probability of failure in U.S. bridge design.

Alternatively, the probabilistic methods can be used to develop a quantity known as the "reliability index" which is somewhat, but not directly, relatable to the probability of failure. Using a reliability-based code in the purest sense, the designer is asked to calculate the value of the reliability index provided by his or her design and then compare that to a code-specified minimum value. Through a process of calibrating load and resistance factors to reliability indexes in simulated trial designs, it is possible to develop a set of load and resistance factors, so that the design process looks very much like the existing LFD methodology. The concept of the reliability index and a process for reverse-engineering load and resistance factors is discussed in Section 5.3.5.

In the case of the LRFD Specifications, some loads and resistances have been modernized as compared with the Standard Specifications. In many cases, the resistances are very similar. Most of the load and resistance factors have been calculated using a statistically based probability method which considers the joint probability of extreme loads and extreme resistance. In the parlance of the LRFD Specifications, "extreme" encompasses both maximum and minimum events.

## 5.3.5 The Probabilistic Basis of the LRFD Specifications

### 5.3.5.1 Introduction to Reliability as a Basis of Design Philosophy

A consideration of probability-based reliability theory can be simplified considerably by initially considering that natural phenomena can be represented mathematically as normal random variables,

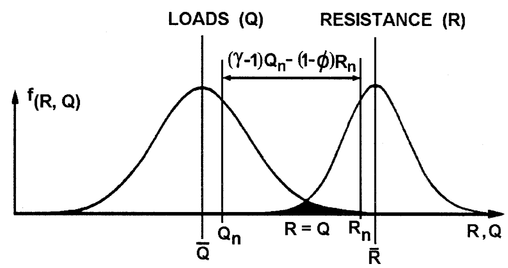


**FIGURE 5.1** Separation of loads and resistance. (*Source:* Kulicki, J.M., et al., NH, Course 13061, Federal Highway Administration, Washington, D.C., 1994.)

as indicated by the well-known bell-shaped curve. This assumption leads to closed-form solutions for areas under parts of this curve, as given in many mathematical handbooks and programmed into many hand calculators.

Accepting the notion that both load and resistance are normal random variables, we can plot the bell-shaped curve corresponding to each of them in a combined presentation dealing with distribution as the vertical axis against the value of load, $Q$, or resistance, $R$, as shown in Figure 5.1 from Kulicki et al. [7]. The mean value of load, $\overline{Q}$, and the mean value of resistance, $\overline{R}$, are also shown. For both the load and the resistance, a second value somewhat offset from the mean value, which is the "nominal" value, or the number that designers calculate the load or the resistance to be, is also shown. The ratio of the mean value divided by the nominal value is called the "bias." The objective of a design philosophy based on reliability theory, or probability theory, is to separate the distribution of resistance from the distribution of load, such that the area of overlap, i.e., the area where load is greater than resistance, is tolerably small. In the particular case of the LRFD formulation of a probability-based specification, load factors and resistance factors are developed together in a way that forces the relationship between the resistance and load to be such that the area of overlap in Figure 5.1 is less than or equal to the value that a code-writing body accepts. Note in Figure 5.1 that it is the nominal load and the nominal resistance, not the mean values, which are factored.

A conceptual distribution of the difference between resistance and loads, combining the individual curves discussed above, is shown in Figure 5.2. It now becomes convenient to define the mean value of resistance minus load as some number of standard deviations, $\beta\sigma$, from the origin. The variable $\beta$ is called the "reliability index" and $\sigma$ is the standard deviation of the quantity $R - Q$. The problem with this presentation is that the variation of the quantity $R - Q$ is not explicitly known. Much is already known about the variation of loads by themselves or resistances by themselves, but the difference between these has not yet been quantified. However, from the probability theory, it is known that if load and resistance are both normal and random variables, then the standard deviation of the difference is

$$\sigma_{(R-Q)} = \sqrt{\sigma_R^2 + \sigma_Q^2} \tag{5.3}$$

Given the standard deviation, and considering Figure 5.2 and the mathematical rule that the mean of the sum or difference of normal random variables is the sum or difference of their individual means, we can now define the reliability index, $\beta$, as

$$\beta = \frac{\overline{R} - \overline{Q}}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{5.4}$$

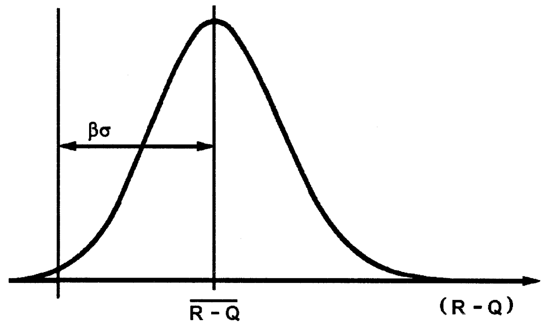


**FIGURE 5.2** Definition of reliability index, β. (*Source:* Kulicki, J.M., et al., NH, Course 13061, Federal Highway Administration, Washington, D.C., 1994.)

Comparable closed-form equations can also be established for other distributions of data, e.g., log-normal distribution. A "trial-and-error" process is used for solving for $\beta$ when the variable in question does not fit one of the already existing closed-form solutions.

The process of calibrating load and resistance factors starts with Eq. (5.4) and the basic design relationship; the factored resistance must be greater than or equal to the sum of the factored loads:

$$\phi R = Q = \Sigma \gamma_i x_i \tag{5.5}$$

Solving for the average value of resistance yields:

$$\bar{R} = \bar{Q} + \beta\sqrt{\sigma_R^2 + \sigma_Q^2} = \lambda R = \frac{1}{\phi}\lambda\Sigma\gamma_i x_i \tag{5.6}$$

By using the definition of bias, indicated by the symbol $\lambda$, Eq. (5.6) leads to the second equality in Eq. (5.6). A straightforward solution for the resistance factor, $\phi$, is

$$\phi = \frac{\lambda\Sigma\gamma_i x_i}{\bar{Q} + \beta\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{5.7}$$

Unfortunately, Eq. (5.7) contains three unknowns, i.e., the resistance factor, $\phi$, the reliability index, $\beta$, and the load factors, $\gamma$.

The acceptable value of the reliability index, $\beta$, must be chosen by a code-writing body. While not explicitly correct, we can conceive of $\beta$ as an indicator of the fraction of times that a design criterion will be met or exceeded during the design life, analogous to using standard deviation as an indication of the total amount of population included or not included by a normal distribution curve. Utilizing this analogy, a $\beta$ of 2.0 corresponds to approximately 97.3% of the values being included under the bell-shaped curve, or 2.7 of 100 values not included. When $\beta$ is increased to 3.5, for example, now only two values in approximately 10,000 are not included.

It is more technically correct to consider the reliability index to be a comparative indicator. One group of bridges having a reliability index that is greater than a second group of bridges also has more safety. Thus, this can be a way of comparing a new group of bridges designed by some new process to a database of existing bridges designed by either ASD or LFD. This is, perhaps, the most correct and most effective use of the reliability index. It is this use which formed the basis for determining the target, or code specified, reliability index, and the load and resistance factors in the LRFD Specifications, as will be discussed in the next two sections.

The probability-based LRFD for bridge design may be seen as a logical extension of the current LFD procedure. ASD does not recognize that various loads are more variable than others. The introduction of the load factor design methodology brought with it the major philosophical change of recognizing that some loads are more accurately represented than others. The conversion to probability-based LRFD methodology could be thought of as a mechanism to select the load and resistance factors more systematically and rationally than was done with the information available when load factor design was introduced.

### 5.3.5.2 Calibration of Load and Resistance Factors

Assuming that a code-writing body has established a target value reliability index $\beta$, usually denoted $\beta_T$, Eq. (5.7) still indicates that both the load and resistance factors must be found. One way to deal with this problem is to select the load factors and then calculate the resistance factors. This process has been used by several code-writing authorities [2–4]. The steps in the process follow:

- Factored loads can be defined as the average value of load, plus some number of standard deviation of the load, as shown as the first part of Eq. (5.6) below.

$$\gamma_i x_i = \bar{x}_i + n\sigma_i = \bar{x}_i + nV_i\bar{x}_i \tag{5.8}$$

  Defining the "variance," $V_i$, as equal to the standard deviation divided by the average value leads to the second half of Eq. (5.8). By utilizing the concept of bias one more time, Eq. (5.6) can now be condensed into Eq. (5.9).

$$\gamma_i = \lambda\left(1 + nV_i\right) \tag{5.9}$$

  Thus, it can be seen that load factors can be written in terms of the bias and the variance. This gives rise to the philosophical concept that load factors can be defined so that all loads have the same probability of being exceeded during the design life. This is not to say that the load factors are identical, just that the probability of the loads being exceeded is the same.

- By using Eq. (5.7) for a given set of load factors, the value of the resistance factor can be assumed for various types of structural members and for various load components, e.g., shear, moment, etc. on the various structural components. Computer simulations of a representative body of structural members can be done, yielding a large number of values for the reliability index.
- Reliability indexes are compared with the target reliability index. If close clustering results, a suitable combination of load and resistance factors has been obtained.
- If close clustering does not result, a new trial set of load factors can be used and the process repeated until the reliability indexes do cluster around, and acceptably close to, the target reliability index.
- The resulting load and resistance factors taken together will yield reliability indexes close to the target value selected by the code-writing body as acceptable.

The outline above assumes that suitable load factors are assumed. If the process of varying the resistance factors and calculating the reliability indexes does not converge to a suitable narrowly grouped set of reliability indexes, then the load factor assumptions must be revised. In fact, several sets of proposed load factors may have to be investigated to determine their effect on the clustering of reliability indexes.

The process described above is very general. To understand how it is used to develop data for a specific situation, the rest of this section will illustrate the application to calibration of the load and resistance factors for the LRFD Specifications. The basic steps were as follows:

- Develop a database of sample current bridges.
- Extract load effects by percentage of total load.
- Develop a simulation bridge set for calculation purposes.
- Estimate the reliability indexes implicit in current designs.
- Revise loads-per-component to be consistent with the LRFD Specifications.
- Assume load factors.
- Vary resistance factors until suitable reliability indexes result.

Approximately 200 representative bridges were selected from various regions of the United States by requesting sample bridge plans from various states. The selection was based on structural type, material, and geographic location to represent a full range of materials and design practices as they vary around the country. Anticipated future trends should also be considered. In the particular case of the LRFD Specifications, this was done by sending questionnaires to various departments of transportation asking them to identify the types of bridges they are expecting to design in the near future.

For each of the bridges in the database, the load indicated by the contract drawings was subdivided by the following characteristic components:

- The dead load due to the weight of factory-made components;
- The dead load of cast-in-place components;
- The dead load due to asphaltic wearing surfaces where applicable;
- The dead weight due to miscellaneous items;
- The live load due to the HS20 loading;
- The dynamic load allowance or impact prescribed in the 1989 AASHTO Specifications.

Full tabulations for all these loads for the full set of bridges in the database are presented in Nowak [8].

Statistically projected live load and the notional values of live load force effects were calculated. Resistance was calculated in terms of moment and shear capacity for each structure according to the prevailing requirements, in this case the AASHTO Standard Specifications for load factor design.

Based on the relative amounts of the loads identified in the preceding section for each of the combination of span and spacing and type of construction indicated by the database, a simulated set of 175 bridges was developed, comprising the following:

- In all; 25 noncomposite steel girder bridge simulations for bending moments and shear with spans of 9, 18, 27, 36, and 60 m and, for each of those spans, spacings of 1.2, 1.8, 2.4, 3.0, and 3.6 m;
- Representative composite steel girder bridges for bending moments and shear having the same parameters as those identified above;
- Representative reinforced concrete T-beam bridges for bending moments and shear having spans of 9, 18, 27, and 39 m, with spacings of 1.2, 1.8, 2.4, and 3.6 m in each span group;
- Representative prestressed concrete I-beam bridges for moments and shear having the same span and spacing parameters as those used for the steel bridges.

Full tabulations of these bridges and their representative amounts of the various loads are presented in Nowak [8].

The reliability indexes were calculated for each simulated and each actual bridge for both shear and moment. The range of reliability indexes which resulted from this phase of the calibration process is presented in Figure 5.3 from Kulicki et al. [7]. It can be seen that a wide range of values was obtained using the current specifications, but this was anticipated based on previous calibration work done for the Ontario Highway Bridge Design Code (OHBDC) [9].

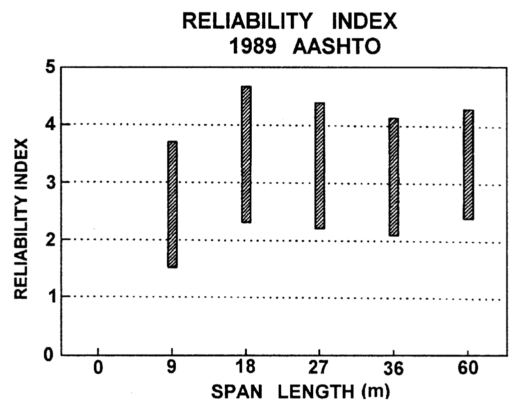


**FIGURE 5.3** Reliability indexes inherent in the 1989 AASHTO Standard Specifications. (*Source:* Kulicki, J.M., et al., NH, Course 13061, Federal Highway Administration, Washington, D.C., 1994.)

**TABLE 5.1** Parameters of Bridge Load Components

| | | | Load Factor | | |
|---|---|---|---|---|---|
| Load Component | Bias Factor | Coefficient of Variation | $n = 1.5$ | $n = 2.0$ | $n = 2.5$ |
| Dead load, shop built | 1.03 | 0.08 | 1.15 | 1.20 | 1.24 |
| Dead load, field built | 1.05 | 0.10 | 1.20 | 1.25 | 1.30 |
| Dead load, asphalt and utilities | 1.00 | 0.25 | 1.375 | 1.50 | 1.65 |
| Live load (with impact) | 1.10–1.20 | 0.18 | 1.40–1.50 | 1.50–1.60 | 1.60–1.70 |

*Source:* Nowak, A.S., Report UMCE 92-25, University of Michigan, Ann Arbor, 1993. With permission.

These calculated reliability indexes, as well as past calibration of other specifications, serve as a basis for selection of the target reliability index, $\beta_T$. A target reliability index of 3.5 was selected for the OHBDC and is under consideration for other reliability-based specifications. A consideration of the data shown in Figure 5.3 indicates that a $\beta$ of 3.5 is representative of past LFD practice. Hence, this value was selected as a target for the calibration of the LRFD Specifications.

### 5.3.5.3 Load and Resistance Factors

The parameters of bridge load components and various sets of load factors, corresponding to different values of the parameter $n$ in Eq. (5.9) are summarized in Table 5.1 from Nowak [8].

Recommended values of load factors correspond to $n = 2$. For simplicity of the designer, one factor is specified for shop-built and field-built components, $\gamma = 1.25$. For $D_3$, weight of asphalt and utilities, $\gamma = 1.50$. For live load and impact, the value of load factor corresponding to n = 2 is $\gamma = 1.60$. However, a more conservative value of $\gamma = 1.75$ is utilized in the LRFD Specifications.

The acceptance criterion in the selection of resistance factors is how close the calculated reliability indexes are to the target value of the reliability index, $\beta_T$. Various sets of resistance factors, $\phi$, are considered. Resistance factors used in the code are rounded off to the nearest 0.05.

Calculations were performed using the load components for each of the 175 simulated bridges using the range of resistance factors shown in Table 5.3. For a given resistance factor, material, span, and girder spacing, the reliability index is computed. Values of $\beta$ were calculated for live-load factors, $\gamma = 1.75$. For comparison, the results are also shown for live-load factor, $\gamma = 1.60$. The calculations are performed for the resistance factors, $\phi$, listed in Table 5.2 from Nowak [8].

Reliability indexes were recalculated for each of the 175 simulated cases and each of the actual bridges from which the simulated bridges were produced. The range of values obtained using the new load and resistance factors is indicated in Figure 5.4.

**TABLE 5.2** Considered Resistance Factors

| Material | Limit State | Resistance Factors, $\phi$ Lower | Upper |
|---|---|---|---|
| Noncomposite steel | Moment | 0.95 | 1.00 |
| | Shear | 0.95 | 1.00 |
| Composite steel | Moment | 0.95 | 1.00 |
| | Shear | 0.95 | 1.00 |
| Reinforced concrete | Moment | 0.85 | 0.90 |
| | Shear | 0.90 | 0.90 |
| Prestressed concrete | Moment | 0.95 | 1.00 |
| | Shear | 0.90 | 0.95 |

*Source:* Nowak, A.S., Report UMCE 92-25, University of Michigan, Ann Arbor, 1993. With permission.

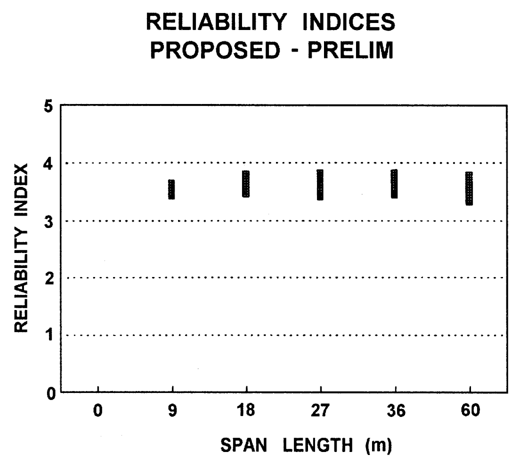


**FIGURE 5.4** Reliability indexes inherent in LRFD Specifications. (*Source:* Kulicki, J.M., et al., NH, Course 13061, Federal Highway Administration, Washington, D.C., 1994.)

Figure 5.4 from Kulicki et al. [7] shows that the new calibrated load and resistance factors and new load models and load distribution techniques work together to produce very narrowly clustered reliability indexes. This was the objective of developing the new factors. Correspondence to a reliability index of 3.5 is something which can now be altered by AASHTO. The target reliability index could be raised or lowered as may be advisable in the future and the factors can be recalculated accordingly. This ability to adjust the design parameters in a coordinated manner is one of the strengths of a probabilistically based reliability design.

## 5.4 Design Objectives

### 5.4.1 Safety

#### 5.4.1.1 Introduction

Public safety is the primary responsibility of the design engineer. All other aspects of design, including serviceability, maintainability, economics, and aesthetics are secondary to the requirement for safety. This does not mean that other objectives are not important, but safety is paramount.

### 5.4.1.2 The Equation of Sufficiency

In design specifications the issue of safety is usually codified by an application of the general statement the design resistances must be greater than, or equal to, the design load effects. In ASD, Eq. (5.1) can be generalized as

$$\Sigma Q_i \le R_E / \text{FS} \tag{5.10}$$

where
$Q_i$ = a load
$R_E$ = elastic resistance
FS = factor of safety

In LFD, Eq. (5.2) can be generalized as

$$\Sigma \gamma_i Q_i \le \phi R \tag{5.11}$$

where
$\gamma_i$ = a load factor
$Q_i$ = a load
$R$ = resistance
$\phi$ = a strength reduction factor

In LRFD, Eq. (5.2) can be generalized as

$$\Sigma \eta_i y_i Q_i \le \phi R_n = R_r \tag{5.12}$$

where
$\eta_i$ = $\eta_D\ \eta_R\ \eta_I$: $\eta_i = \eta_D\ \eta_R\ \eta_I \ge 0.95$ for loads for which a maximum value of $\gamma_i$ is appropriate and $\eta_i = 1/(\eta_I \eta_D \eta_R) \le 1.0$ for loads for which a minimum value of $\gamma_i$ is appropriate
$\gamma_i$ = load factor: a statistically based multiplier on force effects
$\phi$ = resistance factor: a statistically based multiplier applied to nominal resistance
$\eta_i$ = load modifier
$\eta_D$ = a factor relating to ductility
$\eta_R$ = a factor relating to redundancy
$\eta_I$ = a factor relating to operational importance
$Q_i$ = nominal force effect: a deformation, stress, or stress resultant
$R_n$ = nominal resistance: based on the dimensions as shown on the plans and on permissible stresses, deformations, or specified strength of materials
$R_r$ = factored resistance: $\phi R_n$

Eq. (5.12) is applied to each designed component and connection as appropriate for each limit state under consideration.

### 5.4.1.3 Special Requirements of the LRFD Specifications

Comparison of the equation of sufficiency as it was written above for ASD, LFD, and LRFD shows that, as the design philosophy evolved through these three stages, more aspects of the component under design and its relation to its environment and its function to society must be expressly considered. This is not to say that a designer using ASD necessarily considers less than a designer using LFD or LRFD. The specification provisions are the minimum requirements, and prudent designers often consider additional aspects. However, as specifications mature and become more reflective of the real world, additional criteria are often needed to assure adequate safety which may have been provided, albeit nonuniformly, by simpler provisions. Therefore, it is not surprising to find that the LRFD Specifications require explicit consideration of ductility, redundancy, and operational importance in Eq. (5.12), while the Standard Specifications does not.

Ductility, redundancy, and operational importance are significant aspects affecting the margin of safety of bridges. While the first two directly relate to the physical behavior, the last concerns the consequences of the bridge being out of service. The grouping of these aspects is, therefore, arbitrary; however, it constitutes a first effort of codification. In the absence of more precise information, each effect, except that for fatigue and fracture, is estimated as ±5%, accumulated geometrically, a clearly subjective approach. With time, improved quantification of ductility, redundancy, and operational importance, and their interaction, may be attained.

Ductility

The response of structural components or connections beyond the elastic limit can be characterized by either brittle or ductile behavior. Brittle behavior is undesirable because it implies the sudden loss of load-carrying capacity immediately when the elastic limit is exceeded. Ductile behavior is characterized by significant inelastic deformations before any loss of load-carrying capacity occurs. Ductile behavior provides warning of structural failure by large inelastic deformations. Under cyclic loading, large reversed cycles of inelastic deformation dissipate energy and have a beneficial effect on structure response.

If, by means of confinement or other measures, a structural component or connection made of brittle materials can sustain inelastic deformations without significant loss of load-carrying capacity, this component can be considered ductile. Such ductile performance should be verified by experimental testing.

Behavior that is ductile in a static context, but that is not ductile during dynamic response, should also be avoided. Examples of this behavior are shear and bond failures in concrete members and loss of composite action in flexural members.

The ductility capacity of structural components or connections may either be established by full- or large-scale experimental testing, or with analytical models that are based on realistic material behavior. The ductility capacity for a structural system may be determined by integrating local deformations over the entire structural system.

Given proper controls on the innate ductility of basic materials, proper proportioning and detailing of a structural system are the key consideration in ensuring the development of significant, visible, inelastic deformations, prior to failure, at the strength and extreme event limit states.

For the fatigue and fracture limit state for fracture-critical members and for the strength limit state for all members:

$\eta_D$ ≥ 1.05 for nonductile components and connections,
= 1.00 for conventional designs and details complying with these specifications
≥ 0.95 for components and connections for which additional ductility-enhancing measures have been specified beyond those required by these specifications

For all other limit states:

$\eta_D$ = 1.00

Redundancy

Redundancy is usually defined by stating the opposite, e.g., a nonredundant structure is one in which the loss of a component results in collapse or a nonredundant component is one whose loss results in complete or partial collapse. Multiple load path structures should be used, unless there are compelling reasons to the contrary. The LRFD Specifications require additional resistance in order to reduce probability of loss of nonredundant component and to provide additional resistance to accommodate load redistribution.

For the strength limit state:

$\eta_R$ $\geq$ 1.05 for nonredundant members
= 1.00 for conventional levels of redundancy
$\geq$ 0.95 for exceptional levels of redundancy

For all other limit states:

$\eta_R$ = 1.00

The factors currently specified were based solely on judgment and were included to require more explicit consideration of redundancy. Research is under way by Ghosn and Moses [10] to provide more rational requirements based on reliability indexes thought to be acceptable in damaged bridges which must remain in service for a period of about 2 years. The "reverse engineering" concept is being applied to develop values similar in intent to $\eta_R$.

Operational Importance

The concept of operational importance is applied to the strength and extreme event limit states. The owner may declare a bridge, or any structural component or connection, thereof, to be of operational importance. Such classification should be based on social/survival and/or security/defense requirements. If a bridge is deemed of operational importance, $\eta_I$ is taken as $\geq$1.05. Otherwise, $\eta_I$ is taken as 1.0 for typical bridges and may be reduced to 0.95 for relatively less important bridges.

### 5.4.1.4 Design Load Combinations in ASD, LFD, and LRFD

The following permanent and transient loads and forces are considered in the ASD and LFD using the Standard Specifications, and in LRFD using the LRFD Specifications.

The load factors for various loads, making up a design load combination, are indicated in Table 5.4 and Table 5.5 for LRFD and Table 5.6 for ASD and LFD. In the case of the LRFD Specifications, all of the load combinations are related to the appropriate limit state. Any, or all, of the four limit states may be required in the design of any particular component and those which are the minimum necessary for consideration are indicated in the specifications where appropriate. Thus, a design might involve any load combination in Table 5.4.

In the case of ASD or LFD, there is no direct relationship between the load combinations specified in Table 5.6 and limit states, as the design requirements in the Standard Specifications are not organized in that manner. A design by ASD uses those combinations in Table 5.5 indicated for the allowable stress design method as appropriate for the component under consideration. The load combinations indicated for LFD are not used in conjunction with allowable stress design. The opposite is true for LFD.

The application of the load combinations in Table 5.6 for ASD and LFD has been available to bridge designers for decades and is relatively well understood. Numerous textbooks have dealt with these subjects. For this reason, the remainder of this section will deal primarily with the relatively newer LRFD Specifications.

All relevant subsets of the load combinations in Table 5.4 should be investigated. The factors should be selected to produce the total factored extreme force effect. For each load combination, both positive and negative extremes should be investigated. In load combinations where one force effect decreases the effect of another, the minimum value should be applied to load reducing the force effect. For each load combination, every load that is indicated, including all significant effects due to distortion, should be multiplied by the appropriate load factor.

It can be seen in Table 5.4 that some of the load combinations have a choice of two load factors. The larger of the two values for load factors shown for TU, TG, CR, SH, and SE are to be used when calculating deformations; the smaller value should be used when calculating all other force

**TABLE 5.3** Load Designations

| Name of Load | LRFD Designation | Standard of Specification Designation |
|---|---|---|
| | Permanent Loads | |
| Downdrag | DD | |
| Dead load of structural components attachments | DC | D |
| Dead load of wearing surfaces and utilities | DW | D |
| Dead load of earth fill | EF | D |
| Horizontal earth pressure | EH | E |
| Earth surcharge load | ES | E |
| Vertical earth pressure | EV | D |
| | Transient Loads | |
| Vehicular braking force | BR | LF |
| Vehicular centrifugal force | CE | CF |
| Creep | CR | R |
| Vehicular collision force | CT | — |
| Vessel collision force | CV | — |
| Earthquake | EQ | EQ |
| Friction | FR | — |
| Ice load | IC | ICE |
| Vehicular dynamic load allowance | IM | I |
| Vehicular live load | LL | L |
| Live–load surcharge | LS | L |
| Pedestrian live load | PL | L |
| Settlement | SE | — |
| Shrinkage | SH | S |
| Temperature gradient | TG | — |
| Uniform temperature | TU | T |
| Water load and stream pressure | WA | SF |
| Wind on live load | WL | WL |
| Wind load on structure | WS | W |

**TABLE 5.4** Load Combinations and Load Factors in LRFD

| Limit State Load Combinations | DC DD DW EH EV ES | LL IM CE BR PL LS | WA | WS | WL | FR | TU CR SH | TG | SE | Use One of These at a Time | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | EQ | IC | CT | CV |
| Strength I | $\gamma_p$ | 1.75 | 1.00 | — | — | 1.00 | 0.50/1.20 | $\gamma_{TG}$ | $\gamma_{SE}$ | — | — | — | — |
| Strength II | $\gamma_p$ | 1.35 | 1.00 | — | — | 1.00 | 0.50/1.20 | $\gamma_{TG}$ | $\gamma_{SE}$ | — | — | — | — |
| Strength III | $\gamma_p$ | — | 1.00 | 1.40 | — | 1.00 | 0.50/1.20 | $\gamma_{TG}$ | $\gamma_{SE}$ | — | — | — | — |
| Strength IV | | | | | | | | | | | | | |
| EH, EV, ES, DW | $\gamma_p$ | — | 1.00 | — | — | 1.00 | 0.50/1.20 | — | — | — | — | — | — |
| DC only | 1.5 | | | | | | | | | — | — | — | — |
| Strength V | $\gamma_p$ | 1.35 | 1.00 | 0.40 | 0.40 | 1.00 | 0.50/1.20 | $\gamma_{TG}$ | $\gamma_{SE}$ | — | — | — | — |
| Extreme Event I | $\gamma_p$ | $\gamma_{EQ}$ | 1.00 | — | — | 1.00 | — | — | — | 1.00 | — | — | — |
| Extreme Event II | $\gamma_p$ | 0.50 | 1.00 | — | — | 1.00 | — | — | — | — | 1.00 | 1.00 | 1.00 |
| Service I | 1.00 | 1.00 | 1.00 | 0.30 | 0.30 | 1.00 | 1.00/1.20 | $\gamma_{TG}$ | $\gamma_{SE}$ | — | — | — | — |
| Service II | 1.00 | 1.30 | 1.00 | — | — | 1.00 | 1.00/1.20 | — | — | — | — | — | — |
| Service III | 1.00 | 0.80 | 1.00 | — | — | 1.00 | 1.00/1.20 | $\gamma_{TG}$ | $\gamma_{SE}$ | — | — | — | — |
| Fatigue LL, IM and CE only | — | 0.75 | — | — | — | — | — | — | — | — | — | — | — |

**TABLE 5.5** Load Factors for Permanent Loads, $\gamma_p$ in LRFD

| Type of Load | Load Factor | |
|---|---|---|
| | Maximum | Minimum |
| DC: Component and attachments | 1.25 | 0.90 |
| DD: Downdrag | 1.80 | 0.45 |
| DW: Wearing surfaces and utilities | 1.50 | 0.65 |
| EH: Horizontal earth pressure | | |
| • Active | 1.50 | 0.90 |
| • At rest | 1.35 | 0.90 |
| EV: Vertical earth pressure | | |
| • Overall stability | 1.35 | N/A |
| • Retaining structure | 1.35 | 1.00 |
| • Rigid buried structure | 1.30 | 0.90 |
| • Rigid frames | 1.35 | 0.90 |
| • Flexible buried structures other than metal box culverts | 1.95 | 0.90 |
| • Flexible metal box culverts | 1.50 | 0.90 |
| ES: Earth surcharge | 1.50 | 0.75 |

effects. Where movements are calculated for the sizing of expansion dams, the design of bearing, or similar situations where consideration of unexpectedly large movements is advisable, the larger factor should be used. When considering the effect of these loads on forces that are compatibility generated, the lower factor may be used. This latter use requires structural insight.

Consideration of the variability of loads in nature indicates that loads may be either larger or smaller than the nominal load used in the design specifications. While the concept of variability of permanent loads receives little coverage in ASD, it is codified expressly in LFD. Note that in Table 5.6 the LFD load combinations contain a dead load modifier, indicated as $\beta_E$ or $\beta_D$. These $\beta$ terms are not to be confused with the reliability index, heretofore referred to as $\beta$. The purpose of the modifying factors $\beta_E$ and $\beta_D$ is to account for conditions where it is inadvisable to consider either that all of the dead load exists all of the time or that the dead load may be less than the nominal values indicated in the specifications. Thus, for example, the use of the $\beta_D$ factor 0.75 when checking members for minimum axial load maximum moment means when designing columns and those fixtures which abut the columns, such as footings, it is necessary to evaluate not just the maximum bending moment and the maximum axial load, based on assuming that all the elements of a load combination are thought to obtain their maximum values, but also a load combination in which it is assumed that the dead load is lighter than the nominal load. In the case where the majority of the axial load comes from the dead load and the majority of the bending moment comes from lateral load or live load, this modified combination will tend to produce a maximum eccentricity and hence could control the design of columns and footings.

The specified values of $\beta_E$ are given below:

$\beta_E$ 1.00 for vertical and lateral loads on all other structures
$\beta_E$ 1.3 for lateral earth pressure for retaining walls and rigid frames, excluding rigid culverts; for lateral at-rest earth pressures, $\beta_E = 1.15$
$\beta_E$ 0.5 for lateral earth pressure when checking positive moments in rigid frames; this complies with Section 3.20
$\beta_E$ 1.0 for vertical earth pressure
$\beta_D$ 0.75 when checking member for minimum axial load and maximum moment or maximum eccentricity — for column design
$\beta_D$ 1.0 when checking member for maximum axial load and minimum moment — for column design
$\beta_D$ 1.0 for flexural and tension members
$\beta_E$ 1.0 for rigid culverts
$\beta_E$ 1.5 for flexible culverts

**TABLE 5.6** Table of Coefficients $\gamma$ and $\beta$ in ASD and LFD

| | | Col. No. | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 3A | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| | | | $\beta$ Factors | | | | | | | | | | | | | |
| | Group | $\gamma$ | D | $(L+I)_n$ | $(L+I)_p$ | CF | E | B | SF | W | WL | LF | R + S + T | EQ | Ice | % |
| SERVICE LOAD | I | 1.0 | 1 | 1 | 0 | 1 | $\beta_E$ | B | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 100 |
| | IA | 1.0 | 1 | 2 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 150 |
| | IB | 1.0 | 1 | 0 | 1 | 1 | $\beta_E$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | [b] |
| | II | 1.0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 125 |
| | III | 1.0 | 1 | 1 | 0 | 1 | $\beta_E$ | 1 | 1 | 0.3 | 1 | 1 | 0 | 0 | 0 | 125 |
| | IV | 1.0 | 1 | 1 | 0 | 1 | $\beta_E$ | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 125 |
| | V | 1.0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 140 |
| | VI | 1.0 | 1 | 1 | 0 | 1 | $\beta_E$ | 1 | 1 | 0.3 | 1 | 1 | 1 | 0 | 0 | 140 |
| | VII | 1.0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 133 |
| | VIII | 1.0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 140 |
| | IX | 1.0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 150 |
| | X | 1.0 | 1 | 1 | 0 | 0 | $\beta_E$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100 |
| LOAD FACTOR DESIGN | I | 1.3 | $\beta_D$ | 1.67[a] | 0 | 1.0 | $\beta_E$ | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | NOT APPLICABLE |
| | IA | 1.3 | $\beta_D$ | 2.20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | IB | 1.3 | $\beta_D$ | 0 | 1 | 1.0 | $\beta_E$ | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | II | 1.3 | $\beta_D$ | 0 | 0 | 0 | $\beta_E$ | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | |
| | III | 1.3 | $\beta_D$ | 1 | 0 | 1 | $\beta_E$ | 1 | 1 | 0.3 | 1 | 1 | 0 | 0 | 0 | |
| | IV | 1.3 | $\beta_D$ | 1 | 0 | 1 | $\beta_E$ | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | |
| | V | 1.25 | $\beta_D$ | 0 | 0 | 0 | $\beta_E$ | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | |
| | VI | 1.25 | $\beta_D$ | 1 | 0 | 1 | $\beta_E$ | 1 | 1 | 0.3 | 1 | 1 | 1 | 0 | 0 | |
| | VII | 1.3 | $\beta_D$ | 0 | 0 | 0 | $\beta_E$ | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | |
| | VIII | 1.3 | $\beta_D$ | 1 | 0 | 1 | $\beta_E$ | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | |
| | IX | 1.20 | $\beta_D$ | 0 | 0 | 0 | $\beta_E$ | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | |
| | X | 1.30 | 1 | 1.67 | 0 | 0 | $\beta_E$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |

- $(L + I)_n$ = Live load plus impact for AASHTO Highway H or HS loading.
- $(L + I)_p$ = Live load plus impact consistent with the overload criteria of the operation agency.
- % (col. 14) = percentage of basic unit stress.
- No increase in allowable unit stresses shall be permitted for members or connections carrying wind loads only.

[a] 1.25 may be used for design of outside roadway beam when combination of sidewalk live load, and traffic live load plus impact governs the design, but the capacity of the section should not be less than required for highway traffic live load only, using a $\beta$ factor of 1.67. 1.00 may be used for design of deck slab with combination of loads as described in Article 3.24.2.2.

[b] $$\text{Percentage} = \frac{\text{Maximum Unit Stress (Operating Rating)}}{\text{Allowable Basic Unit Stress}} \times 100.$$

The LRFD Specifications recognize the variability of permanent loads by providing both maximum and minimum load factors for the permanent loads, as indicated in Table 5.5. For permanent force effects, the load factor that produces the more critical combination should be selected from Table 5.4. In the application of permanent loads, force effects for each of the specified six load types should be computed separately. Assuming variation of one type of load by span, length, or component within a bridge is not necessary. For each force effect, both extreme combinations may need to be investigated by applying either the high or the low load factor, as appropriate. The algebraic sums of these products are the total force effects for which the bridge and its components should be designed. This reinforces the traditional method of selecting load combinations to obtain realistic extreme effects.

When the permanent load increases the stability or load-carrying capacity of a component or bridge, the minimum value of the load factor for that permanent load should also be investigated. Uplift, which is treated as a separate load case in past editions of the AASHTO Standard Specifications for Highway Bridges, becomes a Strength I load combination. For example, when the dead-load reaction is positive and live load can cause a negative reaction, the load combination for maximum uplift force would be 0.9DC + 0.65DW + 1.75(LL+IM). If both reactions were negative, the load combination would be 1.25DC + 1.50DW + 1.75(LL+IM).

The load combinations for various limit states shown in Table 5.4 are described below.

| | |
|---|---|
| Strength I | Basic load combination relating to the normal vehicular use of the bridge without wind. |
| Strength II | Load combination relating to the use of the bridge by permit vehicles without wind. If a permit vehicle is traveling unescorted, or if control is not provided by the escorts, the other lanes may be assumed to be occupied by the vehicular live load herein specified. For bridges longer than the permit vehicle, addition of the lane load, preceding and following the permit load in its lane, should be considered. |
| Strength III | Load combination relating to the bridge exposed to maximum wind velocity which prevents the presence of significant live load on the bridge. |
| Strength IV | Load combination relating to very high ratios of dead load to live load force effect. This calibration process had been carried out for a large number of bridges with spans not exceeding 60 m. Spot checks had also been made on a few bridges up to 180 m spans. For the primary components of large bridges, the ratio of dead and live load force effects is rather high and could result in a set of resistance factors different from those found acceptable for small- and medium-span bridges. It is believed to be more practical to investigate one more load case, rather than requiring the use of two sets of resistance factors with the load factors provided in Strength I, depending on other permanent loads present. This Load Combination IV is expected to govern when the ratio of dead load to live load force effect exceeds about 7.0. |
| Strength V | Load combination relating to normal vehicular use of the bridge with wind of 90 km/h velocity. |
| Extreme Event I | Load combination relating to earthquake. The designer-supplied live-load factor signifies a low probability of the presence of maximum vehicular live load at the time when the earthquake occurs. In ASD and LFD the live load is ignored when designing for earthquake. |
| Extreme Event II | Load combination relating to reduced live load in combination with a major ice event, or a vessel collision, or a vehicular impact. |
| Service I | Load combination relating to the normal operational use of the bridge with 90 km/h wind. All loads are taken at their nominal values and extreme load conditions are excluded. This combination is also used for checking deflection of certain buried structures and for the investigation of slope stability. |
| Service II | Load combination whose objective is to prevent yielding of steel structures due to vehicular live load, approximately halfway between that used for Service I and Strength I limit state, for which case the effect of wind is of no significance. This load combination corresponds to the overload provision for steel structures in past editions of the AASHTO Standard Specifications for the Design of Highway Bridges. |
| Service III | Load combination relating only to prestressed concrete structures with the primary objective of crack control. The addition of this load combination followed a series of trial designs done by 14 states and several industry groups during 1991 and early 1992. Trial designs for prestressed concrete elements indicated significantly more prestressing would be needed to support the loads specified in the proposed specifications. There is no nationwide physical evidence that these vehicles used to develop the notional live loads have caused detrimental cracking in existing prestressed concrete components. The statistical significance |

of the 0.80 factor on live load is that the event is expected to occur about once a year for bridges with two design lanes, less often for bridges with more than two design lanes, and about once a day for the bridges with a single design lane.

Fatigue — Fatigue and fracture load combination relating to gravitational vehicular live load and dynamic response, consequently BR and PL need not be considered. The load factor reflects a load level which has been found to be representative of the truck population, with respect to large number of return cycles.

### 5.4.2 Serviceability

The LRFD Specification treats serviceability from the view points of durability, inspectibility, maintainability, rideability, deformation control, and future widening.

Contract documents should call for high-quality materials and require that those materials that are subject to deterioration from moisture content and/or salt attack be protected. Inspectibility is to be assured through adequate means for permitting inspectors to view all parts of the structure which have structural or maintenance significance. The provisions related to inspectibility are relatively short, but as all departments of transportation have begun to realize, bridge inspection can be very expensive and is a recurring cost due to the need for biennial inspections. Therefore, the cost of providing walkways and other access means and adequate room for people and inspection equipment to be moved about on the structure is usually a good investment.

Maintainability is treated in the specification in a manner similar to durability; there is a list of desirable attributes to be considered.

The subject of live-load deflections and other deformations remains a very difficult issue. On the one hand, there is very little direct correlation between live-load deflection and premature deterioration of bridges. There is much speculation that "excessive" live-load deflection contributes to premature deck deterioration, but, to date (late 1997), no causative relationship has been statistically established.

Rider comfort is often advanced as a basis for deflection control. Studies in human response to motion have shown that it is not the magnitude of the motion, but rather the acceleration that most people perceive, especially in moving vehicles. Many people have experienced the sensation of being on a bridge and feeling a definite movement, especially when traffic is stopped. This movement is often related to the movement of floor systems, which are really quite small in magnitude, but noticeable nonetheless. There being no direct correlation between magnitude (not acceleration) of movement and discomfort has not prevented the design profession from finding comfort in controlling the gross stiffness of bridges through a deflection limit. As a compromise between the need for establishing comfort levels and the lack of compelling evidence that deflection was a cause of structural distress, the deflection criteria, other than those pertaining to relative deflections of ribs of orthotropic decks and components of some wood decks, were written as voluntary provisions to be activated by those states that so chose. Deflection limits, stated as span divided by some number, were established for most cases, and additional provisions of absolute relative displacement between planks and panels of wooden decks and ribs of orthotropic decks were also added. Similarly, optional criteria were established for a span-to-depth ratio for guidance primarily in starting preliminary designs, but also as a mechanism for checking when a given design deviated significantly from past successful practice.

### 5.4.3 Constructibility

Several new provisions were included in the LRFD Specification related to:

- The need to design bridges so that they can be fabricated and built without undue difficulty and with control over locked-in construction force effects;

- The need to document one feasible method of construction in the contract documents, unless the type of construction is self-evident; and
- A clear indication of the need to provide strengthening and/or temporary bracing or support during erection, but not requiring the complete design thereof.

## References


1. American Association of State Highway and Transportation Officials, *Standard Specifications for Highway Bridges,* 16th ed., AASHTO, Washington, D.C., 1996.
2. American Association of State Highway and Transportation Officials, *Load Resistance Factor Design*, AASHTO, Washington, D.C., 1996.
3. Ontario Ministry of Transportation and Communications, *Ontario Highway Bridge Design Code,* OMTC, Toronto, Ontario, Canada, 1994.
4. Canadian Standards Association, *Canadian Highway Bridge Design Code,* Canadian Standards Association, Rexdale, Ontario, Canada, 1998.
5. Vincent, G. S., Load factor design of steel highway bridges, *AISI Bull.,* 15, March, 1969.
6. American Association of State Highway and Transportation Officials, *Guide Specification and Commentary for Vessel Collision Design of Highway Bridges,* Vol. I: Final Report, AASHTO, Washington, D.C., February 1991.
7. Kulicki, J. M., Mertz, D. R., and Wassef, W. G., LRFD Design of Highway Bridges, NHI Course 13061, Federal Highway Administration, Washington, D.C., 1994.
8. Nowak, A. S., Calibration of LRFD Bridge Design Code, Department of Civil and Environmental Engineering Report UMCE 92-25, University of Michigan, Ann Arbor, 1993.
9. Nowak, A. S. and Lind, N. C., Practical bridge code calibration, *ASCE J. Struct. Div.*, 105 (ST12), 2497–2510, 1979.
10. Ghosn, M. and Moses, F., Redundancy in Highway Bridge Superstructures, Draft Report to NCHRP, February 1997.

Hida, S.E. "Highway Bridge Loads and Load Distribution."
*Bridge Engineering Handbook.*
Ed. Wai-Fah Chen and Lian Duan
Boca Raton: CRC Press, 2000

# 6

# Highway Bridge Loads and Load Distribution



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## 6.1 Introduction

This chapter deals with highway bridge loads and load distribution as specified in the AASHTO Load and Resistance Factor Design (LRFD) Specifications [1]. Stream flow, ice loads, vessel collision loads, loads for barrier design, loads for anchored and mechanically stabilized walls, seismic forces, and loads due to soil–structure interaction will be addressed in subsequent chapters. Load combinations are discussed in Chapter 5.

When proceeding from one component to another in bridge design, the controlling load and the controlling factored load combination will change. For example, permit vehicles, factored and combined for one load group, may control girder design for bending in one location. The standard design vehicular live load, factored and combined for a different load group, may control girder design for shear in another location. Still other loads, such as those due to seismic events, may control column and footing design.

Note that in this chapter, superstructure refers to the deck, beams or truss elements, and any other appurtenances above the bridge soffit. Substructure refers to those components that support loads from the superstructure and transfer load to the ground, such as bent caps, columns, pier walls, footings, piles, pile extensions, and caissons. Longitudinal refers to the axis parallel to the direction of traffic. Transverse refers to the axis perpendicular to the longitudinal axis.

## 6.2 Permanent Loads

The LRFD Specification refers to the weights of the following as "permanent loads":

- The structure
- Formwork which becomes part of the structure
- Utility ducts or casings and contents
- Signs
- Concrete barriers
- Wearing surface and/or potential deck overlay(s)
- Other elements deemed permanent loads by the design engineer and owner
- Earth pressure, earth surcharge, and downdrag

The permanent load is distributed to the girders by assigning to each all loads from superstructure elements within half the distance to the adjacent girder. This includes the dead load of the girder itself and the soffit, in the case of box girder structures. The dead loads due to concrete barrier, sidewalks and curbs, and sound walls, however, may be equally distributed to all girders.

## 6.3 Vehicular Live Loads

The design vehicular live load was replaced in 1993 because of heavier truck configurations on the road today, and because a statistically representative, notional load was needed to achieve a "consistent level of safety." The notional load that was found to best represent "exclusion vehicles," i.e., trucks with loading configurations greater than allowed but routinely granted permits by agency bridge rating personnel, was adopted by AASHTO and named "Highway Load '93" or HL93. The mean and standard deviation of truck traffic was determined and used in the calibration of the load factors for HL93. It is notional in that it does not represent any specific vehicle [2].

The distribution of loads per the LRFD Specification is more complex than in the Standard Specifications for Highway Bridge Design [3]. This change is warranted because of the complexity in bridges today, increased knowledge of load paths, and technology available to be more rational in performing design calculations. The end result will be more appropriately designed structures.

### 6.3.1 Design Vehicular Live Load

The AASHTO "design vehicular live load," HL93, is a combination of a "design truck" or "design tandem" and a "design lane." The design truck is the former Highway Semitrailer 20-ton design truck (HS20-44) adopted by AASHO (now AASHTO) in 1944 and used in the previous Standard Specification. Similarly, the design lane is the HS20 lane loading from the AASHTO Standard Specifications. A shorter, but heavier, design tandem is new to AASHTO and is combined with the design lane if a worse condition is created than with the design truck. Superstructures with very short spans, especially those less than 12 m in length, are often controlled by the tandem combination.

The AASHTO design truck is shown in Figure 6.1. The variable axle spacing between the 145 kN loads is adjusted to create a critical condition for the design of each location in the structure. In the transverse direction, the design truck is 3 m wide and may be placed anywhere in the standard 3.6-m-wide lane. The wheel load, however, may not be positioned any closer than 0.6 m from the lane line, or 0.3 m from the face of curb, barrier, or railing.

The AASHTO design tandem consists of two 110-kN axles spaced at 1.2 m on center. The AASHTO design lane loading is equal to 9.3 N/mm and emulates a caravan of trucks. Similar to the truck loading, the lane load is spread over a 3-m-wide area in the standard 3.6-m lane. The lane loading is not interrupted except when creating an extreme force effect such as in "patch" loading of alternate spans. Only the axles contributing to the extreme being sought are loaded.

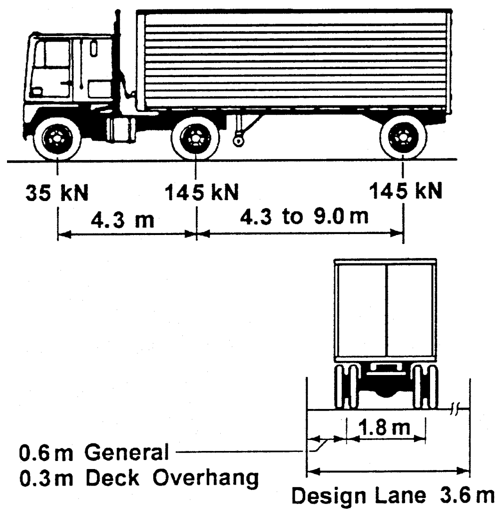



**FIGURE 6.1** AASHTO-LRFD design truck. (AASHTO LRFD Bridge Design Specifications 2nd. ed., American Association of State Highway and Transportation Officials. Washington, D.C., 1998. With permission.)

When checking an extreme reaction at an interior pier or negative moment between points of contraflexure in the superstructure, two design trucks with a 4.3-m spacing between the 145-kN axles are to be placed on the bridge with a minimum of 15 m between the rear axle of the first truck and the lead axle of the second truck. Only 90% of the truck and lane load is used. This procedure differs from the Standard Specification which used shear and moment riders.

### 6.3.2 Permit Vehicles

Most U.S. states have developed their own "Permit Design Vehicle" to account for vehicles routinely granted permission to travel a given route, despite force effects greater than those due the design truck, i.e., the old HS20 loading. California uses anywhere from a 5- to 13-axle design vehicle as shown in Figure 6.2 [4]. Some states use an HS25 design truck, the configuration being identical to the HS20 but axle loads 25% greater.

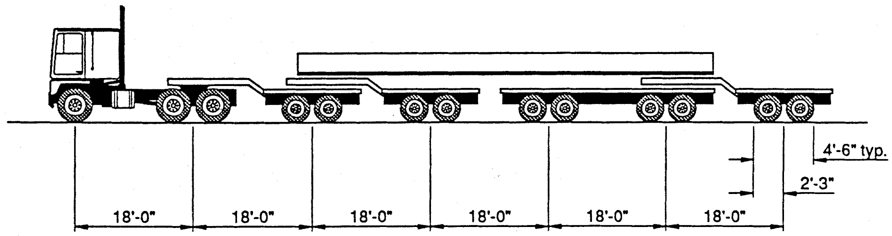
The permit vehicular live load is combined with other loads in the Strength Limit State II as discussed in Chapter 5. Early editions of the AASHTO Specifications expect the design permit vehicle to be preceded and proceeded by a lane load. Furthermore, adjacent lanes may be loaded with the new HL93 load, unless restricted by escort vehicles.

### 6.3.3 Fatigue Loads

For fatigue loading, the LRFD Specification uses the design truck alone with a constant axle spacing of 9 m. The load is placed to produce extreme force effects. In lieu of more exact information, the frequency of the fatigue load for a single lane may be determined by multiplying the average daily truck traffic by $p$, where $p$ is 1.00 in the case of one lane available to trucks, 0.85 in the case of two lanes available to trucks, and 0.80 in the case of three or more lanes available to trucks. If the average daily truck traffic is not known, 20% of the average daily traffic may be used on rural interstate bridges, 15% for other rural and urban interstate bridges, and 10% for bridges in urban areas.

### 6.3.4 Load Distribution for Superstructure Design

Figure 6.3 summarizes load distribution for design of longitudinal superstructure elements. Load distribution tables and the "lever rule" are approximate methods and intended for most designs.




| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| P5 | 26K | 48K | 48K | — | — | — | — | Min. Veh. |
| P7 | 26K | 48K | 48K | 48K | — | — | — | |
| P9 | 26K | 48K | 48K | 48K | 48K | — | — | |
| P11 | 26K | 48K | 48K | 48K | 48K | 48K | — | |
| P13 | 26K | 48K | 48K | 48K | 48K | 48K | 48K | Max. Veh. |

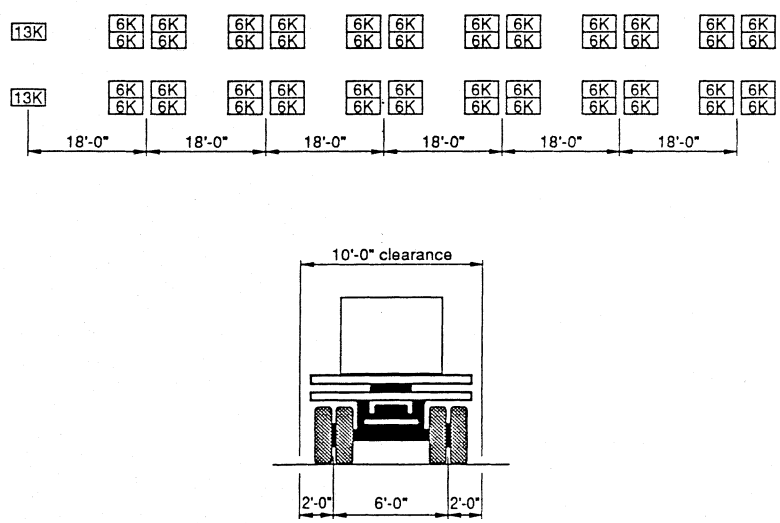



**FIGURE 6.2** Caltrans permit truck. (AASHTO LRFD Bridge Design Specifications 2nd. ed., American Association of State Highway and Transportation Officials. Washington, D.C., 1998. With permission.)

The lever rule considers the slab between two girders to be simply supported. The reaction is determined by summing the reactions from the slabs on either side of the beam under consideration. "Refined analysis" refers to a three-dimensional consideration of the loads and is to be used on more complex structures. In other words, classical force and displacement, finite difference, finite element, folded plate, finite strip, grillage analogy, series/harmonic, or yield line methods are required to obtain load effects for superstructure design.

Note that, by definition of the vehicular design live load, no more than one truck can be in one lane simultaneously, except as previously described to generate maximum reactions or negative moments. After forces have been determined from the longitudinal load distribution and the longitudinal members have been designed, the designer may commence load distribution in the transverse direction for deck and substructure design.

### 6.3.4.1 Decks

Decks may be designed for vehicular live loads using empirical methods or by distributing loads on to "effective strip widths" and analyzing the strips as continuous or simply supported beams.

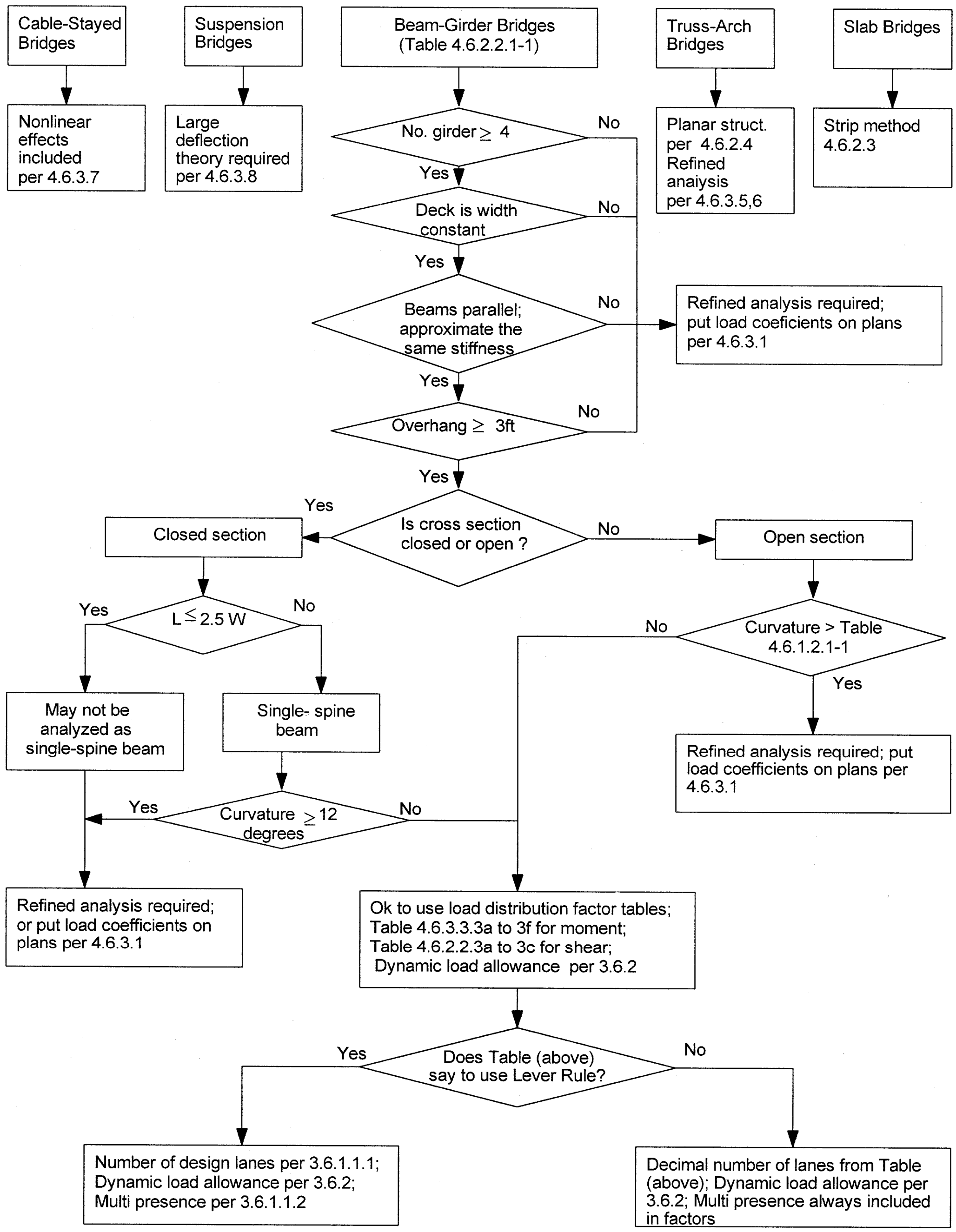



**FIGURE 6.3** Live-load distribution for superstructure design.

Empirical methods rely on transfer of forces by arching of the concrete and shifting of the neutral axis. Loading is discussed in Chapter 24, Bridge Decks and Approach Slabs.

### 6.3.4.2 Beam–Slab Bridges

Approximate methods for load distribution on beam–slab bridges are appropriate for the types of cross sections shown in Table 4.6.2.2.1-1 of the AASHTO LRFD Specification. Load distribution

**TABLE 6.1** Reduction of Load Distribution Factors for Moment in Longitudinal Beams on Skewed Supports

| Type of Superstructure | Applicable Cross Section from Table 4.6.2.2.1-1 | Any Number of Design Lanes Loaded | Range of Applicability |
|---|---|---|---|
| Concrete deck, filled grid, or partially filled grid on steel or concrete beams, concrete T-beams, or double T-sections | a, e, k<br>i, j, if sufficiently connected to act as a unit | $1 - c_1(\tan\theta)^{1.5}$<br>$c_1 = 0.25\left(\frac{K_g}{Lt_s^3}\right)^{0.25}\left(\frac{S}{L}\right)^{0.25}$<br>if $\theta < 30°$, then $c_1 = 0$<br>if $\theta > 60°$, use $\theta = 60°$ | $30° \le \theta \le 60°$<br>$1100 \le S \le 4900$<br>$6000 \le L \le 73{,}000$<br>$N_b \ge 4$ |
| Concrete deck on concrete spread box beams, concrete box beams, and double T-sections used in multibeam decks | b, c, f, g | $1.05 - 0.25 \tan\theta \le 1.0$<br>if $\theta > 60°$, use $\theta = 60°$ | $0 \le \theta \le 60°$ |

*Source*: AASHTO LRFD Bridge Design Specifications, 2nd. ed., American Association of State Highway and Transportation Officials. Washington, D.C., 1998. With permission.

factors, generated from expressions found in AASHTO LRFD Tables 4.6.2.2.2a–f and 4.6.2.2.3a–c, result in a decimal number of lanes and are used for girder design. Three-dimensional effects are accounted for. These expressions are a function of beam area, beam width, beam depth, overhang width, polar moment of inertia, St. Venant's torsional constant, stiffness, beam span, number of beams, number of cells, beam spacing, depth of deck, and deck width. Verification was done using detailed bridge deck analysis, simpler grillage analyses, and a data set of approximately 200 bridges of varying type, geometry, and span length. Limitations on girder spacing, span length, and span depth reflect the limitations of this data set.

The load distribution factors for moment and shear at the obtuse corner are multiplied by skew factors as shown in Tables 6.1 and 6.2, respectively.

### 6.3.4.3 Slab-Type Bridges

Cast-in-place concrete slabs or voided slabs, stressed wood decks, and glued/spiked wood panels with spreader beams are designed for an equivalent width of longitudinal strip per lane for both shear and moment. That width, $E$ (mm), is determined from the formula:

$$E = 250 + 0.42\sqrt{L_1 W_1} \tag{6.1}$$

when one lane is loaded, and

$$E = 2100 + 0.12\sqrt{L_1 W_1} \le W/N_L \tag{6.2}$$

when more than one lane is loaded. $L_1$ is the lesser of the actual span or 18,000 mm, $W_1$ is the lesser of the edge-to-edge width of bridge and 18,000 mm in the case of single-lane loading, and 18,000 mm in the case of multilane loading, and $N_L$ is the numbr of design lanes.

## 6.3.5 Load Distribution for Substructure Design

Bridge substructure includes bent caps, columns, pier walls, pile caps, spread footings, caissons, and piles. These components are designed by placing one or more design vehicular live loads on the traveled way as previously described for maximum reaction and negative bending moment, not exceeding the maximum number of vehicular lanes permitted on the bridge. This maximum may be determined by dividing the width of the traveled way by the standard lane width (3.6 m), and "rounding down," i.e., disregarding any fractional lanes. Note that (1) the traveled way need not be

**TABLE 6.2** Correction Factors for Load Distribution Factors for Support Shear of the Obtuse Corner

| Type of Superstructue | Applicable Cross Section from Table 4.6.2.2.1-1 | Correction Factor | Range of Applicability |
|---|---|---|---|
| Concrete deck, filled grid, or partially filled grid on steel or concrete beams, concrete T-beams or double T-sections | a, e, k<br>i, j, if sufficiently connected to act as a unit | —<br>$1.0+2.0\left(\frac{Lt_s^3}{K_g}\right)^{0.3}\tan\theta$ | $0° \le \theta \le 60°$<br>$1100 \le S \le 4900$<br>$6000 \le L \le 73{,}000$<br>$N_b \ge 4$ |
| Multicell concrete box beams, box sections | d | $1.0+\left[0.25+\frac{L}{70d}\right]\tan\theta$ | $0° \le \theta \le 60°$<br>$1800 \le S \le 4000$<br>$6000 \le L \le 73000$<br>$900 \le d \le 2700$<br>$N_b \ge 3$ |
| Concrete deck on spread concrete box beams | b, c | $1.0+\frac{\sqrt{Ld}}{6S}\tan\theta$ | $0° \le \theta \le 60°$<br>$1800 \le S \le 3500$<br>$6000 \le L \le 43{,}000$<br>$450 \le d \le 1700$<br>$N_b \ge 3$ |
| Concrete box beams used in multibeam decks | f, g | $1.0+\frac{L\sqrt{\tan\theta}}{90d}$ | $0° \le \theta \le 60°$<br>$6000 \le L \le 37{,}000$<br>$430 \le d \le 1500$<br>$900 \le b \le 1500$<br>$5 \le N_b \le 20$ |

*Source*: AASHTO LRFD Bridge Design Specifications, 2nd. ed., American Association of State Highway and Transportation Officials. Washington, D.C., 1998. With permission.

measured from the edge of deck if curbs or traffic barriers will restrict the traveled way for the life of the structure and (2) the fractional number of lanes determined using the previously mentioned load distribution charts for girder design is not used for substructure design.

Figure 6.4 shows selected load configurations for substructure elements. A critical load configuration may result from not using the maximum number of lanes permissible. For example, Figure 6.4a shows a load configuration that may generate the critical loads for bent cap design and Figure 6.4b shows a load configuration that may generate the critical bending moment for column design. Figure 6.4c shows a load configuration that may generate the critical compressive load for design of the piles. Other load configurations will be needed to complete design of a bridge footing. Note that girder locations are often ignored in determination of substructure design moments and shears: loads are assumed to be transferred directly to the structural support, disregarding load transfer through girders in the case of beam–slab bridges. Adjustments are made to account for the likelihood of fully loaded vehicles occurring side-by-side simultaneously. This "multiple presence factor" is discussed in the next section.

In the case of rigid frame structures, bending moments in the longitudinal direction will also be needed to complete column (or pier wall) as well as foundation designs. Load configurations which generate these three cases must be checked:

1. Maximum/minimum axial load with associated transverse and longitudinal moments;
2. Maximum/minimum transverse moment with associated axial load and longitudinal moment;
3. Maximum/minimum longitudinal moment with associated axial load and transverse moment.

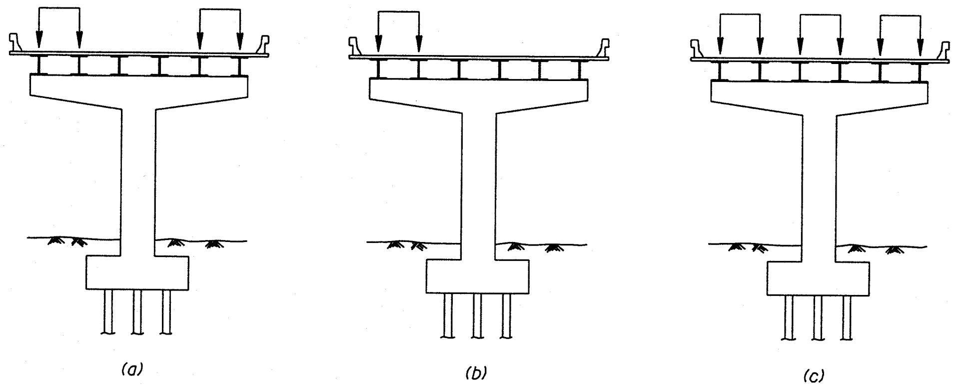



**FIGURE 6.4** Various load configurations for substructure design.

**TABLE 6.3** Multiple Presence Factors

| Number of Loaded Lanes | Multiple Presence Factors *m* |
|---|---|
| 1 | 1.20 |
| 2 | 1.00 |
| 3 | 0.85 |
| >3 | 0.65 |

*Source*: AASHTO LRFD Bridge Design Specifications, 2nd. ed., American Association of State Highway and Transportation Officials. Washington, D.C., 1998. With permission.

If a permit vehicle is also being designed for, then these three cases must also be checked for the load combination associated with Strength Limit State II (discussed in Chapter 5).

## 6.3.6 Multiple Presence of Live-Load Lanes

Multiple presence factors modify the vehicular live loads for the probability that vehicular live loads occur together in a fully loaded state. The factors are shown in Table 6.3.

These factors should be applied prior to analysis or design only when using the lever rule or doing three-dimensional modeling or working with substructures. Sidewalks greater than 600 mm can be treated as a fully loaded lane. If a two-dimensional girder line analysis is being done and distribution factors are being used for a beam-and-slab type of bridge, multiple presence factors are not used because the load distribution factors already consider three-dimensional effects. For the fatigue limit state, the multiple presence factors are also not used.

## 6.3.7 Dynamic Load Allowance

Vehicular live loads are assigned a "dynamic load allowance" load factor of 1.75 at deck joints, 1.15 for all other components in the fatigue and fracture limit state, and 1.33 for all other components and limit states. This factor accounts for hammering when riding surface discontinuities exist, and long undulations when settlement or resonant excitation occurs. If a component such as a footing is completely below grade or a component such as a retaining wall is not subject to vertical reactions from the superstructure, this increase is not taken. Wood bridges or any wood component is factored at a lower level, i.e., 1.375 for deck joints, 1.075 for fatigue, and 1.165 typical, because of the energy-absorbing characteristic of wood. Likewise, buried structures such as culverts are subject to the dynamic load allowance but are a function of depth of cover, $D_E$ (mm):

$$IM = 40(1.0 - 4.1 \times 10^{-4} D_E) \geq 0\% \tag{6.3}$$

### 6.3.8 Horizontal Loads Due to Vehicular Traffic

Substructure design of vertical elements requires that horizontal effects of vehicular live loads be designed for. Centrifugal forces and braking effects are applied horizontally at a distance 1.80 m above the roadway surface. The centrifugal force is determined by multiplying the design truck or design tandem — alone — by the following factor:

$$C = \frac{4v^2}{3gR} \tag{6.4}$$

Highway design speed, $v$, is in m/s; gravitational acceleration, $g$, is 9.807 m/s$^2$; and radius of curvature in traffic lane, $R$, is in m. Likewise, the braking force is determined by multiplying the design truck or design tandem from all lanes likely to be unidirectional in the future, by 0.25. In this case, the lane load is not used because braking effects would be damped out on a fully loaded lane.

## 6.4 Pedestrian Loads

Live loads also include pedestrians and bicycles. The LRFD Specification calls for a $3.6 \times 10^{-3}$ MPa load simultaneous with highway loads on sidewalks wider than 0.6 m. "Pedestrian- or bicycle-only" bridges are to be designed for $4.1 \times 10^{-3}$ MPa. If the pedestrian- or bicycle-only bridge is required to carry maintenance or emergency vehicles, these vehicles are designed for, omitting the dynamic load allowance. Loads due to these vehicles are infrequent and factoring up for dynamic loads is inappropriate.

**TABLE 6.4** Base Wind Pressures, $P_B$, corresponding to $V_B$ = 160 km/h

| Structural Component | Windward Load, MPa | Leeward Load, MPa |
|---|---|---|
| Trusses, columns, and arches | 0.0024 | 0.0012 |
| Beams | 0.0024 | NA |
| Large flat surfaces | 0.0019 | NA |

*Source*: AASHTO LRFD Bridge Design Specifications, 2nd. ed., American Association of State Highway and Transportation Officials. Washington, D.C., 1998. With permission.

## 6.5 Wind Loads

The LRFD Specification provides wind loads as a function of base design wind velocity, $V_B$ equal to 100 mph; and base pressures, $P_B$, corresponding to wind speed $V_B$. Values for $P_B$ are listed in Table 6.4. The design wind pressure, $P_D$, is then calculated as

$$P_D = P_B\left(\frac{V_{DZ}}{V_B}\right)^2 = P_B\frac{V_{DZ}^2}{25{,}600} \tag{6.5}$$

where $V_{DZ}$ is the design wind velocity at design elevation $Z$ in km/h. $V_{DZ}$ is a function of the friction velocity, $V_0$ (km/h), multiplied by the ratio of the actual wind velocity to the base wind velocity both at 10 m above grade, and the natural logarithm of the ratio of height to a meteorological constant length for given surface conditions:

**TABLE 6.5** Values of $V_o$ and $Z_o$ for Various Upstream Surface Conditions

| Condition | Open Country | Suburban | City |
|---|---|---|---|
| $V_o$ (km/h) | 13.2 | 15.2 | 19.4 |
| $Z_o$ (mm) | 70 | 300 | 800 |

*Source*: AASHTO LRFD Bridge Design Specifications, 2nd. ed., American Association of State Highway and Transportation Officials. Washington, D.C., 1998. With permission.

**TABLE 6.6** Temperature Ranges, °C

| Climate | Steel or Aluminum | Concrete | Wood |
|---|---|---|---|
| Moderate | –18 to 50 | –12 to 27 | –12 to 24 |
| Cold | –35 to 50 | –18 to 27 | –18 to 24 |

*Source*: AASHTO LRFD Bridge Design Specifications, 2nd. ed., American Association of State Highway and Transportation Officials. Washington, D.C., 1998. With permission.

$$V_{DZ} = 2.5V_0\left(\frac{V_{10}}{V_B}\right)\ln\left(\frac{Z}{Z_0}\right) \tag{6.6}$$

Values for $V_0$ and $Z_0$ are shown in Table 6.5.

The resultant design pressure is then applied to the surface area of the superstructure as seen in elevation. Solid-type traffic barriers and sound walls are considered as part of the loading surface. If the product of the resultant design pressure and applicable loading surface depth is less than a lineal load of 4.4 N/mm on the windward chord, or 2.2 N/mm on the leeward chord, minimum loads of 4.4 and 2.2 N/mm, respectively, are designed for.

Wind loads are combined with other loads in Strength Limit States III and V, and Service Limit State I, as defined in Chapter 5. Wind forces due to the additional surface area from trucks is accounted for by applying a 1.46 N/mm load 1800 mm above the bridge deck.

Wind loads for substructure design are of two types: loads applied to the substructure and those applied to the superstructure and transmitted to the substructure. Loads applied to the superstructure are as previously described. A base wind pressure of $1.9 \times 10^{-3}$ MPa force is applied directly to the substructure, and is resolved into components (perpendicular to the front and end elevations) when the structure is skewed.

In absence of live loads, an upward load of $9.6 \times 10^{-4}$ MPa is multiplied by the width of the superstructure and applied at the windward quarter point simultaneously with the horizontal wind loads applied perpendicular to the length of the bridge. This uplift load may create a worst condition for substructure design when seismic loads are not of concern.

## 6.6 Effects Due to Superimposed Deformations

Elements of a structure may change size or position due to settlement, shrinkage, creep, or temperature. Changes in geometry cause additional stresses which are of particular concern at connections. Determining effects from foundation settlement are a matter of structural analysis. Effects due to shrinkage and creep are material dependent and the reader is referred to design chapters elsewhere

**TABLE 6.7** Basis of Temperature Gradients

| | Concrete | | 50 mm Asphalt | | 100 mm Asphalt | |
|---|---|---|---|---|---|---|
| Zone | $T_1$ (°C) | $T_2$ (°C) | $T_1$ (°C) | $T_2$ (°C) | $T_1$ (°C) | $T_2$ (°C) |
| 1 | 30 | 7.8 | 24 | 7.8 | 17 | 5 |
| 2 | 25 | 6.7 | 20 | 6.7 | 14 | 5.5 |
| 3 | 23 | 6 | 18 | 6 | 13 | 6 |
| 4 | 21 | 5 | 16 | 5 | 12 | 6 |

*Source:* AASHTO LRFD Bridge Design Specifications, 2nd. ed., American Association of State Highway and Transportation Officials. Washington, D.C., 1998. With permission.

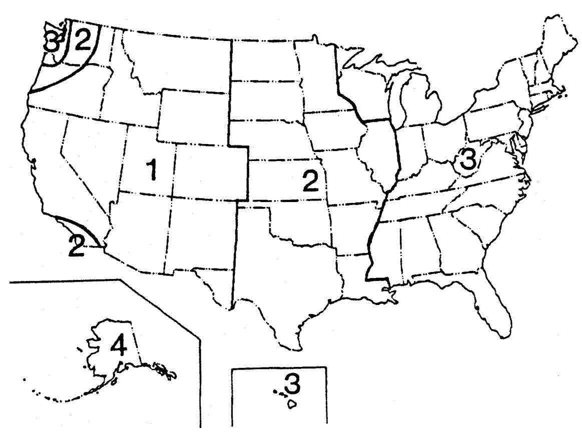



**FIGURE 6.5** Solar radiation zones for the United States. (AASHTO LRFD Bridge Design Specifications 2nd. ed., American Association of State Highway and Transportation Officials. Washington, D.C., 1998. With permission.)

in this book. Temperature effects are dependent on the maximum potential temperature differential from the temperature at time of erection. Upper and lower bounds are shown in Table 6.6, where "moderate" and "cold" climates are defined as having fewer or more than 14 days with an average temperature below 0°C, respectively.

By using appropriate coefficients of thermal expansion, effects from temperature changes are calculated using basic structural analysis. More-refined analysis will consider the time lag between the surface and internal structure temperatures. The LRFD Specification identifies four zones in the United States and provides a linear relationship for the temperature gradient in steel and concrete. See Table 6.7 and Figures 6.5 and 6.6.

## 6.7 Exceptions to Code-Specified Design Loads

The designer is responsible not only for providing plans that accommodate design loads per the referenced Design Specifications, but also for any loads unique to the structure and bridge site. It is also the designer's responsibility to indicate all loading conditions designed for in the contract documents — preferably the construction plans. History seems to indicate that the next generation of bridge engineers will indeed be given the task of "improving" today's new structure. Therefore, the safety of future generations depends on today's designers doing a good job of documentation.

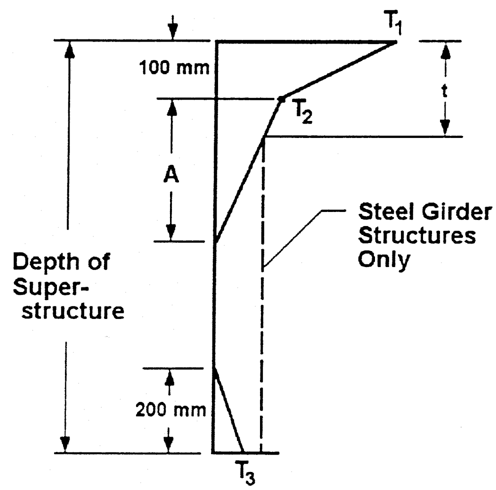



**FIGURE 6.6** Positive vertical temperature gradient in concrete and steel superstructures. (AASHTO LRFD Bridge Design Specifications 2nd. ed., American Association of State Highway and Transportation Officials. Washington, D.C., 1998. With permission.)

## References

1. AASHTO, LRFD Bridge Design Specifications 2nd. ed., American Association of State Highway and Transportation Officials. Washington, D.C., 1998.
2. FHWA Training Course, LRFD Design of Highway Bridges, Vol. 1, 1993.
3. AASHTO, American Association of State Highway and Transportation Officials, Washington, D.C. Officials, *Standard Specifications for Highway Bridges*, 16th ed., 1996, as amended by the Interim Specifications.
4. Caltrans, Bridge Design Specifications, California Department of Transportation, Sacramento, CA, 1999.

Liu, X., Zhang, L. "Structural Theory."
*Bridge Engineering Handbook.*
Ed. Wai-Fah Chen and Lian Duan
Boca Raton: CRC Press, 2000

# 7
# Structural Theory



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## 7.1 Introduction

In this chapter, general forms of three sets of equations required in solving a solid mechanics problem and their extensions into structural theory are presented. In particular, a more generally used method, displacement method, is expressed in detail.

### 7.1.1 Basic Equations: Equilibrium, Compatibility, and Constitutive Law

In general, solving a solid mechanics problem must satisfy equations of equilibrium (static or dynamic), conditions of compatibility between strains and displacements, and stress–strain relations or material constitutive law (see Figure 7.1). The initial and boundary conditions on forces and displacements are naturally included.

From consideration of equilibrium equations, one can relate the stresses inside a body to external excitations, including body and surface forces. There are three equations of equilibrium relating the

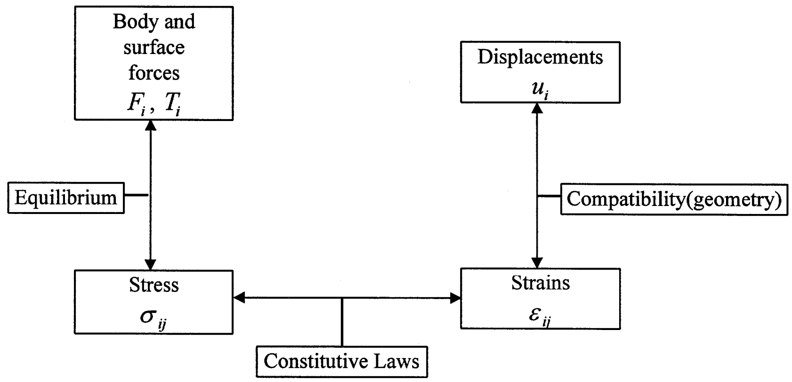



**FIGURE 7.1** Relations of variables in solving a solid mechanics problem.

six components of stress tensor $\sigma_{ij}$ for an infinitesimal material element which will be shown later in Section 7.2.1. In the case of dynamics, the equilibrium equations are replaced by equations of motion, which contain second-order derivatives of displacement with respect to time.

In the same way, taking into account geometric conditions, one can relate strains inside a body to its displacements, by six equations of kinematics expressing the six components of strain ( $\varepsilon_{ij}$ ) in terms of the three components of displacement ( $u_i$ ). These are known as the strain–displacement relations (see Section 7.3.1).

Both the equations of equilibrium and kinematics are valid regardless of the specific material of which the body is made. The influence of the material is expressed by constitutive laws in six equations. In the simplest case, not considering the effects of temperature, time, loading rates, and loading paths, these can be described by relations between stress and strain only.

Six stress components, six strain components, and three displacement components are connected by three equilibrium equations, six kinematics equations, and six constitutive equations. The 15 unknown quantities can be determined from the system of 15 equations.

It should be pointed out that the principle of superposition is valid only when small deformations and elastic materials are assumed.

### 7.1.2 Three Levels: Continuous Mechanics, Finite–Element Method, Beam–Column Theory

In solving a solid mechanics problem, the most direct method solves the three sets of equations described in the previous section. Generally, there are three ways to establish the basic unknowns, namely, the displacement components, the stress components, or a combination of both. The corresponding procedures are called the displacement method, the stress method, or the mixed method, respectively. But these direct methods are only practicable in some simple circumstances, such as those detailed in elastic theory of solid mechanics.

Many complex problems cannot be easily solved with conventional procedures. Complexities arise due to factors such as irregular geometry, nonhomogeneities, nonlinearity, and arbitrary loading conditions. An alternative now available is based on a concept of discretization. The finite-element method (FEM) divides a body into many "small" bodies called finite elements. Formulations by the FEM on the laws and principles governing the behavior of the body usually result in a set of simultaneous equations that can be solved by direct or iterative procedures. And loading effects such as deformations and stresses can be evaluated within certain accuracy. Up to now, FEM has been the most widely used structural analysis method.

In dealing with a continuous beam, the size of the three sets of equations is greatly reduced by assuming characteristics of beam members such as plane sections remain plane. For framed structures or structures constructed using beam–columns, structural mechanics gives them a more pithy and practical analysis.

### 7.1.3 Theoretical Structural Mechanics, Computational Structural Mechanics, and Qualitative Structural Mechanics

Structural mechanics deals with a system of members connected by joints which may be pinned or rigid. Classical methods of structural analysis are based on principles such as the principle of virtual displacement, the minimization of total potential energy, the minimization of total complementary energy, which result in the three sets of governing equations. Unfortunately, conventional methods are generally intended for hand calculations and developers of the FEM took great pains to minimize the amount of calculations required, even at the expense of making the methods somewhat unsystematic. This made the conventional methods unattractive for translation to computer codes.

The digital computer called for a more systematic method of structural analysis, leading to computational structural mechanics. By taking great care to formulate the tools of matrix notation in a mathematically consistent fashion, the analyst achieved a systematic approach convenient for automatic computation: matrix analysis of structures. One of the hallmarks of structural matrix analysis is its systematic nature, which renders digital computers even more important in structural engineering.

Of course, the analyst must maintain a critical, even skeptical, attitude toward computer results. In any event, computer results must satisfy our intuition of what is "reasonable." This qualitative judgment requires that the analyst possess a full understanding of structural behavior, both that being modeled by the program and that which can be expected in the actual structures. Engineers should decide what approximations are reasonable for the particular structure and verify that these approximations are indeed valid, and know how to design the structure so that its behavior is in reasonable agreement with the model adopted to analyze it. This is the main task of a structural analyst.

### 7.1.4 Matrix Analysis of Structures: Force Method and Displacement Method

Matrix analysis of structures was developed in the early 1950s. Although it was initially used on fuselage analysis, this method was proved to be pertinent to any complex structure. If internal forces are selected as basic unknowns, the analysis method is referred to as force method; in a similar way, the displacement method refers to the case where displacements are selected as primary unknowns. Both methods involve obtaining the joint equilibrium equations in terms of the basic internal forces or joint displacements as primary unknowns and solving the resulting set of equations for these unknowns. Having done this, one can obtain internal forces by backsubstitution, since even in the case of the displacement method the joint displacements determine the basic displacements of each member, which are directly related to internal forces and stresses in the member.

A major feature evident in structural matrix analysis is an emphasis on a systematic approach to the statement of the problem. This systematic characteristic together with matrix notation makes it especially convenient for computer coding. In fact, the displacement method, whose basic unknowns are uniquely defined, is generally more convenient than the force method. Most general-purpose structural analysis programs are displacement based. But there are still cases where it may be more desirable to use the force method.

## 7.2 Equilibrium Equations

### 7.2.1 Equilibrium Equation and Virtual Work Equation

For any volume $V$ of a material body having $A$ as surface area, as shown in Figure 7.2, it has the following conditions of equilibrium:

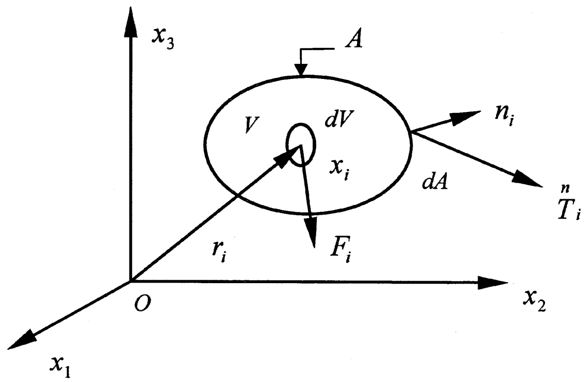



**FIGURE 7.2** Derivation of equations of equilibrium.

*At surface points*

$$T_i = \sigma_{ji} n_j \tag{7.1a}$$

*At internal points*

$$\sigma_{ji,j} + F_i = 0 \tag{7.1b}$$

$$\sigma_{ji} = \sigma_{ij} \tag{7.1c}$$

where $n_i$ represents the components of unit normal vector **n** of the surface; $T_i$ is the stress vector at the point associated with **n**; $\sigma_{ji,j}$ represents the first derivative of $\sigma_{ij}$ with respect to $x_j$; and $F_i$ is the body force intensity. Any set of stresses $\sigma_{ij}$, body forces $F_i$, and external surface forces $T_i$ that satisfies Eqs. (7.1a-c) is a statically admissible set.

Equations (7.1b and c) may be written in $(x,y,z)$ notation as

$$\begin{aligned}
&\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + F_x = 0 \\
&\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + F_y = 0 \\
&\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + F_z = 0
\end{aligned} \tag{7.1d}$$

and

$$\tau_{xy} = \tau_{yx}, \quad \text{etc.} \tag{7.1e}$$

where $\sigma_x$, $\sigma_y$, and $\sigma_z$ are the normal stress in $(x,y,z)$ direction respectively; $\tau_{xy}$, $\tau_{yx}$, and so on, are the corresponding shear stresses in $(x,y,z)$ notation; and $F_x$, $F_y$, and $F_z$ are the body forces in $(x,y,z)$ direction, respectively.

The principle of virtual work has proved a very powerful technique of solving problems and providing proofs for general theorems in solid mechanics. The equation of virtual work uses two

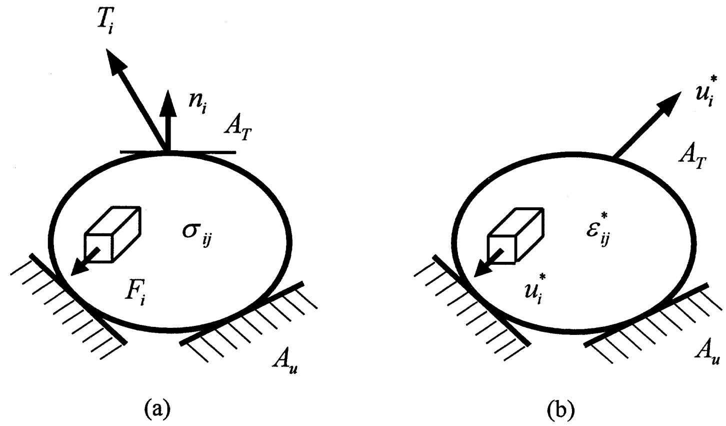



**FIGURE 7.3** Two independent sets in the equation of virtual work.

independent sets of *equilibrium* and *compatible* (see Figure 7.3, where $A_u$ and $A_T$ represent displacement and stress boundary, respectively), as follows:

$$\overbrace{\underbrace{\int_A T_i u_i^* dA + \int_V F_i u_i^* dV = \int_V \sigma_{ij}\varepsilon_{ij}^* dV}_{\text{equilibrium set}}}^{\text{compatible set}} \tag{7.2}$$

or

$$\delta W_{ext} = \delta W_{int} \tag{7.3}$$

which states that the *external* virtual work ( $\delta W_{ext}$ ) equals the *internal* virtual work ( $\delta W_{int}$ ).

Here the integration is over the whole area $A$, or volume $V$, of the body. The stress field $\sigma_{ij}$, body forces $F_i$, and external surface forces $T_i$ are a statically admissible set that satisfies Eqs. (7.1a–c). Similarly, the strain field $\varepsilon_{ij}^*$ and the displacement $u_i^*$ are a compatible kinematics set that satisfies displacement boundary conditions and Eq. (7.16) (see Section 7.3.1). This means the principle of virtual work applies only to small strain or small deformation.

The important point to keep in mind is that, neither the admissible equilibrium set $\sigma_{ij}$, $F_i$, and $T_i$ (Figure 7.3a) nor the compatible set $\varepsilon_{ij}^*$ and $u_i^*$ (Figure 7.3b) need be the actual state, nor need the equilibrium and compatible sets be related to each other in any way. In the other words, these two sets are completely independent of each other.

## 7.2.2 Equilibrium Equation for Elements

For an infinitesimal material element, equilibrium equations have been summarized in Section 7.2.1, which will transfer into specific expressions in different methods. As in ordinary FEM or the displacement method, it will result in the following element equilibrium equations:

$$\{\overline{F}\}^e = [\overline{k}]^e \{\overline{d}\}^e \tag{7.4}$$

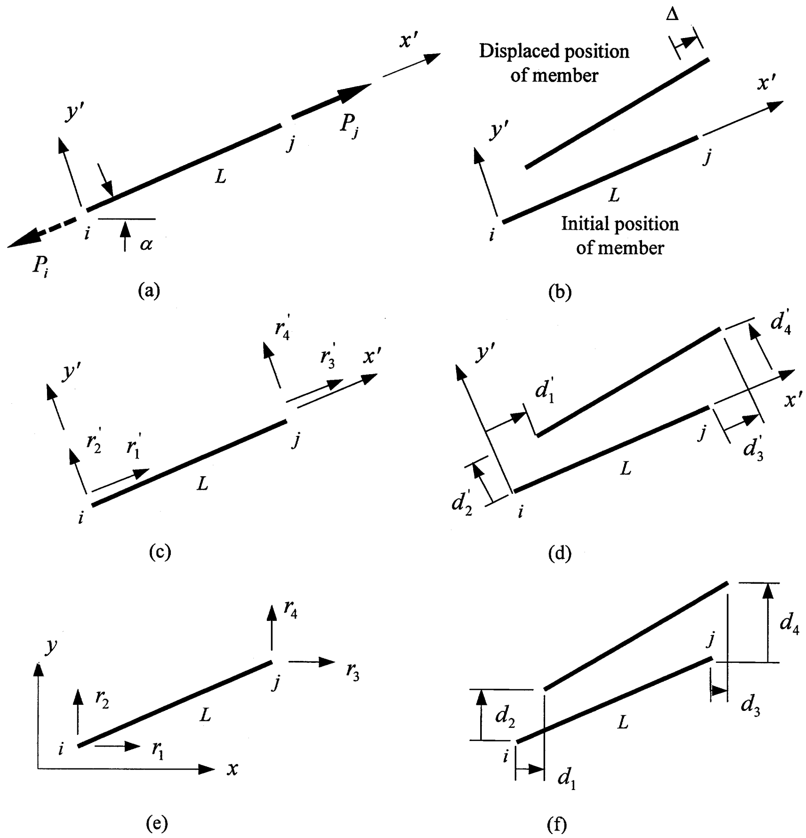



**FIGURE 7.4** Plane truss member–end forces and displacements. (*Source*: Meyers, V.J., *Matrix Analysis of Structures*, New York: Harper & Row, 1983. With permission.)

where $\{\bar{F}\}^e$ and $\{\bar{d}\}^e$ are the element nodal force vector and displacement vector, respectively, while $[\bar{k}]^e$ is element stiffness matrix; the overbar here means in local coordinate system.

In the force method of structural analysis, which also adopts the idea of discretization, it is proved possible to identify a basic set of independent forces associated with each member, in that not only are these forces independent of one another, but also all other forces in that member are directly dependent on this set. Thus, this set of forces constitutes the minimum set that is capable of completely defining the stressed state of the member. The relationship between basic and local forces may be obtained by enforcing overall equilibrium on one member, which gives

$$\{\bar{F}\}^e = [L]\{P\}^e \tag{7.5}$$

where $[L]$ = the element force transformation matrix and $\{P\}^e$ = the element primary forces vector. It is important to emphasize that the physical basis of Eq. (7.5) is member overall equilibrium.

Take a conventional plane truss member for exemplification (see Figure 7.4), one has

$$\{\bar{k}\}^e = \begin{bmatrix} EA/l & 0 & -EA/l & 0 \\ 0 & 0 & 0 & 0 \\ -EA/l & 0 & EA/l & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{7.6}$$

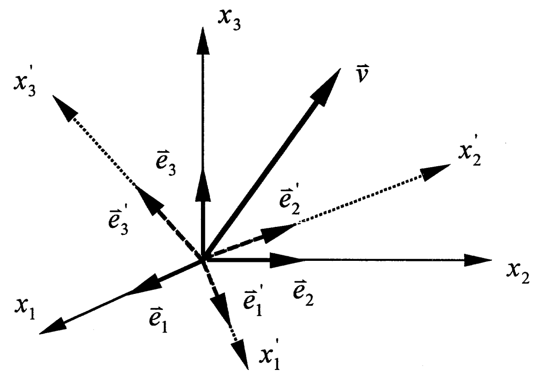



**FIGURE 7.5** Coordinate transformation.

and

$$\begin{aligned} \{\bar{F}\}^e &= \{r_1' \quad r_2' \quad r_3' \quad r_4'\}^T \\ \{\bar{d}\}^e &= \{d_1' \quad d_2' \quad d_3' \quad d_4'\}^T \\ [L] &= \{-1 \quad 0 \quad 1 \quad 0\}^T \\ \{P\}^e &= \{P\} \end{aligned} \tag{7.7}$$

where $EA/l$ = axial stiffness of the truss member and $P$ = axial force of the truss member.

### 7.2.3 Coordinate Transformation

The values of the components of vector **V**, designated by $v_1$, $v_2$, and $v_3$ or simply $v_i$, are associated with the chosen set coordinate axes. Often it is necessary to reorient the reference axes and evaluate new values for the components of **V** in the new coordinate system. Assuming that **V** has components $v_i$ and $v_i'$ in two sets of right-handed Cartesian coordinate systems $x_i$ (old) and $x_i'$ (new) having the same origin (see Figure 7.5), and $\vec{e}_i$, $\vec{e}_i'$ are the unit vectors of $x_i$ and $x_i'$, respectively. Then

$$v_i' = l_{ij} v_j \tag{7.8}$$

where $l_{ji} = \vec{e}_j' \cdot \vec{e}_i = \cos(x_j', x_i)$, that is, the cosines of the angles between $x_i'$ and $x_j$ axes for $i$ and $j$ ranging from 1 to 3; and $[\alpha] = (l_{ij})_{3\times 3}$ is called coordinate transformation matrix from the old system to the new system.

It should be noted that the elements of $l_{ij}$ or matrix $[\alpha]$ are not symmetrical, $l_{ij} \neq l_{ji}$. For example, $l_{12}$ is the cosine of angle from $x_1'$ to $x_2$ and $l_{21}$ is that from $x_2'$ to $x_1$ (see Figure 7.5). The angle is assumed to be measured from the primed system to the unprimed system.

For a plane truss member (see Figure 7.4), the transformation matrix from local coordinate system to global coordinate system may be expressed as

$$[\alpha] = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & \cos\alpha & -\sin\alpha \\ 0 & 0 & \sin\alpha & \cos\alpha \end{bmatrix} \tag{7.9}$$

where $\alpha$ is the inclined angle of the truss member which is assumed to be measured from the global to the local coordinate system.

### 7.2.4 Equilibrium Equation for Structures

For discretized structure, the equilibrium of the whole structure is essentially the equilibrium of each joint. After assemblage,

*For ordinary FEM or displacement method*

$$\{F\} = [K]\{D\} \tag{7.10}$$

*For force method*

$$\{F\} = [A]\{P\} \tag{7.11}$$

where $\{F\}$ = nodal loading vector; $[K]$ = total stiffness matrix; $\{D\}$ = nodal displacement vector; $[A]$ = total forces transformation matrix; $\{P\}$ = total primary internal forces vector.

It should be noted that the coordinate transformation for each element from local coordinates to the global coordinate system must be done before assembly.

In the force method, Eq. (7.11) will be adopted to solve for internal forces of a statically determinate structure. The number of basic unknown forces is equal to the number of equilibrium equations available to solve for them and the equations are linearly independent. For statically unstable structures, analysis must consider their dynamic behavior. When the number of basic unknown forces exceeds the number of equilibrium equations, the structure is said to be statically indeterminate. In this case, some of the basic unknown forces are not required to maintain structural equilibrium. These are "extra" or "redundant" forces. To obtain a solution for the full set of basic unknown forces, it is necessary to augment the set of independent equilibrium equations with elastic behavior of the structure, namely, the force–displacement relations of the structure. Having solved for the full set of basic forces, we can determine the displacements by backsubstitution.

### 7.2.5 Influence Lines and Surfaces

In the design and analysis of bridge structures , it is necessary to study the effects intrigued by loads placed in various positions. This can be done conveniently by means of diagrams showing the effect of moving a unit load across the structures. Such diagrams are commonly called influence lines (for framed structures) or influence surfaces (for plates). Observe that whereas a moment or shear diagram shows the variation in moment or shear along the structure due to some particular position of load, an influence line or surface for moment or shear shows the variation of moment or shear at a *particular* section due to a unit load placed anywhere along the structure.

Exact influence lines for statically determinate structures can be obtained analytically by statics alone. From Eq. (7.11), the total primary internal forces vector $\{P\}$ can be expressed as

$$\{P\} = [A]^{-1}\{F\} \tag{7.12}$$

by which given a unit load at one node, the excited internal forces of all members will be obtained, and thus Eq. (7.12) gives the analytical expression of influence lines of all member internal forces for discretized structures subjected to moving nodal loads.

For statically indeterminate structures, influence values can be determined directly from a consideration of the geometry of the deflected load line resulting from imposing a unit deformation corresponding to the function under study, based on the principle of virtual work. This may better be demonstrated by a two-span continuous beam shown in Figure 7.6, where the influence line of internal bending moment $M_B$ at section $B$ is required.

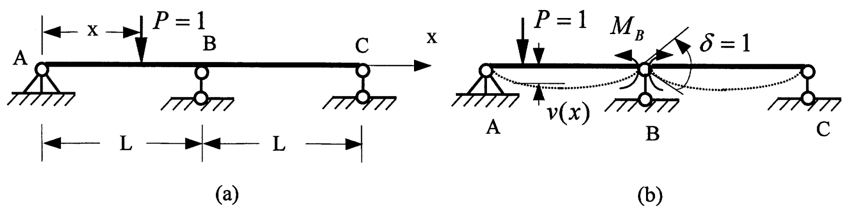



**FIGURE 7.6** Influence line of a two-span continuous beam.

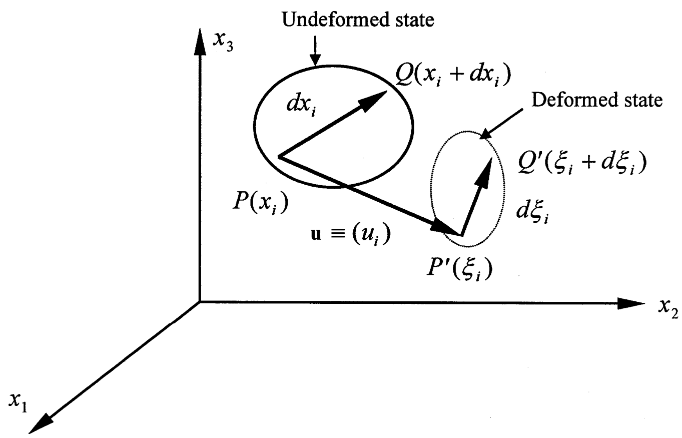



**FIGURE 7.7** Deformation of a line element for Lagrangian and Eluerian variables.

Cutting section $B$ to expose $M_B$ and give it a unit relative rotation $\delta = 1$ (see Figure 7.6) and employing the principle of virtual work gives

$$M_B \cdot \delta = -P \cdot v(x) \tag{7.13}$$

Therefore,

$$M_B = -v(x) \tag{7.14}$$

which means the influence value of $M_B$ equals to the deflection $v(x)$ of the beam subjected to a unit rotation at joint $B$ (represented by dashed line in Figure 7.6b). Solving for $v(x)$ can be carried out easily referring to material mechanics.

## 7.3 Compatibility Equations

### 7.3.1 Large Deformation and Large Strain

Strain analysis is concerned with the study of deformation of a continuous body which is unrelated to properties of the body material. In general, there are two methods of describing the deformation of a continuous body, Lagrangian and Eulerian. The Lagrangian method employs the coordinates of each particle in the initial position as the independent variables. The Eulerian method defines the independent variables as the coordinates of each material particle at the time of interest.

Let the coordinates of material particle $P$ in a body in the initial position be denoted by $x_i$ ( $x_1, x_2, x_3$) referred to the fixed axes $x_i$, as shown in Figure 7.7. And the coordinates of the particle after deformation are denoted by $\xi_i$ ( $\xi_1, \xi_2, \xi_3$ ) with respect to axes $x_i$. As for the independent variables, Lagrangian formulation uses the coordinates ( $x_i$) while Eulerian formulation employs the coordinates ( $\xi_i$). From motion analysis of line element $PQ$ (see Figure 7.7), one has

*For Lagrangian formulation, the Lagrangian strain tensor is*

$$\varepsilon_{ij} = \frac{1}{2}\,(u_{i,j} + u_{j,i} + u_{r,i}u_{r,j}) \tag{7.15}$$

where $u_{i,j} = \partial u_i / \partial x_j$ and all quantities are expressed in terms of ( $x_i$ ).

*For Eulerian formulation, the Eulerian strain tensor is*

$$E_{ij} = \frac{1}{2}\,(u_{i/j} + u_{j/i} + u_{r/i}u_{r/j}) \tag{7.16}$$

where $u_{i/j} = \partial u_i / \partial \xi_j$ and all quantities are described in terms of ( $\xi_i$ ).

If the displacement derivatives $u_{i,j}$ and $u_{i/j}$ are not so small that their nonlinear terms cannot be neglected, it is called large deformation, and the solving of $u_i$ will be rather difficult since the nonlinear terms appear in the governing equations.

If both the displacements and their derivatives are small, it is immaterial whether the derivatives in Eqs. (7.15) and (7.16) are calculated using the ( $x_i$ ) or the ( $\xi_i$ ) variables. In this case both Lagrangian and Eulerian descriptions yield the same strain–displacement relationship:

$$\varepsilon_{ij} = E_{ij} = \frac{1}{2}\,(u_{i,j} + u_{j,i}) \tag{7.17}$$

which means small deformation, the most common in structural engineering.

For given displacements ( $u_i$ ) in strain analysis, the strain components ( $\varepsilon_{ij}$ ) can be determined from Eq. (7.17). For prescribed strain components ( $\varepsilon_{ij}$ ), some restrictions must be imposed on it in order to have single-valued continuous displacement functions $u_i$, since there are six equations for three unknown functions. Such restrictions are called compatibility conditions, which for a simply connected region may be written as

$$\varepsilon_{ij,kl} + \varepsilon_{kl,ij} - \varepsilon_{ik,jl} - \varepsilon_{ji,ik} = 0 \tag{7.18a}$$

or, expanding these expressions in the (*x, y, z*) notations, it gives

$$\begin{gathered}
\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = 2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y} \\
\frac{\partial^2 \varepsilon_y}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial y^2} = 2\frac{\partial^2 \varepsilon_{yz}}{\partial y \partial z} \\
\frac{\partial^2 \varepsilon_z}{\partial x^2} + \frac{\partial^2 \varepsilon_x}{\partial z^2} = 2\frac{\partial^2 \varepsilon_{zx}}{\partial z \partial x} \\
\frac{\partial}{\partial x}\left(-\frac{\partial \varepsilon_{yz}}{\partial x} + \frac{\partial \varepsilon_{zx}}{\partial y} + \frac{\partial \varepsilon_{xy}}{\partial z}\right) = \frac{\partial^2 \varepsilon_x}{\partial y \partial z} \\
\frac{\partial}{\partial y}\left(-\frac{\partial \varepsilon_{zx}}{\partial y} + \frac{\partial \varepsilon_{xy}}{\partial z} + \frac{\partial \varepsilon_{yz}}{\partial x}\right) = \frac{\partial^2 \varepsilon_y}{\partial z \partial x} \\
\frac{\partial}{\partial z}\left(-\frac{\partial \varepsilon_{xy}}{\partial z} + \frac{\partial \varepsilon_{yz}}{\partial x} + \frac{\partial \varepsilon_{zx}}{\partial y}\right) = \frac{\partial^2 \varepsilon_z}{\partial x \partial y}
\end{gathered} \tag{7.18b}$$

Any set of strains $\varepsilon_{ij}$ and displacements $u_i$, that satisfies Eqs. (7.17) and (7.18a) or (7.18b), as well as displacement boundary conditions, is a kinematics admissible set, or a compatible set.

### 7.3.2 Compatibility Equation for Elements

For ordinary FEM, compatibility requirements are self-satisfied in the formulating procedure. As for equilibrium equations, a basic set of independent displacements can be identified for each member, and the kinematics relationships between member basic displacements and member–end displacements of one member can be given as follows:

$$\{\Delta\}^e = [L]^T\{\bar{d}\}^e \tag{7.19}$$

where $\{\Delta\}^e$ is element primary displacement vector, $[L]$ and $\{\bar{d}\}^e$ have been shown in Section 7.2.2. For plane truss member, $\{\Delta\}^e = \{\Delta\}$, where $\Delta$ is the relative displacement of the member (see Figure 7.5). It should also be noted that the physical basis of Eq. (7.19) is the overall compatibility of the element.

### 7.3.3 Compatibility Equation for Structures

For the whole structure, one has the following equation after assembly process:

$$\{\Delta\} = [A]^T\{D\} \tag{7.20}$$

where $\{\Delta\}$ = total primary displacement vector; $\{D\}$ = total nodal displacement vector; and $[A]^T$ = the transposition of $[A]$ described in Section 7.2.4.

A statically determinate structure is kinematically determinate. Given a set of basic member displacements, there are a sufficient number of compatibility relationships available to allow the structure nodal displacements to be determined. In addition to their application to settlement and fabrication error loading, thermal loads can also be considered for statically determinate structures. External forces on a structure cause member distortions and, hence, nodal displacements, but before such problems can be solved, the relationships between member forces and member distortions must be developed. These will be shown in Section 7.5.1.

### 7.3.4 Contragredient Law

During the development of the equilibrium and compatibility relationships, it has been noticed that various corresponding force and displacement transformations are the transposition of each other, as shown not only in Eqs. (7.5) and (7.19) of element equilibrium and compatibility relations, but also in Eqs. (7.11) and (7.20) of global equilibrium and compatibility relations, although each pair of these transformations was obtained independently of the other in the development. These special sets of relations are termed the contragredient law which was established on the basis of virtual work concepts. Therefore, after a particular force transformation matrix is obtained, the corresponding displacement transformation matrix would be immediately apparent, and it remains valid to the contrary.

## 7.4 Constitutive Equations

### 7.4.1 Elasticity and Plasticity

A material body will produce deformation when subjected to external excitations. If upon the release of applied actions the body recovers its original shape and size, it is called an *elastic* material, or

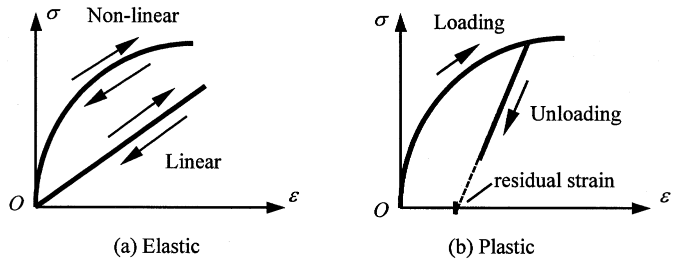



**FIGURE 7.8** Sketches of behavior of elastic and plastic materials.

one can say the material has the characteristic of *elasticity.* Otherwise, it is a *plastic* material or a material with *plasticity.* For an elastic body, the current state of stress depends only on the current state of deformation; that is, the constitutive equations for elastic material are given by

$$\sigma_{ij} = F_{ij}(\varepsilon_{kl}) \tag{7.21}$$

where $F_{ij}$ is called the elastic response function. Thus, the elastic material behavior described by Eq. (7.21) is reversible and path independent (see Figure 7.8a), in which case the material is usually termed *Cauchy elastic* material.

Reversibility and path independence are not exhibited by plastic materials (see Figure 7.8b). In general, a plastic material does not return to its original shape; *residual* deformation and stresses remain inside the body even when all external tractions are removed. As a result, it is necessary for plasticity to extend the elastic stress–strain relations into the plastic range where permanent plastic stain is possible. It makes the solution of a solid mechanics problem more complicated.

## 7.4.2 Linear Elastic and Nonlinear Elastic Behavior

Just as the term *linear* implies, linear elasticity means the elastic response function $F_{ij}$ of Eq. (7.21) is a linear function, whose most general form for a Cauchy elastic material is given by

$$\sigma_{ij} = B_{ij} + C_{ijkl}\varepsilon_{kl} \tag{7.22}$$

where $B_{ij}$ = components of initial stress tensor corresponding to the *initial strain-free* state (i.e., $\varepsilon_{ij}$ = 0), and $C_{ijkl}$ = tensor of material elastic constants.

If it is assumed that $B_{ij}$ = 0, Eq. (7.22) will be reduced to

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{7.23}$$

which is often referred to as the generalized Hook's law.

For an *isotropic* linear elastic material, the elastic constants in Eq. (7.23) must be the same for all directions and thus $C_{ijkl}$ must be an isotropic fourth-order tensor, which means that there are only two independent material constants. In this case, Eq. (7.23) will reduce to

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} \tag{7.24}$$

where λ and μ are the two material constants, usually called *Lame's constants*; $\delta_{ij}$ = *Kronecker delta* and $\varepsilon_{kk}$ = the summation of the diagonal terms of $\varepsilon_{ij}$ according to the *summation convention,* which means that, whenever a subscript occurs twice in the same term, it is understood that the subscript is to be summed from 1 to 3.

If the elastic response function $F_{ij}$ in Eq. (7.21) is not linear, it is called nonlinear elastic, and the material exhibits nonlinear mechanical behavior even when sustaining small deformation. That is, the material elastic "constants" do not remain constant any more, whereas the deformation can still be reversed completely.

### 7.4.3 Geometric Nonlinearity

Based on the sources from which it arises, nonlinearity can be categorized into material nonlinearity (including nonlinear elasticity and plasticity) and geometric nonlinearity. When the nonlinear terms in the strain–displacement relations cannot be neglected (see Section 7.3.1) or the deflections are large enough to cause significant changes in the structural geometry, it is termed geometric nonlinearity. It is also called large deformation, and the principle of superposition derived from small deformations is no longer valid. It should be noted that for accumulated large displacements with small deformations, it could be linearized by a step-by-step procedure.

According to the different choice of reference frame, there are two types of Lagrangian formulation: the total Lagrangian formulation, which takes the original unstrained configuration as the reference frame, and the updated Lagrangian formulation based on the latest-obtained configuration, which are usually carried out step by step. Whatever formulation one chooses, a geometric stiffness matrix or initial stress matrix will be introduced into the equations of equilibrium to take account of the effects of the initial stresses on the stiffness of the structure. These depend on the magnitude or conditions of loading and deformations, and thus cause the geometric nonlinearity. In beam–column theory, this is well known as the second-order or the $P$–$\Delta$ effect. For detailed discussions, see Chapter 36.

## 7.5 Displacement Method

### 7.5.1 Stiffness matrix for elements

In displacement method, displacement components are taken as primary unknowns. From Eqs. (7.5) and (7.19) the equilibrium and compatibility requirements on elements have been acquired. For a statically determinate structure, no subsidiary conditions are needed to obtain internal forces under nodal loading or the displaced position of the structure given the basic distortion such as support settlement or fabrication errors. For a statically indeterminate structure, however, supplementary conditions, namely, the constitutive law of materials constructing the structure, should be incorporated for the solution of internal forces as well as nodal displacements.

From structural mechanics, the basic stiffness relationships for a member between basic internal forces and basic member–end displacements can be expressed as

$$\{P\}^e = [k]^e \{\Delta\}^e \tag{7.25}$$

where $[k]^e$ is the element basic stiffness matrix, which can be termed $[EA/l]$ for a conventional plane truss member (see Figure 7.4).

Substitution of Eqs. (7.19) and (7.25) into Eq. (7.5) yields

$$\begin{aligned}\{\bar{F}\}^e &= [L][k]^e[L]^T\{\bar{d}\}^e \\ &= [\bar{k}]^e\{\bar{d}\}^e\end{aligned} \tag{7.26}$$

where

$$[\bar{k}]^e = [L][k]^e[L]^T \tag{7.27}$$

is called the element stiffness matrix, the same as in Eq. (7.4). It should be kept in mind that the element stiffness matrix $[\bar{k}]^e$ is symmetric and singular, since given the member–end forces, member–end displacements cannot be determined uniquely because the member may undergo rigid body movement.

### 7.5.2 Stiffness Matrix for Structures

Our final aim is to obtain equations that define approximately the behavior of the whole body or structure. Once the element stiffness relations of Eq. (7.26) is established for a generic element, the global equations can be constructed by an assembling process based on the law of compatibility and equilibrium, which are generally expressed in matrix notation as

$$\{F\} = [K]\{D\} \tag{7.28}$$

where $[K]$ is the stiffness matrix for the whole structure. It should be noted that the basic idea of assembly involves a minimization of *total* potential energy, and the assembled stiffness matrix $[K]$ is *symmetric* and *banded* or *sparsely populated*.

Eq. (7.28) tells us the capabilities of a structure to withstand applied loading rather than the true behavior of the structure if boundary conditions are not introduced. In other words, without boundary conditions, there can be an infinite number of possible solutions since stiffness matrix $[K]$ is singular; that is, its determinant vanishes. Hence, Eqs. (7.28) should be modified to reflect boundary conditions and the final modified equations are expressed by inserting overbars as

$$\{\bar{F}\} = [\bar{K}]\{\bar{D}\} \tag{7.29}$$

### 7.5.3 Matrix Inversion

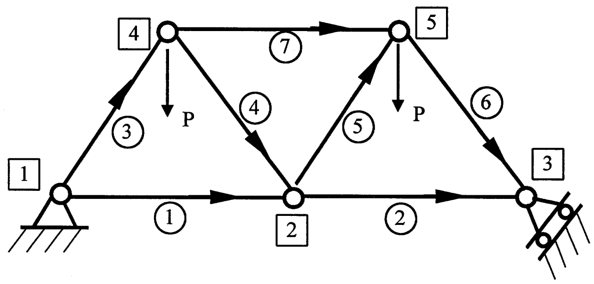
It has been shown that sets of simultaneous algebraic equations are generated in the application of both the displacement method and the force method in structural analysis, which are usually linear. The coefficients of the equations are constant and do not depend on the magnitude or conditions of loading and deformations, since linear Hook's law is generally assumed valid and small strains and deformations are used in the formulation. Solving Eq. (7.29) is, namely, to invert the modified stiffness matrix $[\bar{K}]$. This requires tremendous computational efforts for large-scale problems. The equations can be solved by using direct, iterative, or other methods. Two steps of elimination and backsubstitution are involved in the direct procedures, among which are Gaussian elimination and a number of its modifications. These are some of the most widely used sets of direct methods because of their better accuracy and small number of arithmetic operations.

### 7.5.4 Special Consideration

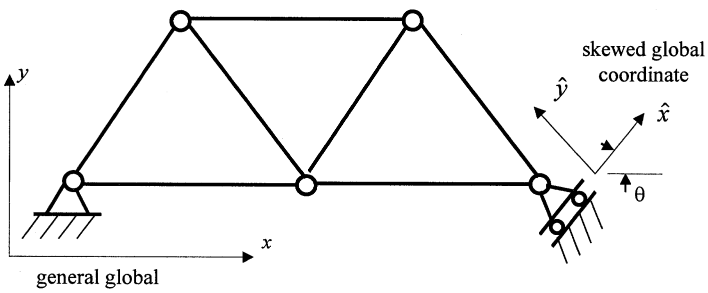
In practice, a variety of special circumstances, ranging from loading to internal member conditions and supporting conditions, should be given due consideration in structural analysis.

Initially strains, which are not directly associated with stresses, result from two causes, thermal loading or fabrication error. If the member with initial strains is unconstrained, there will be a set of initial member–end displacements associated with these initial strains, but nevertheless no initial member–end forces. For a member constrained to act as part of a structure, the general member force–displacement relationships will be modified as follows:

$$\{\bar{F}\}^e = [\bar{k}]^e\left(\{\bar{d}\}^e - \{\bar{d}_0\}^e\right) \tag{7.30a}$$




(a) Dimensions and Loading




(b) Mixed coordinate model

**FIGURE 7.9** Plane truss with skewed support.

or

$$\{\overline{F}\}^e = [\overline{k}]^e \{\overline{d}\}^e + \{R_{F0}\}^e \tag{7.30b}$$

where

$$\{R_{F0}\}^e = -[\overline{k}]^e \{\overline{d}_0\}^e \tag{7.31}$$

are fixed-end forces, and $\{\overline{d}_0\}^e$ a vector of initial member–end displacements for the member.

It is interesting to note that a support settlement may be regarded as an initial strain. Moreover, initial strains including thermal loading and fabrication errors, as well as support settlements, can all be treated as external excitations. Hence, the corresponding fixed-end forces as well as the equivalent nodal loading can be obtained which makes the conventional procedure described previously still practicable.

For a skewed support which provides a constraint to the structure in a nonglobal direction, the effect can be given due consideration by adapting a skewed global coordinate (see Figure 7.9) by introducing a skewed coordinate at the skewed support. This can perhaps be better demonstrated by considering a specific example of a plane truss shown in Figure 7.9. For members jointed at a skewed support, the coordinate transformation matrix will takes the form of

$$[\alpha] = \begin{bmatrix} \cos\alpha_i & -\sin\alpha_i & 0 & 0 \\ \sin\alpha_i & \cos\alpha_i & 0 & 0 \\ 0 & 0 & \cos\alpha_j & -\sin\alpha_j \\ 0 & 0 & \sin\alpha_j & \cos\alpha_j \end{bmatrix} \tag{7.32}$$

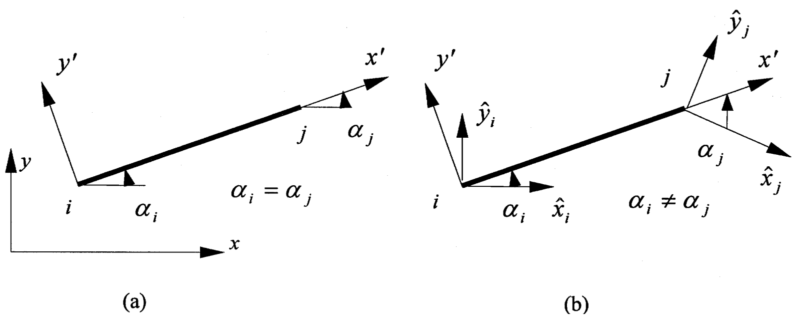



**FIGURE 7.10** Plane truss member coordinate transformation. (a) Normal global coordinate; (b) skewed global coordinate.

where $\alpha_i$ and $\alpha_j$ are inclined angles of truss member in skewed global coordinate (see Figure 7.10), say, for member 2 in Figure 7.9, $\alpha_i = 0$ and $\alpha_j = -\theta$.

For other special members such as inextensional or variable cross section ones, it may be necessary or convenient to employ special member force–displacement relations in structural analysis. Although the development and programming of a stiffness method general enough to take into account all these special considerations is formidable, more important perhaps is that the application of the method remains little changed. For more details, readers are referred to Reference. [5].

## 7.6 Substructuring and Symmetry Consideration

For highly complex or large-scale structures, one is required to solve a very large set of simultaneous equations, which are sometimes restricted by the computation resources available. In that case, special data-handling schemes like static condensation are needed to reduce the number of unknowns by appropriately numbering nodal displacement components and disposition of element force–displacement relations. Static condensation is useful in dynamic analysis of framed structures since the rotatory moment of inertia is usually neglected.

Another scheme physically partitions the structure into a collection of smaller structures called "substructures," which can be processed by parallel computers. In static analysis, the first step of substructuring is to introduce imaginary fixed inner boundaries, and then release all inner boundaries simultaneously, which gives rise to a subsequent analysis of these substructure series in a smaller scale. It is essentially the patitioning of Eq. (7.28) as follows. For the $r$th substructure, one has

*Case* $(\alpha)$ : *Introducing inner fixed boundaries*

$$\begin{bmatrix} K_{bb} & K_{bi} \\ K_{ib} & K_{ii} \end{bmatrix}^{(r)} \begin{Bmatrix} 0 \\ D_i^{\alpha} \end{Bmatrix}^{(r)} = \begin{Bmatrix} F_b^{\alpha} \\ F_i \end{Bmatrix}^{(r)} \tag{7.33}$$

*Case* $(\beta)$ : *Releasing all inner fixed boundaries*

$$\begin{bmatrix} K_{bb} & K_{bi} \\ K_{ib} & K_{ii} \end{bmatrix}^{(r)} \begin{Bmatrix} D_b \\ D_i^{\beta} \end{Bmatrix}^{(r)} = \begin{Bmatrix} F_b^{\beta} \\ 0 \end{Bmatrix}^{(r)} \tag{7.34}$$

where subscripts $b$ and $i$ denote inner fixed and free nodes, respectively.

Combining Eqs. (7.33) and (7.34) gives the force–displacement relations for enlarged elements—substructures which may be expressed as

$$\left[K_b\right]^{(r)}\left\{D_b\right\}^{(r)} = \left\{F_b\right\}^{(r)} \tag{7.35}$$

which is analogous to Eq. (7.26) and $\left\{F_b\right\}^{(r)} = \left\{F_b^{(r)}\right\} - \left[K_{bi}^{(r)}\right]\left[K_{ii}^{(r)}\right]^{-1}\left\{F_i^{(r)}\right\}$. And thereby the conventional procedure is still valid.

Similarly, in the cases of structural symmetry of geometry and material, proper consideration of loading symmetry and antisymmetry can give rise to a much smaller set of governing equations.

For more details, please refer to the literature on structural analysis.

## References


1. Chen, W.F. and Saleeb, A.F., *Constitutive Eqs. for Engineering Materials*, Vols. 1 & 2, Elsevier Science Ltd., New York, 1994.
2. Chen, W.F., *Plasticity in Reinforced Concrete*, McGraw-Hill, New York, 1982.
3. Chen, W.F. and T. Atsuta, *Theory of Beam-Columns*, Vol. 1, *In-Plane Behavior and Design*, McGraw-Hill, New York, 1976.
4. Desai, C.S., *Elementary Finite Element Method*, Prentice-Hall, Englewood Cliffs, NJ, 1979.
5. Meyers, V.J., *Matrix Analysis of Structures*, Harper & Row, New York, 1983.
6. Michalos, J., *Theory of Structural Analysis and Design*, Ronald Press, New York, 1958.
7. Hjelmstad, K.D., *Fundamentals of Structural Mechanics*, Prentice-Hall College Div., Upper Saddle River, NJ, 1996.
8. Fleming, J.F., *Analysis of Structural Systems*, Prentice-Hall College Div., Upper Saddle River, NJ, 1996.
9. Dadeppo, D.A. *Introduction to Structural Mechanics and Analysis*, Prentice-Hall College Div., Upper Saddle River, NJ, 1998.

Krimotat, A., Sheng, L. "Structural Modeling."
*Bridge Engineering Handbook.*
Ed. Wai-Fah Chen and Lian Duan
Boca Raton: CRC Press, 2000

# 8

# Structural Modeling



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## 8.1 Introduction

Prior to construction of any structural system, an extensive engineering design and analysis process must be undertaken. During this process, many engineering assumptions are routinely used in the application of engineering principles and theories to practice. A subset of these assumptions is used in a multitude of analytical methods available to structural analysts. In the modern engineering office, with the proliferation and increased power of personal computers, increasing numbers of engineers depend on structural analysis computer software to solve their engineering problems. This modernization of the engineering design office, coupled with an increased demand placed on the accuracy and efficiency of structural designs, requires a more-detailed understanding of the basic principles and assumptions associated with the use of modern structural analysis computer programs. The most popular of these programs are GT STRUDL, STAADIII, SAP2000, as well as some more powerful and complex tools such as ADINA, ANSYS, NASTRAN, and ABAQUS.

The objective of the analysis effort is to investigate the most probable responses of a bridge structure due to a range of applied loads. The results of these investigations must then be converted to useful design data, thereby providing designers with the information necessary to evaluate the performance of the bridge structure and to determine the appropriate actions in order to achieve the most efficient design configuration. Additionally, calculation of the structural system capacities is an important aspect in determining the most reliable design alternative. Every effort must be made to ensure that all work performed during any analytical activity enables designers to produce a set of quality construction documents including plans, specifications, and estimates.

The purpose of this chapter is to present basic modeling principles and suggest some guidelines and considerations that should be taken into account during the structural modeling process. Additionally, some examples of numerical characterizations of selected bridge structures and their components are provided. The outline of this chapter follows the basic modeling process. First, the selection of modeling methodology is discussed, followed by a description of the structural geometry, definition of the material and section properties of the components making up the structure, and description of the boundary conditions and loads acting on the structure.

## 8.2 Theoretical Background

Typically, during the analytical phase of any bridge design, finite-element-based structural analysis programs are used to evaluate the structural integrity of the bridge system. Most structural analysis programs employ sound, well-established finite-element methodologies and algorithms to solve the analytical problem. Others employ such methods as moment distribution, column analogy, virtual work, finite difference, and finite strip, to name a few. It is of utmost importance for the users of these programs to understand the theories, assumptions, and limitations of numerical modeling using the finite-element method, as well as the limitations on the accuracy of the computer systems used to execute these programs. Many textbooks [1, 4, 6] are available to study the theories and application of finite-element methodologies to practical engineering problems. It is strongly recommended that examination of these textbooks be made prior to using finite-element-based computer programs for any project work. For instance, when choosing the types of elements to use from the finite-element library, the user must consider some important factors such as the basic set of assumptions used in the element formulation, the types of behavior that each element type captures, and the limitations on the physical behavior of the system.

Other important issues to consider include numerical solution techniques used in matrix operations, computer numerical precision limitations, and solution methods used in a given analysis. There are many solution algorithms that employ direct or iterative methods, and sparse solver technology for solving the same basic problems; however, selecting these solution methods efficiently requires the user to understand the best conditions in which to apply each method and the basis or assumptions involved with each method. Understanding the solution parameters such as tolerances for iterative methods and how they can affect the accuracy of a solution are also important, especially during the nonlinear analysis process.

Dynamic analysis is increasingly being required by many design codes today, especially in regions of high seismicity. Response spectrum analysis is frequently used and easily performed with today's analysis tools; however, a basic understanding of structural dynamics is crucial for obtaining the proper results efficiently and interpreting analysis responses. Basic linear structural dynamics theory can be found in many textbooks [2,3]. While many analysis tools on the market today can perform very sophisticated analyses in a timely manner, the user too must be more savvy and knowledgeable to control the overall analysis effort and optimize the performance of such tools.

## 8.3 Modeling

### 8.3.1 Selection of Modeling Methodology

The technical approach taken by the engineer must be based on a philosophy of providing practical analysis in support of the design effort. Significant importance must be placed on the analysis procedures by the entire design team. All of the analytical modeling, analysis, and interpretation of results must be based on sound engineering judgment and a solid understanding of fundamental engineering principles. Ultimately, the analysis must validate the design.

Many factors contribute to determination of the modeling parameters. These factors should reflect issues such as the complexity of the structure under investigation, types of loads being examined, and, most importantly, the information needed to be obtained from the analysis in the most efficient and "design-friendly" formats. This section presents the basic principles and considerations for structural modeling. It also provides examples of modeling options for the various bridge structure types.

A typical flowchart of the analysis process is presented in Figure 8.1. The technical approach to computer modeling is usually based on a logical progression. The first step in achieving a reliable computer model is to define a proper set of material and soil properties, based on published data and site investigations. Second, critical components are assembled and tested numerically where

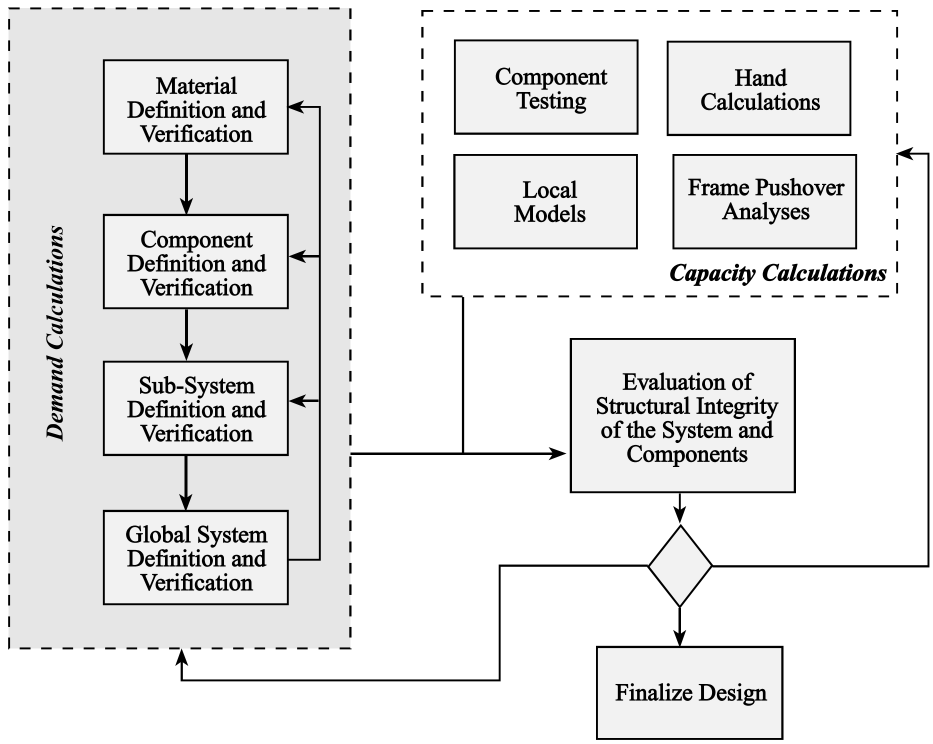


**FIGURE 8.1** Typical analysis process.

validation of the performance of these components is considered important to the global model response. Closed-form solutions or available test data are used for these validations.

The next step is the creation and numerical testing of subsystems such as the bridge towers, superstructure elements, or individual frames. Again, as in the previous step, simple procedures are used in parallel to validate computer models. Last, a full bridge model consisting of the bridge subsystems is assembled and exercised. This final global model should include appropriate representation of construction sequence, soil and foundation boundary conditions, structural component behavior, and connection details.

Following the analysis and after careful examination of the analytical results, the data is postprocessed and provided to the designers for the purpose of checking the design and determining suitable design modifications, as necessary. Postprocessing might include computation of deck section resultant forces and moments, determination of extreme values of displacements for columns or towers and deck, and recovery of forces of constraint between structural components. The entire process may be repeated to validate any modifications made, depending on the nature and significance of such modifications.

An important part of the overall analytical procedure is determination of the capacities of the structural members. A combination of engineering calculations, computer analyses, and testing is utilized in order to develop a comprehensive set of component and system capacities. The evaluation of the structural integrity of the bridge structure, its components, and their connections are then conducted by comparing capacities with the demands calculated from the structural analysis.

Depending on the complexity of the structure under investigation and the nature of applied loads, two- or three-dimensional models can be utilized. In most cases, beam elements can be used to model structural elements of the bridge (Figure 8.2), so the component responses are presented in the form of force and moment resultants. These results are normally associated with individual element coordinate systems, thus simplifying the evaluations of these components. Normally, these

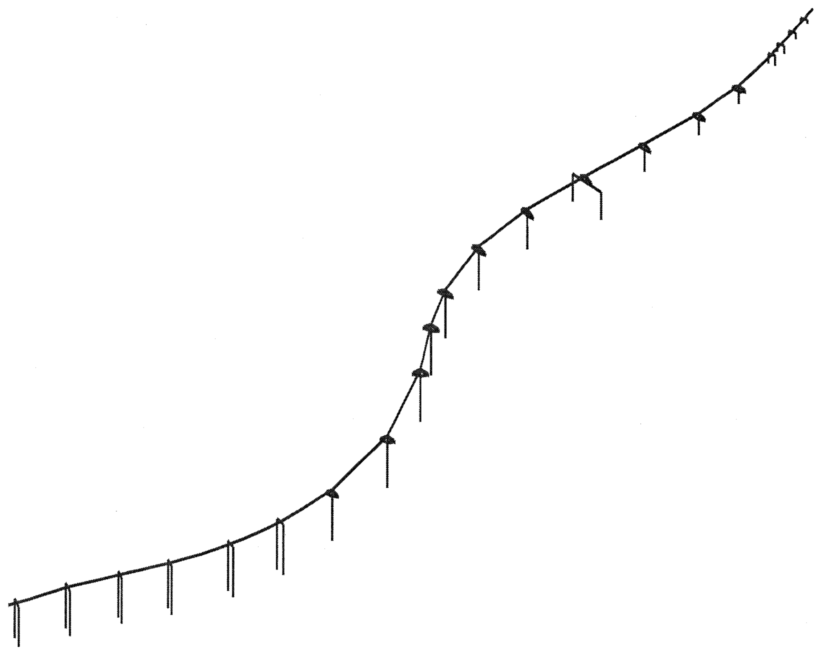

**FIGURE 8.2** Typical beam model.

force resultants describe axial, shear, torsion, and bending actions at a given model location. Therefore, it is very important during the initial modeling stages to determine key locations of interest, so the model can be assembled such that important results can be obtained at these locations. While it is convenient to use element coordinate systems for the evaluation of the structural integrity of individual components, nodal results such as displacements and support reactions are usually output in the global coordinate systems. Proper refinement of the components must also be considered since different mesh size can sometimes cause significant variations in results. A balance between mesh refinement and reasonable element aspect ratios must be maintained so that the behavioral characteristics of the computer model is representative of the structure it simulates. Also, mesh refinement considerations must be made in conjunction with the cost to model efficiency. Higher orders of accuracy in modeling often come at a cost of analysis turnaround time and overall model efficiency. The analyst must use engineering judgment to determine if the benefits of mesh refinement justify the costs. For example, for the convenience in design of bridge details such as reinforcement bar cutoff, prestressing cable layouts, and section changes, the bridge superstructure is usually modeled with a high degree of refinement in the dead- and live-load analyses to achieve a well-defined force distribution. The same refinement may not be necessary in a dynamic analysis. Quite often, coarser models (at least four elements per span for the superstructure and three elements per column) are used in the dynamic analyses. These refinements are the minimum guidelines for discrete lumped mass models in dynamic analysis to maintain a reasonable mass distribution during the numerical solution process.

For more complex structures with complicated geometric configurations, such as curved plate girder bridges (Figure 8.3), or bridges with highly skewed supports (Figure 8.4), more-detailed finite-element models should be considered, especially if individual components within the superstructure need to be evaluated, which could not be facilitated with a beam superstructure representation. With the increasing speed of desktop computers, and advances in finite-element modeling tools, these models are becoming increasingly more popular. The main reason for their increased popularity is the improved accuracy, which in turn results in more efficient and cost-effective design.

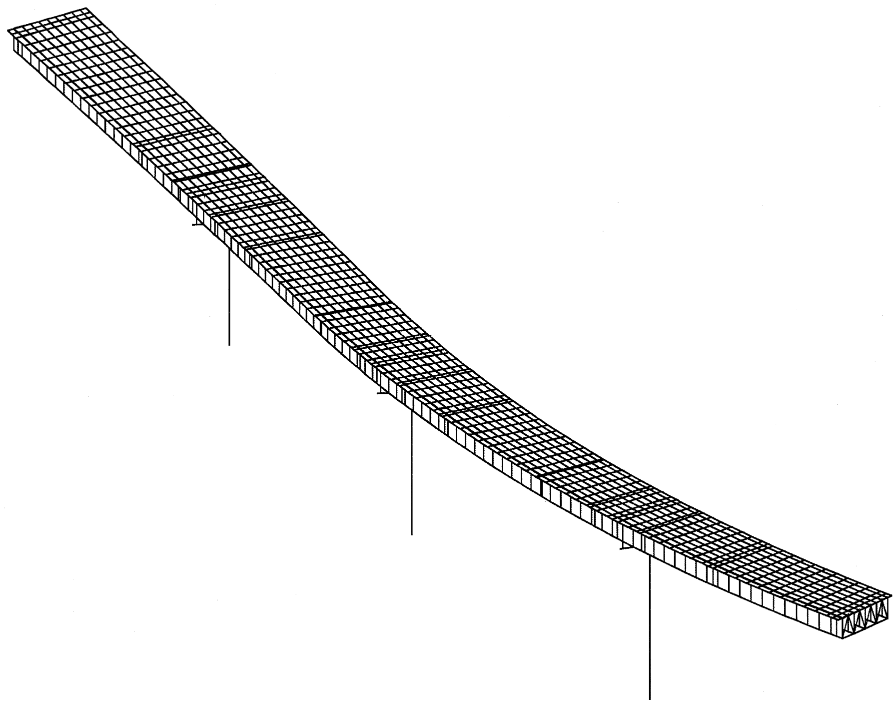

**FIGURE 8.3** Steel plate-girder bridge—finite-element model.

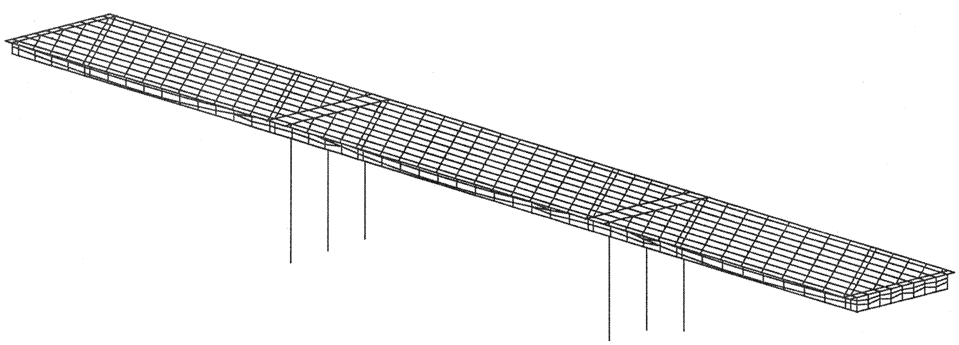

**FIGURE 8.4** Concrete box girder with 45° skewed supports finite-element model.

More complex models, however, require a significantly higher degree of engineering experience and expertise in the theory and application of the finite-element method. In the case of a complex model, the engineer must determine the degree of refinement of the model. This determination is usually made based on the types of applied loads as well as the behavioral characteristics of the structure being represented by the finite-element model. It is important to note that the format of the results obtained from detailed models, such as shell and three-dimensional (3D) continuum models) is quite different from the results obtained from beam (or stick) models. Stresses and strains are obtained for each of the bridge components at a much more detailed level; therefore, calculation of a total force applied to the superstructure, for example, becomes a more difficult, tedious task. However, evaluation of local component behavior, such as cross frames, plate girder sections, or bridge deck sections, can be accomplished directly from the analysis results of a detailed finite-element model.

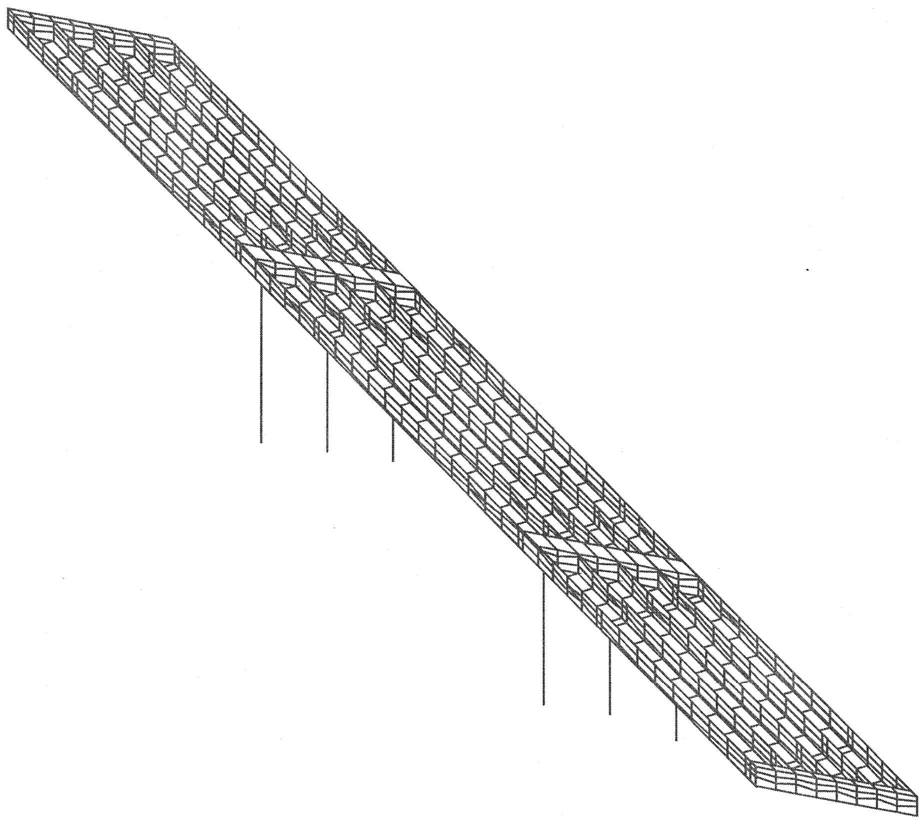

FIGURE 8.5 Concrete box-girder modeling example (deck elements not shown).

### 8.3.2 Geometry

After selecting an appropriate modeling methodology, serious considerations must be given to proper representation of the bridge geometric characteristics. These geometric issues are directly related to the behavioral characteristics of the structural components as well as the overall global structure. The considerations must include not only the global geometry of the bridge structure, i.e., horizontal alignment, vertical elevation, superelevation of the roadway, and severity of the support skews, but local geometric characterizations of connection details of individual bridge components as well. Such details include representations of connection regions such as column-to-cap beam, column-to-box girder, column-to-pile cap, cap beam to superstructure, cross frames to plate girder, gusset plates to adjacent structural elements, as well as various bearing systems commonly used in bridge engineering practice. Some examples of some modeling details are demonstrated in Figures 8.5 through 8.11.

Specifically, Figure 8.5 demonstrates how a detailed model of a box girder bridge structure can be assembled via use of shell elements (for girder webs and soffit), truss elements (for post-tensioning tendons), 3D solid elements (for internal diaphragms), and beam elements (for columns). Figure 8.6 illustrates some details of the web, deck, and abutment modeling for the same bridge structure. Additionally, spring elements are used to represent abutment support conditions for the vertical as well as back-wall directions. An example of a column and its connection to the superstructure in an explicit finite-element model is presented in the Figure 8.7. Three elements are used to represent the full length of the column. A set of rigid links connects the superstructure to each of the supporting columns (Figures 8.8 and 8.9). This is necessary to properly transmit bending

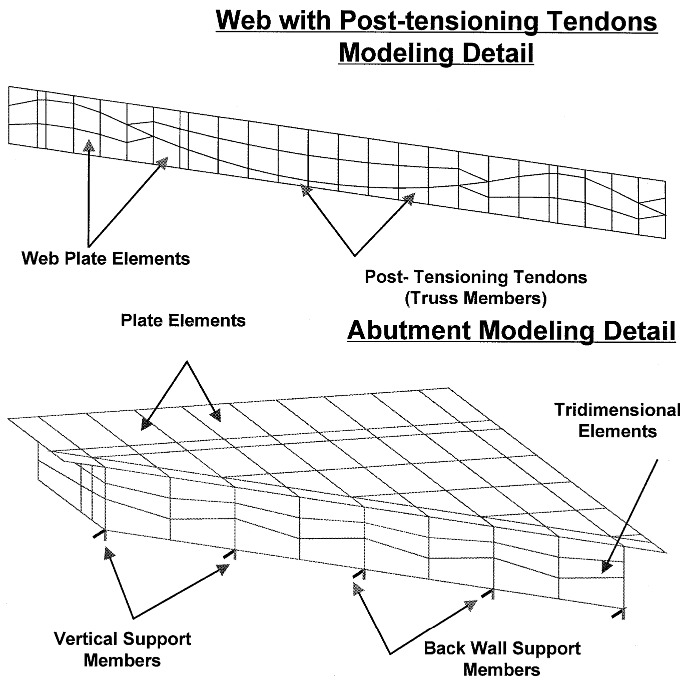


**FIGURE 8.6** Selected modeling details.

action of these components, since the beam elements (columns) are characterized by six degrees of freedom per node, while 3D solids (internal diaphragms) carry only three degrees of freedom per node (translations only). In this example post-tensioning tendons are modeled explicitly, via truss elements with the proper drape shape (Figure 8.9). This was done so that accurate post-tensioning load application was achieved and the effects of the skews were examined in detail. However, when beam models are used for the dynamic analysis (Figure 8.2), special attention must be given to the beam column joint modeling. For a box girder superstructure, since cap beams are monolithic to the superstructure, considerations must be given to capture proper dynamic behavior of this detail through modification of the connection properties. It is common to increase the section properties of the cap beam embedded in the superstructure to simulate high stiffness of this connection.

Figure 8.10 illustrates the plate girder modeling approach for a section of superstructure. Plate elements are used to model deck sections and girder webs, while beams are used to characterize flanges, haunches, cross frame members, as well as columns and cap beams (Figure 8.11). Proper offsets are used to locate the centerlines of these components in their proper locations.

### 8.3.3 Material and Section Properties

One of the most important aspects of capturing proper behavior of the structure is the determination of the material and section properties of its components. Reference [5] is widely used for calculating section properties for a variety of cross-sectional geometry. For 3D solid finite element, the material constitutive law is the only thing to specify whereas for other elements consideration of modification of material properties are needed to match the actual structural behavior. Most structural theories are based on homogeneous material such as steel. While this means structural behavior can be directly calculated using the actual material and section properties, it also indicates that nonhomogeneous material such as reinforced concrete may subject certain limitation. Because of the composite nonlinear performance nature of reinforced concrete, section properties need to be adjusted for the objective of analysis. For elastic analysis, if strength requirement is the objective, section

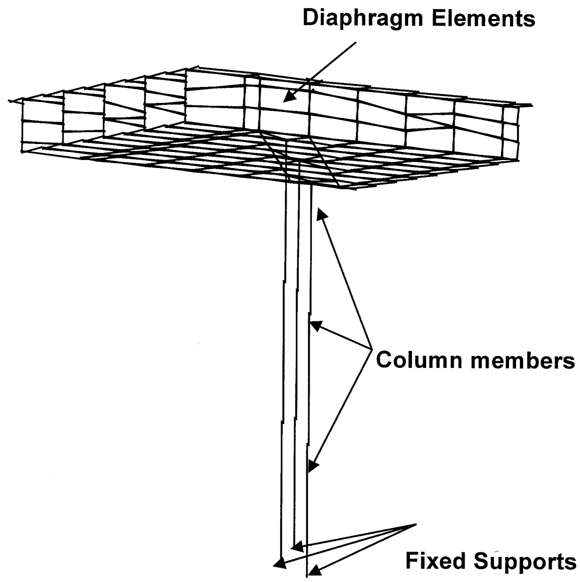


**FIGURE 8.7** Bent region modeling detail.

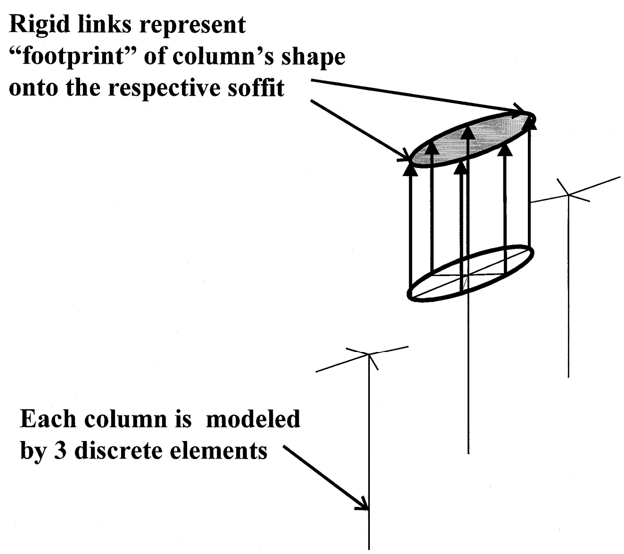


**FIGURE 8.8** Column-to-superstructure connection modeling detail.

properties are less important as long as relative stiffness is correct. Section properties become most critical when structure displacement and deformation are objectives. Since concrete cracks beyond certain deformation, section properties need to be modified for this behavior. In general, if ultimate deformation is expected, then effective stiffness should be the consideration in section properties. It is common to use half value of the moment of inertia for reinforced concrete members and full value for prestressed concrete members. To replicate a rigid member behavior such as cap beams, section properties need to be amplified 100 times to eliminate local vibration problems in dynamic analysis.

Nonlinear behaviors are most difficult to handle in both complex and simple finite-element models. When solid elements are used, the constitutive relationships describing material behavior

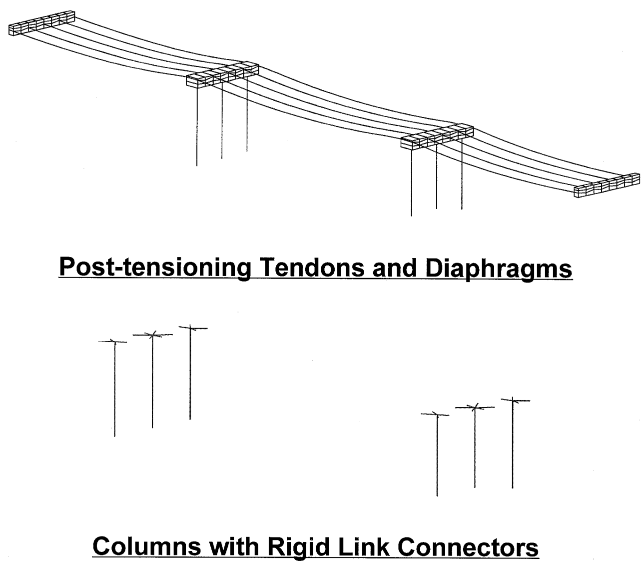


**FIGURE 8.9** Post-tensioning tendons, diaphragms, and column-to-diaphragm connection modeling examples.

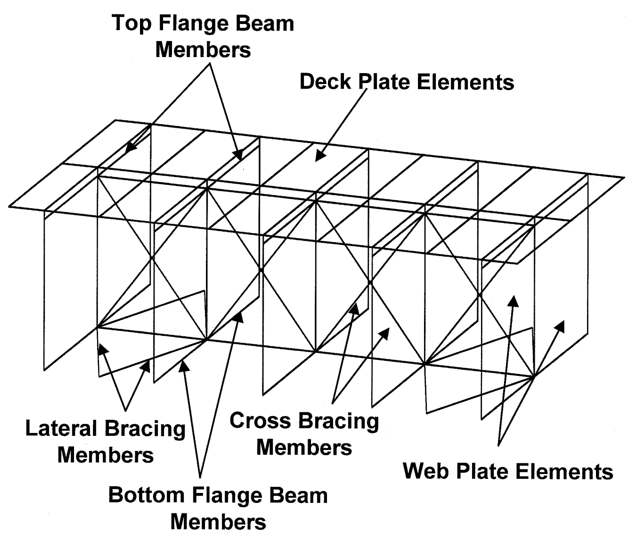


**FIGURE 8.10** Plate girder superstructure modeling example.

should be utilized. These properties should be calibrated by the data obtained from the available test experiments. For beam–column-type elements, however, it is essential that the engineer properly estimates performance of the components either by experiments or theoretical detailed analysis. Once member performance is established, a simplified inelastic model can be used to simulate the expected member behavior. Depending on the complexity of the member, bilinear, or multilinear material representations may be used extensively. If member degradation needs to be incorporated in the analysis, then the Takeda model may be used. While a degrading model can correlate theoretical behavior with experimental results very well, elastic–plastic or bilinear models can give the engineer a good estimate of structural behavior without detailed material property parameters.

When a nonlinear analysis is performed, the engineer needs to understand the sensitivity issue raised by such analysis techniques. Without a good understanding of member behavior, it is very

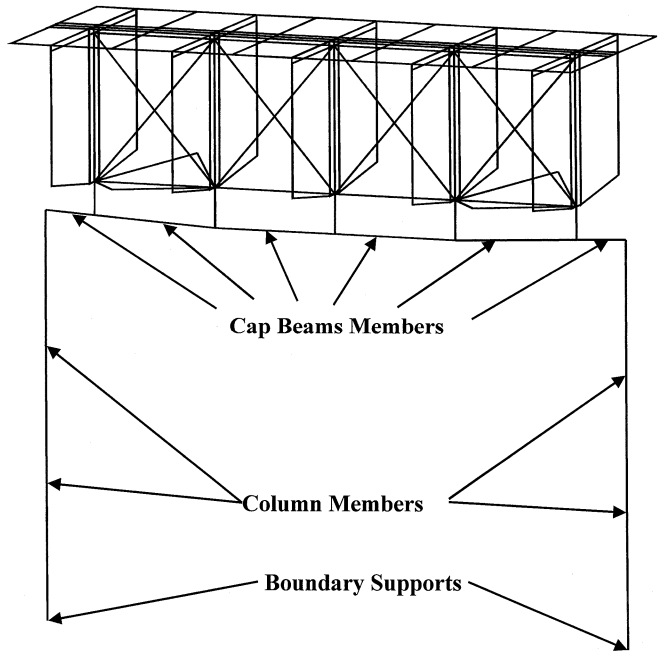


**FIGURE 8.11** Plate girder bent region modeling example.

easy to fall into the "garbage in, garbage out" mode of operation. It is essential for the engineer to verify member behavior with known material properties before any production analyses are conducted. For initial design, all material properties should be based on the nominal values. However, it is important to verify the design with the expected material properties.

## 8.3.4 Boundary Conditions

Another key ingredient for the success of the structural analysis is the proper characterization of the boundary conditions of the structural system. Conditions of the columns or abutments at the support (or ground) points must be examined by engineers and properly implemented into the structural analysis model. This can be accomplished via several means based on different engineering assumptions. For example, during most of the static analysis, it is common to use a simple representation of supports (e.g., fixed, pinned, roller) without characterizations of the soil/foundation stiffness. However, for a dynamic analysis, proper representation of the soil/foundation system is essential (Figure 8.12). Most finite-element programs will accept a $[6 \times 6]$ stiffness matrix input for such system. Other programs require extended $[12 \times 12]$ stiffness matrix input describing the relationship between the ground point and the base of the columns. Prior to using these matrices, it is important that the user investigate the internal workings of the finite-element program, so the proper results are obtained by the analysis.

In some cases it is necessary to model the foundation/soil system with greater detail. Nonlinear modeling of the system can be accomplished via nonlinear spring/damper representation (Figure 8.13) or, in the extreme case, by explicit modeling of subsurface elements and plasticity-based springs representing surrounding soil mass (Figure 8.14). It is important that if this degree of detail is necessary, the structural engineer works very closely with the geotechnical engineers to determine proper properties of the soil springs. **As a general rule it is essential to set up small models to test behavior and check the results via hand calculations.**

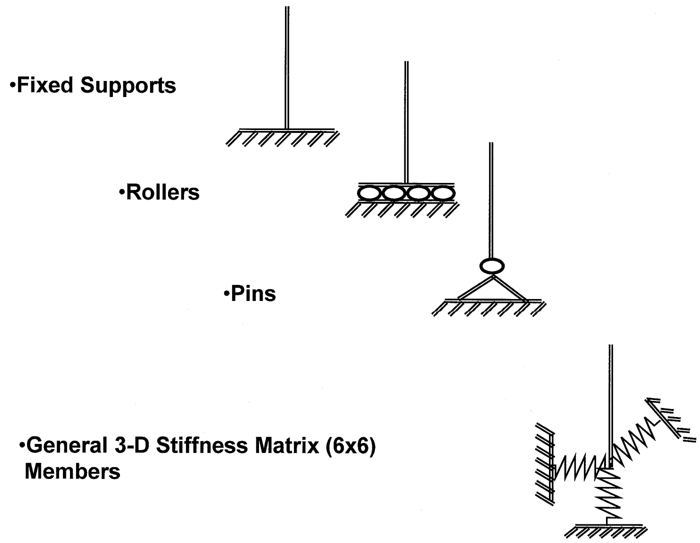


**FIGURE 8.12** Examples of foundation modeling.

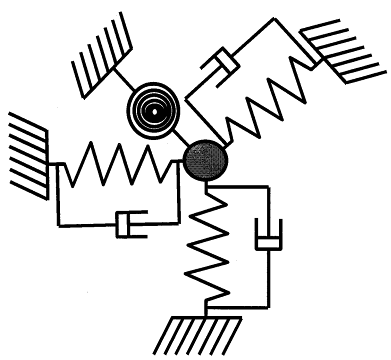

**FIGURE 8.13** Nonlinear spring/damper model.

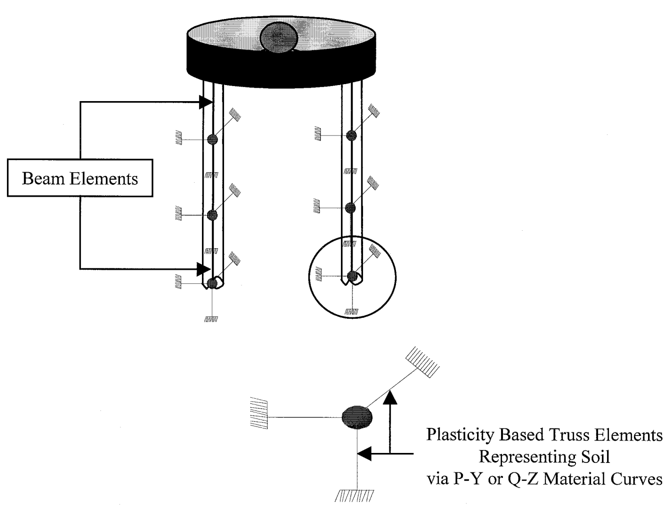


**FIGURE 8.14** Soil–structure interaction modeling.

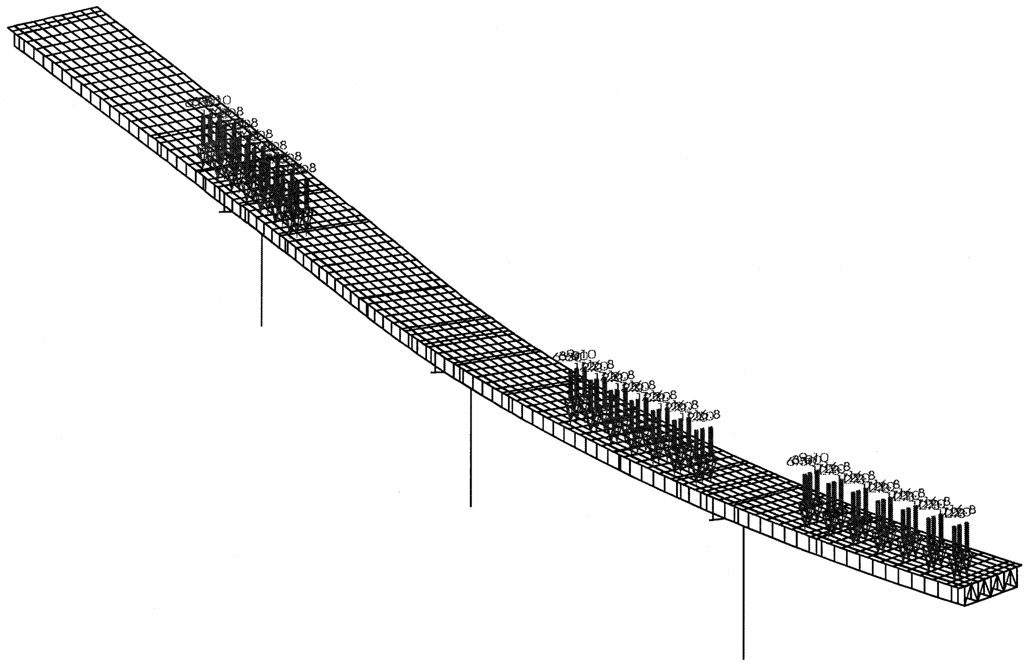

FIGURE 8.15 Truck load application example.

### 8.3.5 Loads

During engineering design activities, computer models are used to evaluate bridge structures for various service loads, such as traffic, wind, thermal, construction, and other service loads. These service loads can be represented by a series of static load cases applied to the structural model. Some examples of application of the truck loads are presented in Figures 8.15 and 8.16.

In many cases, especially in high seismic zones, dynamic loads control many bridge design parameters. In this case, it is very important to understand the nature of these loads, as well as the theory that governs the behavior of structural systems subjected to these dynamic loads. In high seismic zones, a multimode response spectrum analysis is required to evaluate the dynamic response of bridge structures. In this case, the response spectrum loading is usually described by the relationship of the structural period vs. ground acceleration, velocity, or displacement for a given structural damping. In some cases, usually for more complex bridge structures, a time history analysis is required. During these analytical investigations, a set of time history loads (normally, displacement or acceleration vs. time) is applied to the boundary nodes of the structure. Reference [3] is the most widely used theoretical reference related to the seismic analysis methodology for either response spectrum or time history analysis.

## 8.4 Summary

In summary, the analysis effort should support the overall design effort by verifying the design and addressing any issues with respect to the efficiency and the viability of the design. Before modeling commences, the engineer must define the scope of the problem and ask what key results and types of data he or she is interested in obtaining from the analytical model. With these basic parameters in mind, the engineer can then apply technical knowledge to formulate the simplest, most elegant model to represent the structure properly and provide the range of solutions that are accurate and fundamentally sound. The engineer must bound the demands on the structure by looking at limiting

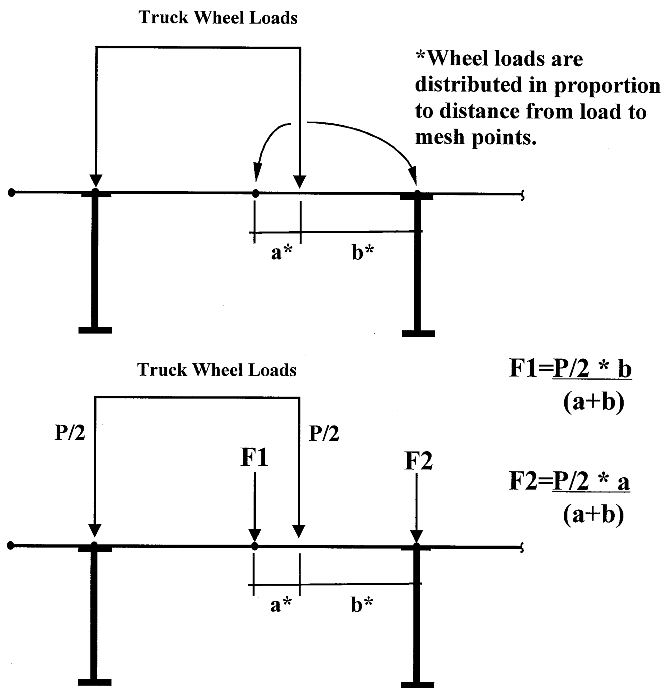


**FIGURE 8.16** Equivalent truck load calculation example.

load cases and modifying the structure parameters, such as boundary conditions or material properties. Rigorous testing of components, hand calculations, local modeling, and sound engineering judgment must be used to validate the analytical model at all levels. Through a rigorous analytical methodology and proper use of today's analytical tools, structural engineers can gain a better understanding of the behavior of the structure, evaluate the integrity of the structure, and validate and optimize the structural design.

## References

1. Bathe, K.J., *Finite Element Procedures*, 2nd ed., Prentice-Hall, Englewood Cliffs, NJ, 1996.
2. Chopra, A. K., *Dynamics of Structures, Theory and Applications to Earthquake Engineering*, Prentice-Hall, Englewood Cliffs, NJ, 1995.
3. Clough, R. W. and J. Penzien, *Dynamics of Structures*, 2nd ed., McGraw-Hill, New York, 1993.
4. Priestley, M.J.N., F. Seible, and G.M. Calvi, *Seismic Design and Retrofit of Bridges*, John Wiley & Sons, New York, 1996.
5. Young, W.C., *Roark's Formulas for Stress and Strain*, 6th ed., McGraw-Hill, New York, 1989.
6. Zienkiewicz, O.C. and R.L.Taylor, *The Finite Element Method*, Vol. 1, *Basic Formulation and Linear Problems*, 4th ed., McGraw-Hill, Berkshire, England, 1994.

Lyang, J., Lee, D., Kung, J. "Reinforced Concrete Bridges."
*Bridge Engineering Handbook.*
Ed. Wai-Fah Chen and Lian Duan
Boca Raton: CRC Press, 2000

# Section II
## Superstructure Design

# 9

# Reinforced Concrete Bridges



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## 9.1 Introduction

The raw materials of concrete, consisting of water, fine aggregate, coarse aggregate, and cement, can be found in most areas of the world and can be mixed to form a variety of structural shapes. The great availability and flexibility of concrete material and reinforcing bars have made the reinforced concrete bridge a very competitive alternative. Reinforced concrete bridges may consist of precast concrete elements, which are fabricated at a production plant and then transported for erection at the job site, or cast-in-place concrete, which is formed and cast directly in its setting location. Cast-in-place concrete structures are often constructed monolithically and continuously. They usually provide a relatively low maintenance cost and better earthquake-resistance performance. Cast-in-place concrete structures, however, may not be a good choice when the project is on a fast-track construction schedule or when the available falsework opening clearance is limited. In this chapter, various structural types and design considerations for conventional cast-in-place, reinforced concrete highway bridge are discussed. Two design examples of a simply supported slab bridge and a two-span box girder bridge are also presented. All design specifications referenced in this chapter are based on 1994 AASHTO LRFD (Load and Resistance Factor Design) Bridge Design Specifications [1].

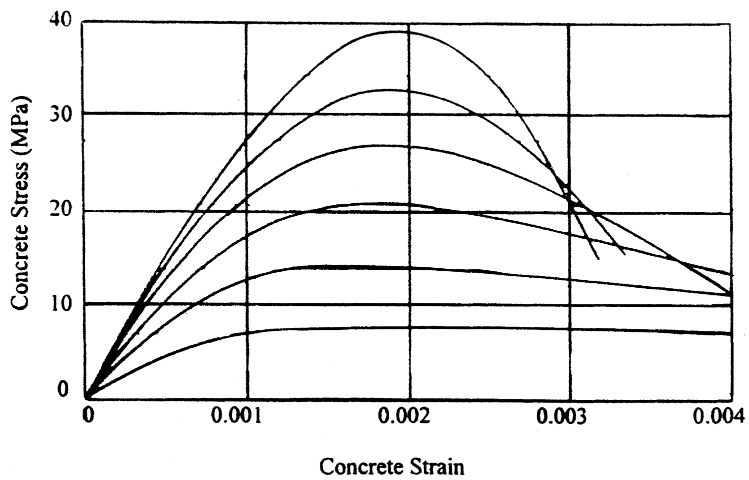



**FIGURE 9.1** Typical stress–strain curves for concrete under uniaxial compression loading.

## 9.2 Materials

### 9.2.1 Concrete

#### 1 Compressive Strength

The compressive strength of concrete ($f_c'$) at 28 days after placement is usually obtained from a standard 150-mm-diameter by 300-mm-high cylinder loaded longitudinally to failure. Figure 9.1 shows typical stress–strain curves from unconfined concrete cylinders under uniaxial compression loading. The strain at the peak compression stress $f_c'$ is approximately 0.002 and maximum usable strain is about 0.003. The concrete modulus of elasticity, $E_c$, may be calculated as

$$E_c = 0.043\gamma_c^{1.5}\sqrt{f_c'} \text{ MPa} \tag{9.1}$$

where $\gamma_c$ is the density of concrete (kg/m$^3$) and $f_c'$ is the specified strength of concrete (MPa). For normal-weight concrete ($\gamma_c = 2300$ kg/m$^3$), $E_c$ may be calculated as $4800\sqrt{f_c'}$ MPa.

The concrete compressive strength or class of concrete should be specified in the contract documents for each bridge component. A typical specification for different classes of concrete and their corresponding specified compressive strengths is shown in Table 9.1. These classes are intended for use as follows:

- Class A concrete is generally used for all elements of structures and specially for concrete exposed to salt water.
- Class B concrete is used in footings, pedestals, massive pier shafts, and gravity walls.
- Class C concrete is used in thin sections under 100 mm in thickness, such as reinforced railings and for filler in steel grid floors.
- Class P concrete is used when strengths exceeding 28 MPa are required.
- Class S concrete is used for concrete deposited under water in cofferdams to seal out water.

Both concrete compressive strengths and water–cement ratios are specified in Table 9.1 for different concrete classes. This is because the water–cement ratio is a dominant factor contributing to both durability and strength, while simply obtaining the required concrete compressive strength to satisfy the design assumptions may not ensure adequate durability.

**TABLE 9.1** Concrete Mix Characteristics by Class[1]

| Class of Concrete | Minimum Cement Content (kg/m³) | Maximum Water–Cement Ratio (kg/kg) | Air Content Range, % | Coarse Aggregate per AASHTO M43 (square size of openings, mm) | 28-day Compressive Strength, $f'_c$ MPa |
|---|---|---|---|---|---|
| A | 362 | 0.49 | — | 25 to 4.75 | 28 |
| A(AE) | 362 | 0.45 | $6.0 \pm 1.5$ | 25 to 4.75 | 28 |
| B | 307 | 0.58 | — | 50 to 4.75 | 17 |
| B(AE) | 307 | 0.55 | $5.0 \pm 1.5$ | 50 to 4.75 | 17 |
| C | 390 | 0.49 | — | 12.5 to 4.75 | 28 |
| C(AE) | 390 | 0.45 | $7.0 \pm 1.5$ | 12.5 to 4.75 | 28 |
| P | 334 | 0.49 | As specified elsewhere | 25 to 4.75 or 19 to 4.75 | As specified elsewhere |
| S | 390 | 0.58 | — | 25 to 4.75 | — |
| Low-density | 334 | As specified in the contract documents | | | |

*Notes*:
1. AASHTO Table C5.4.2.1-1 (From AASHTO LRFD Bridge Design Specifications, ©1994 by the American Association of State Highway and Transportation Officials, Washington, D.C. With permission.)
2. Concrete strengths above 70 MPa need to have laboratory testing verification. Concrete strengths below 16 MPa should not be used.
3. The sum of portland cement and other cementitious materials should not exceed 475 kg/m3.
4. Air-entrained concrete (AE) can improve durability when subjected to freeze–thaw action and to scaling caused by chemicals applied for snow and ice removal.

## 2. Tensile Strength

The tensile strength of concrete can be measured directly from tension loading. However, fixtures for holding the specimens are difficult to apply uniform axial tension loading and sometimes will even introduce unwanted secondary stresses. The direct tension test method is therefore usually used to determine the cracking strength of concrete caused by effects other than flexure. For most regular concrete, the direct tensile strength may be estimated as 10% of the compressive strength.

The tensile strength of concrete may be obtained indirectly by the split tensile strength method. The splitting tensile stress ($f_s$) at which a cylinder is placed horizontally in a testing machine and loaded along a diameter until split failure can be calculated as

$$f_s = 2P/(\pi LD) \tag{9.2}$$

where $P$ is the total applied load that splits the cylinder, $L$ is the length of cylinder, and $D$ is the diameter of the cylinder.

The tensile strength of concrete can also be evaluated by means of bending tests conducted on plain concrete beams. The flexural tensile stress, known as the modulus of rupture ($f_r$) is computed from the flexural formula $M/S$, where $M$ is the applied failure bending moment and $S$ is the elastic section modulus of the beam. Modulus of rupture ($f_r$) in MPa can be calculated as

$$f_r = \begin{cases} 0.63\sqrt{f'_c} & \text{for normal-weight concrete} \\ 0.52\sqrt{f'_c} & \text{for sand–low-density concrete} \\ 0.45\sqrt{f'_c} & \text{for all–low-density concrete} \end{cases} \tag{9.3}$$

**TABLE 9.2** Steel Deformed Bar Sizes and Weight (ASTM A615M and A706M)

| Bar Number | Nominal Dimensions | | |
|---|---|---|---|
| | Diameter, mm | Area, mm$^2$ | Unit Weight, kg/m |
| 10 | 9.5 | 71 | 0.560 |
| 13 | 12.7 | 129 | 0.994 |
| 16 | 15.9 | 199 | 1.552 |
| 19 | 19.1 | 284 | 2.235 |
| 22 | 22.2 | 387 | 3.042 |
| 25 | 25.4 | 510 | 3.973 |
| 29 | 28.7 | 645 | 5.060 |
| 32 | 32.3 | 819 | 6.404 |
| 36 | 35.8 | 1006 | 7.907 |
| 43 | 43.0 | 1452 | 11.38 |
| 57 | 57.3 | 2581 | 20.24 |

Both the splitting tensile stress ($f_s$) and flexural tensile stress ($f_r$) overestimate the tensile cracking stress determined by a direct tension test. However, concrete in tension is usually ignored in strength calculations of reinforced concrete members because the tensile strength of concrete is low. The modulus of elasticity for concrete in tension may be assumed to be the same as in compression.

### 3. Creep and Shrinkage

Both creep and shrinkage of concrete are time-dependent deformations and are discussed in Chapter 10.

## 9.2.2 Steel Reinforcement

Deformed steel bars are commonly employed as reinforcement in most reinforced concrete bridge construction. The surface of a steel bar is rolled with lugs or protrusions called deformations in order to restrict longitudinal movement between the bars and the surrounding concrete. Reinforcing bars, rolled according to ASTM A615/A615M specifications (billet steel) [2], are widely used in construction. ASTM A706/A706M low-alloy steel deformed bars (Grade 420 only) [2] are specified for special applications where extensive welding of reinforcement or controlled ductility for earthquake-resistant, reinforced concrete structures or both are of importance.

### 1. Bar Shape and Size

Deformed steel bars are approximately numbered based on the amount of millimeters of the nominal diameter of the bar. The nominal dimensions of a deformed bar are equivalent to those of a plain round bar which has the same mass per meter as the deformed bar. Table 9.2 lists a range of deformed bar sizes according to the ASTM specifications.

### 2. Stress–Strain Curve

The behavior of steel reinforcement is usually characterized by the stress–strain curve under uniaxial tension loading. Typical stress–strain curves for steel Grade 300 and 420 are shown in Figure 9.2. The curves exhibit an initial linear elastic portion with a slope calculated as the modulus of elasticity of steel reinforcement $E_s$ = 200,000 MPa; a yield plateau in which the strain increases (from $\varepsilon_y$ to $\varepsilon_h$) with little or no increase in yield stress ($f_y$); a strain-hardening range in which stress again increases with strain until the maximum stress ($f_u$) at a strain ($\varepsilon_u$) is reached; and finally a range in which the stress drops off until fracture occurs at a breaking strain of $\varepsilon_b$.

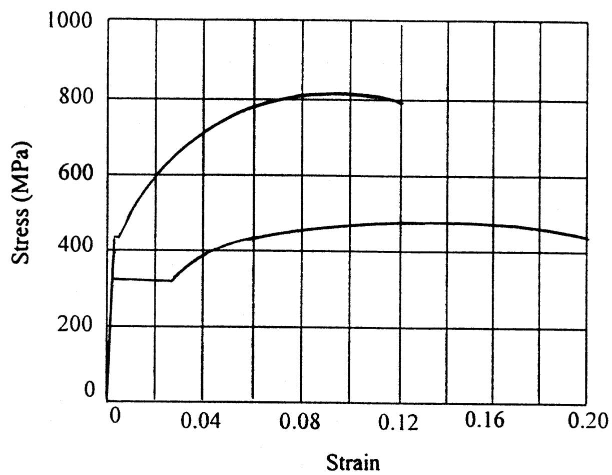



**FIGURE 9.2** Typical stress–strain curves for steel reinforcement.

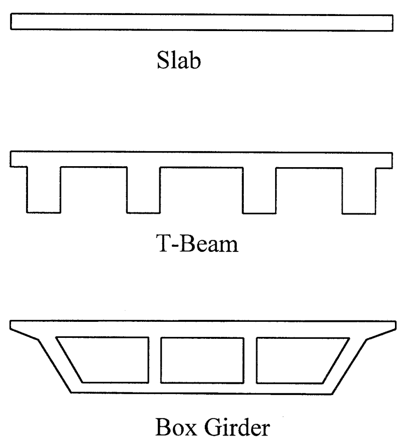



**FIGURE 9.3** Typical reinforced concrete sections in bridge superstructures.

## 9.3 Bridge Types

Reinforced concrete sections, used in the bridge superstructures, usually consist of slabs, T-beams (deck girders), and box girders (Figure 9.3). Safety, cost-effectiveness, and aesthetics are generally the controlling factors in the selection of the proper type of bridges [3]. Occasionally, the selection is complicated by other considerations such as the deflection limit, life-cycle cost, traffic maintenance during construction stages, construction scheduling and worker safety, feasibility of falsework layout, passage of flood debris, seismicity at the site, suitability for future widening, and commitments made to officials and individuals of the community. In some cases, a prestressed concrete or steel bridge may be a better choice.

### 9.3.1 Slab Bridges

Longitudinally reinforced slab bridges have the simplest superstructure configuration and the neatest appearance. They generally require more reinforcing steel and structural concrete than do girder-type bridges of the same span. However, the design details and formworks are easier and less expensive. It has been found economical for simply supported spans up to 9 m and for continuous spans up to 12 m.

### 9.3.2 T-Beam Bridges

The T-beam construction consists of a transversely reinforced slab deck which spans across to the longitudinal support girders. These require a more-complicated formwork, particularly for skewed bridges, compared to the other superstructure forms. T-beam bridges are generally more economical for spans of 12 to 18 m. The girder stem thickness usually varies from 35 to 55 cm and is controlled by the required horizontal spacing of the positive moment reinforcement. Optimum lateral spacing of longitudinal girders is typically between 1.8 and 3.0 m for a minimum cost of formwork and structural materials. However, where vertical supports for the formwork are difficult and expensive, girder spacing can be increased accordingly.

### 9.3.3 Box-Girder Bridges

Box-girder bridges contain top deck, vertical web, and bottom slab and are often used for spans of 15 to 36 m with girders spaced at 1.5 times the structure depth. Beyond this range, it is probably more economical to consider a different type of bridge, such as post-tensioned box girder or steel girder superstructure. This is because of the massive increase in volume and materials. They can be viewed as T-beam structures for both positive and negative moments. The high torsional strength of the box girder makes it particularly suitable for sharp curve alignment, skewed piers and abutments, superelevation, and transitions such as interchange ramp structures.

## 9.4 Design Considerations

### 9.4.1 Basic Design Theory

The AASHTO LRFD Specifications (1994) [1] were developed in a reliability-based limit state design format. Limit state is defined as the limiting condition of acceptable performance for which the bridge or component was designed. In order to achieve the objective for a safe design, each bridge member and connection is required to examine some, or all, of the service, fatigue, strength, and extreme event limit states. All applicable limit states shall be considered of equal importance. The basic requirement for bridge design in the LRFD format for each limit state is as follows:

$$\eta \sum \gamma_i Q_i \leq \phi R_n \tag{9.4}$$

where $\eta$ = load modifier to account for bridge ductility, redundancy, and operational importance, $\gamma_i$ = load factor for load component *i*, $Q_i$ = nominal force effect for load component *i*, $\phi$ = resistance factor, and $R_n$ = nominal resistance. The margin of safety for a bridge design is provided by ensuring the bridge has sufficient capacity to resist various loading combinations in different limit states.

The load factors, $\gamma$, which often have values larger than one, account for the loading uncertainties and their probabilities of occurrence during bridges design life. The resistance factors, $\phi$, which are typically less than unity at the strength limit state and equal to unity for all other limit states, account for material variabilities and model uncertainties. Table 9.3 lists the resistance factors in the strength limit state for conventional concrete construction. The load modifiers, $\eta$, which are equal to unity for all non-strength-limit states, account for structure ductility, redundancy, and operational importance. They are related to the bridge physical strength and the effects of a bridge being out of service. Detailed load resistance factor design theory and philosophy are discussed in Chapter 5.

**TABLE 9.3** Resistance Factors $\phi$ in the Strength Limit State for Conventional Construction

| Strength Limit State | Resistance Factors $\phi$ |
|---|---|
| For flexural and tension of reinforced concrete | 0.90 |
| For shear and torsion | |
| Normal weight concrete | 0.90 |
| Lightweight concrete | 0.70 |
| For axial compression with spirals and ties | 0.75 |
| (except for Seismic Zones 3 and 4 at the extreme event limit state) | |
| For bearing on concrete | 0.79 |
| For compression in strut-and-tie models | 0.70 |

*Notes*:
1. AASHTO 5.5.4.2.1 (From AASHTO LRFD Bridge Design Specifications, ©1994 by the American Association of State Highway and Transportation Officials, Washington, D.C. With permission.)
2. For compression members with flexural, the value of $\phi$ may be increased linearly to the value for flexural as the factored axial load resistance, $\phi P_n$, decreases from 0.10 $f_c' A_g$ to 0.

## 9.4.2 Design Limit States

### 1. Service Limit States

For concrete structures, service limit states correspond to the restrictions on cracking width and deformations under service conditions. They are intended to ensure that the bridge will behave and perform acceptably during its service life.

a. Control of Cracking

Cracking may occur in the tension zone for reinforced concrete members due to the low tensile strength of concrete. Such cracks may occur perpendicular to the axis of the members under axial tension or flexural bending loading without significant shear force, or inclined to the axis of the members with significant shear force. The cracks can be controlled by distributing steel reinforcements over the maximum tension zone in order to limit the maximum allowable crack widths at the surface of the concrete for given types of environment. The tensile stress in the steel reinforcement ($f_s$) at the service limit state should not exceed

$$f_{sa} = \frac{Z}{\left(d_c A\right)^{1/3}} \le 0.6 f_y \tag{9.5}$$

where $d_c$ (mm) is the concrete cover measured from extreme tension fiber to the center of the closest bars and should not to be taken greater than 50 mm; $A$ (mm$^2$) is the concrete area having the same centroid as the principal tensile reinforcement divided by the number of bars; $Z$ (N/mm) should not exceed 30,000 for members in moderate exposure conditions, 23,000 in severe exposure conditions, and 17,500 for buried structures. Several smaller tension bars at moderate spacing can provide more effective crack control by increasing $f_{sa}$ rather than installing a few larger bars of equivalent area.

When flanges of reinforced concrete T-beams and box girders are in tension, the flexural tension reinforcement should be distributed over the lesser of the effective flange width or a width equal to 1/10 of the span in order to avoid the wide spacing of the bars. If the effective flange width exceeds 1/10 of the span length, additional longitudinal reinforcement, with an area not less than 0.4% of the excess slab area, should be provided in the outer portions of the flange.

For flexural members with web depth exceeding 900 mm, longitudinal skin reinforcements should be uniformly distributed along both side faces for a height of $d/2$ nearest the flexural tension reinforcement for controlling cracking in the web. Without such auxiliary steel, the width of the

**TABLE 9.4** Traditional Minimum Depths for Constant Depth Superstructures

| | Minimum Depth (Including Deck) | |
|---|---|---|
| Bridge Types | Simple Spans | Continuous Spans |
| Slabs | $\frac{1.2(S+3000)}{30}$ | $\frac{(S+3000)}{30} \geq 165$ mm |
| T-beams | $0.070L$ | $0.065L$ |
| Box beams | $0.060L$ | $0.055L$ |
| Pedestrian structure beams | $0.035L$ | $0.033L$ |

*Notes:*
1. AASHTO Table 2.5.2.6.3-1 (From AASHTO LRFD Bridge Design Specifications, ©1994 by the American Association of State Highway and Transportation Officials, Washington, D.C. With permission.)
2. $S$ (mm) is the slab span length and $L$ (mm) is the span length.
3. When variable-depth members are used, values may be adjusted to account for change in relative stiffness of positive and negative moment sections.

cracks in the web may greatly exceed the crack widths at the level of the flexural tension reinforcement. The area of skin reinforcement ($A_{sk}$) in mm²/mm of height on each side face should satisfy

$$A_{sk} \geq 0.001\,(d_e - 760) \leq \frac{A_s}{1200} \tag{9.6}$$

where $d_e$ (mm) is the flexural depth from extreme compression fiber to the centroid of the tensile reinforcement and $A_s$ (mm²) is the area of tensile reinforcement and prestressing steel. The maximum spacing of the skin reinforcement shall not exceed $d/6$ or 300 mm.

b. Control of Deformations

Service-load deformations in bridge elements need to be limited to avoid the structural behavior which differs from the assumed design conditions and to ease the psychological effects on motorists. Service-load deformations may not be a potential source of collapse mechanisms but usually cause some undesirable effects, such as the deterioration of wearing surfaces and local cracking in concrete slab which could impair serviceability and durability. AASHTO LRFD [1] provides two alternative criteria for controlling the deflections:

*Limiting Computed Deflections* (*AASHTO 2.5.2.6.2*):

| | |
|---|---|
| Vehicular load, general | Span length/800 |
| Vehicular and/or pedestrian loads | Span length/1000 |
| Vehicular load on cantilever arms | Span length/300 |
| Vehicular and/or pedestrian loads on cantilever arms | Span length/1000 |

*Limiting Span-to-Depth Ratios* (*AASHTO 2.5.2.6.3*): For superstructures with constant depth, Table 9.4 shows the typical minimum depth recommendation for a given span length.

Deflections of bridges can be estimated in two steps: (1) instantaneous deflections which occur at the first loading and (2) long-time deflections which occur with time due to the creep and shrinkage of the concrete.

Instantaneous deflections may be computed by using the elastic theory equations. The modulus of elasticity for concrete can be calculated from Eq. (9.1). The moment of inertia of a section can be taken as either the uncracked gross moment of inertia ($I_g$) for uncracked elements or the effective moment of inertia ($I_e$) for cracked elements. The effective moment of inertia can be calculated as

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g \tag{9.7}$$

and

$$M_{cr} = f_r \frac{I_g}{y_t} \tag{9.8}$$

where $M_{cr}$ is the moment at first cracking, $f_r$ is the modulus of rupture, $y_t$ is the distance from the neutral axis to the extreme tension fiber, $I_{cr}$ is the moment of inertia of the cracked section transformed to concrete (see Section 9.4.6), and $M_a$ is the maximum moment in a component at the stage for which deformation is computed. For prismatic members, the effective moment of inertia may be calculated at midspan for simple or continuous bridges and at support for cantilevers. For continuous nonprismatic members, the moment of inertia may be calculated as the average of the critical positive and negative moment sections.

Long-time deflections may be calculated as the instantaneous deflection multiplied by the following:

| | |
|---|---|
| If the instantaneous deflection is based on $I_g$: | 4.0 |
| If the instantaneous deflection is based on $I_e$: | $3.0–1.2\left(A_s'/A_s\right) \geq 1.6$ |

where $A_s'$ is area of compression reinforcement and $A_s$ is the area of tension reinforcement.

### 2. Fatigue Limit States

Fatigue limit states are used to limit stress in steel reinforcements to control concrete crack growth under repetitive truck loading in order to prevent early fracture failure before the design service life of a bridge. Fatigue loading consists of one design truck with a constant spacing of 9000 mm between the 145-kN axles. Fatigue is considered at regions where compressive stress due to permanent loads is less than two times the maximum tensile live-load stress resulting from the fatigue-load combination. Allowable fatigue stress range in straight reinforcement is limited to

$$f_f = 145 - 0.33 f_{min} + 55\left(\frac{r}{h}\right) \tag{9.9}$$

where $f_{min}$ (MPa) is the minimum stress in reinforcement from fatigue loading (positive for tension and negative for compression stress) and $r/h$ is the ratio of the base radius to the height of rolled-on transverse deformations (0.3 may be used if the actual value in not known).

The cracked section properties should be used for fatigue. Gross section properties may be used when the sum of stresses, due to unfactored permanent loads, plus 1.5 times the fatigue load is not to exceed the tensile stress of $0.25\sqrt{f_c'}$ .

### 3. Strength Limit States and Extreme Event Limit States

For reinforced concrete structures, strength and extreme event limit states are used to ensure that strength and stability are provided to resist specified statistically significant load combinations. A detailed discussion for these limit states is covered in Chapter 5.

## 9.4.3 Flexural Strength

Figure 9.4 shows a doubly reinforced concrete beam when flexural strength is reached and the depth of neutral axis falls outside the compression flange ($c > h_f$). Assume that both tension and compression steel are yielding and the concrete compression stress block is in a rectangular shape$\varepsilon_{cu}$ is the maximum strain at the extreme concrete compression fiber and is about 0.003 for unconfined concrete.

Concrete compression force in the web;

$$C_w = 0.85\, f_c'\, ab_w = 0.85\beta_1\, f_c'\, cb_w \tag{9.10}$$

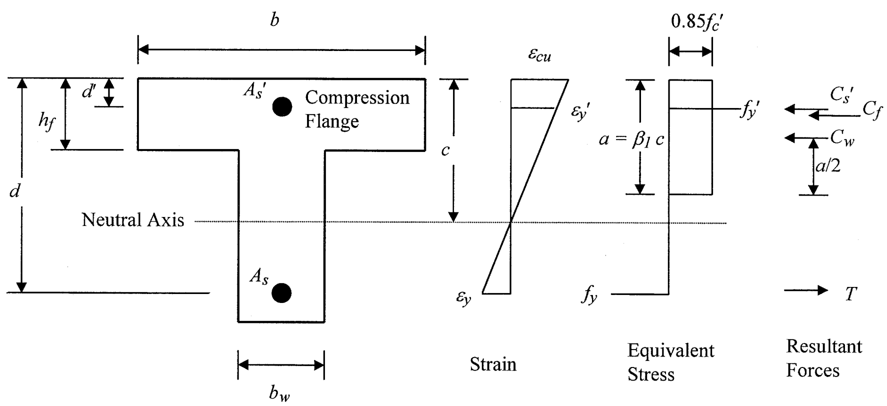



**FIGURE 9.4** Reinforced concrete beam when flexural strength is reached.

where

$$a = c\ \beta_1 \tag{9.11}$$

Concrete compression force in the flange:

$$C_f = 0.85\beta_1\ f_c'\ (b - b_w)h_f \tag{9.12}$$

Compression force in the steel:

$$C_s' = A_s' f_y' \tag{9.13}$$

Tension force in the steel:

$$T = A_s f_y \tag{9.14}$$

From the equilibrium of the forces in the beam, we have

$$C_w + C_f + C_s' = T \tag{9.15}$$

The depth of the neutral axis can be solved as

$$c = \frac{A_s f_y - A_s' f_y' - 0.85\beta_1 f_c'(b - b_w)h_f}{0.85\beta_1 f_c' b_w} \geq h_f \tag{9.16}$$

The nominal flexural strength is

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) + A_s' f_y'\left(\frac{a}{2} - d'\right) + 0.85\beta_1 f_c'(b - b_w)h_f\left(\frac{a}{2} - \frac{h_f}{2}\right) \tag{9.17}$$

where $A_s$ is the area of tension steel, $A_s'$ is the area of compression steel, $b$ is the width of the effective flange, $b_w$ is the width of the web, $d$ is the distance between the centroid of tension steel and the most compressed concrete fiber, $d'$ is the distance between the centroid of compression steel and the most compressed concrete fiber, and $h_f$ is the thickness of the effective flange. The concrete stress factor, $\beta_1$ can be calculated as

$$
\beta_1 = \begin{cases} 0.85 & \text{for } f_c' \le 28 \text{ MPa} \\ 0.85 - 0.05\left(\dfrac{f_c' - 28}{7}\right) & \text{for } 28 \text{ MPa} \le f_c' \le 56 \text{ MPa} \\ 0.65 & \text{for } f_c' \ge 56 \text{ MPa} \end{cases} \tag{9.18}
$$

Limits for reinforcement are

- Maximum tensile reinforcement:

$$
\frac{c}{d} \le 0.42 \tag{9.19}
$$

When Eq. (9.19) is not satisfied, the reinforced concrete sections become overreinforced and will have sudden brittle compression failure if they are not well confined.

- Minimum tensile reinforcement:

$$
\rho_{\min} \ge 0.03 \frac{f_c'}{f_y'}, \quad \text{where } \rho_{\min} = \text{ratio of tension steel to gross area} \tag{9.20}
$$

When Eq. (9.20) is not satisfied, the reinforced concrete sections become underreinforced and will have sudden tension steel fracture failure.

The strain diagram can be used to verify compression steel yielding assumption.

$$
f_s' = f_y' \quad \text{if} \quad \varepsilon_s' = \varepsilon_{cu}\left(\frac{c - d'}{c}\right) \ge \frac{f_y'}{E_s} \tag{9.21}
$$

If compression steel is not yielding as checked from Eqs. (9.21). The depth of neutral axis, $c$, and value of nominal flexural strength, $M_n$, calculated from Eqs. (9.16) and (9.17) are incorrect. The actual forces applied in compression steel reinforcement can be calculated as

$$
C_s' = A_s' f_s' = A_s \varepsilon_s' E_s' = A_s \varepsilon_{cu}\left(\frac{d - c}{c}\right) E_s' \tag{9.22}
$$

The depth of neutral axis, $c$, can be solved by substituting Eqs. (9.22) into forces equilibrium Eq. (9.15). The flexural strength, $M_n$, can then be obtained from Eq. (9.17) with the actual applied compression steel forces. In a typical beam design, the tension steel will always be yielding and the compression steel is close to reaching yielding strength as well.

If the depth of the neutral axis falls within the compression flange ($x \le h_f$) or for sections without compression flange, then the depth of the neutral axis, $c$, and the value of nominal flexural strength, $M_n$, can be calculated by setting $b_w$ equal to $b$.

## 9.4.4 Shear Strength

### 1. Strut-and-Tie Model

The strut-and-tie model should be used for shear and torsion designs of bridge components at locations near discontinuities, such as regions adjacent to abrupt changes in the cross section, openings, and dapped ends. The model should also be used for designing deep footings and pile

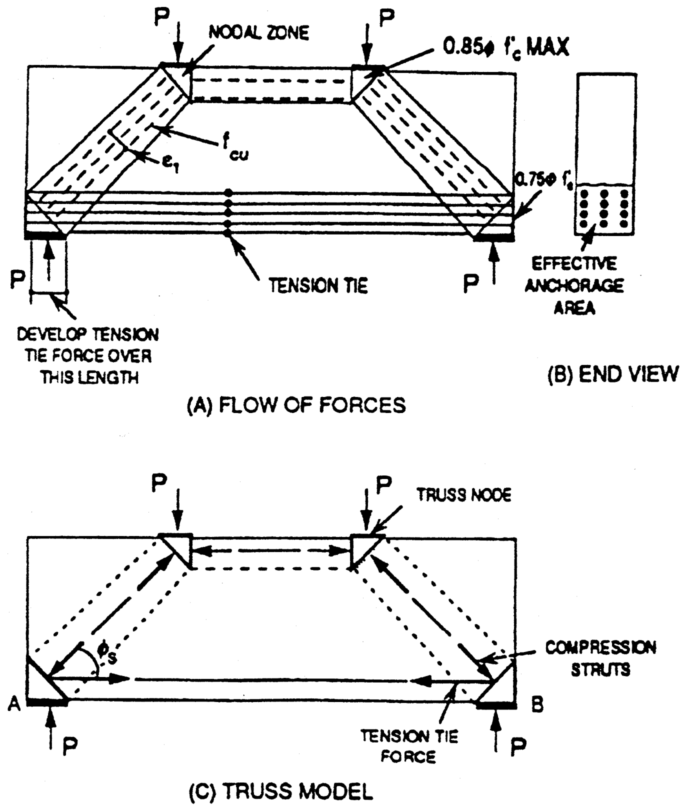



**FIGURE 9.5** Strut-and-tie model for a deep beam. (*Source*: AASHTO LRFD Bridge Design Specifications, Figure 5.6.3.2-1, © 1994 by the American Association of State Highway and Transportation Officials, Washington, D.C. With permission.)

caps or in other situations where the distance between the centers of the applied load and the supporting reactions is less than about twice the member thickness. Figure 9.5 shows a strut-and-tie model for a deep beam that is composed of steel tension ties and concrete compressive struts. These are interconnected at nodes to form a truss capable of carrying all applied loads to the supports.

## 2. Sectional Design Model

The sectional design model can be used for the shear and torsion design for regions of bridge members where plane sections remain plane after loading. It was developed by Collins and Mitchell [4] and is based on the modified compression field theory. The general shear design procedure for reinforced concrete members, containing transverse web reinforcement, is as follows:

- Calculate the effective shear depth $d_v$:
  Effective shear depth is calculated between the resultants of the tensile and compressive forces due to flexure. This should not be less than the greater of $0.9d_e$ or $0.72h$, where $d_e$ is the effective depth from extreme compression fiber to the centroid of the tensile reinforcement and $h$ is the overall depth of a member.
- Calculate shear stress:

$$v = \frac{V_u}{\phi b_v d_v} \tag{9.23}$$

where $b_v$ is the equivalent web width and $V_u$ is the factored shear demand envelope from the strength limit state.

- Calculate $v/f_c'$, if this ratio is greater than 0.25, then a larger web section needs to be used.
- Assume an angle of inclination of the diagonal compressive stresses, θ, and calculate the strain in the flexural tension reinforcement:

$$\varepsilon_x = \frac{\dfrac{M_u}{d_v} + 0.5V_u \cot\theta}{E_s A_s} \tag{9.24}$$

where $M_u$ is the factored moment demand. It is conservative to take $M_u$ enveloped from the strength limit state that will occur at that section, rather than a moment coincident with $V_u$.

- Use the calculated $v/f_c'$ and $\varepsilon_x$ to find θ from Figure 9.6 and compare it with the value assumed. Repeat the above procedure until the assumed θ is reasonably close to the value found from Figure 9.6. Then record the value of β, a factor which indicates the ability of diagonally cracked concrete to transmit tension.
- Calculate the required transverse web reinforcement strength, $V_s$:

$$V_s = \frac{V_u}{\phi} - V_c = \frac{V_u}{\phi} - 0.083\beta\sqrt{f_c'}\, b_v d_v \tag{9.25}$$

where $V_c$ is the nominal concrete shear resistance.

- Calculate the required spacing for the transverse web reinforcement:

$$s \le \frac{A_v f_y d_v \cot\theta}{V_s} \tag{9.26}$$

where $A_v$ is the area of a transverse web reinforcement within distance $s$.

Check for the minimum transverse web reinforcement requirement:

$$A_v \ge 0.083\sqrt{f_c'}\,\frac{b_v S}{f_y} \quad \text{or} \quad s \le \frac{A_v f_y}{0.083\sqrt{f_c'}\, b_v} \tag{9.27}$$

Check for the maximum spacing requirement for transverse web reinforcements:

$$\text{if } V_u < 0.1\, f_c'\, b_v d_v, \quad \text{then } s \le 0.8 d_v \le 600 \text{ mm} \tag{9.28}$$

$$\text{if } V_u \ge 0.1\, f_c'\, b_v d_v, \quad \text{then } s \le 0.4 d_v \le 300 \text{ mm} \tag{9.29}$$

- Check the adequacy of the longitudinal reinforcements to avoid yielding due to the combined loading of moment, axial load, and shear.

$$A_s f_y \ge \frac{M_u}{d_v \phi} + \left(\frac{V_u}{\phi} - 0.5\, V_s\right)\cot\theta \tag{9.30}$$

If the above equation is not satisfied, then you need either to add more longitudinal reinforcement or to increase the amount of transverse web reinforcement.

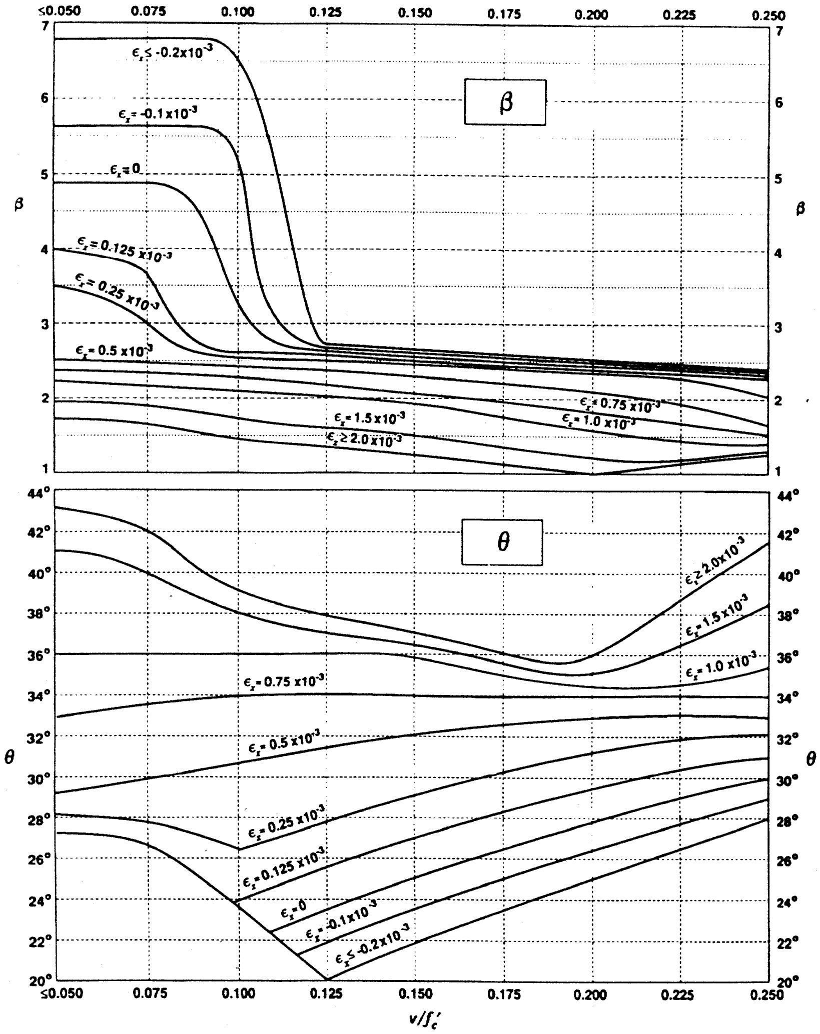



**FIGURE 9.6** Values of θ and β for sections with transverse web reinforcement. (*Source*: AASHTO LRFD Bridge Design Specifications, Figure 5.8.3.4.2-1, ©1994 by the American Association of State Highway and Transportation Officials, Washington, D.C. With permission.)

### 9.4.5 Skewed Concrete Bridges

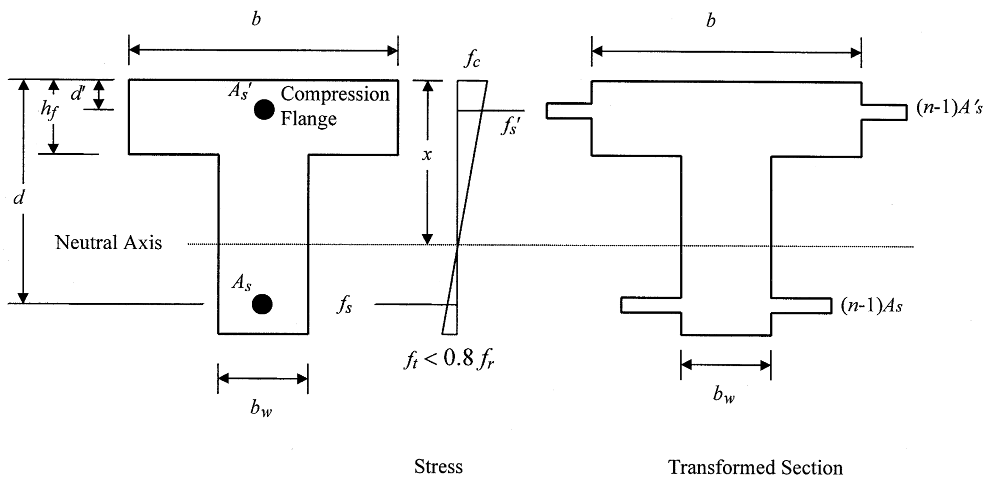
Shear, in the exterior beam at the obtuse corner of the bridge, needs to be adjusted when the line of support is skewed. The value of the correction factor obtained from AASHTO Table 4.6.2.2.3c-1, needs to be applied to live-load distribution factors for shear. In determining end shear in multibeam bridges, all beams should be treated like the beam at the obtuse corner, including interior beams.

Moment load distribution factors in longitudinal beams on skew supports may be reduced according to AASHTO Table 4.6.2.2.2e-1, when the line supports are skewed and the difference between skew angles of two adjacent lines of supports does not exceed 10°.

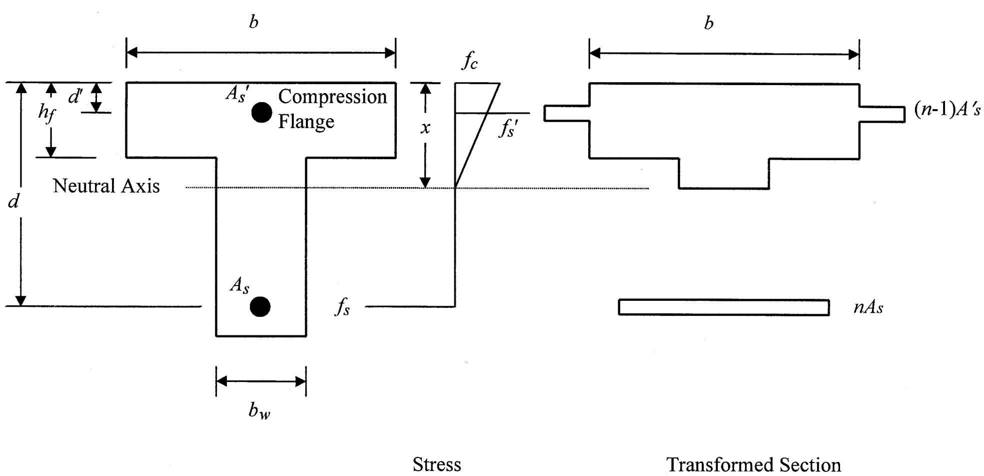
### 9.4.6 Design Information

#### 1. Stress Analysis at Service Limit States [5]

A reinforced concrete beam subject to flexural bending moment is shown in Figure 9.7 and $x$ is the distance between the neutral axis and the extreme compressed concrete fiber. Assume the neutral axis




(a) Stress and Transformed Section Before Cracking




(b) Stress and Transformed Section After Cracking

**FIGURE 9.7** Reinforced concrete beam for working stress analysis.

falls within the web ($x > h_f$) and the stress in extreme tension concrete fiber is greater than 80% of the concrete modulus of rupture ($f_t \geq 0.8\, f_r$). The depth of neutral axis, $x$, can be solved through the following quadratic equation by using the cracked transformed section method (seeFigure 9.7).

$$b(x)\left(\frac{x}{2}\right)-\left(b-b_w\right)\left(x-h_f\right)\left(\frac{x-h_f}{2}\right)+(n-1)A_s'\left(x-d'\right)=nA_s(d-x) \tag{9.31}$$

$$x=\sqrt{B^2+C}-B \tag{9.32}$$

where

$$B = \frac{1}{b_w}\left[h_f\left(b - b_w\right) + nA_s + (n-1)A_s'\right] \tag{9.33}$$

$$C = \frac{2}{b_w}\left[\frac{h_f^2}{2}\left(b - b_w\right) + ndA_s + (n-1)d'A_s'\right] \tag{9.34}$$

and the moment of inertia of the cracked transformed section about the neutral axis:

$$I_{\text{cr}} = \frac{1}{3}bx^3 - \frac{1}{3}\left(b - b_w\right)\left(x - h_f\right)^3 + nA_s(d-x)^2 + (n-1)A_s'(x-d')^2 \tag{9.35}$$

if the calculated neutral axis falls within the compression flange ($x \le h_f$) or for sections without compression flange, the depth of neutral axis, $x$, and cracked moment of inertia, $I_{\text{cr}}$, can be calculated by setting $b_w$ equal to $b$.

*Stress in extreme compressed concrete fiber*:

$$f_c = \frac{Mx}{I_{\text{cr}}} \tag{9.36}$$

*Stress in compression steel*:

$$f_s' = \frac{nM(x-d')}{I_{\text{cr}}} = nf_c\left(1 - \frac{d'}{x}\right) \tag{9.37}$$

*Stress in tension steel*:

$$f_s = \frac{nM(d-x)}{I_{cr}} = nf_c\left(\frac{d}{x} - 1\right) \tag{9.38}$$

where

$$n = \frac{E_s}{E_c} \tag{9.39}$$

and $M$ is moment demand enveloped from the service limit state.

## 2. Effective Flange Width (AASHTO 4.6.2.6)

When reinforced concrete slab and girders are constructed monolithically, the effective flange width ($b_{\text{eff}}$) of a concrete slab, which will interact with girders in composite action, may be calculated as

*For interior beams*:

$$b_{\text{eff}}^I = \text{the smallest of}\begin{cases}\dfrac{l_{\text{eff}}}{4} \\ 12t_s + b_w \\ \text{the average spacing of adjacent beams}\end{cases} \tag{9.40}$$

**TABLE 9.5** Cover for Unprotected Main Reinforcing Steel (mm)

| Situation | Cover (mm) |
|---|---|
| Direct exposure to salt water | 100 |
| Cast against earth | 75 |
| Coastal | 75 |
| Exposure to deicing salt | 60 |
| Deck surface subject to tire stud or chain wear | 60 |
| Exterior other than above | 50 |
| Interior other than above | |
| • Up to No. 36 Bar | 40 |
| • No. 43 and No. 57 Bars | 50 |
| Bottom of CIP slab | |
| • Up to No. 36 Bar | 25 |
| • No. 43 and No. 57 Bars | 50 |

*Notes:*

1. Minimum cover to main bars, including bars protected by epoxy coating, shall be 25 mm.
2. Cover to epoxy-coated steel may be used as interior exposure situation.
3. Cover to ties and stirrups may be 12 mm less than the value specified here, but shall not be less than 25 mm.
4. Modification factors for water:cement ratio, $w/c$, shall be the following:

   for $w/c \leq 0.40$ modification factor = 0.8
   for $w/c \geq 0.40$ modification factor = 1.2

*Source:* AASHTO Table C5.12.3-1. (From AASHTO LRFD Bridge Design Specifications, ©1994 by the American Association of State Highway and Transportation Officials, Washington, D.C. With permission.)

*For exterior beams*:

$$b_{\text{eff}}^{E} = \frac{1}{2} b_{\text{eff}}^{I} + \text{the smallest of} \begin{cases} \dfrac{l_{\text{eff}}}{8} \\ 6t_s + \dfrac{b_w}{2} \\ \text{the width of overhang} \end{cases} \tag{9.41}$$

where the effective span length ($l_{\text{eff}}$) may be calculated as the actual span for simply supported spans. Also, the distance between the points of permanent load inflection for continuous spans of either positive or negative moments ($t_s$) is the average thickness of the slab, and $b_w$ is the greater of web thickness or one half the width of the top flange of the girder.

### 3. Concrete Cover (AASHTO 5.12.3)

Concrete cover for unprotected main reinforcing steel should not be less than that specified in Table 9.5 and modified for the water:cement ratio.

## 9.4.7 Details of Reinforcement

Table 9.6 shows basic tension, compression, and hook development length for Grade 300 and Grade 420 deformed steel reinforcement (AASHTO 5.11.2). Table 9.7 shows the minimum center-to-center spacing between parallel reinforcing bars (AASHTO 5.10.3).

**TABLE 9.6** Basic Rebar Development Lengths for Grade 300 and 420 (AASHTO 5.11.2)

| | $f_c'$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Bar | 28 MPa | | | 35 MPa | | | 42 MPa | | |
| Size | Tension | Compression | Hook | Tension | Compression | Hook | Tension | Compression | Hook |
| Grade 300, $f_y$ = 300 MPa | | | | | | | | | |
| 13 | 230 | 175 | 240 | 230 | 170 | 215 | 230 | 170 | 200 |
| 16 | 290 | 220 | 300 | 290 | 210 | 270 | 290 | 210 | 245 |
| 19 | 345 | 260 | 365 | 345 | 255 | 325 | 345 | 255 | 295 |
| 22 | 440 | 305 | 420 | 400 | 295 | 375 | 400 | 295 | 345 |
| 25 | 580 | 350 | 480 | 520 | 335 | 430 | 475 | 335 | 395 |
| 29 | 735 | 395 | 545 | 655 | 380 | 485 | 600 | 380 | 445 |
| 32 | 930 | 440 | 610 | 835 | 430 | 550 | 760 | 430 | 500 |
| 36 | 1145 | 490 | 680 | 1020 | 475 | 605 | 935 | 475 | 555 |
| 43 | 1420 | 585 | 815 | 1270 | 570 | 730 | 1160 | 570 | 665 |
| 57 | 1930 | 780 | 1085 | 1725 | 765 | 970 | 1575 | 760 | 885 |
| Grade 420, $f_y$ = 420 MPa | | | | | | | | | |
| 13 | 320 | 245 | 255 | 320 | 235 | 225 | 320 | 235 | 210 |
| 16 | 405 | 305 | 320 | 405 | 295 | 285 | 405 | 295 | 260 |
| 19 | 485 | 365 | 380 | 485 | 355 | 340 | 485 | 355 | 310 |
| 22 | 615 | 425 | 445 | 560 | 410 | 395 | 560 | 410 | 360 |
| 25 | 810 | 485 | 505 | 725 | 470 | 455 | 665 | 470 | 415 |
| 29 | 1025 | 550 | 570 | 920 | 530 | 510 | 840 | 530 | 465 |
| 32 | 1300 | 615 | 645 | 1165 | 600 | 575 | 1065 | 600 | 525 |
| 36 | 1600 | 685 | 710 | 1430 | 665 | 635 | 1305 | 665 | 580 |
| 43 | 1985 | 820 | 855 | 1775 | 795 | 765 | 1620 | 795 | 700 |
| 57 | 2700 | 1095 | 1140 | 2415 | 1060 | 1020 | 2205 | 1060 | 930 |

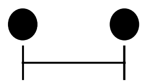
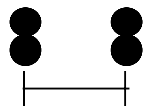
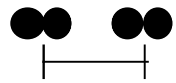
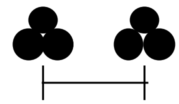
*Notes*:
1. Numbers are rounded up to nearest 5 mm.
2. Basic hook development length has included reinforcement yield strength modification factor.
3. Minimum tension development length (AASHTO 5.11.2.1). Maximum of (1) basic tension development length times appropriate modification factors (AASHTO 5.11.2.1.2 and 5.11.2.1.3) and (2) 300 mm.
4. Minimum compression development length (AASHTO 5.11.2.2). Maximum of (1) basic compression development length times appropriate modification factors (AASHTO 5.11.2.2.2) and (2) 200 mm.
5. Minimum hook development length (AASHTO 5.11.2.4). Maximum of (1) basic hook development length times appropriate modification factors (AASHTO 5.11.2.4.2), (2) eight bar diameters, and (3) 150 mm.

Except at supports of simple spans and at the free ends of cantilevers, reinforcement (AASHTO 5.11.1.2) should be extended beyond the point at which it is no longer required to resist the flexural demand for a distance of

$$\text{the largest of } \begin{cases} \text{the effective depth of the member} \\ \text{15 times the nominal diameter of a bar} \\ \text{0.05 times the clear span length} \end{cases} \tag{9.42}$$

Continuing reinforcement shall extend not less than the development length beyond the point where bent or terminated tension reinforcement is no longer required for resisting the flexural demand.

**TABLE 9.7** Minimum Rebar Spacing for CIP Concrete (mm) (AASHTO 5.10.3)

| Bar Size | Minimum Spacing | | | |
|---|---|---|---|---|
| |  |  |  |  |
| 13 | 51 | 51 | 63 | 63 |
| 16 | 54 | 56 | 70 | 70 |
| 19 | 57 | 68 | 76 | 83 |
| 22 | 60 | 78 | 82 | 96 |
| 25 | 64 | 90 | 90 | 110 |
| 29 | 72 | 101 | 101 | 124 |
| 32 | 81 | 114 | 114 | 140 |
| 36 | 90 | 127 | 127 | 155 |
| 43 | 108 | 152 | 152 | |
| 57 | 143 | 203 | 203 | |

*Notes:*
1. Clear distance between bars should not be less than 1.5 times the maximum size of the course aggregate.
2. Note 1 does not need to be verified when maximum size of the course aggregate grading is less than 25 mm.
3. Bars spaced less than $3d_b$ on center require modification of development length (AASHTO 5.11.2.1.2).

For negative moment reinforcement, in addition to the above requirement for bar cutoff, it must be extended to a length beyond the inflection point for a distance of

$$\text{the largest of}\begin{cases}\text{the effective depth of the member}\\ \text{12 times the nominal diameter of a bar}\\ \text{0.0625 times the clear span length}\end{cases}\tag{9.43}$$

## 9.5 Design Examples

### 9.5.1 Solid Slab Bridge Design

Given

A simple span concrete slab bridge with clear span length ($S$) of 9150 mm is shown in Figure 9.8. The total width ($W$) is 10,700 mm, and the roadway is 9640 wide ($W_R$) with 75 mm ($d_W$) of future wearing surface.

*The material properties are as follows*: Density of wearing surface $\rho_w = 2250$ kg/m$^3$; concrete density $\rho_c = 2400$ kg/m$^3$; concrete strength $f_c' = 28$ MPa, $E_c = 26\,750$ MPa; reinforcement $f_y = 420$ MPa, $E_s = 200{,}000$ MPa; $n = 8$.

Requirements

Design the slab reinforcement base on AASHTO-LRFD (1994) Strength I and Service I (cracks) Limit States.

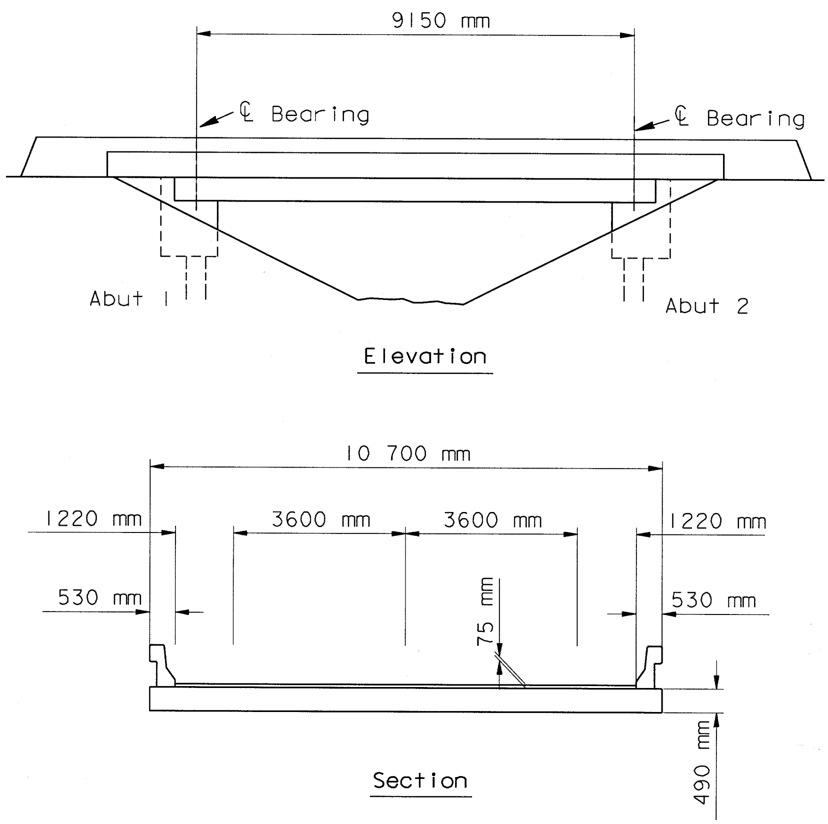



**FIGURE 9.8** Solid slab bridge design example.

Solution

1. **Select Deck Thickness (Table 9.4)**

$$h_{min} = 1.2\left(\frac{S+3000}{30}\right) = 1.2\left(\frac{9150+3000}{30}\right) = 486\,\text{mm}$$

Use $h = 490$ mm

2. **Determine Live Load Equivalent Strip Width (AASHTO 4.6.2.3 and 4.6.2.1.4b)**
   a. *Interior strip width*:
      i. Single-lane loaded:

$$E_{\text{interior}} = 250 + 0.42\sqrt{L_1 W_1}$$

$L_1$ = lesser of actual span length and 18,000 mm

$W_1$ = lesser of actual width or 9000 mm for single lane loading or 18,000 mm for multilane loading

$$E_{\text{interior}} = 250 + 0.42\sqrt{(9150)(9000)} = 4061\ \text{mm}$$

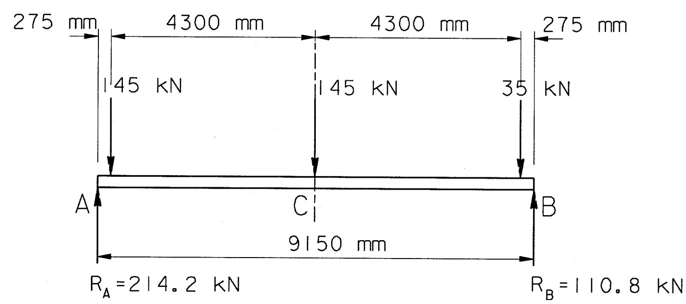



**FIGURE 9.9** Position of design truck for maximum moment.

ii. Multilane loaded:

$$N_L = INT\left(\frac{W}{3600}\right) = INT\left(\frac{10,700}{3600}\right) = 2$$

$$\frac{W}{N_L} = \frac{10,700}{2} = 5350 \text{ mm}$$

$$E_{\text{interior}} = 2100 + 0.12\sqrt{L_1 W_1} = 2100 + \sqrt{(9150)(10,700)} = 3287 \text{ mm} < 5350 \text{ mm}$$

$$\underline{\text{Use } E_{\text{interior}} = 3287 \text{ mm}}$$

b. *Edge strip width*:

$E_{\text{edge}}$ = the distance between the edge of the deck and the inside face of the barrier + 300 mm + ½ strip width < full strip or 1800 mm

$$E_{\text{edge}} = 530 + 300 + \frac{3287}{2} = 2324 \text{ mm} > 1800 \text{ mm}$$

$$\underline{\text{Use } E_{\text{edge}} = 1800 \text{ mm}}$$

**3. Dead Load**

Slab: $W_{\text{slab}} = (0.49)\ (2400)\ (9.81)\ (10^{-3}) = 11.54\ \text{kN/m}^2$

Future wearing: $W_{\text{fw}} = (0.075)\ (2250)\ (9.81)\ (10^{-3}) = 1.66\ \text{kN/m}^2$

Assume 0.24 $m^3$ concrete per linear meter of concrete barrier

Concrete barrier: $W_{\text{barrier}} = (0.24)\ (2400)\ (9.81)\ (10^{-3}) = 5.65\ \text{kN/m}^2$

**4. Calculate Live-Load Moments**

Moment at midspan will control the design.

a. *Moment due to the design truck* (see Figure 9.9):

$$M_{\text{LL-Truck}} = (214.2)\ (4.575) — (145)\ (4.3) = 356.47\ \text{kN·m}$$

b. *Moment due to the design tandem* (see Figure 9.10):

$$M_{\text{LL-Tandem}} = (95.58)\ (4.575) = 437.28\ \text{kN·m.}$$

<u>Design Tandem Controls</u>

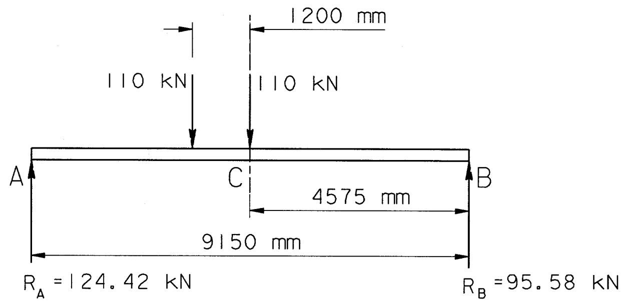



**FIGURE 9.10** Position of tandem for maximum moment.

c. *Moment due to lane load*:

$$M_{\text{LL-Lane}} = \frac{(9.3)(9.15)^2}{8} = 97.32 \text{ kN·m}$$

**5. Determine Load Factors (AASHTO Table 3.4.1-1) and Load Combinations (AASHTO 1.3.3-5)**

a. *Strength I Limit State load factors*:

Weight of superstructure (DC): 1.25
Weight of wearing surface (DW): 1.50
Live Load (LL): 1.75
$\eta_d = 0.95$, $\eta_R = 1.05$, $\eta_I = 0.95$
$\eta = (0.95)(1.05)(0.95) = 0.948 \le 0.95$

$\underline{\text{Use } \eta = 0.95}$

b. *Interior strip moment* (1 m wide) (AASHTO 3.6.2.1 and 3.6.1.2.4):

Dynamic load factor IM = 0.33

Lane load $$M_{\text{LL-Lane}} = \left(\frac{97.32}{3.287}\right) = 29.61 \text{ kN·m}$$

Live load $$M_{\text{LL+IM}} = (1 + 0.33)\left(\frac{437.28}{3.287}\right) + 29.61 = 206.54 \text{ kN·m}$$

Future wearing $$M_{\text{DW}} = \frac{W_{fw}L^2}{8} = \frac{(1.66)(9.15)^2}{8} = 17.37 \text{ kN·m}$$

Dead load $$M_{DC} = \frac{W_{\text{slab}}L^2}{8} = \frac{(11.54)(9.15)^2}{8} = 120.77 \text{ kN·m}$$

Factored moment
$$\begin{aligned} M_U &= \eta[1.25(M_{\text{DC}}) + 1.50(M_{\text{DW}}) + 1.75(M_{\text{LL+IM}})] \\ &= (0.95)\,[1.25\,(120.77) + (1.50)\,(17.37) + (1.75)\,(206.54)] \\ &= 511.54 \text{ kN·m} \end{aligned}$$

c. *Edge strip moment* (1 m wide) (AASHTO Table 3.6.1.1.2-1):

End strip is limited to half lane width, use multiple presence factor 1.2 and half design lane load.

Lane load $M_{\text{LL-Lane}} = (1.2)\left(\frac{1}{2}\right)\left(\frac{97.3}{1.8}\right) = 32.44 \text{ kN·m}$

Live load $M_{\text{LL+IM}} = (1 + 0.33)(1.2)\left(\frac{1}{2}\right)\left(\frac{437.28}{1.8}\right) + 32.44 = 226.3 \text{ kN·m}$

Dead load $M_{\text{DC}} = \left(11.54 + \frac{5.65}{1.8}\right)\left(\frac{9.15^2}{8}\right) = 153.63 \text{ kN·m}$

Future wearing $M_{\text{DW}} = (1.66)\left(\frac{1.8 - 0.53}{1.8}\right)\left(\frac{9.15^2}{8}\right) = 12.25 \text{ kN·m}$

Factored moment $M_U = (0.95)[(1.25)(153.63) + (1.50)(12.25) + (1.75)(226.3)] = 579.12 \text{ kN·m}$

**6. Reinforcement Design**

a. *Interior strip*:

Assume No. 25 bars, $d = 490 - 25 - \left(\frac{25}{2}\right) = 452.5$ mm.

The required reinforcements are calculated using Eqs. (9.11), (9.16), and (9.17).

Neglect the compression steel and set $b_w = b$ for sections without compression flange.

$$M_u = \phi A_s f_y\left(d - \frac{a}{2}\right) \quad \text{and} \quad a = c\beta_1 = \frac{A_s f_y}{0.85 f_c' b_w}$$

$A_s$ can be solved by substituting $a$ into $M_u$ or

$$R_u = \frac{M_u}{\phi b d^2} = \frac{511.54 \times 10^6}{(0.9)(1000)(452.5)^2} = 2.766 \text{ N/mm}$$

$$m = \frac{f_y}{(0.85) f_c'} = \frac{420}{(0.85)(28)} = 17.647$$

$$\rho = \frac{1}{m}\left[1 - \sqrt{1 - \frac{2mR_u}{f_y}}\right] = \frac{1}{17.647}\left[1 - \sqrt{1 - \frac{2(17.647)(2.776)}{420}}\right] = 0.00705$$

Required reinforced steel $A_s = \rho bd = (0.00705)(1000)(452.5) = 3189 \text{ mm}^2/\text{m}$.

Maximum allowed spacing of No. 25 bar = 510/3189 = 0.160 m.

Try No. 25 bars at 150 mm.

i. Check limits for reinforcement:

$\beta_1 = 0.85$ for $f_c' = 28$ MPa; see Eq. (9.18)

$$c = \frac{A_s f_y}{0.85 \beta_1 f_c' b_w} = \frac{(510)(420)}{0.85(0.85)(28)(150)} = 70.6 \text{ mm}$$

from Eqs. (9.19),

$$\frac{c}{d} = \frac{70.79}{452.5} = 0.156 \le 0.42 \qquad \text{OK}$$

from Eqs. (9.20),

$$\rho_{\min} = \frac{510}{(150)(452.5)} = 0.007\ 51 \geq (0.03)\left(\frac{28}{420}\right) = 0.002 \qquad \text{OK}$$

ii. Check crack control:

Service load moment $M_{sa}$ = $1.0[1.0(M_{DC}) + 1.0(M_{DW}) + 1.0(M_{LL+IM})]$
= [120.77 + 17.37 + (176.93 + 29.61)]
= 344.68 kN·m

$$0.8f_r = 0.8(0.63\sqrt{f_c'}) = 0.8(0.63)\sqrt{28} = 2.66 \text{ MPa}$$

$$f_c = \frac{M_{sa}}{S} = \frac{344{,}680}{\frac{1}{6}(490)^2} = 8.61\text{MPa} \geq 0.8f_r;\text{, Section is cracked}$$

Cracked moment of inertia can be calculated by using Eqs. (9.32) to (9.35).

$n$ = 8, $b$ = 150.0 mm, $A_s$ = 510 mm, $d$ = 452.5 mm.

$$B = \frac{1}{b}(nA_s) = \frac{1}{150}(8)(510) = 27.2$$

$$C = \frac{2}{b}(ndA_s) = \frac{2}{150}(8)(452.5)(510) = 24616$$

$$x = \sqrt{B^2 + C} - B = \sqrt{(27.2)^2 + (24616)} - (27.2) = 132 \text{ mm}$$

$$I_{cr} = \frac{1}{3}bx^3 + nA_s(d-x)^2 = \frac{1}{3}(150)(132)^3 + (8)(510)(452.5-132)^2 = 534.1\times10^6\text{mm}^4$$

From Eq. (9.38) $f_s = n\dfrac{M_{sa}(d-x)}{I_{cr}} = (8)\dfrac{(344{,}680)(452.5-132)}{534.1\times10^6} = 248 \text{ MPa}$

Allowable tensile stress in the reinforcement can be calculated from Eq. (9.5) with $Z$ = 23,000 N/mm for moderate exposure and

$$d_c = 25 + \frac{25}{2} = 37.5 \text{ mm}$$

$$A = 2d_c \times \text{bar spacing} = (2)(37.5)(150) = 11{,}250 \text{ mm}$$

$$f_{sa} = \frac{Z}{(d_cA)^{1/3}} \leq 0.6f_y$$

$$f_{sa} = \frac{23{,}000}{[(37.5)(11{,}250)]^{1/3}} = \frac{23{,}000}{75} = 307 \text{ MPa} \geq 0.6f_y = 0.6\ (420) = 252 \text{ MPa}$$

$$f_s = 248\text{ MPa} \leq f_{sa} = 252\text{ MPa}, \qquad \text{OK}$$

Use No. 25 Bar @150 mm for interior strip

b. *Edge strip*:

By similar procedure, Edge Strip Use No. 25 bar at 125 mm

7. **Determine Distribution Reinforcement (AASHTO 5.14.4.1)**
The bottom transverse reinforcement may be calculated as a percentage of the main reinforcement for positive moment:

$$\frac{1750}{\sqrt{L}} \leq 50\%, \text{ that is, } \frac{1750}{\sqrt{9150}} = 18.3\% \leq 50\%$$

a. *Interior strip*:
Main reinforcement: No. 25 at150 mm,

$$A_s = \frac{510}{150} = 3.40 \ \text{mm}^2/\text{mm}.$$

Required transverse reinforcement = (0.183)(3.40) = 0.622 $\text{mm}^2$/mm

$\underline{\text{Use No. 16 @ 300 mm}}$ transverse bottom bars,

$$A_s = \frac{199}{300} = 0.663 \ \text{mm}^2/\text{mm}$$

b. *End strip*:
Main reinforcement: No. 25 at 125 mm,

$$A_s = \frac{510}{125} = 4.08 \ \text{mm}^2/\text{mm}$$

Required transverse reinforcement = (0.183)(4.08) = 0.746 $\text{mm}^2$/mm

$\underline{\text{Use No. 16 at 250 mm,}}$ $A_s$ = 0.79 $\text{mm}^2$/mm.

For construction consideration, $\underline{\text{Use No. 16 @250 mm}}$ across entire width of the bridge.

8. **Determine Shrinkage and Temperature Reinforcement (AASHTO 5.10.8)**
Temperature

$$A_s \geq 0.75\frac{A_g}{f_y} = 0.75\frac{(1)(490)}{420} = 0.875 \ \text{mm}^2/\text{mm in each direction}$$

Top layer = 0.875/2 = 0.438 $\text{mm}^2$/mm

$\underline{\text{Use No. 13 @ 300 mm}}$ transverse top bars, $A_s$ = 0.430 $\text{mm}^2$/mm

9. **Design Sketch**
See Figure 9.11 for design sketch in transverse section.

10. **Summary**
To complete the design, loading combinations for all limit states need to be checked. Design practice should also give consideration to long-term deflection, cracking in the support area for longer or continuous spans. For large skew bridges, alteration in main rebar placement is essential.

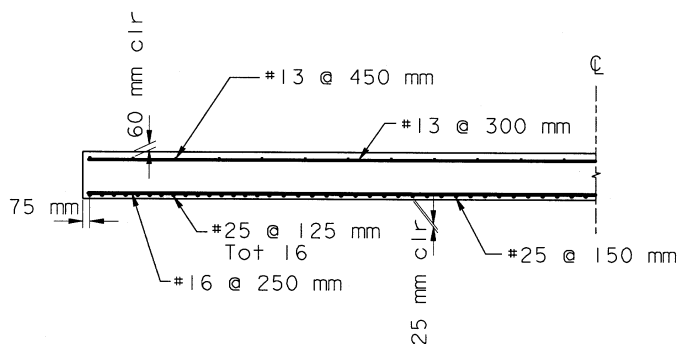



**FIGURE 9.11** Slab reinforcement detail.

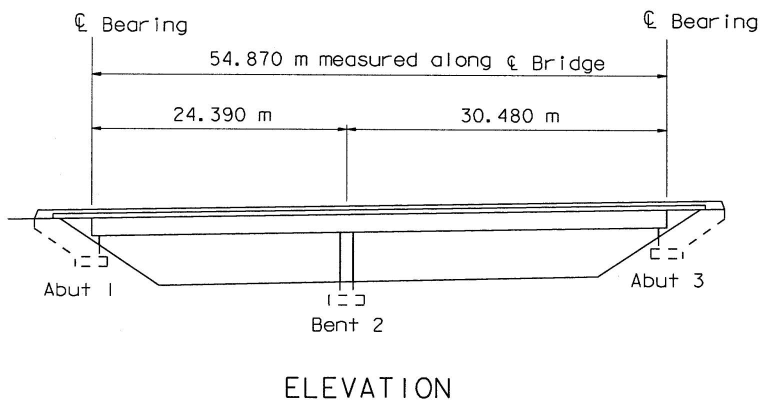



**FIGURE 9.12** Two-span reinforced box girder bridge.

## 9.5.2 Box-Girder Bridge Design

Given

A two-span continuous cast-in-place reinforced concrete box girder bridge, with span length of 24 390 mm ($L_1$) and 30 480 mm ($L_2$), is shown in Figure 9.12. The total superstructure width ($W$) is 10 800 mm, and the roadway width ($W_R$) is 9730 mm with 75 mm ($d_W$) thick of future wearing surface.

*The material properties are assumed as follows*: Density of wearing surface $\rho_w = 2250$ kg/m³; concrete density $\rho_c = 2400$ kg/m³; concrete strength $f'_c = 28$ MPa, $E_c = 26\,750$ MPa; reinforcement $f_y = 420$ MPa, $E_s = 200\,000$ MPa.

Requirements

Design flexural and shear reinforcements for an exterior girder based on AASHTO-LRFD (1994) Limit State Strength I, Service I (cracks and deflection), and Fatigue Limit States.

Solution

1. **Determine Typical Section (see Figure 9.13)**
   a. *Section dimensions*:
      Try the following dimensions:
      *Overall Structural Thickness, h* = $\underline{1680 \text{ mm}}$ (Table 9.4)
      *Effective length, s* = 2900 – 205 = 2695 mm
      *Design depth* (*deck slab*),

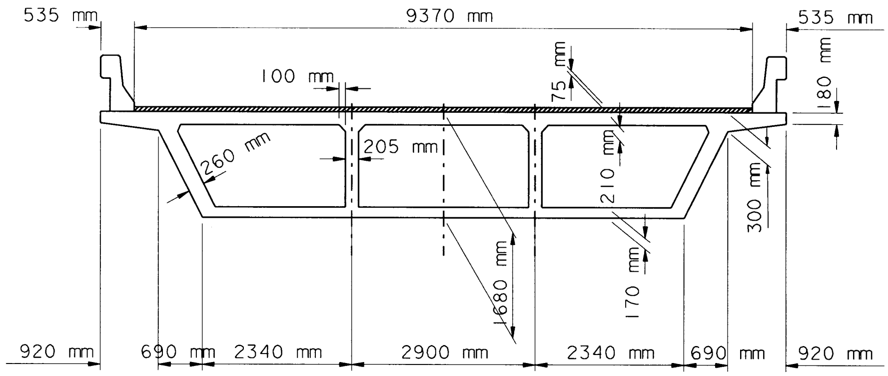



**FIGURE 9.13** Typical section.

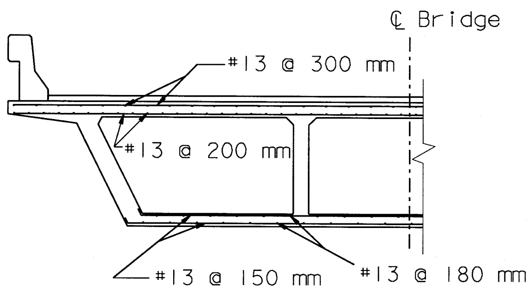



**FIGURE 9.14** Slab reinforcement.

$$t_{\text{top}} = \underline{210 \text{ mm}}$$

$$> \frac{1}{20}(2900 - 205 - 100 \cdot 2) = 124.8 \text{ mm} \quad \text{(AASHTO 5.14.1.3)}$$

$$\frac{s}{t_{top}} = \frac{2695}{210} = 12.8 < 18 \quad \text{(AASHTO 9.7.2.4)}$$

*Bottom flange depth,*

$$t_{\text{bot}} = \underline{170 \text{ mm}} > 140 \text{ mm (AASHTO 5.14.1.3)}$$

$$> \frac{1}{16}(2900 - 205 - 100 \cdot 2) = 156 \text{ mm} \quad \text{(AASHTO 5.14.1.3)}$$

*Web thickness,* $b_w = \underline{205 \text{ mm}}$ > 200 mm for ease of construction (AASHTO 5.14.1.3)

b. *Deck slab reinforcement*:
The detail slab design procedure is covered in Chapter 15 of this handbook. The slab design for this example, using the empirical method, is shown in Figure 9.14.

**2. Calculate Design Loads**

The controlling load case is assumed to be Strength Limit State I.

a. *Permanent load*:
It is assumed that the self-weight of the box girder and the future wearing surface are equally distributed to each girder. The weight of the barrier rails is, however, distributed to the exterior girders only.
Dead load of box girder = (0.000 023 57)(4 938 600) = 116.4 N/mm
Dead load of the concrete barriers = 5.65(2) = 11.3 N/mm
Dead load of the future wearing surface = (0.0000221)(729 750) = 16.12 N/mm

b. *Live loads*:
i. Vehicle live loads:
A standard design truck (AASHTO 3.6.1.2.2), a standard design tandem (AASHTO 3.6.1.2.3), and the design lane load (AASHTO 3.6.1.2.4) are used to compute the extreme force effects.
ii. Multiple presence factors (AASHTO 3.6.1.1.2 and AASHTO Table 3.6.1.1.2-1):
No. of traffic lanes = INT (9730/3600) = 2 lanes
The multiple presence factor, m = 1.0
iii. Dynamic load allowance (AASHTO 3.6.2.1 and AASHTO Table 3.6.2.1-1):
IM = 15% for Fatigue and Fracture Limit State
IM = 33% for Other Limit States

c. *Load modifiers*:
For Strength Limit State:

$$\eta_D = 0.95; \quad \eta_R = 0.95; \quad \eta_I = 1.05; \quad \text{and} \quad \eta = \eta_D\eta_R\eta_I = 0.95 \text{ (AASHTO 1.3.2)}$$

For Service Limit State:

$$\eta_D = 1.0; \quad \eta_R = 1.0; \quad \eta_I = 1.0; \quad \text{and} \quad \eta = \eta_D\eta_R\eta_I = 1.0 \text{ (AASHTO 1.3.2)}$$

d. *Load factors*:

$$\gamma_{DC} = 0.9 \sim 1.25; \quad \gamma_{DW} = 0.65 \sim 1.50; \quad \gamma_{LL} = 1.75$$

e. *Distribution factors for live-load moment and shear* (AASHTO 4.6.2.2.1):
i. Moment distribution factor for exterior girders:
For Span 1 and Span 2:

$$W_e = \frac{2900}{2} + 1211 = 2661 \text{ mm} < S = 2900 \text{ mm}$$

$$g_m^E = \frac{W_e}{4300} = \frac{2661}{4300} = 0.619$$

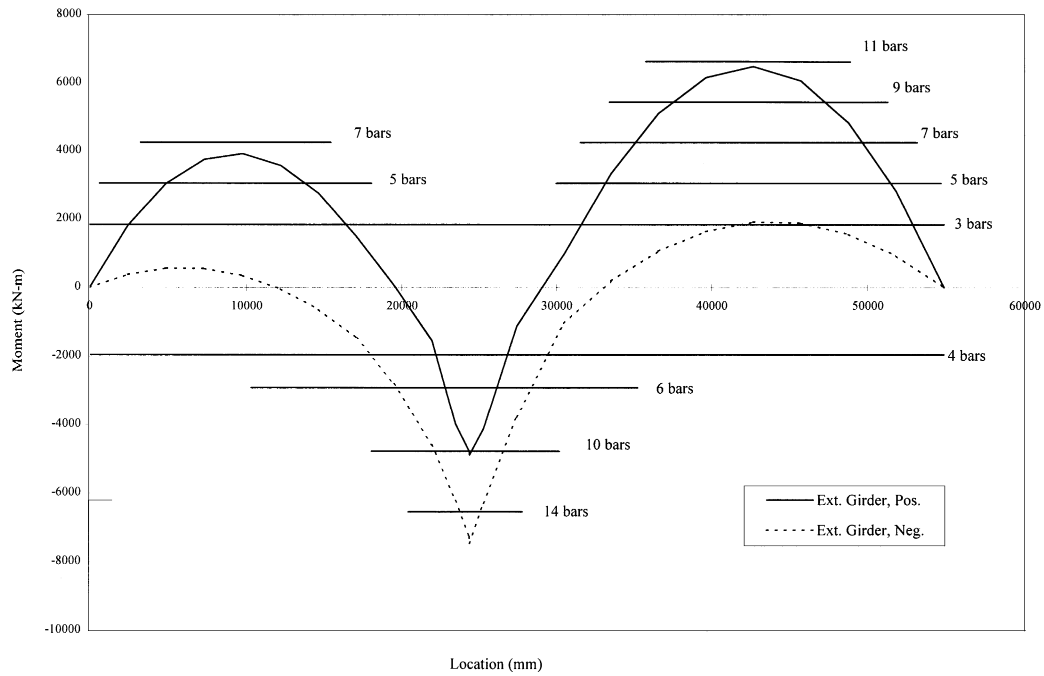



**FIGURE 9.15** Design moment envelope and provided moment capacity with reinforcement cut-off.

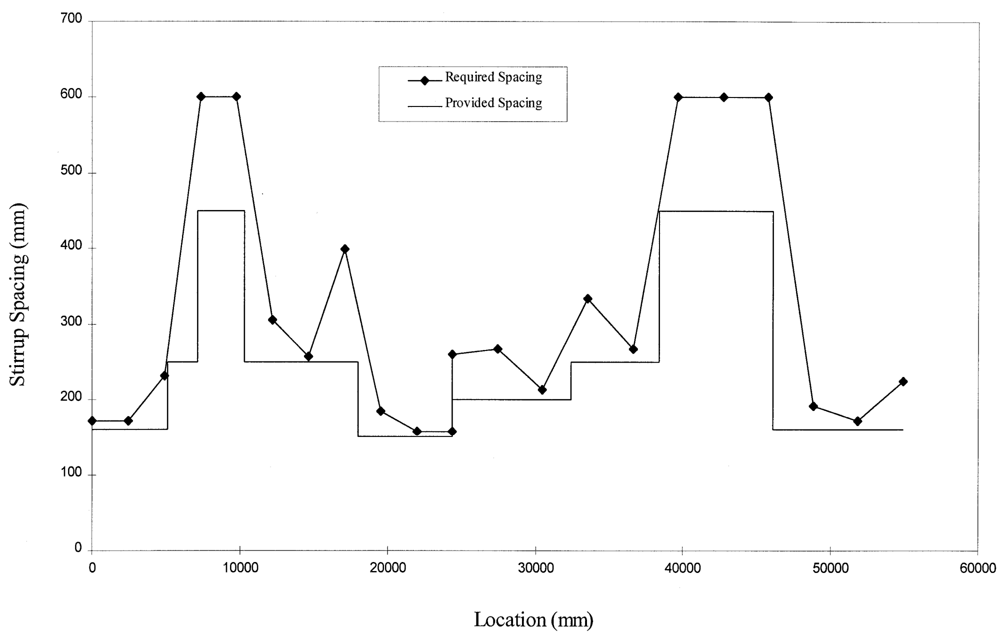



**FIGURE 9.16** Shear reinforcement spacing for the exterior girder.

**TABLE 9.8** Moment Envelope Summary for Exterior Girder at Every 1/10 of Span Length of Span 1 and Span 2

| Span | Distance (mm) | Unfactored Moment Envelope (kN-m) | | | | | | | | | | | | Factored Moment Envelope (kN-m) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | One Design Lane Load | | One Truck | | Train | Live Load Envelope | | Exterior Girder | | | | | Exterior Girder | |
| | | Positive | Negative | Positive | Negative | Negative | Positive | Negative | DC | DW | LL (Pos.) | | LL (Neg.) | Positive | Negative |
| 0.0 $L_1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 |
| 0.1 $L_1$ | 2439 | 216 | –55 | 568 | –93 | –93 | 971 | –179 | 607 | 70 | 601 | | –111 | 1819 | 378 |
| 0.2 $L_1$ | 4878 | 377 | –110 | 955 | –187 | –187 | 1647 | –359 | 1005 | 116 | 1020 | | –222 | 3053 | 561 |
| 0.3 $L_1$ | 7317 | 482 | –165 | 1203 | –281 | –281 | 2082 | –539 | 1196 | 137 | 1289 | | –333 | 3758 | 553 |
| 0.4 $L_1$ | 9756 | 533 | –220 | 1303 | –375 | –375 | 2266 | –719 | 1178 | 135 | 1403 | | –445 | 3923 | 350 |
| 0.5 $L_1$ | 12195 | 528 | –275 | 1278 | –468 | –469 | 2228 | –897 | 951 | 109 | 1379 | | –556 | 3577 | –43 |
| 0.6 $L_1$ | 14634 | 467 | –331 | 1157 | –562 | –563 | 2006 | –1078 | 517 | 59 | 1242 | | –668 | 2762 | –632 |
| 0.7 $L_1$ | 17073 | 352 | –386 | 912 | –656 | –657 | 1565 | –1258 | –126 | –15 | 969 | | –779 | 1494 | –1466 |
| 0.8 $L_1$ | 19512 | 181 | –441 | 570 | –750 | –751 | 939 | –1439 | –976 | –113 | 581 | | –890 | 62 | –2800 |
| 0.9 $L_1$ | 21951 | 38 | –580 | 220 | –844 | –1055 | 331 | –1785 | –2035 | –234 | 205 | | –1105 | –1545 | –4587 |
| 0.96 $L_1$ | 23414 | –216 | –748 | 29 | –903 | –1396 | –177 | –2344 | –2810 | –324 | –110 | | –1451 | –3981 | –6211 |
| 1.0 $L_1$ | 24390 | –326 | –878 | 0 | –938 | –1616 | –326 | –2725 | –3303 | –380 | –202 | | –1686 | –4799 | –7267 |
| 0.0 $L_2$ | 24390 | –281 | –904 | 0 | –1059 | –1643 | –281 | –2780 | –3400 | –392 | –174 | | –1721 | –4885 | –7457 |
| 0.03 $L_2$ | 25304 | –273 | –754 | 0 | –868 | –1433 | –273 | –2394 | –2835 | –327 | –169 | | –1482 | –4112 | –6295 |
| 0.1 $L_2$ | 27438 | 41 | –466 | 224 | –527 | –960 | 339 | –1569 | –1597 | –184 | 210 | | –971 | –1130 | –3772 |
| 0.2 $L_2$ | 30486 | 202 | –234 | 640 | –468 | –468 | 1053 | –856 | –119 | –14 | 652 | | –530 | 974 | –1042 |
| 0.3 $L_2$ | 33534 | 470 | –196 | 1072 | –410 | –410 | 1896 | –741 | 1035 | 119 | 1173 | | –459 | 3349 | 195 |
| 0.4 $L_2$ | 36582 | 662 | –168 | 1403 | –351 | –351 | 2528 | –635 | 1862 | 214 | 1565 | | –393 | 5118 | 1071 |
| 0.5 $L_2$ | 39630 | 768 | –140 | 1591 | –293 | –293 | 2884 | –530 | 2365 | 272 | 1785 | | –328 | 6163 | 1645 |
| 0.6 $L_2$ | 42678 | 787 | –112 | 1640 | –234 | –234 | 2968 | –423 | 2543 | 292 | 1837 | | –262 | 6490 | 1919 |
| 0.7 $L_2$ | 45726 | 720 | –84 | 1532 | –176 | –176 | 2758 | –318 | 2395 | 275 | 1707 | | –197 | 6073 | 1890 |
| 0.8 $L_2$ | 48774 | 566 | –56 | 1210 | –117 | –117 | 2175 | –212 | 1922 | 221 | 1347 | | –131 | 4835 | 1562 |
| 0.9 $L_2$ | 51822 | 326 | –28 | 708 | –59 | –59 | 1268 | –106 | 1123 | 129 | 785 | | –66 | 2822 | 930 |
| 1.0 $L_2$ | 54870 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 |

**TABLE 9.9** Shear Envelope Summary at Every 1/10 of Span 1 and Span 2

| | | Unfactored Shear Envelope (kN) | | | | | | | | | | Factored Shear Envelope (kN) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | One Design Lane Load | | One Truck | | Live Load Envelope | | Exterior Girder | | | | Exterior Girder | |
| Span | Distance (mm) | Positive | Negative | Positive | Negative | Positive | Negative | DC | DW | LL (Pos.) | LL (Neg.) | Positive | Negative |
| 0.0 $L_1$ | 0 | 100 | –23 | 272 | –38 | 462 | –74 | 292 | 34 | 353 | –56 | 981 | 176 |
| 0.1 $L_1$ | 2439 | 77 | –23 | 233 | –38 | 387 | –74 | 206 | 24 | 296 | –56 | 770 | 97 |
| 0.2 $L_1$ | 4878 | 55 | –23 | 195 | –38 | 314 | –74 | 120 | 14 | 240 | –56 | 562 | 18 |
| 0.3 $L_1$ | 7317 | 32 | –23 | 159 | –70 | 243 | –116 | 36 | 4 | 186 | –89 | 358 | –115 |
| 0.4 $L_1$ | 9756 | 9 | –23 | 124 | –105 | 174 | –163 | –50 | –6 | 133 | –124 | 175 | –274 |
| 0.5 $L_1$ | 12195 | –13 | –36 | 92 | –143 | 109 | –226 | –136 | –16 | 84 | –173 | 14 | –471 |
| 0.6 $L_1$ | 14634 | –23 | –59 | 64 | –178 | 62 | –396 | –221 | –26 | 48 | –226 | –126 | –675 |
| 0.7 $L_1$ | 17073 | –23 | –81 | 41 | –212 | 32 | –363 | –306 | –35 | 24 | –278 | –243 | –875 |
| 0.8 $L_1$ | 19512 | –23 | –104 | 23 | –243 | 8 | –427 | –391 | –45 | 6 | –327 | –353 | –1072 |
| 0.9 $L_1$ | 21951 | –23 | –127 | 8 | –271 | –12 | –487 | –477 | –55 | –9 | –373 | –660 | –1264 |
| 1.0 $L_1$ | 24390 | –23 | –149 | 0 | –295 | –23 | –541 | –563 | –65 | –16 | –414 | –787 | –1449 |
| 0.0 $L_2$ | 24390 | 171 | 9 | 299 | 0 | 569 | 9 | 645 | 74 | 442 | 6 | 1606 | 882 |
| 0.1 $L_2$ | 27438 | 143 | 9 | 277 | –3 | 511 | 5 | 539 | 62 | 383 | 4 | 1365 | 734 |
| 0.2 $L_2$ | 30486 | 115 | 9 | 250 | –15 | 448 | –11 | 432 | 50 | 335 | –8 | 1141 | 386 |
| 0.3 $L_2$ | 33534 | 86 | 9 | 220 | –38 | 379 | –42 | 325 | 38 | 284 | –31 | 910 | 249 |
| 0.4 $L_2$ | 36582 | 58 | 9 | 186 | –59 | 305 | –69 | 218 | 25 | 229 | –52 | 675 | 115 |
| 0.5 $L_2$ | 39630 | 30 | 9 | 149 | –87 | 228 | –107 | 112 | 13 | 171 | –80 | 435 | –30 |
| 0.6 $L_2$ | 42678 | 9 | –8 | 111 | –119 | 157 | –166 | 5 | 1 | 117 | –125 | 202 | –202 |
| 0.7 $L_2$ | 45726 | 9 | –36 | 75 | –154 | 109 | –241 | –102 | –12 | 81 | –180 | 41 | –437 |
| 0.8 $L_2$ | 48774 | 9 | –65 | 48 | –192 | 73 | –320 | –209 | –24 | 55 | –240 | –103 | –681 |
| 0.9 $L_2$ | 51822 | 9 | –93 | 20 | –231 | 36 | –400 | –316 | –36 | 27 | –300 | –248 | –925 |
| 1.0 $L_2$ | 54870 | 9 | –121 | 19 | –272 | 34 | –483 | –422 | –49 | 26 | –362 | –348 | –1171 |

ii. Shear distribution factor for exterior girders:

| Design Lane | Span 1 | Span 2 |
|---|---|---|
| One design lane loaded | $g_v^E = \dfrac{0.5(1015+2815)}{\left(\dfrac{\sqrt{5}}{2}\right)(2884)} = 0.594$ | $g_v^E = \dfrac{0.5(1015+2815)}{\left(\dfrac{\sqrt{5}}{2}\right)(2884)} = 0.594$ |
| Two or more design lanes loaded | $d_e = 1066 - 535 = 531 < 1500$ <br> $e = 0.64 + \dfrac{531}{3800} = 0.78$ <br> $g_v^E = 0.78\left(\dfrac{2900}{2200}\right)^{0.9}\left(\dfrac{1680}{24,385}\right)^{0.1}$ <br> $= 0.765$ | $d_e = 1066 - 535 = 531 < 1500$ <br> $e = 0.64 + \dfrac{531}{3800} = 0.78$ <br> $g_v^E = 0.78\left(\dfrac{2900}{2200}\right)^{0.9}\left(\dfrac{1680}{30,480}\right)^{0.1}$ <br> $= 0.749$ |
| Govern | 0.765 | 0.749 |

f. *Factored moment envelope and shear envelope*:
The moment and shear envelopes for the exterior girder, unfactored and factored based on Strength Limit State I, are listed in Tables 9.8 and 9.9. Figures 9.15 and 9.16 show the envelope diagram for moments and shears based on Strength Limit State I, respectively.

**3. Flexural Design**

a. *Determine the effective flange width* (Section 9.4.6):

i. Effective compression flange for positive moments:

Span 1:

For interior girder,

$$b_{\text{top}}^I = \text{the smallest of} \begin{cases} \dfrac{1}{4}L_{1,\text{eff}} = \dfrac{1}{4}(0.65)(24,390) = 3963 \text{ mm} \\ 12t_{\text{top}} + b_w = 12(210) + (205) = \underline{2725 \text{ mm}} \ \textit{governs} \\ \textit{the average spacing of adjacent beams} = 2900 \text{ mm} \end{cases}$$

For exterior girder,

$$b_{top}^E = \frac{1}{2}b_{top}^I + \text{the smallest of}$$

$$\begin{cases} \dfrac{1}{8}L_{1,eff} = \dfrac{1}{8}(0.65)(24,390) = 1982 \text{ mm} \\ 6t_{\text{top}} + \dfrac{1}{2}b_w = (6)(210) + \dfrac{1}{2}(291) = 1405 \text{ mm} \\ \text{the width of the overhang} = 920 + \dfrac{291}{2} = \underline{1065 \text{ mm}} \ \ \textit{governs} \end{cases}$$

$$= \frac{1}{2}(2724) + 1065$$

$$= 2427 \text{ mm}$$

Span 2: The effective flange widths for Span 2 turns out to be the same as those in Span 1.

ii. Effective compression flange for negative moments:
Span 1:
For interior girder,

$$b_{\text{bot}}^{I} = \text{the smallest of} \begin{cases} \dfrac{1}{4}L_{\text{eff}} = \dfrac{1}{4}[(0.5)(24\ 390) + (0.25)(30\ 480) = 4954 \text{ mm} \\ 12t_{\text{bot}} + b_w = 12(170) + (205) = \underline{2245 \text{ mm}} \ \ \textit{governs} \\ \textit{the average spacing of adjacent beams} = 2900 \text{ mm} \end{cases}$$

For exterior girder,

$$b_{\text{bot}}^{E} = \frac{1}{2}b_{\text{bot}}^{I} + \text{the smallest of}$$

$$\begin{cases} \dfrac{1}{8}L_{\text{eff}} = \dfrac{1}{8}[(0.5)(24,390) + (0.25)(30,480) = 2477 \text{ mm} \\ 6t_{\text{bot}} + \dfrac{1}{2}b_w = (6)(170) + \dfrac{1}{2}(291) = 1166 \text{ mm} \\ \text{the width of the overhang} = 0 + \dfrac{291}{2} = \underline{146 \text{ mm}} \ \ \textit{governs} \end{cases}$$

$$= \frac{1}{2}(2245) + 146$$

$$= 1268 \text{ mm}$$

Span 2: The effective flange widths are the same as those in Span 1.

b. *Required flexural reinforcement*:
The required reinforcements are calculated using Eqs. (9.16) and (9.17), neglecting the compression steel
The minimum reinforcement required, based on Eq. (9.20), is

$$\rho_{\min} \geq 0.03\frac{f_c'}{f_y} = 0.03\left(\frac{28}{420}\right) = 0.002$$

$$A_g(\text{Exterior girder}) = 1\ 103\ 530 \text{ mm}^2$$

$$A_{s_{\min}}(\text{Exterior girder}) = (0.002)(1\ 103\ 530) = 2207 \text{ mm}^2$$

Use $A_{s_{\min}} = \underline{2500 \text{ mm}^2}$
The required and provided reinforcements for sections located at 1⁄10 of each span interval and the face of the bent cap are listed in Table 9.10.

**TABLE 9.10** Section Reinforcement Design for Exterior Girder

| | | Positive Moment | | | | | Negative Moment | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Section | Distance from Abut. 1 (mm) | $M_u$ (kN-m) | $A_s$ Required (mm$^2$) | No. of Reinf. Bars Use #36 | $A_s$ (provided) (mm$^2$) | $\phi M_n$ (provided) (kN-m) | $M_u$ (kN-m) | $A_s$ Required (mm$^2$) | No. of Reinf. Bars Use #32 | $A_s$ (Provided) (mm$^2$) | $\phi M_n$ (Provided) (kN-m) |
| 0.0 $L_1$ | 0 | 0 | 0 | 3 | 3018 | 1841 | 0 | 0 | 4 | 3276 | 1954 |
| 0.1 $L_1$ | 2439 | 1819 | 2979 | 3 | 3018 | 1841 | 0 | 0 | 4 | 3276 | 1954 |
| 0.2 $L_1$ | 4878 | 3053 | 5020 | 5 | 5030 | 3055 | 0 | 0 | 4 | 3276 | 1954 |
| 0.3 $L_1$ | 7317 | 3758 | 6194 | 7 | 7042 | 4257 | 0 | 0 | 4 | 3276 | 1954 |
| 0.4 $L_1$ | 9756 | 3923 | 6469 | 7 | 7042 | 4257 | 0 | 0 | 4 | 3276 | 1954 |
| 0.5 $L_1$ | 12195 | 3577 | 5892 | 7 | 7042 | 4257 | 43 | 72 | 6 | 4914 | 2910 |
| 0.6 $L_1$ | 14634 | 2762 | 4537 | 5 | 5030 | 3055 | 632 | 1048 | 6 | 4914 | 2910 |
| 0.7 $L_1$ | 17073 | 1494 | 2444 | 3 | 3018 | 1841 | 1466 | 2448 | 6 | 4914 | 2910 |
| 0.8 $L_1$ | 19512 | 62 | 101 | 3 | 3018 | 1841 | 2800 | 4724 | 6 | 4914 | 2910 |
| 0.9 $L_1$ | 21951 | 0 | 0 | 3 | 3018 | 1841 | 4587 | 7848 | 10 | 8190 | 4780 |
| 0.96 $L_1$ | 23414 | 0 | 0 | 3 | 3018 | 1841 | 6211 | 10767 | 14 | 11466 | 6544 |
| 0.03 $L_2$ | 25304 | 0 | 0 | 3 | 3018 | 1841 | 6295 | 10920 | 14 | 11466 | 6544 |
| 0.1 $L_2$ | 27438 | 0 | 0 | 3 | 3018 | 1841 | 3772 | 6412 | 10 | 8190 | 4780 |
| 0.2 $L_2$ | 30486 | 974 | 1590 | 3 | 3018 | 1841 | 1042 | 1735 | 6 | 4914 | 2910 |
| 0.3 $L_2$ | 33534 | 3349 | 5512 | 7 | 7042 | 4257 | 0 | 0 | 6 | 4914 | 2910 |
| 0.4 $L_2$ | 36582 | 5118 | 8475 | 9 | 9054 | 5449 | 0 | 0 | 4 | 3276 | 1954 |
| 0.5 $L_2$ | 39630 | 6163 | 10243 | 11 | 11066 | 6629 | 0 | 0 | 4 | 3276 | 1954 |
| 0.6 $L_2$ | 42678 | 6490 | 10799 | 11 | 11066 | 6629 | 0 | 0 | 4 | 3276 | 1954 |
| 0.7 $L_2$ | 45726 | 6073 | 10091 | 11 | 11066 | 6629 | 0 | 0 | 4 | 3276 | 1954 |
| 0.8 $L_2$ | 48774 | 4835 | 7999 | 9 | 9054 | 5449 | 0 | 0 | 4 | 3276 | 1954 |
| 0.9 $L_2$ | 51822 | 2822 | 4637 | 5 | 5030 | 3055 | 0 | 0 | 4 | 3276 | 1954 |
| 1.0 $L_2$ | 54870 | 0 | 0 | 3 | 3018 | 1841 | 0 | 0 | 4 | 3276 | 1954 |

c. *Reinforcement layout*:

i. Reinforcement cutoff (Section 9.4.7):

- The extended length at cutoff for positive moment reinforcement, No. 36, is

$$\text{the largest of} \begin{cases} \text{Effective depth of the section} \ = \ \underline{1625 \text{ mm}} & \textit{governs} \\ 15 \ d_b = 537 \text{ mm} \\ 0.05 \text{ of span length} \ = \ 0.05 \ (24 \ 390) = 1220 \text{ mm} \end{cases}$$

From Table 9.6, the stagger lengths for No. 36 and No. 32 bars are

$$l_d \text{ of No. 36 bars } = 1600 \text{ mm}$$

$$l_d \text{ of No. 32 bars } = 1300 \text{ mm}$$

- The extended length at cutoff for negative moment reinforcement, No. 32, is

$$\text{the largest of} \begin{cases} \text{Effective depth of the section} \ = \ \underline{1601 \text{ mm}} & \textit{governs} \\ 15 \ d_b = 485 \text{ mm} \\ 0.05 \text{ of span length} \ = \ 0.05 \ (30 \ 480) = 1524 \text{ mm} \end{cases}$$

- Negative moment reinforcements, in addition to the above requirement for bar cutoff, have to satisfy Eq. (9.43). The extended length beyond the inflection point has to be the largest of the following:

$$\begin{cases} d = \underline{1601 \text{ mm}} \quad \textit{governs for Span 1} \\ 12 d_b = 387.6 \text{ mm} \\ 0.0625 \times (\text{clear span length}) = (0.0625)(24 \ 390) = 1524 \text{ mm} \\ \quad \text{or} \qquad\qquad\qquad\qquad = (0.0625)(30 \ 480) = \underline{1905 \text{ mm}} \quad \textit{governs for Span 2} \end{cases}$$

ii. Reinforcement distribution (Section 9.4.2):

$$\frac{1}{10} (\text{average adjacent span length}) \ = \ \frac{1}{10} (30 \ 480 + 24 \ 385) = 2743 \text{ mm}$$

$$b_{\text{top}}^E = 2427 \text{ mm} < \ 2743 \text{ mm}$$

All tensile reinforcements should be distributed within the effective tension flange width.

iii. Side reinforcements in the web, Eq. (9.6)

$$A_{sk} \geq 0.001(d_e - 760) = 0.001(1625 - 760) = 0.865 \text{ mm}^2/\text{mm of height}$$

$$A_{sk} \leq \frac{A_s}{1200} = \frac{13{,}462}{1200} = 11.21 \text{ mm}^2/\text{mm of height}$$

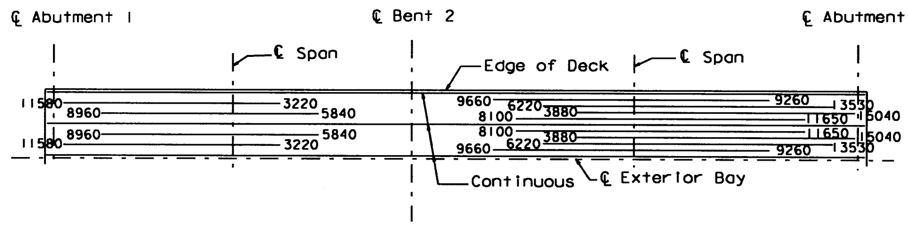



**FIGURE 9.17** Bottom slab reinforcement of exterior girder.

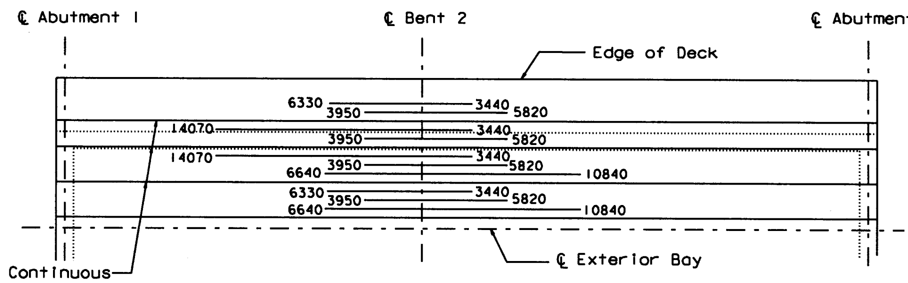



**FIGURE 9.18** Top deck reinforcement of exterior girder.

$$A_{sk} = 0.865(250) = 216 \text{ mm}^2 \quad \underline{\text{Use No. 19 at 250 mm on each side face of the web}}$$

The reinforcement layout for bottom slab and top deck of exterior girder are shown in Figure 9.17 and 9.18, respectively. The numbers next to the reinforcing bars indicate the bar length extending beyond either the centerline of support or span.

**4. Shear Design**

From Table 9.9, it is apparent that the maximum shear demand is located at the critical section near Bent 2 in Span 2.

a. *Determine the critical section near Bent 2 in Span 2:*

$$A_s = 11{,}466 \text{ mm}^2,\ b = 1268 \text{ mm}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{(11{,}466)(420)}{0.85(28)(1268)} = 160 \text{ mm}$$

$$d_v = \text{ the largest of } \begin{cases} d_e - \dfrac{a}{2} = 1601 - \dfrac{160}{2} = \underline{1521 \text{ mm}} \ \textit{governs} \\ 0.9 d_e = 0.9(1601) = 1441 \text{ mm} \\ 0.72h = 0.72(1680) = 1210 \text{ mm} \end{cases}$$

The critical section is at a distance of $d_v$ from the face of the support, i.e., distance between centerline of Bent 2 and the critical section = 600 + 1521 = 2121 mm = $0.07 L_2$.

b. *At the above section, find $M_u$ and $V_u$, using interpolation from Tables 9.8 and 9.9:*

$$M_u = 3772 + (6295 - 3772)(0.03/0.07) = 4853 \text{ kN·m}$$

$$V_u = 1365 + (1606 - 1365)(0.03/0.1) = 1437 \text{ kN}$$

$$v = \frac{V_u}{\phi_v b_v d_v} = \frac{1437 \cdot (1000)}{(0.9)(291)(1521)} = 3.61 \text{ MPa}$$

$$\frac{v}{f'_c} = \frac{3.61}{28} = 0.129 < 0.25 \quad \text{O.K.}$$

c. *Determine θ and β, and required shear reinforcement spacing*:
Try θ = 37.5°, cot θ = 1.303, $A_s$ = 11 466 mm², $E_c$ = 200 GPa, from Eq. (9.24).

$$\varepsilon_x = \frac{\dfrac{4\ 853\ 000}{1521} + 0.5(1437)(1.303)}{200(11\ 466)} = 1.80 \times 10^{-3}$$

From Figure 9.6, we obtain $\theta = 37.5°$, which agrees with the assumption.
Use θ = 37.5°, β = 1.4, from Eq. (9.25)

$$V_s = \frac{1437}{0.9} - 0.083(1.4)\sqrt{28}(291)(1521) \times 10^{-3} = 1325 \text{ kN}$$

Use No. 16 rebars, $A_v$ = 199(2) = 398 mm², from Eq. (9.26)

$$\text{Required spacing, } s \le \frac{(398)(420)(1521)}{1325 \times 10^3}(1.303) = 250 \text{ mm}$$

d. *Determine the maximum spacing required*:
Note that $V_u$ = 1437 kN > 0.1 $f'_c b'_v d_v$ = 0.1(28)(291)(1521) × 10⁻³ = 1239 kN
From Eqs. (9.27) and (9.29):

$$s_{\max} = \text{ the smallest of } \begin{cases} \dfrac{(398)(420)}{0.083\sqrt{28}(291)} = 1307 \text{ mm} \\ 0.4(1521) = 608 \text{ mm} \\ \underline{300 \text{ mm}} \quad \text{governs} \end{cases}$$

Use $\underline{s = 250 \text{ mm}}$ < 300 mm <u>OK.</u>

e. *Check the adequacy of the longitudinal reinforcements, using Eq. (9.30)*:

$$A_s f_y = (11\ 466)(420) = 4\ 815\ 720 \text{ N}$$

$$\frac{M_u}{d_v \phi_f} + \left(\frac{V_u}{\phi_v} - 0.5V_s\right)\cot\theta = \frac{4853 \times 10^6}{(1521)(0.9)} + \left(\frac{1437 \times 10^3}{0.9} - 0.5(1325 \times 10^3)\right)(1.303)$$

$$= 4\ 762\ 401 \text{ N} < A_s f_y \quad \text{O.K.}$$

Using the above procedure, the shear reinforcements, i.e., stirrups in the web, for each section can be obtained. Figure 9.16 shows the shear reinforcements required and provided in the exterior girder for both spans.

6. **Crack Control Check (Section 9.4.2)**

For illustration purpose, we select the section located at midspan of Span 1 in this example, i.e. at $0.5\,L_1$

a. *Check if the section is cracked*:

Service load moment, $M_{pos}$ = (1.0)($M_{DC}$ + $M_{DW}$ + $M_{LL+IM}$)
= (1.0)(951 + 109 + 1379)
= 2439 kN-m

Modulus of rupture $f_r = 0.63\sqrt{f'_c} = 0.63\sqrt{28} = 3.33$ MPa, $0.8 f_r = 2.66$ MPa

$b_{top}$ = 2427 mm, $b_{bot}$ = 1268 mm, obtain

$I_g = 4.162 \times 10^{11}$ mm$^4$ and $\bar{y} = 655$ mm,

where $\bar{y}$ is the distance from the most compressed concrete fiber to the neutral axis

$$S = \frac{I_g}{(d-\bar{y})} = \frac{4.162\times 10^{11}}{(1680-655)} = 4.06\times 10^8 \text{ mm}^3$$

$$f_c = \frac{M_{pos}}{S} = \frac{2439\times 10^6}{4.06\times 10^8} = 6.01 \text{ MPa} > 0.8 f_r = 2.66 \text{ MPa}$$

The section is cracked.

b. *Calculate tensile stress of the reinforcement*:

Assuming the neutral axis is located in the web, thus applying Eqs. (9.31) through (9.34) with $A_s$ = 7042 mm$^2$, $A'_s$ = 0, and $\beta_1$ = 0.85, solve for $x$

$$x = 239 \text{ mm} > h_f = b_{top} = 210 \text{ mm} \quad \text{O.K.}$$

From Eq. (9.35), obtain

$$I_{cr} = \frac{1}{3}(2427)(239)^3 - \frac{1}{3}(2427-291)(239-210)^3 + 7(7042)(1625-239)^2$$

$$= 1.057\times 10^{11} \text{ mm}^4$$

and from Eq. (9.38), the tensile stress in the longitudinal reinforcement is

$$f_s = \frac{7(2439\times 10^6)(1625-239)}{1.057\times 10^{11}} = 224 \text{ MPa}$$

c. *The allowable stress can be obtained using Eq. (9.5), with $Z$ = 30 000 for moderate exposure and $d_c$ = 50 mm*

$$f_{sa} = \frac{Z}{(d_c A)^{1/3}} = \frac{30{,}000}{\left((50)\,\dfrac{(50\cdot 2\cdot 1268)}{7}\right)^{1/3}} = 310 \text{ MPa} > 0.6 f_y = 252 \text{ MPa}$$

Use $f_{sa}$ = 252 MPa > $f_s$ = 223 MPa     O.K.

The other sections can be checked following the same procedure described above.

7. **Check Deflection Limit**

Based on the Service Limit State, we can compute the $I_e$ for sections at 1/10 of the span length interval. For illustration, let the section be at $0.4 L_2$

Deflection distribution factor = (no. of design lanes)/(no. of supporting beams) = 2/4 = 0.5
Note that $b_{\text{top}}$ = 2424 mm, $t_{\text{top}}$ = 210 mm, $b_w$ = 291 mm, $h$ = 1680 mm, $d$ = 1625 mm, $b_{\text{bot}}$ = 1268 mm, $t_{\text{bot}}$ = 170 mm, and neglecting compression steel

$$A_g = (2427)(210) + (1680 - 210 - 170)(291) + (1268)(170) = 1\ 103\ 530 \text{ mm}^2$$

$$y_t = \left[\frac{(509\ 670)\left(1680 - \frac{210}{2}\right) + (378\ 300)\left(170 + \frac{1300}{2}\right) + (215\ 560)\left(\frac{170}{2}\right)}{1\ 103\ 530}\right] = 1025 \text{ mm}$$

$$I_g = \frac{1}{12}(2427)(210)^3 + (509\ 670)(550)^2 + \frac{1}{12}(291)(1300)^3 + (378\ 300)(205)^2$$

$$+\frac{1}{12}(1268)(170)^3 + (215\ 560)(940)^2$$

$$= 4.16 \times 10^{11} \text{ mm}^4$$

$$M_{\text{cr}} = f_r \frac{I_g}{y_t} = (3.33)\frac{4.16 \times 10^{11}}{1025} = 1.35 \times 10^9 \text{ N} - \text{mm}$$

Use Eqs. (9.31) throuth (9.35) to solve for x and $I_{\text{cr}}$, with $A_s$ = 9054 mm² and $A_s' = 0$, we obtain

$$x = 272 \text{ mm}, \quad I_{\text{cr}} = 1.32 \times 10^{11} \text{ mm}^4$$

From Table 9.8:

$$M_a = 1862 + 214 + (0.5)(1565) = 2859 \text{ kN-m}$$

$$\frac{M_{cr}}{M_a} = \frac{1.35 \times 10^9}{2.86 \times 10^9} = 0.47$$

$$I_e = \left(\frac{M_{\text{cr}}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{\text{cr}}}{M_a}\right)^3\right] I_{\text{cr}} = (0.47)^3(4.16 \times 10^{11}) + \left[1 - (0.47)^3\right](1.32 \times 10^{11}) = 1.61 \times 10^{11} \text{ mm}^4$$

The above computation can be repeated to obtain $I_e$ for other sections. It is assumed that the maximum deflection occurs where the maximum flexural moment is. To be conservative, the minimum $I_e$ is used to calculate the deflection.

$$\Delta_{\max} = \begin{cases} 19 \text{ mm} & \text{truck load} \\ 13 \text{ mm} & \text{lane + 25\% of truck load} \end{cases} < \frac{L_2}{800} = \frac{30\ 480}{800} = 38 \text{ mm} \quad \text{O.K.}$$

**8. Check Fatigue Limit State**

For illustration purpose, check the bottom reinforcements for the section at $0.7L_1$. For positive moment at this section, $A_s$ = 4024 mm², $A_s'$ = 4095 mm², $d$ = 1625 mm, and $d'$ = 79 mm. Note that the maximum positive moment due to the assigned truck is 757 kN-m, while the largest negative moment 598 kN-m.

$$M_{\max} \text{ due to fatigue load} = 0.75(0.619)(757)(1 + 0.15) = 404 \text{ kN-m}$$

Use Eqs. (9.31) through (9.35) and (9.38) to obtain the maximum tensile stress in the main bottom reinforcements as

$$f_{\max} = 64 \text{ MPa}$$

The negative moment at this section is

$$M_{\min} \text{ due to fatigue load} = 0.75(0.619)(-598)(1 + 0.15) = -319 \text{ kN-m}$$

Using Eqs. (9.31) through (9.35) and (9.38), with $A_s = 4095 \text{ mm}^2$, $A_s' = 4024 \text{ mm}^2$, $d = 1601$ mm, and $d' = 55$ mm, we obtain the maximum compressive stress in the main bottom reinforcements as

$$f_{\min} = -7.0 \text{ MPa}$$

Thus, the stress range for fatigue

$$f_{\max} - f_{\min} = 64 - (-7.0) = 71 \text{ MPa}$$

From Eq. (9.9), allowable stress range

$$f_r = 145 - 0.33(-7.0) + 55(0.3) = 164 \text{ MPa} > 71 \text{ MPa} \qquad \text{OK}$$

Other sections can be checked in the same fashion described above.

9. Summary
   The purpose of the above example is mainly to illustrate the design procedure for flexural and shear reinforcement for the girder. It should be noted that, in reality, the controlling load case may not be the Strength Limit State; therefore, all the load cases specified in the AASHTO should be investigated for a complete design. It should also be noted that the interior girder design can be achieved by following the similar procedures described herein.

## References


1. AASHTO, *AASHTO LRFD Bridge Design Specifications*, American Association of State Highway and Transportation Officials, Washington, D.C., 1994.
2. ASTM, *Annual Book of ASTM Standards*, American Society for Testing and Materials, Philadelphia, 1996.
3. Caltrans, *Bridge Design Aids Manual*, California Department of Transportation, Sacramento, 1994.
4. Collins, M. P. and Mitchell, D., *Prestressed Concrete Structures*, Prentice-Hall, Englewood Cliffs, NJ, 1991.
5. Caltrans, *Bridge Design Practices*, California Department of Transportation, Sacramento, 1995.
6. ACI Committee 318, Building Code Requirements for Reinforced Concrete (ACI 318-95), American Concrete Institute, 1995.
7. Barker, R. M. and Puckett, J. A., *Design of Highway Bridges*, John Wiley & Sons, New York, 1997.
8. Park, R. and Paulay, T., *Reinforced Concrete Structures*, John Wiley & Sons, New York, 1975.
9. Xanthakos, P. P., *Theory and Design of Bridges*, John Wiley & Sons, New York, 1994.

Duan, L., Chen, K., Tan, A. "Prestressed Concrete Bridges."
*Bridge Engineering Handbook.*
Ed. Wai-Fah Chen and Lian Duan
Boca Raton: CRC Press, 2000

# 10

# Prestressed Concrete Bridges



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## 10.1 Introduction

Prestressed concrete structures, using high-strength materials to improve serviceability and durability, are an attractive alternative for long-span bridges, and have been used worldwide since the 1950s. This chapter focuses only on conventional prestressed concrete bridges. Segmental concrete bridges will be discussed in Chapter 11. For more detailed discussion on prestressed concrete, references are made to textbooks by Lin and Burns [1], Nawy [2], Collins and Mitchell [3].

### 10.1.1 Materials

#### 10.1.1.1 Concrete

A 28-day cylinder compressive strength ( $f'_c$ ) of concrete 28 to 56 MPa is used most commonly in the United States. A higher early strength is often needed, however, either for the fast precast method used in the production plant or for the fast removal of formwork in the cast-in-place method. The modulus of elasticity of concrete with density between 1440 and 2500 kg/m$^3$ may be taken as

$$E_c = 0.043\, w_c \sqrt{f'_c} \qquad (10.1)$$

where $w_c$ is the density of concrete (kg/m$^3$). Poisson's ratio ranges from 0.11 to 0.27, but 0.2 is often assumed.

The modulus of rupture of concrete may be taken as [4]

$$f_r = \begin{cases} 0.63\sqrt{f_c'} & \text{for normal weight concrete — flexural} \\ 0.52\sqrt{f_c'} & \text{for sand - lightweight concrete — flexural} \\ 0.44\sqrt{f_c'} & \text{for all - lightweight concrete — flexural} \\ 0.1 f_c' & \text{for direct tension} \end{cases} \tag{10.2}$$

Concrete shrinkage is a time-dependent material behavior and mainly depends on the mixture of concrete, moisture conditions, and the curing method. Total shrinkage strains range from 0.0004 to 0.0008 over the life of concrete and about 80% of this occurs in the first year.

For moist-cured concrete devoid of shrinkage-prone aggregates, the strain due to shrinkage $\varepsilon_{sh}$ may be estimated by [4]

$$\varepsilon_{sh} = -k_s k_h \left(\frac{t}{35+t}\right) 0.51\times 10^{-3} \tag{10.3}$$

$$K_s = \left[\frac{\dfrac{t}{26e^{0.0142(V/S)}+t}}{\dfrac{t}{45+t}}\right]\left[\frac{1064-3.7(V/S)}{923}\right] \tag{10.4}$$

where $t$ is drying time (days); $k_s$ is size factor and $k_h$ is humidity factors may be approximated by $K_n$ = (140-H)/70 for $H$ < 80%; $K_n$ = 3(100-H)/70 for $H \geq$ 80%; and $V/S$ is volume to surface area ratio. If the moist-cured concrete is exposed to drying before 5 days of curing, the shrinkage determined by Eq. (10.3) should be increased by 20%.

For stem-cured concrete devoid of shrinkage-prone aggregates:

$$\varepsilon_{sh} = -k_s k_h \left(\frac{t}{55+t}\right) 0.56\times 10^{-3} \tag{10.5}$$

Creep of concrete is a time-dependent inelastic deformation under sustained load and depends primarily on the maturity of the concrete at the time of loading. Total creep strain generally ranges from about 1.5 to 4 times that of the "instantaneous" deformation. The creep coefficient may be estimated as [4]

$$\psi(t,t_1) = 3.5K_c K_f \left(1.58-\frac{H}{120}\right) t_i^{-0.118} \frac{(t-t_i)^{0.6}}{10+(t-t_i)^{0.6}} \tag{10.6}$$

$$K_f = \frac{62}{42+f_c'} \tag{10.7}$$

$$K_s = \left[\frac{\dfrac{t}{26e^{0.0142(V/S)}+t}}{\dfrac{t}{45+t}}\right]\left[\frac{1.8+1.77e^{-0.0213(V/S)}}{2.587}\right] \tag{10.8}$$

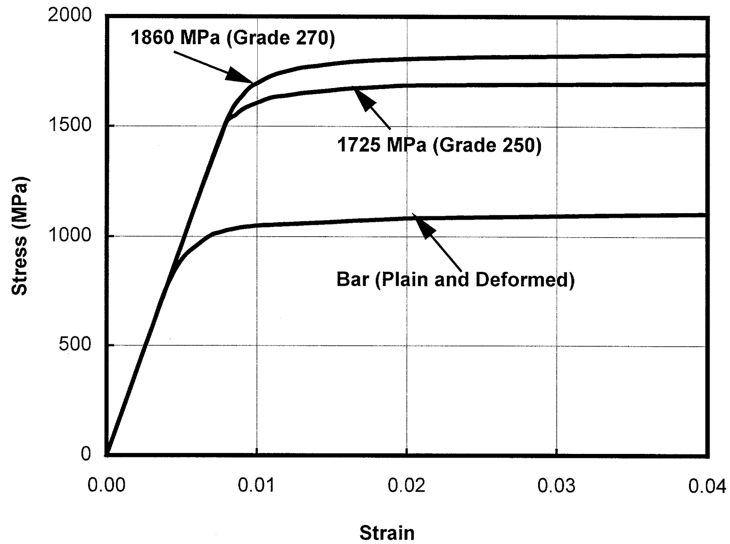


**FIGURE 10.1** Typical stress–strain curves for prestressing steel.

where $H$ is relative humidity (%); $t$ is maturity of concrete (days); $t_i$ is age of concrete when load is initially applied (days); $K_c$ is the effect factor of the volume-to-surface ratio; and $K_f$ is the effect factor of concrete strength.

Creep, shrinkage, and modulus of elasticity may also be estimated in accordance with CEB-FIP Mode Code [15].

### 10.1.1.2 Steel for Prestressing

Uncoated, seven-wire stress-relieved strands (AASHTO M203 or ASTM A416), or low-relaxation seven-wire strands and uncoated high-strength bars (AASHTO M275 or ASTM A722) are commonly used in prestresssed concrete bridges. Prestressing reinforcement, whether wires, strands, or bars, are also called *tendons*. The properties for prestressing steel are shown in Table 10.1.

**TABLE 10.1** Properties of Prestressing Strand and Bars

| Material | Grade and Type | Diameter (mm) | Tensile Strength $f_{pu}$ (MPa) | Yield Strength $f_{py}$ (MPa) | Modulus of Elasticity $E_p$ (MPa) |
|---|---|---|---|---|---|
| Strand | 1725 MPa (Grade 250) | 6.35–15.24 | 1725 | 80% of $f_{pu}$ except 90% of $f_{pu}$ | 197,000 |
| | 1860 MPa (Grade 270) | 10.53–15.24 | 1860 | for low relaxation strand | |
| Bar | Type 1, Plain | 19 to 25 | 1035 | 85% of $f_{pu}$ | |
| | Type 2, Deformed | 15 to 36 | 1035 | 80% of $f_{pu}$ | 207,000 |

Typical stress–strain curves for prestressing steel are shown in Figure 10.1. These curves can be approximated by the following equations:

*For Grade 250 [5]:*

$$f_{ps} = \begin{cases} 197{,}000\,\varepsilon_{ps} & \text{for } \varepsilon_{ps} \le 0.008 \\ 1710 - \dfrac{0.4}{\varepsilon_{ps} - 0.006} < 0.98 f_{pu} & \text{for } \varepsilon_{ps} > 0.008 \end{cases} \tag{10.9}$$

*For Grade 270 [5]*:

$$f_{ps} = \begin{cases} 197{,}000\,\varepsilon_{ps} & \text{for } \varepsilon_{ps} \le 0.008 \\ 1848 - \dfrac{0.517}{\varepsilon_{ps} - 0.0065} < 0.98 f_{pu} & \text{for } \varepsilon_{ps} > 0.008 \end{cases} \tag{10.10}$$

*For Bars*:

$$f_{ps} = \begin{cases} 207{,}000\,\varepsilon_{ps} & \text{for } \varepsilon_{ps} \le 0.004 \\ 1020 - \dfrac{0.192}{\varepsilon_{ps} - 0.003} < 0.98 f_{pu} & \text{for } \varepsilon_{ps} > 0.004 \end{cases} \tag{10.11}$$

#### 10.1.1.3 Advanced Composites for Prestressing

Advanced composites–fiber-reinforced plastics (FPR) with their high tensile strength and good corrosion resistance work well in prestressed concrete structures. Application of advanced composites to prestressing have been investigated since the 1950s [6–8]. Extensive research has also been conducted in Germany and Japan [9]. The Ulenbergstrasse bridge, a two-span (21.3 and 25.6 m) solid slab using 59 fiberglass tendons, was built in 1986 in Germany. It was the first prestressed concrete bridge to use advanced composite tendons in the world [10].

FPR cables and rods made of ararmid, glass, and carbon fibers embedded in a synthetic resin have an ultimate tensile strength of 1500 to 2000 MPa, with the modulus of elasticity ranging from 62,055 MPa to 165,480 MPa [9]. The main advantages of FPR are (1) a high specific strength (ratio of strength to mass density) of about 10 to 15 times greater than steel; (2) a low modulus of elasticity making the prestress loss small; (3) good performance in fatigue; tests show [11] that for CFRP, at least three times the higher stress amplitudes and higher mean stresses than steel are achieved without damage to the cable over 2 million cycles.

Although much effort has been given to exploring the use of advanced composites in civil engineering structures (see Chapter 51) and the cost of advanced composites has come down significantly, the design and construction specifications have not yet been developed. Time is still needed for engineers and bridge owners to realize the cost-effectiveness and extended life expectancy gained by using advanced composites in civil engineering structures.

#### 10.1.1.4 Grout

For post-tensioning construction, when the tendons are to be bound, grout is needed to transfer loads and to protect the tendons from corrosion. Grout is made of water, sand, and cements or epoxy resins. AASHTO-LRFD [4] requires that details of the protection method be indicated in the contract documents. Readers are referred to the *Post-Tensioning Manual* [12].

### 10.1.2 Prestressing Systems

There are two types of prestressing systems: pretensioning and post-tensioning systems. Pretensioning systems are methods in which the strands are tensioned before the concrete is placed. This method is generally used for mass production of pretensioned members. Post-tensioning systems are methods in which the tendons are tensioned after concrete has reached a specified strength. This technique is often used in projects with very large elements (Figure 10.2). The main advantage of post-tensioning is its ability to post-tension both precast and cast-in-place members. Mechanical prestressing–jacking is the most common method used in bridge structures.

**FIGURE 10.2** A post–tensioned box–girder bridge under construction.

## 10.2 Section Types

### 10.2.1 Void Slabs

Figure 10.3a shows FHWA [13] standard precast prestressed voided slabs. Sectional properties are listed in Table 10.2. Although the cast-in-place prestressed slab is more expensive than a reinforced concrete slab, the precast prestressed slab is economical when many spans are involved. Common spans range from 6 to 15 m. Ratios of structural depth to span are 0.03 for both simple and continuous spans.

### 10.2.2 I-Girders

Figures 10.3b and c show AASHTO standard I-beams [13]. The section properties are given in Table 10.3. This bridge type competes well with steel girder bridges. The formwork is complicated, particularly for skewed structures. These sections are applicable to spans 9 to 36 m. Structural depth-to-span ratios are 0.055 for simple spans and 0.05 for continuous spans.

### 10.2.3 Box Girders

Figure 10.3d shows FHWA [13] standard precast box sections. Section properties are given in Table 10.4. These sections are used frequently for simple spans of over 30 m and are particularly suitable for widening bridges to control deflections.

The box-girder shape shown in Figure 10.3e is often used in cast-in-place prestressed concrete bridges. The spacing of the girders can be taken as twice the depth. . This type is used mostly for spans of 30 to 180 m. Structural depth-to-span ratios are 0.045 for simple spans, and 0.04 for continuous spans. The high torsional resistance of the box girder makes it particularly suitable for curved alignment (Figure 10.4) such as those needed on freeway ramps.

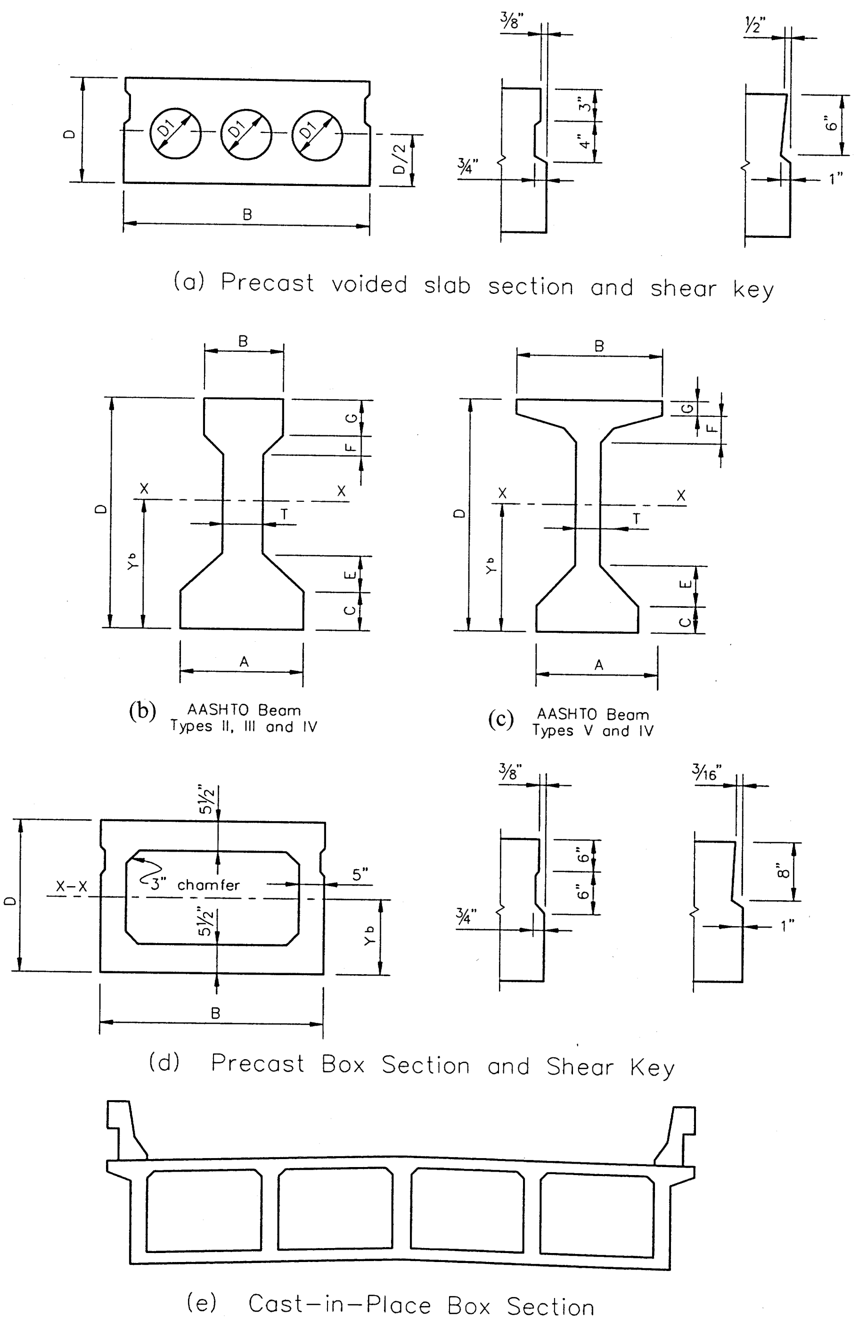


**FIGURE 10.3** Typical cross sections of prestressed concrete bridge superstructures.

## 10.3 Losses of Prestress

*Loss of prestress* refers to the reduced tensile stress in the tendons. Although this loss does affect the service performance (such as camber, deflections, and cracking), it has no effect on the ultimate strength of a flexural member unless the tendons are unbounded or the final stress is less than $0.5f_{pu}$ [5]. It should be noted, however, that an accurate estimate of prestress loss is more pertinent in some prestressed concrete members than in others. Prestress losses can be divided into two categories:

**TABLE 10.2** Precast Prestressed Voided Slabs Section Properties (Fig. 10.3a)

| Span Range, ft (m) | Section Dimensions: Width $B$ in. (mm) | Depth $D$ in. (mm) | $D1$ in. (mm) | $D2$ in. (mm) | Section Properties: $A$ in.$^2$ (mm$^2$ 10$^6$) | $I_x$ in.$^4$ (mm$^4$ 10$^9$) | $S_x$ in.$^3$ (mm$^3$ 10$^6$) |
|---|---|---|---|---|---|---|---|
| 25 | 48 | 12 | 0 | 0 | 576 | 6,912 | 1,152 |
| (7.6) | (1,219) | (305) | (0) | (0) | (0.372) | (2.877) | (18.878) |
| 30~35 | 48 | 15 | 8 | 8 | 569 | 12,897 | 1,720 |
| (10.1~10.70) | (1,219) | (381) | (203) | (203) | (0.362) | (5.368) | (28.185) |
| 40~45 | 48 | 18 | 10 | 10 | 628 | 21,855 | 2,428 |
| (12.2~13.7) | (1,219) | (457) | (254) | (254) | (0.405) | (10.097) | (310.788) |
| 50 | 48 | 21 | 12 | 10 | 703 | 34,517 | 3,287 |
| (15.2) | (1,219) | (533) | (305) | (254) | (0.454) | (1.437) | (53.864) |

**TABLE 10.3** Precast Prestressed I-Beam Section Properties (Figs. 10.3b and c)

| AASHTO Beam Type | Section Dimensions, in. (mm): Depth $D$ | Bottom Width $A$ | Web Width $T$ | Top Width $B$ | $C$ | $E$ | $F$ | $G$ |
|---|---|---|---|---|---|---|---|---|
| II | 36 (914) | 18 (457) | 6 (152) | 12 (305) | 6 (152) | 6 (152) | 3 (76) | 6 (152) |
| III | 45 (1143) | 22 (559) | 7 (178) | 16 (406) | 7 (178) | 7.5 (191) | 4.5 (114) | 7 (178) |
| IV | 54 (1372) | 26 (660) | 8 (203) | 20 (508) | 8 (203) | 9 (229) | 6 (152) | 8 (203) |
| V | 65 (1651) | 28 (711) | 8 (203) | 42 (1067) | 8 (203) | 10 (254) | 3 (76) | 5 (127) |
| VI | 72 (1829) | 28 (711) | 8 (203) | 42 (1067) | 8 (203) | 10 (254) | 3 (76) | 5 (127) |

| AASHTO Beam Type | Section Properties: $A$ in.$^2$ (mm$^2$ 10$^b$) | $Y_b$ in. (mm) | $I_x$ in.$^4$ (mm$^4$ 10$^9$) | $S_b$ in.$^3$ (mm$^4$ 10$^6$) | $S_t$ in.$^3$ (mm$^4$ 10$^6$) | Span Ranges, ft (m) |
|---|---|---|---|---|---|---|
| II | 369 | 15.83 | 50,980 | 3220 | 2528 | 40 ~ 45 |
| | (0.2381) | (402.1) | (21.22) | (52.77) | (41.43) | (12.2 ~ 13.7) |
| III | 560 | 20.27 | 125,390 | 6186 | 5070 | 50 ~ 65 |
| | (0.3613) | (514.9) | (52.19) | (101.38) | (83.08) | (15.2 ~ 110.8) |
| IV | 789 | 24.73 | 260,730 | 10543 | 8908 | 70 ~ 80 |
| | (0.5090) | (628.1) | (108.52) | (172.77) | (145.98) | (21.4 ~ 24.4) |
| V | 1013 | 31.96 | 521,180 | 16307 | 16791 | 90 ~ 100 |
| | (0.6535) | (811.8) | (216.93) | (267.22) | (275.16) | (27.4 ~ 30.5) |
| VI | 1085 | 36.38 | 733,340 | 20158 | 20588 | 110 ~ 120 |
| | (0.7000) | (924.1) | (305.24) | (330.33) | (337.38) | (33.5 ~ 36.6) |

- Instantaneous losses including losses due to anchorage set ($\Delta f_{pA}$), friction between tendons and surrounding materials ($\Delta f_{pF}$), and elastic shortening of concrete ($\Delta f_{pES}$) during the construction stage;
- Time-dependent losses including losses due to shrinkage ($\Delta f_{pSR}$), creep ($\Delta f_{pCR}$), and relaxation of the steel ($\Delta f_{pR}$) during the service life.

The total prestress loss ($\Delta f_{pT}$) is dependent on the prestressing methods.

*For pretensioned members*:

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR} \tag{10.12}$$

*For post-tensioned members*:

$$\Delta f_{pT} = \Delta f_{pA} + \Delta f_{pF} + \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR} \tag{10.13}$$

**FIGURE 10.4** Prestressed box–girder bridge (I-280/110 Interchange, CA).

**TABLE 10.4** Precast Prestressed Box Section Properties (Fig. 10.3d)

| | Section Dimensions | | Section Properties | | | | |
|---|---|---|---|---|---|---|---|
| Span ft (m) | Width $B$ in. (mm) | Depth $D$ in. (mm) | $A$ in.$^2$ (mm$^2$ $10^6$) | $Y_b$ in. (mm) | $I_x$ in$^4$ (mm$^4$ $10^9$) | $S_b$ in.$^3$ (mm$^3$ $10^6$) | $S_t$ in.$^3$ (mm$^3$ $10^6$) |
| 50 | 48 | 27 | 693 | 13.37 | 65,941 | 4,932 | 4,838 |
| (15.2) | (1,219) | (686) | (0.4471) | (3310.6) | (27.447) | (80.821) | (710.281) |
| 60 | 48 | 33 | 753 | 16.33 | 110,499 | 6,767 | 6,629 |
| (18.3) | (1,219) | (838) | (0.4858) | (414.8) | (45.993) | (110.891) | (108.630) |
| 70 | 48 | 39 | 813 | 110.29 | 168,367 | 8,728 | 8,524 |
| (21.4) | (1,219) | (991) | (0.5245) | (490.0) | (70.080) | (143.026) | (1310.683) |
| 80 | 48 | 42 | 843 | 20.78 | 203,088 | 9,773 | 9,571 |
| (24.4) | (1,219) | (1,067) | (0.5439) | (527.8) | (84.532) | (160.151) | (156.841) |

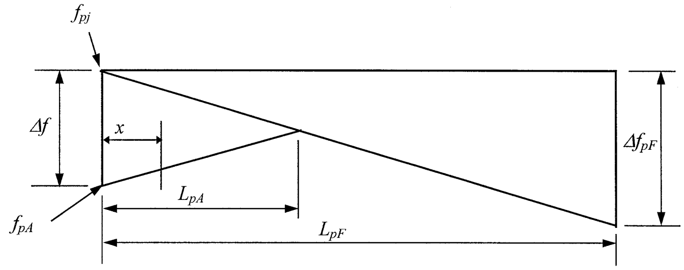


**FIGURE 10.5** Anchorage set loss model.

**TABLE 10.5** Friction Coefficients for Post-Tensioning Tendons

| Type of Tendons and Sheathing | Wobble Coefficient $K$ (1/mm) × ($10^{-6}$) | Curvature Coefficient μ (1/rad) |
|---|---|---|
| Tendons in rigid and semirigid galvanized ducts, seven-wire strands | 0.66 | 0.05 ~ 0.15 |
| Pregreased tendons, wires and seven-wire strands | 0.98 ~ 6.6 | 0.05 ~ 0.15 |
| Mastic-coated tendons, wires and seven-wire strands | 3.3 ~ 6.6 | 0.05 ~ 0.15 |
| Rigid steel pipe deviations | 66 | 0.25, lubrication required |

*Source*: AASHTO LRFD Bridge Design Specifications, 1st Ed., American Association of State Highway and Transportation Officials. Washington, D.C. 1994. With permission.

## 10.3.1 Instantaneous Losses

### 10.3.1.1 Anchorage Set Loss

As shown in Figure 10.5, assuming that the anchorage set loss changes linearly within the length ($L_{pA}$), the effect of anchorage set on the cable stress can be estimated by the following formula:

$$\Delta f_{pA} = \Delta f\left(1 - \frac{x}{L_{pA}}\right) \tag{10.14}$$

$$L_{pA} = \sqrt{\frac{E\,(\Delta L)\,L_{pF}}{\Delta f_{pF}}} \tag{10.15}$$

$$\Delta f = \frac{2\,\Delta f_{pF}\,L_{pA}}{L_{pF}} \tag{10.16}$$

where $\Delta L$ is the thickness of anchorage set; $E$ is the modulus of elasticity of anchorage set; $\Delta f$ is the change in stress due to anchor set; $L_{pA}$ is the length influenced by anchor set; $L_{pF}$ is the length to a point where loss is known; and $x$ is the horizontal distance from the jacking end to the point considered.

### 10.3.1.2 Friction Loss

For a post-tensioned member, friction losses are caused by the tendon profile *curvature effect* and the local deviation in tendon profile *wobble effects*. AASHTO-LRFD [4] specifies the following formula:

$$\Delta f_{pF} = f_{pj}\left(1 - e^{-(Kx + \mu\alpha)}\right) \tag{10.17}$$

where $K$ is the wobble friction coefficient and μ is the curvature friction coefficient (see Table 10.5); $x$ is the length of a prestressing tendon from the jacking end to the point considered; and α is the sum of the absolute values of angle change in the prestressing steel path from the jacking end.

### 10.3.1.3 Elastic Shortening Loss $\Delta f_{pES}$

The loss due to elastic shortening can be calculated using the following formula [4]:

$$\Delta f_{pES} = \begin{cases} \dfrac{E_p}{E_{ci}}\,f_{cgp} & \text{for pretensioned members} \\[2ex] \dfrac{N-1}{2N}\,\dfrac{E_p}{E_{ci}}\,f_{cgp} & \text{for post-tensioned members} \end{cases} \tag{10.18}$$

**TABLE 10.6** Lump Sum Estimation of Time-Dependent Prestress Losses

| Type of Beam Section | Level | For Wires and Strands with $f_{pu}$ = 1620, 1725, or 1860 MPa | For Bars with $f_{pu}$ = 1000 or 1100 MPa |
|---|---|---|---|
| Rectangular beams and solid slab | Upper bound | 200 + 28 PPR | 130 + 41 PPR |
| | Average | 180 + 28 PPR | |
| Box girder | Upper bound | 145 + 28 PPR | 100 |
| | Average | 130 + 28 PPR | |
| I-girder | Average | $230\left[1.0-0.15\frac{f_c'-41}{41}\right]+41\,\text{PPR}$ | 130 + 41 PPR |
| Single–T, double–T hollow core and voided slab | Upper bound | $230\left[1.0-0.15\frac{f_c'-41}{41}\right]+41\,\text{PPR}$ | $230\left[1.0-0.15\frac{f_c'-41}{41}\right]+41\,\text{PPR}$ |
| | Average | $230\left[1.0-0.15\frac{f_c'-41}{41}\right]+41\,\text{PPR}$ | |

*Note*:
1. PPR is partial prestress ratio = $(A_{ps}f_{py})/(A_{ps}f_{py} + A_s f_y)$.
2. For low-relaxation strands, the above values may be reduced by
   - 28 MPa for box girders
   - 41 MPa for rectangular beams, solid slab and I-girders, and
   - 55 MPa for single–T, double–T, hollow–core and voided slabs.

*Source*: AASHTO LRFD Bridge Design Specifications, 1st Ed., American Association of State Highway and Transportation Officials. Washington, D.C. 1994. With permission.

where $E_{ci}$ is modulus of elasticity of concrete at transfer (for pretensioned members) or after jacking (for post-tensioned members); $N$ is the number of identical prestressing tendons; and $f_{cgp}$ is sum of the concrete stress at the center of gravity of the prestressing tendons due to the prestressing force at transfer (for pretensioned members) or after jacking (for post-tensioned members) and the self-weight of members at the section with the maximum moment. For post-tensioned structures with bonded tendons, $f_{cgp}$ may be calculated at the center section of the span for simply supported structures, at the section with the maximum moment for continuous structures.

## 10.3.2 Time-Dependent Losses

### 10.3.2.1 Lump Sum Estimation

AASHTO-LRFD [4] provides the approximate lump sum estimation (Table 10.6) of time-dependent loses $\Delta f_{pTM}$ resulting from shrinkage and creep of concrete, and relaxation of the prestressing steel. While the use of lump sum losses is acceptable for "average exposure conditions," for unusual conditions, more-refined estimates are required.

### 10.3.2.2 Refined Estimation

a. *Shrinkage Loss*: Shrinkage loss can be determined by formulas [4]:

$$\Delta f_{pSR} = \begin{cases} 93 - 0.85H & \text{for pretensioned members} \\ 11 - 1.03H & \text{for post-tensioned members} \end{cases} \tag{10.19}$$

where $H$ is average annual ambient relative humidity (%).

b. *Creep Loss*: Creep loss can be predicted by [4]:

$$\Delta f_{pCR} = 12 f_{cgp} - 7\Delta f_{cdp} \geq 0 \tag{10.20}$$

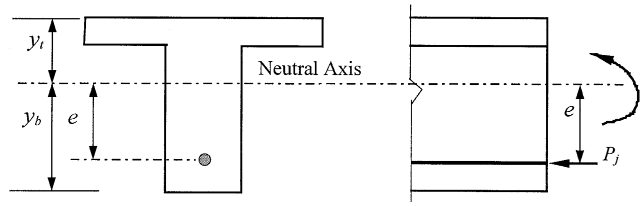


**FIGURE 10.6** Prestressed concrete member section at Service Limit State.

where $f_{cgp}$ is concrete stress at center of gravity of prestressing steel at transfer, and $\Delta f_{cdp}$ is concrete stress change at center of gravity of prestressing steel due to permanent loads, except the load acting at the time the prestressing force is applied.

c. *Relaxation Loss*: The total relaxation loss ($\Delta f_{pR}$) includes two parts: relaxation at time of transfer $\Delta f_{pR1}$ and after transfer $\Delta f_{pR2}$. For a pretensioned member initially stressed beyond 0.5 $f_{pu}$, AASHTO-LRFD [4] specifies

$$\Delta f_{pR1} = \begin{cases} \dfrac{\log 24t}{10}\left[\dfrac{f_{pi}}{f_{py}} - 0.55\right] f_{pi} & \text{for stress-relieved strand} \\ \dfrac{\log 24t}{40}\left[\dfrac{f_{pi}}{f_{py}} - 0.55\right] f_{pi} & \text{for low-relaxation strand} \end{cases} \tag{10.21}$$

For stress-relieved strands

$$\Delta f_{pR2} = \begin{cases} 138 - 0.4\Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR}) & \text{for pretensioning} \\ 138 - 0.3\Delta f_{pF} - 0.4\Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR}) & \text{for post-tensioning} \end{cases} \tag{10.22}$$

where $t$ is time estimated in days from testing to transfer. For low-relaxation strands, $\Delta f_{pR2}$ is 30% of those values obtained from Eq. (10.22).

## 10.4 Design Considerations

### 10.4.1 Basic Theory

Compared with reinforced concrete, the main distinguishing characteristics of prestressed concrete are that

- The stresses for concrete and prestressing steel and deformation of structures at each stage, i.e., during prestressing, handling, transportation, erection, and the service life, as well as stress concentrations, need to be investigated on the basis of elastic theory.
- The prestressing force is determined by concrete stress limits under service load.
- Flexure and shear capacities are determined based on the ultimate strength theory.

For the prestressed concrete member section shown in Figure 10.6, the stress at various load stages can be expressed by the following formula:

$$f = \frac{P_j}{A} \pm \frac{P_j e y}{I} \pm \frac{My}{I} \tag{10.23}$$

**TABLE 10.7** Stress Limits for Prestressing Tendons

| Stress Type | Prestressing Method | Prestressing Tendon Type: Stress Relieved Strand and Plain High-Strength Bars | Low Relaxation Strand | Deformed High-Strength Bars |
|---|---|---|---|---|
| At jacking, $f_{pj}$ | Pretensioning | $0.72f_{pu}$ | $0.78f_{pu}$ | — |
| | Post-tensioning | $0.76f_{pu}$ | $0.80f_{pu}$ | $0.75f_{pu}$ |
| After transfer, $f_{pt}$ | Pretensioning | $0.70f_{pu}$ | $0.74f_{pu}$ | — |
| | Post-tensioning — at anchorages and couplers immediately after anchor set | $0.70f_{pu}$ | $0.70f_{pu}$ | $0.66f_{pu}$ |
| | Post-tensioning — general | $0.70f_{pu}$ | $0.74f_{pu}$ | $0.66f_{pu}$ |
| At Service Limit State, $f_{pc}$ | After all losses | $0.80f_{py}$ | $0.80f_{py}$ | $0.80f_{py}$ |

*Source:* AASHTO LRFD Bridge Design Specifications, 1st Ed., American Association of State Highway and Transportation Officials. Washington, D.C. 1994. With permission.

**TABLE 10.8** Temporary Concrete Stress Limits at Jacking State before Losses due to Creep and Shrinkage — Fully Prestressed Components

| Stress Type | Area and Condition | Stress (MPa) |
|---|---|---|
| Compressive | Pretensioned | $0.60\, f'_{ci}$ |
| | Post-tensioned | $0.55\, f'_{ci}$ |
| Tensile | Precompressed tensile zone without bonded reinforcement | N/A |
| | Area other than the precompressed tensile zones and without bonded auxiliary reinforcement | $0.25\sqrt{f'_{ci}} \le 1.38$ |
| | Area with bonded reinforcement which is sufficient to resist 120% of the tension force in the cracked concrete computed on the basis of uncracked section | $0.58\sqrt{f'_{ci}}$ |
| | Handling stresses in prestressed piles | $0.415\sqrt{f'_{ci}}$ |

*Note:* Tensile stress limits are for nonsegmental bridges only.
*Source:* AASHTO LRFD Bridge Design Specifications, 1st Ed., American Association of State Highway and Transportation Officials. Washington, D.C. 1994. With permission.

where $P_j$ is the prestress force; $A$ is the cross-sectional area; $I$ is the moment of inertia; $e$ is the distance from the center of gravity to the centroid of the prestressing cable; $y$ is the distance from the centroidal axis; and $M$ is the externally applied moment.

*Section properties* are dependent on the prestressing method and the load stage. In the analysis, the following guidelines may be useful:

- Before bounding of the tendons, for a post-tensioned member, the net section should be used theoretically, but the gross section properties can be used with a negligible tolerance.
- After bounding of tendons, the transformed section should be used, but gross section properties may be used approximately.
- At the service load stage, transformed section properties should be used.

## 10.4.2 Stress Limits

The stress limits are the basic requirements for designing a prestressed concrete member. The purpose for stress limits on the prestressing tendons is to mitigate tendon fracture, to avoid inelastic tendon deformation, and to allow for prestress losses. Tables 10.7 lists the AASHTO-LRFD [4] stress limits for prestressing tendons.

**TABLE 10.9** Concrete Stress Limits at Service Limit State after All Losses — Fully Prestressed Components

| Stress Type | Area and Condition | | Stress (MPa) |
|---|---|---|---|
| Compressive | Nonsegmental bridge at service stage | | $0.45\,f_c'$ |
| | Nonsegmental bridge during shipping and handling | | $0.60\,f_c'$ |
| | Segmental bridge during shipping and handling | | $0.45\,f_c'$ |
| Tensile | Precompressed tensile zone assuming uncracked section | With bonded prestressing tendons other than piles | $0.50\sqrt{f_c'}$ |
| | | Subjected to severe corrosive conditions | $0.25\sqrt{f_c'}$ |
| | | With unbonded prestressing tendon | No tension |

*Note:* Tensile stress limits are for nonsegmental bridges only.
*Source*: AASHTO LRFD Bridge Design Specifications, 1st Ed., American Association of State Highway and Transportation Officials. Washington, D.C. 1994. With permission.

The purpose for stress limits on the concrete is to ensure no overstressing at jacking and after transfer stages and to avoid cracking (fully prestressed) or to control cracking (partially prestressed) at the service load stage. Tables 10.8 and 10.9 list the AASHTO-LRFD [4] stress limits for concrete.

A prestressed member that does not allow cracking at service loads is called a fully prestressed member, whereas one that does is called a partially prestressed member. Compared with full prestress, partial prestress can minimize camber, especially when the dead load is relatively small, as well as provide savings in prestressing steel, in the work required to tension, and in the size of end anchorages and utilizing cheaper mild steel. On the other hand, engineers must be aware that partial prestress may cause earlier cracks and greater deflection under overloads and higher principal tensile stresses under service loads. Nonprestressed reinforcement is often needed to provide higher flexural strength and to control cracking in a partially prestressed member.

## 10.4.3 Cable Layout

A cable is a group of prestressing tendons and the center of gravity of all prestressing reinforcement. It is a general design principle that the maximum eccentricity of prestressing tendons should occur at locations of maximum moments. Although straight tendons (Figure 10.7a) and harped multi-straight tendons (Figure 10.7b and c) are common in the precast members, curved tendons are more popular for cast-in-place post-tensioned members. Typical cable layouts for bridge superstructures are shown in Figure 10.7.

To ensure that the tensile stress in extreme concrete fibers under service does not exceed code stress limits [4, 14], cable layout envelopes are delimited. Figure 10.8 shows limiting envelopes for simply supported members. From Eq. (10.23), the stress at extreme fiber can be obtained

$$f = \frac{P_j}{A} \pm \frac{P_j e C}{I} \pm \frac{MC}{I} \tag{10.24}$$

where $C$ is the distance of the top or bottom extreme fibers from the center gravity of the section ($y_b$ or $y_t$ as shown in Figure 10.6).

When no tensile stress is allowed, the limiting eccentricity envelope can be solved from Eq. (10.24) with

$$e_{limit} = \frac{I}{AC} \pm \frac{M}{IP_j} \tag{10.25}$$

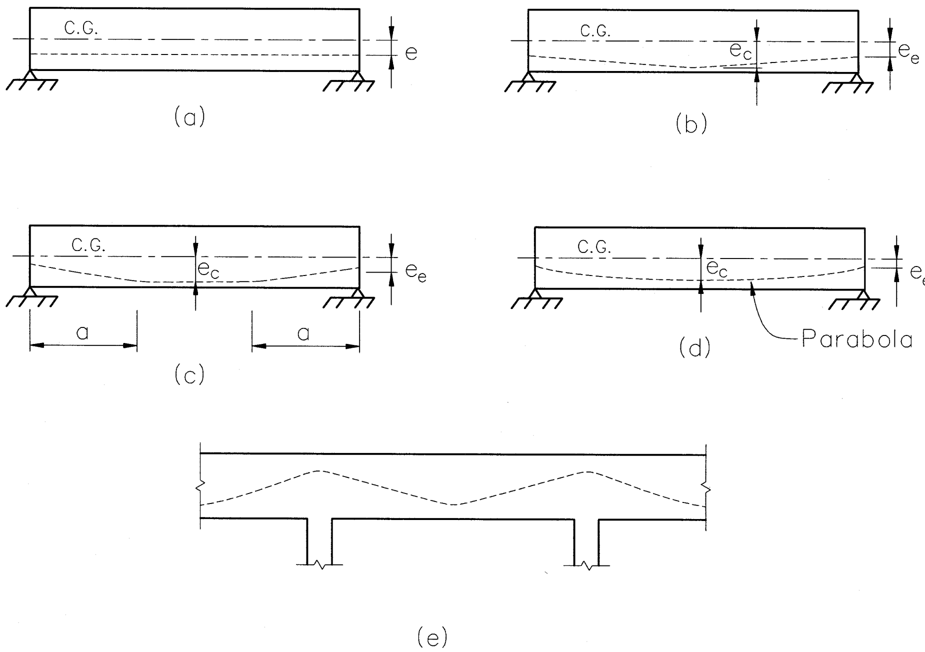


**FIGURE 10.7** Cable layout for bridge superstructures.

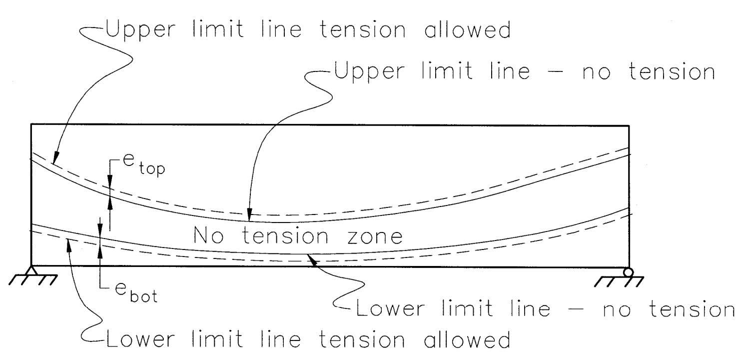


**FIGURE 10.8** Cable layout envelopes.

For limited tension stress $f_t$, additional eccentricities can be obtained:

$$e' = \frac{f_t I}{P_j C} \tag{10.26}$$

### 10.4.4 Secondary Moments

The primary moment ($M_1 = P_j e$) is defined as the moment in the concrete section caused by the eccentricity of the prestress for a statically determinate member. The secondary moment $M_s$ (Figure 10.9d) is defined as moment induced by prestress and structural continuity in an indeterminate member. Secondary moments can be obtained by various methods. The resulting moment is simply the sum of the primary and secondary moments.

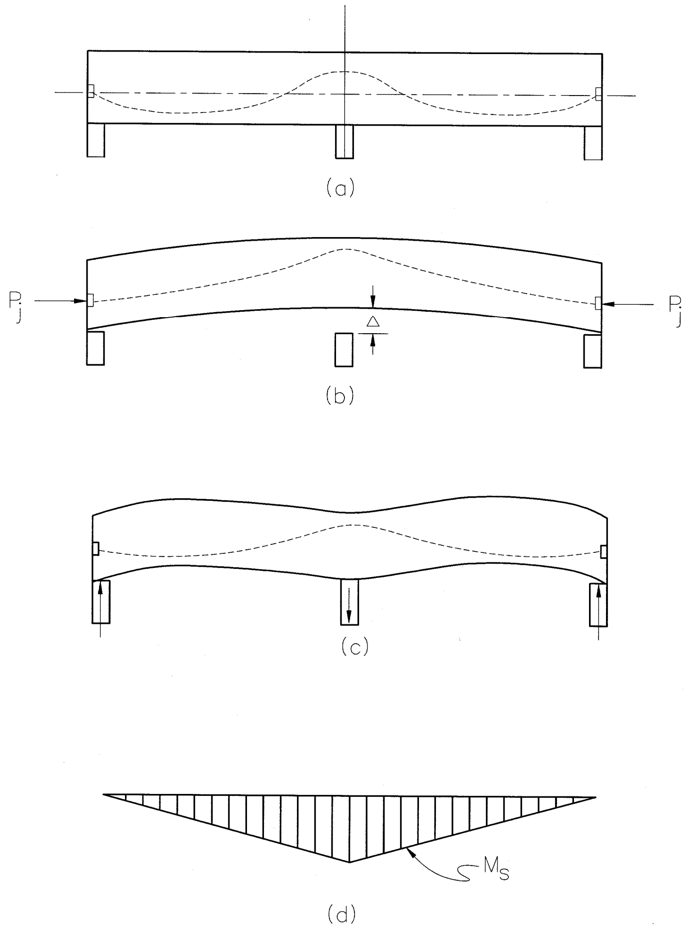


**FIGURE 10.9** Secondary moments.

### 10.4.5 Flexural Strength

Flexural strength is based on the following assumptions [4]:

- For members with bonded tendons, strain is linearly distributed across a section; for members with unbonded tendons, the total change in tendon length is equal to the total change in member length over the distance between two anchorage points.
- The maximum usable strain at extreme compressive fiber is 0.003.
- The tensile strength of concrete is neglected.
- A concrete stress of 0.85 $f_c'$ is uniformly distributed over an equivalent compression zone.
- Nonprestressed reinforcement reaches the yield strength, and the corresponding stresses in the prestressing tendons are compatible based on plane section assumptions.

For a member with a flanged section (Figure 10.10) subjected to uniaxial bending, the equations of equilibrium are used to give a nominal moment resistance of

$$M_n = A_{ps} f_{ps} \left( d_p - \frac{a}{2} \right) + A_s f_y \left( d_s - \frac{a}{2} \right) - A_s' f_y' \left( d_s' - \frac{a}{2} \right) + 0.85 f_c' (b - b_w) \beta_1 h_f \left( \frac{a}{2} - \frac{h_f}{2} \right) \tag{10.27}$$

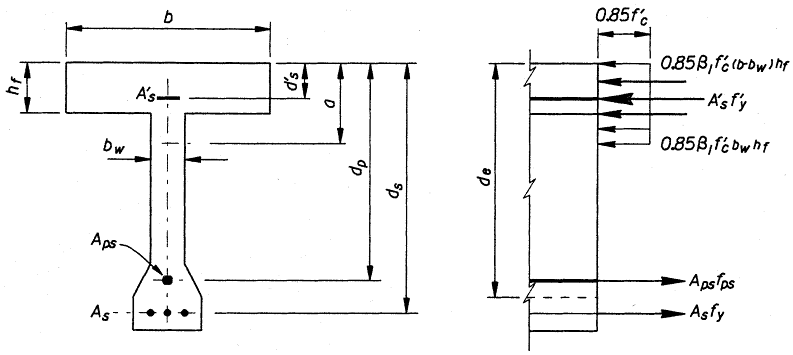


**FIGURE 10.10** A flanged section at nominal moment capacity state.

$$a = \beta_1 c \tag{10.28}$$

*For bonded tendons*:

$$c = \frac{A_{ps} f_{pu} + A_s f_y - A'_s f'_y - 0.85\beta_1 f'_c (b - b_w) h_f}{0.85\beta_1 f'_c b_w + k A_{ps} \dfrac{f_{pu}}{d_p}} \geq h_f \tag{10.29}$$

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right) \tag{10.30}$$

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right) \tag{10.31}$$

$$0.85 \geq \beta_1 = 0.85 - \frac{(f'_c - 28)(0.05)}{7} \geq 0.65 \tag{10.32}$$

where $A$ represents area; $f$ is stress; $b$ is the width of the compression face of member; $b_w$ is the web width of a section; $h_f$ is the compression flange depth of the cross section; $d_p$ and $d_s$ are distances from extreme compression fiber to the centroid of prestressing tendons and to centroid of tension reinforcement, respectively; subscripts $c$ and $y$ indicate specified strength for concrete and steel, respectively; subscripts $p$ and $s$ mean prestressing steel and reinforcement steel, respectively; subscripts $ps$, $py$, and $pu$ correspond to states of nominal moment capacity, yield, and specified tensile strength of prestressing steel, respectively; superscript $'$ represents compression. The above equations also can be used for rectangular section in which $b_w = b$ is taken.

*For unbound tendons*:

$$c = \frac{A_{ps} f_{pu} + A_s f_y - A'_s f'_y - 0.85\beta_1 f'_c (b - b_w) h_f}{0.85\beta_1 f'_c b_w} \geq h_f \tag{10.33}$$

$$f_{ps} = f_{pe} + \Omega_u E_p \varepsilon_{cu}\left(\frac{d_p}{c} - 1.0\right)\frac{L_1}{L_2} \leq 0.94 f_{py} \tag{10.34}$$

where $L_1$ is length of loaded span or spans affected by the same tendons; $L_2$ is total length of tendon between anchorage; $\Omega_u$ is the bond reduction coefficient given by

$$\Omega_u = \begin{cases} \dfrac{3}{L/d_p} & \text{for uniform and near third point loading} \\ \dfrac{1.5}{L/d_p} & \text{for near midspan loading} \end{cases} \tag{10.35}$$

in which $L$ is span length.

*Maximum reinforcement limit*:

$$\frac{c}{d_e} \leq 0.42 \tag{10.36}$$

$$d_e = \frac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y} \tag{10.37}$$

*Minimum reinforcement limit*:

$$\phi M_n \geq 1.2 \ M_{cr} \tag{10.38}$$

in which $\phi$ is flexural resistance factor 1.0 for prestressed concrete and 0.9 for reinforced concrete; $M_{cr}$ is the cracking moment strength given by the elastic stress distribution and the modulus of rupture of concrete.

$$M_{cr} = \frac{I}{y_t}\left(f_r + f_{pe} - f_d\right) \tag{10.39}$$

where $f_{pe}$ is compressive stress in concrete due to effective prestresses; and $f_d$ is stress due to unfactored self-weight; both $f_{pe}$ and $f_d$ are stresses at extreme fiber where tensile stresses are produced by externally applied loads.

### 10.4.6 Shear Strength

The shear resistance is contributed by the concrete, the transverse reinforcement and vertical component of prestressing force. The modified compression field theory-based shear design strength [3] was adopted by the AASHTO-LRFD [4] and has the formula:

$$V_n = \text{the lesser of} \begin{cases} V_c + V_s + V_p \\ 0.25 f_c' b_v d_v + V_p \end{cases} \tag{10.40}$$

where

$$V_c = 0.083 \beta \sqrt{f_c'}\, b_v d_v \tag{10.41}$$

$$V_s = \frac{A_v f_y d_v (\cos\theta + \cot\alpha)\sin\alpha}{s} \tag{10.42}$$

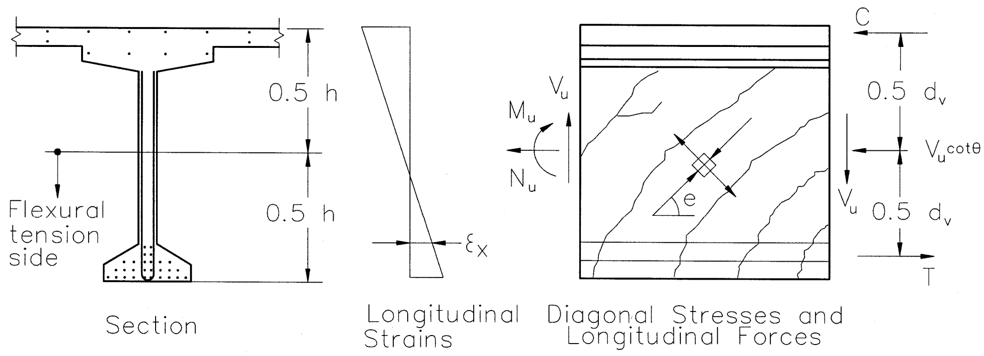


**FIGURE 10.11** Illustration of $A_c$ for shear strength calculation. (*Source:* AASHTO LRFD Bridge Design Specifications, 1st Ed., American Association of State Highway and Transportation Officials. Washington, D.C. 1994. With permission.)

**TABLE 10.10** Values of θ and β for Sections with Transverse Reinforcement

| $\frac{v}{f'_c}$ | Angle (degree) | $\varepsilon_x \times 1000$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | –.02 | –0.15 | –0.1 | 0 | 0.125 | 0.25 | 0.50 | 0.75 | 1.00 | 1.50 | 2.00 |
| ≤ 0.05 | θ | 27.0 | 27.0 | 27.0 | 27.0 | 27.0 | 28.5 | 29.0 | 33.0 | 36.0 | 41.0 | 43.0 |
| | β | 6.78 | 6.17 | 5.63 | 4.88 | 3.99 | 3.49 | 2.51 | 2.37 | 2.23 | 1.95 | 1.72 |
| 0.075 | θ | 27.0 | 27.0 | 27.0 | 27.0 | 27.0 | 27.5 | 30.0 | 33.5 | 36.0 | 40.0 | 42.0 |
| | β | 6.78 | 6.17 | 5.63 | 4.88 | 3.65 | 3.01 | 2.47 | 2.33 | 2.16 | 1.90 | 1.65 |
| 0.100 | θ | 23.5 | 23.5 | 23.5 | 23.5 | 24.0 | 26.5 | 30.5 | 34.0 | 36.0 | 38.0 | 39.0 |
| | β | 6.50 | 5.87 | 5.31 | 3.26 | 2.61 | 2.54 | 2.41 | 2.28 | 2.09 | 1.72 | 1.45 |
| 0.127 | θ | 20.0 | 21.0 | 22.0 | 23.5 | 26.0 | 28.0 | 31.5 | 34.0 | 36.0 | 37.0 | 38.0 |
| | β | 2.71 | 2.71 | 2.71 | 2.60 | 2.57 | 2.50 | 2.37 | 2.18 | 2.01 | 1.60 | 1.35 |
| 0.150 | θ | 22.0 | 22.5 | 23.5 | 25.0 | 27.0 | 29.0 | 32.0 | 34.0 | 36.0 | 36.5 | 37.0 |
| | β | 2.66 | 2.61 | 2.61 | 2.55 | 2.50 | 2.45 | 2.28 | 2.06 | 1.93 | 1.50 | 1.24 |
| 0.175 | θ | 23.5 | 24.0 | 25.0 | 26.5 | 28.0 | 30.0 | 32.5 | 34.0 | 35.0 | 35.5 | 36.0 |
| | β | 2.59 | 2.58 | 2.54 | 2.50 | 2.41 | 2.39 | 2.20 | 1.95 | 1.74 | 1.35 | 1.11 |
| 0.200 | θ | 25.0 | 25.5 | 26.5 | 27.5 | 29.0 | 31.0 | 33.0 | 34.0 | 34.5 | 35.0 | 36.0 |
| | β | 2.55 | 2.49 | 2.48 | 2.45 | 2.37 | 2.33 | 2.10 | 1.82 | 1.58 | 1.21 | 1.00 |
| 0.225 | θ | 26.5 | 27.0 | 27.5 | 29.0 | 30.5 | 32.0 | 33.0 | 34.0 | 34.5 | 36.5 | 39.0 |
| | β | 2.45 | 2.38 | 2.43 | 2.37 | 2.33 | 2.27 | 1.92 | 1.67 | 1.43 | 1.18 | 1.14 |
| 0.250 | θ | 28.0 | 28.5 | 29.0 | 30.0 | 31.0 | 32.0 | 33.0 | 34.0 | 35.5 | 38.5 | 41.5 |
| | β | 2.36 | 2.32 | 2.36 | 2.30 | 2.28 | 2.01 | 1.64 | 1.52 | 1.40 | 1.30 | 1.25 |

(*Source:* AASHTO LRFD Bridge Design Specifications, 1st Ed., American Association of State Highway and Transportation Officials. Washington, D.C. 1994. With permission.)

where $b_v$ is the effective web width determined by subtracting the diameters of ungrouted ducts or one half the diameters of grouted ducts; $d_v$ is the effective depth between the resultants of the tensile and compressive forces due to flexure, but not to be taken less than the greater of $0.9d_e$ or $0.72h$; $A_v$ is the area of transverse reinforcement within distance $s$; $s$ is the spacing of stirrups; α is the angle of inclination of transverse reinforcement to longitudinal axis; β is a factor indicating ability of diagonally cracked concrete to transmit tension; θ is the angle of inclination of diagonal compressive stresses (Figure 10.11). The values of β and θ for sections with transverse reinforcement are given in Table 10.10. In using this table, the shear stress $v$ and strain $\varepsilon_x$ in the reinforcement on the flexural tension side of the member are determined by

$$v = \frac{V_u - \phi V_p}{\phi b_v d_v} \tag{10.43}$$

$$\varepsilon_x = \frac{\dfrac{M_u}{d_v} + 0.5N_u + 0.5V_u \cot\theta - A_{ps} f_{po}}{E_s A_s + E_p A_{ps}} \leq 0.002 \qquad (10.44)$$

where $M_u$ and $N_u$ are factored moment and axial force (taken as positive if compressive) associated with $V_u$ and $f_{po}$ is stress in prestressing steel when the stress in the surrounding concrete is zero and can be conservatively taken as the effective stress after losses $f_{pe}$. When the value of $\varepsilon_x$ calculated from the above equation is negative, its absolute value shall be reduced by multiplying by the factor $F_\varepsilon$, taken as

$$F_\varepsilon = \frac{E_s A_s + E_p A_{ps}}{E_c A_c + E_s A_s + E_p A_{ps}} \qquad (10.45)$$

where $E_s$, $E_p$, and $E_c$ are modulus of elasticity for reinforcement, prestressing steel, and concrete, respectively; $A_c$ is area of concrete on the flexural tension side of the member as shown in Figure 10.11.

*Minimum transverse reinforcement*:

$$A_{v\min} = 0.083\sqrt{f'_c}\,\frac{b_v s}{f_y} \qquad (10.46)$$

*Maximum spacing of transverse reinforcement*:

$$\text{For } V_u < 0.1 f'_c b_v d_v \qquad s_{\max} = \text{the smaller of} \begin{cases} 0.8 d_v \\ 600 \text{ mm} \end{cases} \qquad (10.47)$$

$$\text{For } V_u \geq 0.1 f'_c b_v d_v \qquad s_{\max} = \text{the smaller of} \begin{cases} 0.4 d_v \\ 300 \text{ mm} \end{cases} \qquad (10.48)$$

### 10.4.7 Camber and Deflections

As opposed to load deflection, camber is usually referred to as reversed deflection and is caused by prestressing. A careful evaluation of camber and deflection for a prestressed concrete member is necessary to meet serviceability requirements. The following formulas developed by the moment–area method can be used to estimate midspan immediate camber for simply supported members as shown in Figure 10.7.

*For straight tendon (Figure 10.7a)*:

$$\Delta = \frac{L^2}{8E_c I} M_e \qquad (10.49)$$

*For one-point harping tendon (Figure 10.7b)*:

$$\Delta = \frac{L^2}{8E_c I}\left(M_c + \frac{2}{3} M_e\right) \qquad (10.50)$$

*For two-point harping tendon* (*Figure 10.7c*):

$$\Delta = \frac{L^2}{8E_cI}\left(M_c + M_e - \frac{M_e}{3}\left(\frac{2a}{L}\right)^2\right) \tag{10.51}$$

*For parabola tendon* (*Figure 10.7d*):

$$\Delta = \frac{L^2}{8E_cI}\left(M_e + \frac{5}{6}M_c\right) \tag{10.52}$$

where $M_e$ is the primary moment at end, $P_j e_{\text{end}}$, and $M_c$ is the primary moment at midspan $P_j e_c$. Uncracked gross section properties are often used in calculating camber. For deflection at service loads, cracked section properties, i.e., moment of inertia $I_{cr}$, should be used at the post-cracking service load stage. It should be noted that long term effect of creep and shrinkage shall be considered in the final camber calculations. In general, final camber may be assumed 3 times as great as immediate camber.

### 10.4.8 Anchorage Zones

In a pretensioned member, prestressing tendons transfer the compression load to the surrounding concrete over a length $L_t$ gradually. In a post-tensioned member, prestressing tendons transfer the compression directly to the end of the member through bearing plates and anchors. The anchorage zone, based on the principle of St. Venant, is geometrically defined as the volume of concrete through which the prestressing force at the anchorage device spreads transversely to a more linear stress distribution across the entire cross section at some distance from the anchorage device [4].

For design purposes, the anchorage zone can be divided into general and local zones [4]. The region of tensile stresses is the general zone. The region of high compressive stresses (immediately ahead of the anchorage device) is the local zone. For the design of the general zone, a "strut-and-tie model," a refined elastic stress analysis or approximate methods may be used to determine the stresses, while the resistance to bursting forces is provided by reinforcing spirals, closed hoops, or anchoraged transverse ties. For the design of the local zone, bearing pressure is a major concern. For detailed requirements, see AASHTO-LRFD [4].

## 10.5 Design Example

### Two-Span Continuous Cast-in-Place Box-Girder Bridge

Given

A two-span continuous cast-in-place prestressed concrete box-girder bridge has two equal spans of length 48 m with a single-column bent. The superstructure is 10.4 m wide. The elevation view of the bridge is shown in Figure 10.12a.

*Material*:

Initial concrete: $f'_{ci} = 24$ MPa, $E_{ci} = 24{,}768$ MPa
Final concrete: $f'_c = 28$ MPa, $E_c = 26{,}752$ MPa
Prestressing steel: $f_{pu} = 1860$ MPa low relaxation strand, $E_p = 197{,}000$ MPa
Mild steel: $f_y = 400$ MPa, $E_s = 200{,}000$ MPa

*Prestressing*:

Anchorage set thickness = 10 mm
Prestressing stress at jacking $f_{pj} = 0.8\ f_{pu} = 1488$ MPa
The secondary moments due to prestressing at the bent are $M_{DA} = 1.118\ P_j$, $M_{DG} = 1.107\ P_j$

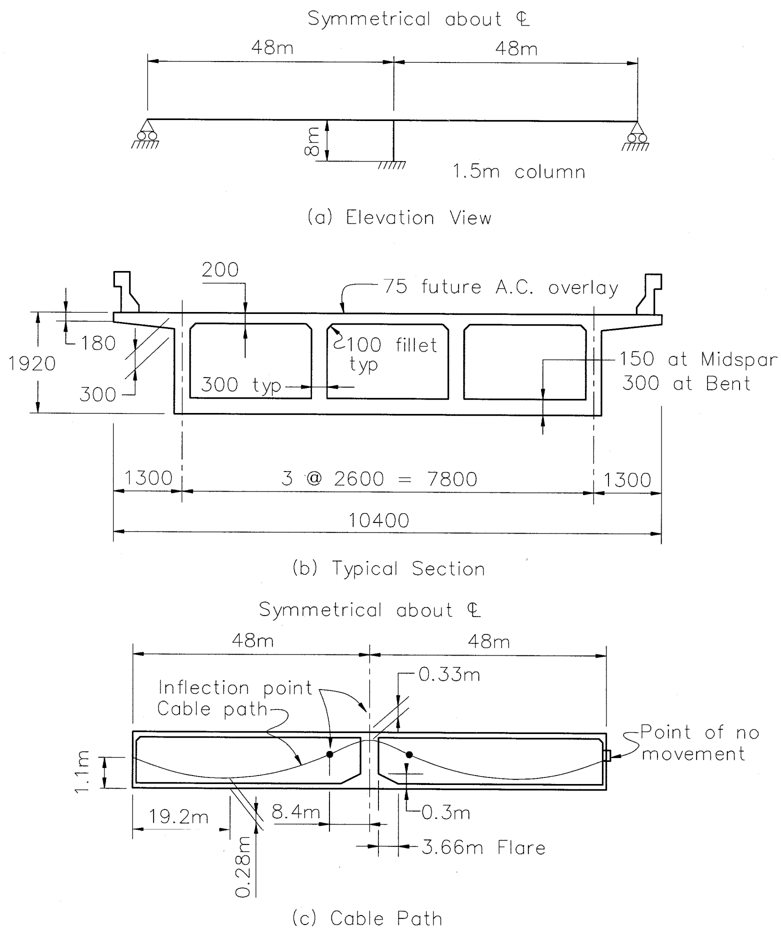


**FIGURE 10.12** A two–span continuous prestressed concrete box–girder bridge.

*Loads*:

Dead Load = self-weight + barrier rail + future wearing 75 mm AC overlay
Live Load = AASHTO HL-93 Live Load + dynamic load allowance

*Specification*:

AASHTO-LRFD [4] (referred as AASHTO in this example)

Requirements

1. Determine cross section geometry
2. Determine longitudinal section and cable path
3. Calculate loads
4. Calculate live load distribution factors for interior girder
5. Calculate unfactored moment and shear demands for interior girder
6. Determine load factors for Strength Limit State I and Service Limit State I
7. Calculate section properties for interior girder
8. Calculate prestress losses
9. Determine prestressing force $P_j$ for interior girder

10. Check concrete strength for interior girder — Service Limit State I
11. Flexural strength design for interior girder — Strength Limit State I
12. Shear strength design for interior girder — Strength Limit State I

Solution

1. **Determine Cross Section Geometry**
   a. *Structural depth — d*:
      For prestressed continuous spans, the structural depth $d$ can be determined using a depth-to-span ratio ($d/L$) of 0.04 (AASHTO LRFD Table 2.5.2.6.3-1).

$$d = 0.04L = 0.04(48) = 1.92 \text{ m}$$

   b. *Girder spacing — S*:
      The spacing of girders is generally taken no more than twice their depth.

$$S_{max} < 2\,d = 2\,(1.92) = 3.84 \text{ m}$$

      By using an overhang of 1.2 m, the center-to-center distance between two exterior girders is 10.4 m – (2)(1.2 m) = 8 m.

      Try three girders and two bays, $S = 8$ m/2 = 4 m > 3.84 m NG

      Try four girders and three bays, $S = 8$ m/3 = 2.67 m < 3.84 m OK

      Use a girder spacing $S = 2.6$ m

   c. *Typical section*:
      From past experience and design practice, we select that a thickness of 180 mm at the edge and 300 mm at the face of exterior girder for the overhang. The web thickness is chosen to be 300 mm at normal section and 450 mm at the anchorage end. The length of the flare is usually taken as 1⁄10 of the span length, say 4.8 m. The deck and soffit thickness depends on the clear distance between adjacent girders; 200 and 150 mm are chosen for the deck and soffit thickness, respectively. The selected box-girder section configurations for this example are shown in Figure 10.12b. The section properties of the box girder are as follows:

| Properties | Midspan | Bent (face of support) |
|---|---|---|
| $A$ (m$^2$) | 5.301 | 6.336 |
| $I$ (m$^4$) | 2.844 | 3.513 |
| $y_b$ (m) | 1.102 | 0.959 |

2. **Determine Longitudinal Section and Cable Path**
   To lower the center of gravity of the superstructure at the face of the bent cap in the CIP post-tensioned box girder, the thickness of soffit is flared to 300 mm as shown in Figure 10.12c. A cable path is generally controlled by the maximum dead-load moment and the position of the jack at the end section. Maximum eccentricities should occur at points of maximum dead load moments and almost no eccentricity should be present at the jacked end section. For this example, the maximum dead-load moments occur at three locations: at the bent cap, at the locations close to 0.4$L$ for Span 1 and 0.6$L$ for Span 2. A parabolic cable path is chosen as shown in Figure 10.12c.

3. **Calculate Loads**
   a. *Component dead load — DC*:
      The component dead load $DC$ includes all structural dead loads with the exception of the future wearing surface and specified utility loads. For design purposes, two parts of the $DC$ are defined as:

$DC1$ — girder self-weight (density 2400 kg/m$^3$) acting at the prestressing stage
$DC2$ — barrier rail weight (11.5 kN/m) acting at service stage after all losses.

b. *Wearing surface load — DW*:
The future wearing surface of 75 mm with a density 2250 kg/m$^3$

$$\begin{aligned} DW &= \text{(deck width – barrier width) (thickness of wearing surface) (density)} \\ &= [10.4 \text{ m} - 2(0.54 \text{ m})](0.075 \text{ m})(2250 \text{ kg/m}^3)(9.8066 \text{ m/s}^2) = 15{,}423 \text{ N/m} \\ &= 15.423 \text{ kN/m} \end{aligned}$$

c. *Live-Load LL and Dynamic Load Allowance — IM*:
The design live load $LL$ is the AASHTO HL-93 vehicular live loading. To consider the wheel-load impact from moving vehicles, the dynamic load allowance $IM$ = 33% [AASHTO LRFD Table 3.6.2.1-1] is applied to the design truck.

4. Calculate Live Load Distribution Factors
AASHTO [1994] recommends that approximate methods be used to distribute live load to individual girders (AASHTO-LRFD 4.6.2.2.2). The dimensions relevant to this prestressed box girder are: depth $d$ = 1920 mm, number of cells $N_c$ = 3, spacing of girders $S$ = 2600 mm, span length $L$ = 48,000 mm, half of the girder spacing plus the total overhang $W_e$ = 2600 mm, and the distance between the center of an exterior girder and the interior edge of a barrier $d_e$ = 1300 – 535 = 765 m. This box girder is within the range of applicability of the AASHTO approximate formulas. The live-load distribution factors are calculated as follows:

a. *Live-load distribution factor for bending moments*:
i. Interior girder (AASHTO Table 4.6.2.2.2b-1):
• One design lane loaded:

$$\begin{aligned} g_M &= \left(1.75 + \frac{S}{1100}\right)\left(\frac{300}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45} \\ &= \left(1.75 + \frac{2600}{1100}\right)\left(\frac{300}{48{,}000}\right)^{0.35}\left(\frac{1}{3}\right)^{0.45} = 0.425 \text{ lanes} \end{aligned}$$

• Two or more design lanes loaded:

$$\begin{aligned} g_M &= \left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{430}\right)\left(\frac{1}{L}\right)^{0.25} \\ &= \left(\frac{13}{3}\right)^{0.3}\left(\frac{2600}{430}\right)\left(\frac{1}{48{,}000}\right)^{0.25} = 0.634 \text{ lanes} \qquad \text{(controls)} \end{aligned}$$

ii. Exterior girder (AASHTO Table 4.6.2.2.2d-1):

$$g_M = \frac{W_e}{4300} = \frac{2600}{4300} = 0.605 \text{ lanes}$$

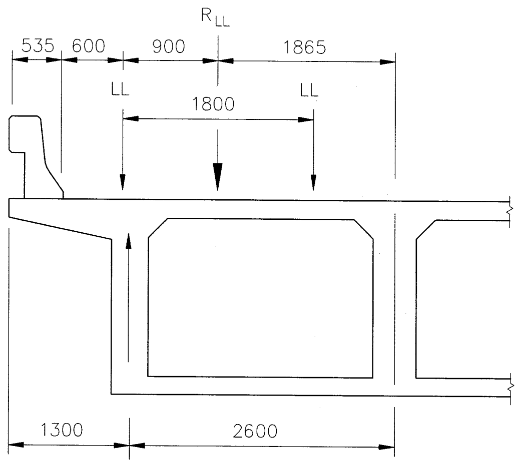


**FIGURE 10.13** Live–load distribution for exterior girder — lever rule.

b. *Live-load distribution factor for shear*:
   i. Interior girder (AASHTO Table 4.62.2.3a-1):
      • One design lane loaded:

$$g_V = \left(\frac{S}{2900}\right)^{0.6}\left(\frac{d}{L}\right)^{0.1}$$

$$= \left(\frac{2600}{2900}\right)^{0.6}\left(\frac{1920}{48{,}000}\right)^{0.1} = 0.679 \ \text{lanes}$$

      • Two or more design lanes loaded:

$$g_V = \left(\frac{S}{2200}\right)^{0.9}\left(\frac{d}{L}\right)^{0.1}$$

$$= \left(\frac{2600}{2200}\right)^{0.9}\left(\frac{1920}{48{,}000}\right)^{0.1} = 0.842 \ \text{lanes} \qquad \text{(controls)}$$

   ii. Exterior girder (AASHTO Table 4.62.2.3b-1):
      • One design lane loaded — Lever rule:
      The lever rule assumes that the deck in its transverse direction is simply supported by the girders and uses statics to determine the live-load distribution to the girders. AASHTO-LRFD [4] also requires that when the lever rule is used, the multiple presence factor $m$ should apply. For a one design lane loaded, $m = 1.2$. The lever rule model for the exterior girder is shown in Figure 10.13. From static equilibrium:

$$R = \frac{965 + 900}{2600} = 0.717$$

$$g_v = mR = 1.2(0.717) = 0.861 \qquad \text{(controls)}$$

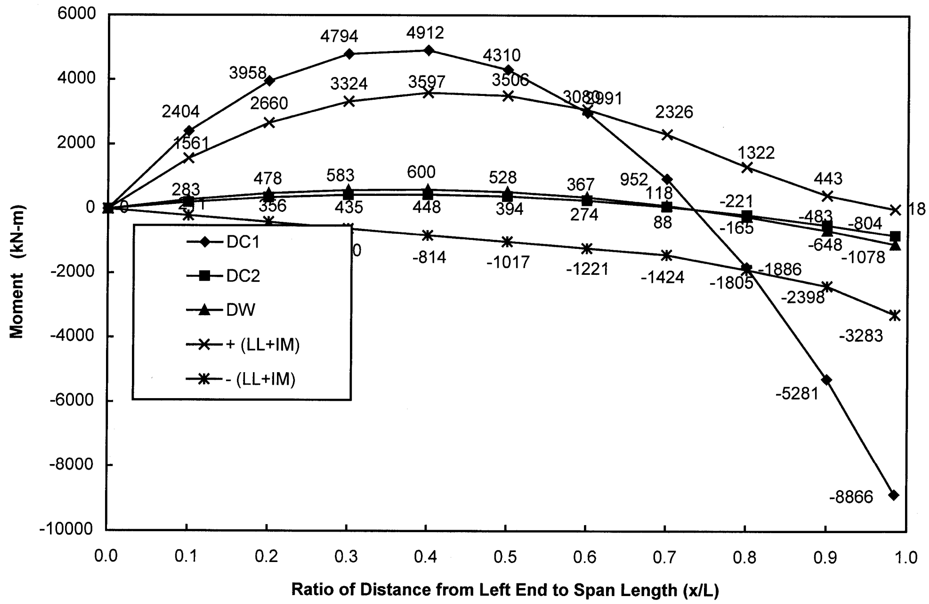


**FIGURE 10.14** Moment envelopes for Span 1.

- Two or more design lanes loaded — Modify interior girder factor by *e*:

$$g_V = e g_{V(\text{interior girder})} = \left(0.64 + \frac{d_e}{3800}\right) g_{V(\text{interior girder})}$$

$$= \left(0.64 + \frac{765}{3800}\right)(0.842) = 0.708 \text{ lanes}$$

- The live load distribution factors at the strength limit state:

| Strength Limit State I | Interior Girder | Exterior Girder |
|---|---|---|
| *Bending moment* | 0.634 lanes | 0.605 lanes |
| *Shear* | 0.842 lanes | 0.861 lanes |

5. **Calculate Unfactored Moments and Shear Demands for Interior Girder**
It is practically assumed that all dead loads are carried by the box girder and equally distributed to each girder. The live loads take forces to the girders according to live load distribution factors (AASHTO Article 4.6.2.2.2). Unfactored moment and shear demands for an interior girder are shown in Figures 10.14 and 10.15. Details are listed in Tables 10.11 and 10.12. Only the results for Span 1 are shown in these tables and figures since the bridge is symmetrical about the bent.

6. **Determine Load Factors for Strength Limit State I and Service Limit State I**
a. *General design equation* (ASHTO Article 1.3.2):

$$\eta \sum \gamma_i Q_i \le \phi R_n \tag{10.53}$$

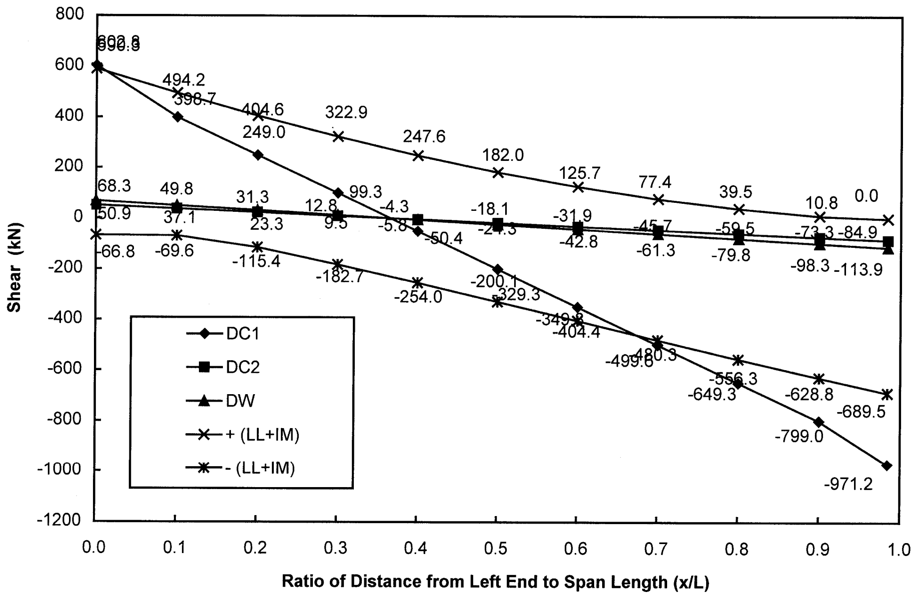


**FIGURE 10.15** Shear envelopes for Span 1

**TABLE 10.11** Moment and Shear due to Unfactored Dead Load for the Interior Girder

| | | Unfactored Dead Load | | | | | |
|---|---|---|---|---|---|---|---|
| | | *DC*1 | | *DC*2 | | DW | |
| Span | Location (*x/L*) | $M_{DC1}$ (kN-m) | $V_{DC1}$ (kN) | $M_{DC2}$ (kN-m) | $V_{DC2}$ (kN) | $M_{DW}$ (kN-m) | $V_{DW}$ (kN) |
| | 0.0 | 0 | 603 | 0 | 51 | 0 | 68 |
| | 0.1 | 2404 | 399 | 211 | 37 | 283 | 50 |
| | 0.2 | 3958 | 249 | 356 | 23 | 478 | 31 |
| | 0.3 | 4794 | 99 | 435 | 10 | 583 | 13 |
| 1 | 0.4 | 4912 | –50 | 448 | –4 | 600 | –6 |
| | 0.5 | 4310 | –200 | 394 | –18 | 528 | –24 |
| | 0.6 | 2991 | –350 | 274 | –32 | 367 | –43 |
| | 0.7 | 952 | –500 | 88 | –46 | 118 | –61 |
| | 0.8 | –1805 | –649 | –165 | –59 | –221 | –80 |
| | 0.9 | –5281 | –799 | –483 | –73 | –648 | –98 |
| | Face of column | –8866 | –971 | –804 | –85 | –1078 | –114 |

*Note*:
1. *DC*1 — interior girder self-weight.
2. *DC*2 — barrier self-weight.
3. *DW* — wearing surface load.
4. Moments in Span 2 are symmetrical about the bent.
5. Shears in span are anti-symmetrical about the bent.

where $\gamma_i$ are load factors and $\phi$ a resistance factor; $Q_i$ represents force effects; $R_n$ is the nominal resistance; $\eta$ is a factor related to the ductility, redundancy, and operational importance of that being designed and is defined as:

$$\eta = \eta_D \eta_R \eta_I \geq 0.95 \tag{10.54}$$

**TABLE 10.12** Moment and Shear Envelopes and Associated Forces for the Interior Girder due to AASHTO HL-93 Live Load

| Span | Location ($x/L$) | Positive Moment and Associated Shear | | Negative Moment and Associated Shear | | Shear and Associated Moment | |
|---|---|---|---|---|---|---|---|
| | | $M_{LL+IM}$ (kN-m) | $V_{LL+IM}$ (kN) | $V_{LL+IM}$ (kN) | $M_{LL+IM}$ (kN-m) | $V_{LL+IM}$ (kN) | $M_{LL+IM}$ (kN-m) |
| | 0.0 | 0 | 259 | 0 | –255 | 497 | 0 |
| | 0.1 | 1561 | 312 | –203 | –42 | 416 | 1997 |
| | 0.2 | 2660 | 249 | –407 | –42 | 341 | 3270 |
| | 0.3 | 3324 | 47 | –610 | –42 | 272 | 3915 |
| 1 | 0.4 | 3597 | 108 | –814 | –42 | –214 | 3228 |
| | 0.5 | 3506 | –25 | –1017 | –42 | –277 | 3272 |
| | 0.6 | 3080 | –81 | –1221 | –42 | –341 | 2771 |
| | 0.7 | 2326 | –258 | –1424 | –42 | –404 | 1956 |
| | 0.8 | 1322 | –166 | –1886 | –68 | –468 | 689 |
| | 0.9 | 443 | –112 | –2398 | –141 | –529 | –945 |
| | Face of column | 18 | –97 | –3283 | –375 | –581 | –1850 |

*Note:*
1. $LL + IM$ — AASHTO HL-93 live load plus dynamic load allowance.
2. Moments in Span 2 are symmetrical about the bent.
3. Shears in Span 2 are antisymmetrical about the bent.
4. Live load distribution factors are considered.

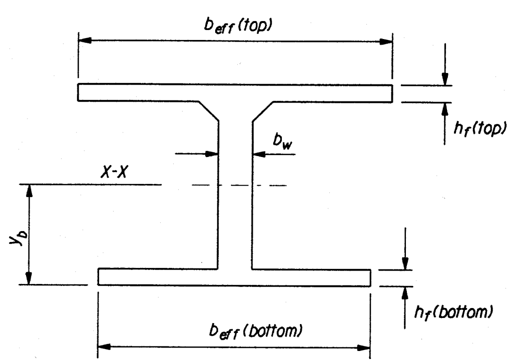


**FIGURE 10.16** Effective flange width of interior girder.

For this bridge, the following values are assumed:

| Limit States | Ductility $\eta_D$ | Redundancy $\eta_R$ | Importance $\eta_I$ | $\eta$ |
|---|---|---|---|---|
| *Strength limit state* | 0.95 | 0.95 | 1.05 | 0.95 |
| *Service limit state* | 1.0 | 1.0 | 1.0 | 1.0 |

b. *Load factors and load combinations*:
The load factors and combinations are specified as (AASHTO Table 3.4.1-1):
Strength Limit State I: $1.25(DC1 + DC2) + 1.5(DW) + 1.75(LL + IM)$
Service Limit State I: $DC1 + DC2 + DW + (LL + IM)$

7. **Calculate Section Properties for Interior Girder**
For an interior girder as shown in Figure 10.16, the effective flange width $b_{\text{eff}}$ is determined (AASHTO Article 4.6.2.6) by:

**TABLE 10.13** Effective Flange Width and Section Properties for Interior Girder

| Location | Dimension | Mid span | Bent (face of support) |
|---|---|---|---|
| Top flange | $h_f$ (mm) | 200 | 200 |
| | $L_{eff}/4$ (mm) | 9,000 | 11,813 |
| | $12h_f + b_w$ (mm) | 2,700 | 2,700 |
| | $S$ (mm) | 2,600 | 2,600 |
| | $b_{eff}$ (mm) | **2,600** | **2,600** |
| Bottom flange | $h_f$ (mm) | 150 | 300 |
| | $L_{eff}/4$ (mm) | 9,000 | 11,813 |
| | $12h_f + b_w$ (mm) | 2,100 | 3,900 |
| | $S$ (mm) | 2,600 | 2,600 |
| | $b_{eff}$ (mm) | **2,100** | **2,600** |
| Area | $A$ (m²) | 1.316 | 1.736 |
| Moment of inertia | $I$ (m⁴) | 0.716 | 0.968 |
| Center of gravity | $y_b$ (m) | 1.085 | 0.870 |

*Note*: $L_{eff}$ = 36.0 m for midspan; $L_{eff}$ = 47.25 m for the bent; $b_w$ = 300 mm.

$$b_{eff} = \text{the lesser of} \begin{cases} \dfrac{L_{eff}}{4} \\ 12h_f + b_w \\ S \end{cases} \tag{10.55}$$

where $L_{eff}$ is the effective span length and may be taken as the actual span length for simply supported spans and the distance between points of permanent load inflection for continuous spans; $h_f$ is the compression flange thickness and $b_w$ is the web width; and $S$ is the average spacing of adjacent girders. The calculated effective flange width and the section properties are shown in Table 10.13 for the interior girder.

**8. Calculate Prestress Losses**

For a CIP post-tensioned box girder, two types of losses, instantaneous losses (friction, anchorage set, and elastic shortening) and time-dependent losses (creep and shrinkage of concrete, and relaxation of prestressing steel), are significant. Since the prestress losses are not symmetrical about the bent for this bridge, the calculation is performed for both spans.

a. *Frictional loss* $\Delta f_{pF}$:

$$\Delta f_{pF} = f_{pj}\left(1 - e^{-(Kx + \mu\alpha)}\right) \tag{10.56}$$

where $K$ is the wobble friction coefficient = 6.6 × $10^{-7}$/mm and $\mu$ is the coefficient of friction = 0.25 (AASHTO Article 5.9.5.2.2b); $x$ is the length of a prestressing tendon from the jacking end to the point considered; $\alpha$ is the sum of the absolute values of angle change in the prestressing steel path from the jacking end.

For a parabolic cable path (Figure 10.17), the angle change is $\alpha = 2e_p/L_p$, where $e_p$ is the vertical distance between two control points and $L_p$ is the horizontal distance between two control points. The details are given in Table 10.14.

b. *Anchorage set loss* $\Delta f_{pA}$:

For an anchor set thickness of $\Delta L$ = 10 mm and $E$ = 200,000 MPa, consider the point $D$ where $L_{pF}$ = 48 m and $\Delta f_{pF}$ = 96.06 MPa:

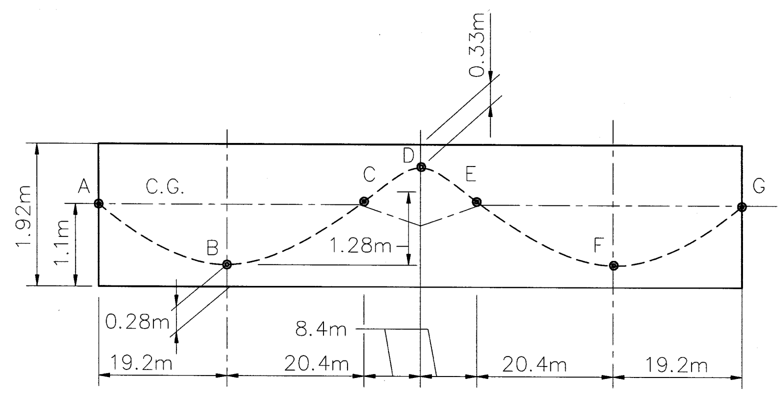


**FIGURE 10.17** Parabolic cable path.

**TABLE 10.14** Prestress Frictional Loss

| Segment | $e_p$ (mm) | $L_p$ (m) | $\alpha$ (rad) | $\Sigma\alpha$ (rad) | $\Sigma L_b$ (m) | Point | $\Delta f_{pF}$ (Mpa) |
|---|---|---|---|---|---|---|---|
| A | 0.00 | 0 | 0 | 0 | 0 | A | 0.00 |
| AB | 820 | 19.2 | 0.0854 | 0.0854 | 19.2 | B | 31.44 |
| BC | 926 | 20.4 | 0.0908 | 0.1762 | 39.6 | C | 64.11 |
| CD | 381 | 8.4 | 0.0908 | 0.2669 | 48.0 | D | 96.06 |
| DE | 381 | 8.4 | 0.0908 | 0.3577 | 56.4 | E | 127.28 |
| EF | 926 | 20.4 | 0.0908 | 0.4484 | 76.8 | F | 157.81 |
| FG | 820 | 19.2 | 0.0854 | 0.5339 | 96.0 | G | 185.91 |

$$L_{pA} = \sqrt{\frac{E\,(\Delta L)\,L_{pF}}{\Delta f_{pF}}} = \sqrt{\frac{200{,}000(10)(48{,}000)}{96.06}} = 31\ 613\ \text{mm} = 31.6\ \text{m} < 48\ \text{m} \qquad \text{OK}$$

$$\Delta f = \frac{2\,\Delta f_{pF}\,L_{pA}}{L_{pF}} = \frac{2(96.06)(31.6)}{48} = 126.5\ \text{MPa}$$

$$\Delta f_{pA} = \Delta f\left(1 - \frac{x}{L_{pA}}\right) = 126.5\left(1 - \frac{x}{31.6}\right)$$

c. *Elastic shortening loss* $\Delta f_{pES}$:
The loss due to elastic shortening in post-tensioned members is calculated using the following formula (AASHTO Article 5.9.5.2.3b):

$$\Delta f_{pES} = \frac{N-1}{2N}\frac{E_p}{E_{ci}} f_{cgp} \tag{10.57}$$

To calculate the elastic shortening loss, we assume that the prestressing jack force for an interior girder $P_j$ = 8800 kN and the total number of prestressing tendons $N$ = 4. $f_{cgp}$ is calculated for face of support section:

$$f_{cgp} = \frac{P_j}{A} + \frac{P_j e^2}{I_x} + \frac{M_{DC1} e}{I_x}$$

$$= \frac{8800}{1.736} + \frac{8800\,(0.714)^2}{0.968} + \frac{(-8866)(0.714)}{0.968}$$

$$= 5069 + 4635 - 6540 = 3164 \text{ kN/m}^2 = 3.164 \text{ MPa}$$

$$\Delta f_{pES} = \frac{N-1}{2N}\frac{E_p}{E_{ci}} f_{cgp} = \frac{4-1}{2(4)}\frac{197{,}000}{24768}(3.164) = 9.44 \text{ MPa}$$

d. *Time-dependent losses* $\Delta f_{pTM}$:
AASHTO provides a table to estimate the accumulated effect of time-dependent losses resulting from the creep and shrinkage of concrete and the relaxation of the steel tendons. From AASHTO Table 5.9.5.3-1:

$$\Delta f_{pTM} = 145 \text{ MPa} \quad \text{(upper bound)}$$

e. *Total losses* $\Delta f_{pT}$:

$$\Delta f_{pT} = \Delta f_{pF} + \Delta f_{pA} + \Delta f_{pES} + \Delta f_{pTM}$$

Details are given in Table 10.15.

**9. Determine Prestressing Force $P_j$ for Interior Girder**

Since the live load is not in general equally distributed to girders, the prestressing force $P_j$ required for each girder may be different. To calculate prestress jacking force $P_j$, the initial prestress force coefficient $F_{pCI}$ and final prestress force coefficient $F_{pCF}$ are defined as:

$$F_{pCI} = 1 - \frac{\Delta f_{pF} + \Delta f_{pA} + \Delta f_{pES}}{f_{pj}} \tag{10.58}$$

$$F_{pCF} = 1 - \frac{\Delta f_{pT}}{f_{pj}} \tag{10.59}$$

The secondary moment coefficients are defined as:

$$M_{sC} = \begin{cases} \dfrac{x}{L}\dfrac{M_{DA}}{P_j} & \text{for Span 1} \\[2ex] \left(1 - \dfrac{x}{L}\right)\dfrac{M_{DG}}{P_j} & \text{for Span 2} \end{cases} \tag{10.60}$$

where $x$ is the distance from the left end for each span. The combined prestressing moment coefficients are defined as:

$$M_{psCI} = F_{pCI}(e) + M_{sC} \tag{10.61}$$

**TABLE 10.15** Cable Path and Prestress Losses

| Span | Location ($x/L$) | Prestress Losses (MPa) $\Delta f_{pF}$ | $\Delta f_{pA}$ | $\Delta f_{pES}$ | $\Delta f_{pTM}$ | $\Delta f_{pT}$ | Force Coefficient $F_{pCI}$ | $F_{pCF}$ |
|---|---|---|---|---|---|---|---|---|
| | 0.0 | 0.00 | 126.50 | | | 280.94 | 0.909 | 0.811 |
| | 0.1 | 7.92 | 107.28 | | | 269.65 | 0.916 | 0.819 |
| | 0.2 | 15.80 | 88.07 | | | 258.31 | 0.924 | 0.826 |
| | 0.3 | 23.64 | 68.85 | | | 246.94 | 0.931 | 0.834 |
| | 0.4 | 31.44 | 49.64 | | | 235.52 | 0.939 | 0.842 |
| 1 | 0.5 | 39.19 | 30.42 | 9.44 | 145.00 | 224.06 | 0.947 | 0.849 |
| | 0.6 | 46.91 | 11.21 | | | 212.56 | 0.955 | 0.857 |
| | 0.7 | 54.58 | 0.00 | | | 209.02 | 0.957 | 0.860 |
| | 0.8 | 62.21 | 0.00 | | | 216.65 | 0.952 | 0.854 |
| | 0.9 | 77.89 | 0.00 | | | 232.33 | 0.941 | 0.844 |
| | 1.0 | 96.06 | 0.00 | | | 250.50 | 0.929 | 0.832 |
| | 0.0 | 96.06 | | | | 250.50 | 0.929 | 0.832 |
| | 0.1 | 113.99 | | | | 268.43 | 0.917 | 0.820 |
| | 0.2 | 129.10 | | | | 283.54 | 0.907 | 0.809 |
| | 0.3 | 136.33 | | | | 290.77 | 0.902 | 0.805 |
| 2 | 0.4 | 143.53 | 0.00 | 9.44 | 145.00 | 297.97 | 0.897 | 0.800 |
| | 0.5 | 150.69 | | | | 305.13 | 0.892 | 0.795 |
| | 0.6 | 157.81 | | | | 312.25 | 0.888 | 0.790 |
| | 0.7 | 164.89 | | | | 319.33 | 0.883 | 0.785 |
| | 0.8 | 171.94 | | | | 326.38 | 0.878 | 0.781 |
| | 0.9 | 178.94 | | | | 333.38 | 0.873 | 0.776 |
| | 1.0 | 185.91 | | | | 340.35 | 0.869 | 0.771 |

*Note:* $F_{pCI} = 1 - \dfrac{\Delta f_{pF} + \Delta f_{pA} + \Delta f_{pES}}{f_{pj}}$

$F_{pCF} = 1 - \dfrac{\Delta f_{pT}}{f_{pj}}$

$$M_{psCF} = F_{pCF}(e) + M_{sC} \tag{10.62}$$

where $e$ is the distance between the cable and the center of gravity of a cross section; positive values of $e$ indicate that the cable is above the center of gravity, and negative ones indicate the cable is below the center of gravity of the section.

The prestress force coefficients and the combined moment coefficients are calculated and tabled in Table 10.16. According to AASHTO, the prestressing force $P_j$ can be determined using the concrete tensile stress limit in the precompression tensile zone (see Table 10.5):

$$f_{DC1} + f_{DC2} + f_{DW} + f_{LL+IM} + f_{psF} \geq -0.5\sqrt{f_c'} \tag{10.63}$$

in which

$$f_{DC1} = \frac{M_{DC1}C}{I_x} \tag{10.64}$$

$$f_{DC2} = \frac{M_{DC2}C}{I_x} \tag{10.65}$$

**TABLE 10.16** Prestress Force and Moment Coefficients

| Span | Location ($x/L$) | Cable Path $e$ (m) | Force Coefficients $F_{pCI}$ | Force Coefficients $F_{pCF}$ | Moment Coefficients (m) $F_{pCI}e$ | Moment Coefficients (m) $F_{pCF}e$ | Moment Coefficients (m) $M_{sC}$ | Moment Coefficients (m) $M_{psCI}$ | Moment Coefficients (m) $M_{psCF}$ |
|---|---|---|---|---|---|---|---|---|---|
| | 0.0 | 0.015 | 0.909 | 0.811 | 0.014 | 0.012 | 0.000 | 0.014 | 0.012 |
| | 0.1 | –0.344 | 0.916 | 0.819 | –0.315 | –0.281 | 0.034 | –0.281 | –0.247 |
| | 0.2 | –0.600 | 0.924 | 0.826 | –0.554 | –0.496 | 0.068 | –0.486 | –0.428 |
| | 0.3 | –0.754 | 0.931 | 0.834 | –0.702 | –0.629 | 0.102 | –0.600 | –0.526 |
| | 0.4 | –0.805 | 0.939 | 0.842 | –0.756 | –0.678 | 0.136 | –0.620 | –0.541 |
| 1 | 0.5 | –0.754 | 0.947 | 0.849 | –0.714 | –0.640 | 0.171 | –0.543 | –0.470 |
| | 0.6 | –0.600 | 0.955 | 0.857 | –0.573 | –0.514 | 0.205 | –0.368 | –0.310 |
| | 0.7 | –0.344 | 0.957 | 0.860 | –0.329 | –0.295 | 0.239 | –0.090 | –0.057 |
| | 0.8 | 0.015 | 0.952 | 0.884 | 0.014 | 0.013 | 0.273 | 0.287 | 0.286 |
| | 0.9 | 0.377 | 0.941 | 0.844 | 0.355 | 0.318 | 0.307 | 0.662 | 0.625 |
| | 1.0 | 0.717 | 0.929 | 0.832 | 0.666 | 0.596 | 0.341 | 1.007 | 0.937 |
| | 0.0 | 0.717 | 0.929 | 0.832 | 0.666 | 0.596 | 0.347 | 1.013 | 0.943 |
| | 0.1 | 0.377 | 0.917 | 0.820 | 0.346 | 0.309 | 0.312 | 0.658 | 0.622 |
| | 0.2 | 0.015 | 0.907 | 0.809 | 0.014 | 0.012 | 0.278 | 0.291 | 0.290 |
| | 0.3 | –0.344 | 0.902 | 0.805 | –0.310 | –0.277 | 0.243 | –0.067 | –0.034 |
| | 0.4 | –0.600 | 0.897 | 0.800 | –0.538 | –0.480 | 0.208 | –0.330 | –0.272 |
| 2 | 0.5 | –0.754 | 0.892 | 0.795 | –0.673 | –0.599 | 0.174 | –0.499 | –0.426 |
| | 0.6 | –0.805 | 0.888 | 0.790 | –0.715 | –0.636 | 0.139 | –0.576 | –0.497 |
| | 0.7 | –0.754 | 0.883 | 0.785 | –0.665 | –0.592 | 0.104 | –0.561 | –0.488 |
| | 0.8 | –0.600 | 0.878 | 0.781 | –0.527 | –0.468 | 0.069 | –0.457 | –0.399 |
| | 0.9 | –0.344 | 0.873 | 0.776 | –0.300 | –0.267 | 0.035 | –0.266 | –0.232 |
| | 1.0 | 0.015 | 0.869 | 0.771 | 0.013 | 0.012 | 0.000 | 0.013 | 0.012 |

*Note:* $e$ is distance between cable path and central gravity of the interior girder cross section, positive means cable is above the central gravity, and negative indicates cable is below the central gravity.

$$f_{DW} = \frac{M_{DW}C}{I_x} \tag{10.66}$$

$$f_{LL+IM} = \frac{M_{LL+IM}C}{I_x} \tag{10.67}$$

$$f_{psF} = \frac{P_{pe}}{A} + \frac{(P_{pe}e)C}{I_x} + \frac{M_sC}{I_x} = \frac{F_{pCF}P_j}{A} + \frac{M_{psCF}P_jC}{I_x} \tag{10.68}$$

where $C$ ( = $y_b$ or $y_t$) is the distance from the extreme fiber to the center of gravity of the cross section. $f'_c$ is in MPa and $P_{pe}$ is the effective prestressing force after all losses have been incurred. From Eqs. (10.63) and (10.68), we have

$$P_j = \frac{-f_{DC1} - f_{DC2} - f_{DW} - f_{LL+IM} - 0.5\sqrt{f'_c}}{\dfrac{F_{pCF}}{A} + \dfrac{M_{psCF}C}{I_x}} \tag{10.69}$$

Detailed calculations are given in Table 10.17. Most critical points coincide with locations of maximum eccentricity: 0.4$L$ in Span 1, 0.6$L$ in Span 2, and at the bent. For this bridge, the controlling section is through the right face of the bent. Herein, $P_j$ = 8741 kN. Rounding $P_j$ up to 8750 kN gives a required area of prestressing steel of $A_{ps} = P_j/f_{pj}$ = 8750/1488 (1000) = 5880 mm$^2$.

**TABLE 10.17** Determination of Prestressing Jacking Force for an Interior Girder

| Span | Location ($x/L$) | Top Fiber Stress (MPa) $f_{DC1}$ | $f_{DC2}$ | $f_{DW}$ | $f_{LL+IM}$ | Top Fiber Jacking Force, $P_j$ (kN) | Bottom Fiber Stress (MPa) $f_{DC1}$ | $f_{DC2}$ | $f_{DW}$ | $f_{LL+IM}$ | Jacking Force $P_j$(kN) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.0 | 0.000 | 0.000 | 0.000 | 0.000 | — | 0.000 | 0.000 | 0.000 | 0.000 | 0 |
| | 0.1 | 2.803 | 0.246 | 0.330 | 1.820 | — | −3.642 | −0.320 | −0.429 | −2.365 | 4405 |
| | 0.2 | 4.616 | 0.415 | 0.557 | 3.103 | — | −5.998 | −0.540 | −0.724 | −4.032 | 6778 |
| | 0.3 | 5.591 | 0.507 | 0.680 | 3.876 | — | −7.265 | −0.659 | −0.884 | −5.037 | 7824 |
| | 0.4 | 5.728 | 0.522 | 0.700 | 4.195 | — | −7.442 | −0.678 | −0.910 | −5.450 | 8101 |
| 1 | 0.5 | 5.027 | 0.459 | 0.616 | 4.089 | — | −6.532 | −0.597 | −0.800 | −5.313 | 7807 |
| | 0.6 | 3.488 | 0.319 | 0.428 | 3.591 | — | −4.532 | −0.415 | −0.557 | −4.667 | 6714 |
| | 0.7 | 1.110 | 0.102 | 0.137 | 2.712 | — | −1.443 | −0.133 | −0.178 | −3.524 | 3561 |
| | 0.8 | −2.105 | −0.192 | −0.258 | 1.542 | 2601 | 2.736 | 0.250 | 0.335 | −2.004 | — |
| | 0.9 | −6.159 | −0.565 | −0.756 | 0.516 | 5567 | 8.003 | 0.733 | 0.982 | −0.671 | — |
| | 1.0 | −9.617 | −0.872 | −1.169 | 0.020 | 8406 | 7.968 | 0.722 | 0.969 | −0.016 | — |
| | 0.0 | −9.617 | −0.872 | −1.169 | 0.020 | 8370 | 7.968 | 0.722 | 0.969 | −0.016 | — |
| | 0.1 | −.6.159 | −0.564 | −0.756 | 0.516 | 5661 | 8.003 | 0.733 | 0.982 | −0.671 | — |
| | 0.2 | −2.105 | −0.192 | −0.258 | 1.542 | 2681 | 2.736 | 0.250 | 0.335 | −2.004 | — |
| | 0.3 | 1.110 | 0.102 | 0.137 | 2.712 | — | −1.443 | −0.133 | −0.178 | −3.524 | 3974 |
| | 0.4 | 3.488 | 0.319 | 0.428 | 3.591 | — | −4.532 | −0.415 | −0.557 | −4.667 | 7381 |
| 2 | 0.5 | 5.027 | 0.459 | 0.616 | 4.089 | — | −6.532 | −0.597 | −0.800 | −5.313 | 8483 |
| | 0.6 | 5.728 | 0.522 | 0.700 | 4.195 | — | −7.443 | −0.678 | −0.910 | −5.450 | **8741** |
| | 0.7 | 5.591 | 0.507 | 0.680 | 3.876 | — | −7.265 | −0.659 | −0.884 | −5.037 | 8382 |
| | 0.8 | 4.616 | 0.415 | 0.557 | 3.103 | — | −5.998 | −0.540 | −0.724 | −4.032 | 7220 |
| | 0.9 | 2.803 | 0.246 | 0.330 | 1.820 | — | −3.642 | −0.320 | −0.429 | −2.365 | 4666 |
| | 1.0 | 0.000 | 0.000 | 0.000 | 0.000 | — | 0.000 | 0.000 | 0.000 | 0.000 | 0 |

*Notes*:
1. Positive stress indicates compression and negative stress indicates tension.
2. $P_j$ are obtained by Eq. (10.69).

**10. Check Concrete Strength for Interior Girder — Service Limit State I**

Two criteria are imposed on the level of concrete stresses when calculating required concrete strength (AASHTO Article 5.9.4.2):

$$\begin{cases} f_{DC1} + f_{psI} \le 0.55 f'_{ci} & \text{at prestressing state} \\ f_{DC1} + f_{DC2} + f_{DW} + f_{LL+IM} + f_{psF} \le 0.45 f'_c & \text{at service state} \end{cases} \tag{10.70}$$

$$f_{psI} = \frac{P_{jI}}{A} + \frac{(P_{jI}e)C}{I_x} + \frac{M_{sI}C}{I_x} = \frac{F_{pCI}P_j}{A} + \frac{M_{psCI}P_jC}{I_x} \tag{10.71}$$

The concrete stresses in the extreme fibers (after instantaneous losses and final losses) are given in Tables 10.18. and 10.19. For the initial concrete strength in the prestressing state, the controlling location is the top fiber at 0.8$L$ section in Span 1. From Eq. (10.70), we have

$$f'_{ci,reg} \ge \frac{f_{DC1} + f_{psI}}{0.55} = \frac{7.15}{0.55} = 13\,\text{MPa}$$

$$\therefore \quad \underline{\text{use } f'_{ci} = 24 \text{ MPa}} \qquad \text{OK}$$

**TABLE 10.18** Concrete Stresses after Instantaneous Losses for the Interior Girder

| Span | Location ($x/L$) | Top Fiber Stress (MPa) $f_{DC1}$ | $F_{pCI^*}$ $P_j/A$ | $M_{psCI^*}$ $P_j^* Y_t/I$ | $f_{psI}$ | Total Initial Stress | Bottom Fiber Stress (MPa) $f_{DC1}$ | $F_{pCI^*}$ $P_j/A$ | $M_{psCI^*}$ $P_j^* Y_t/I$ | $f_{psI}$ | Total Initial Stress |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.0 | 0.00 | 6.04 | 0.14 | 6.18 | 6.18 | 0.00 | 6.04 | –0.18 | 5.86 | 5.86 |
| | 0.1 | 2.80 | 6.09 | –2.87 | 3.23 | 6.03 | –3.64 | 6.09 | 3.72 | 9.82 | 6.17 |
| | 0.2 | 4.62 | 6.14 | –4.96 | 1.18 | 5.80 | –6.00 | 6.14 | 6.45 | 12.59 | 6.59 |
| | 0.3 | 5.59 | 6.19 | –6.12 | 0.07 | 5.66 | –7.27 | 6.19 | 7.95 | 14.15 | 6.88 |
| | 0.4 | 5.73 | 6.24 | –6.32 | –0.08 | 5.65 | –7.44 | 6.24 | 8.22 | 14.46 | 7.02 |
| 1 | 0.5 | 5.03 | 6.30 | –5.54 | 0.75 | 5.78 | –6.53 | 6.30 | 7.20 | 13.50 | 6.97 |
| | 0.6 | 3.49 | 6.35 | –3.76 | 2.59 | 6.08 | –4.53 | 6.35 | 4.88 | 11.23 | 6.70 |
| | 0.7 | 1.11 | 6.36 | –0.92 | 5.44 | 6.55 | –1.44 | 6.36 | 1.20 | 7.56 | 6.12 |
| | 0.8 | –2.11 | 6.33 | 2.93 | 9.26 | **7.15** | 2.74 | 6.33 | –3.81 | 2.52 | 5.26 |
| | 0.9 | –6.16 | 6.26 | 6.76 | 13.02 | 6.86 | 8.00 | 6.26 | –8.78 | –2.52 | 5.48 |
| | 1.0 | –9.62 | 4.68 | 9.56 | 14.24 | 4.62 | 7.97 | 4.68 | –7.92 | –3.24 | 4.73 |
| | 0.0 | –9.62 | 4.68 | 9.62 | 14.30 | 4.68 | 7.97 | 4.68 | –7.97 | –3.28 | 4.68 |
| | .1 | –6.16 | 6.10 | 6.72 | 12.82 | 6.66 | 8.00 | 6.10 | –8.73 | –2.63 | 5.37 |
| | 0.2 | –2.11 | 6.03 | 2.97 | 9.00 | 6.90 | 2.74 | 6.03 | –3.86 | 2.17 | 4.90 |
| | 0.3 | 1.11 | 6.00 | –0.69 | 5.31 | 6.42 | –1.44 | 6.00 | 0.89 | 6.89 | 5.45 |
| | 0.4 | 3.49 | 5.97 | –3.37 | 2.60 | 6.08 | –4.53 | 5.97 | 4.38 | 10.34 | 5.81 |
| 2 | 0.5 | 5.03 | 5.93 | –5.09 | 0.84 | 5.87 | –6.53 | 5.93 | 6.62 | 12.55 | 6.02 |
| | 0.6 | 5.73 | 5.90 | –5.87 | 0.03 | 5.75 | –7.44 | 5.90 | 7.63 | 13.54 | 6.09 |
| | 0.7 | 5.59 | 5.87 | –5.73 | 0.14 | 5.73 | –7.27 | 5.87 | 7.44 | 13.31 | 6.05 |
| | 0.8 | 4.62 | 5.84 | –4.67 | 1.71 | 5.79 | –6.00 | 5.84 | 6.07 | 11.90 | 5.91 |
| | 0.9 | 2.80 | 5.81 | –2.71 | 3.10 | 5.90 | –3.64 | 5.81 | 3.52 | 9.33 | 5.69 |
| | 1.0 | 0.00 | 5.78 | 0.13 | 5.91 | 5.91 | 0.00 | 5.78 | –0.17 | 5.60 | 5.60 |

*Note:* Positive stress indicates compression and negative stress indicates tension

For the final concrete strength at the service limit state, the controlling location is in the top fiber at 0.6$L$ section in Span 2. From Eq. (10.70), we have

$$f'_{c,req} \geq \frac{f_{DC1} + f_{DC2} + f_{DW} + f_{LL+IM} + f_{psF}}{0.45} = \frac{11.32}{0.45} = 21.16 \text{ MPa} < 28 \text{MPa}$$

$$\therefore \quad \underline{\text{choose } f'_c = 28 \text{MPa}} \qquad \text{OK}$$

**11. Flexural Strength Design for Interior Girder — Strength Limit State I**

AASHTO [4] requires that for the Strength Limit State I

$$M_u \leq \phi M_n$$

$$M_u = \eta \sum \gamma_i M_i = 0.95[1.25(M_{DC1} + M_{DC2}) + 1.5M_{DW} + 1.75M_{LLH}] + M_{ps}$$

where $\phi$ is the flexural resistance factor 1.0 and $M_{ps}$ is the secondary moment due to prestress. Factored moment demands $M_u$ for the interior girder in Span 1 are calculated in Table 10.20. Although the moment demands are not symmetrical about the bent (due to different secondary prestress moments), the results for Span 2 are similar and the differences will not be considered in this example. The detailed calculations for the flexural resistance $\phi M_n$ are shown in Table 10.21. It is seen that no additional mild steel is required.

**TABLE 10.19** Concrete Stresses after Total Losses for the Interior Girder

| Span | Location ($x/L$) | Top Fiber Stress (MPa) | | | | | Bottom Fiber Stress (MPa) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $f_{LOAD}$ | $F_{pCF*}$ $P_{j/A}$ | $M_{psCF*}$ $P_{j*Yt/I}$ | $f_{psF}$ | Total Final Stress | $f_{LOAD}$ | $F_{pCF*}$ $P_{j/A}$ | $M_{psCF*}$ $P_{j*yb}/I$ | $f_{psF}$ | Total Final Stress |
| | 0.0 | 0.00 | 5.39 | 0.12 | 5.52 | 5.52 | 0.00 | 5.39 | –0.16 | 5.23 | 5.23 |
| | 0.1 | 5.20 | 5.44 | –2.52 | 2.92 | 8.12 | –6.76 | 5.44 | 3.28 | 8.72 | 1.97 |
| | 0.2 | 8.69 | 5.49 | –4.36 | 1.13 | 9.82 | –11.29 | 5.49 | 5.67 | 11.16 | –0.13 |
| | 0.3 | 10.66 | 5.55 | –5.37 | 0.17 | 10.83 | –13.85 | 5.55 | 6.98 | 12.52 | –1.32 |
| | 0.4 | 11.14 | 5.60 | –5.52 | 0.07 | 11.22 | –14.48 | 5.60 | 7.18 | 12.77 | –1.71 |
| 1 | 0.5 | 10.19 | 5.65 | –4.79 | 0.85 | 11.05 | –13.24 | 5.65 | 6.23 | 11.88 | –1.37 |
| | 0.6 | 7.83 | 5.70 | –3.16 | 2.54 | 10.37 | –10.17 | 5.70 | 4.11 | 9.81 | –0.36 |
| | 0.7 | 4.06 | 5.71 | –0.58 | 5.14 | 9.20 | –5.28 | 5.71 | 0.75 | 6.47 | 1.19 |
| | 0.8 | –4.75 | 5.68 | 2.91 | 8.60 | 3.84 | 6.18 | 5.68 | –3.79 | 1.89 | 8.07 |
| | 0.9 | –10.28 | 5.61 | 6.38 | 11.99 | 1.72 | 13.35 | 5.61 | –8.29 | –2.68 | 10.67 |
| | 1.0 | –15.22 | 4.19 | 8.90 | 13.09 | –2.13 | 12.61 | 4.19 | –7.37 | –3.18 | 9.43 |
| | 0.0 | –15.22 | 4.19 | 8.95 | 13.14 | –2.07 | 12.61 | 4.19 | –7.42 | –3.23 | 9.38 |
| | 0.1 | –10.28 | 5.45 | 6.34 | 11.79 | 1.52 | 13.35 | 5.45 | –8.24 | –2.79 | 10.56 |
| | 0.2 | –4.75 | 5.38 | 2.96 | 8.34 | 3.58 | 6.18 | 5.38 | –3.84 | 1.54 | 7.72 |
| | 0.3 | 4.06 | 5.35 | 0.34 | 5.01 | 9.07 | –5.28 | 5.35 | 0.45 | 5.80 | 0.52 |
| | 0.4 | 7.83 | 5.32 | –2.77 | 2.55 | 10.37 | –10.17 | 5.32 | 3.60 | 8.92 | –1.25 |
| 2 | 0.5 | 10.19 | 5.29 | –4.34 | 0.94 | 11.13 | –13.24 | 5.29 | 5.64 | 10.93 | –2.31 |
| | 0.6 | 11.14 | 5.25 | –5.07 | 0.18 | **11.32** | –14.48 | 5.25 | 6.59 | 11.85 | **–2.63** |
| | 0.7 | 10.66 | 5.22 | –4.98 | 0.24 | 10.90 | –13.85 | 5.22 | 6.47 | 11.69 | –2.15 |
| | 0.8 | 8.69 | 5.19 | –4.07 | 1.12 | 9.81 | –11.29 | 5.19 | 5.29 | 10.48 | –0.81 |
| | 0.9 | 5.20 | 5.16 | –2.37 | 2.79 | 7.99 | –6.76 | 5.16 | 3.08 | 8.24 | 1.48 |
| | 1.0 | 0.00 | 5.13 | 0.12 | 5.25 | 5.25 | 0.00 | 5.13 | –0.15 | 4.97 | 4.97 |

*Notes:*
1. $f_{LOAD} = f_{DC1} + f_{DC2} + f_{DW} + f_{LL+IM}$.
2. Positive stress indicates compression and negative stress indicates tension.

**TABLE 10.20** Factored Moments for an Interior Girder

| Span | Location ($x/L$) | $M_{DC1}$ (kN-m) Dead Load 1 | $M_{DC2}$ (kN-m) Dead Load 2 | $M_{DW}$ (kN-m) Wearing Surface | $M_{LL+IM}$ (kN-m) | | $M_{ps}$ (kN-m) P/S | $M_u$ (kN-m) | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Positive | Negative | | Positive | Negative |
| | 0.0 | 0 | 0 | 0 | 0 | 0 | 0 | **0** | 0 |
| | 0.1 | 2404 | 211 | 283 | 1561 | –203 | 298 | **6,402** | 3,469 |
| | 0.2 | 3958 | 356 | 478 | 2660 | –407 | 597 | **10,824** | 5,725 |
| | 0.3 | 4794 | 435 | 583 | 3324 | –610 | 895 | **13,462** | 6,922 |
| | 0.4 | 4912 | 448 | 600 | 3597 | –814 | 1194 | **14,393** | 7,060 |
| 1 | 0.5 | 4310 | 395 | 528 | 3506 | –1017 | 1492 | **13,660** | 6,140 |
| | 0.6 | 2991 | 274 | 367 | 3080 | –1221 | 1790 | **11,310** | 4,161 |
| | 0.7 | 952 | 88 | 118 | 2326 | –1424 | 2089 | **7,358** | 1,124 |
| | 0.8 | –1805 | –165 | –221 | 1322 | –1886 | 2387 | 1,931 | **3,403** |
| | 0.9 | –5281 | –483 | –648 | 443 | –2398 | 2685 | –4.348 | **9,071** |
| | 1.0 | –8866 | –804 | –1078 | 18 | –3283 | 2984 | –10,005 | **–15,492** |

*Note:* $M_u = 0.95[1.25(M_{DC1} + M_{DC2}) + 1.5M_{DW} + 1.75M_{LL+IM}] + M_{ps}$.

### 12. Shear Strength Design for Interior Girder — Strength Limit State I

AASHTO [4] requires that for the strength limit state I

$$V_u \le \phi V_n$$

$$V_u = \eta \sum \gamma_i V_i = 0.95[1.25(V_{DC1} + V_{DC2}) + 1.5V_{DW} + 1.75V_{LL+IM}] + V_{ps}$$

**TABLE 10.21** Flexural Strength Design for Interior Girder — Strength Limit State I

| Span | Location ($x/L$) | $A_{ps}$ mm² | $d_p$ mm | $A_s$ mm² | $d_s$ mm | $b$ mm | $c$ mm | $f_{ps}$ Mpa | $d_e$ mm | $a$ mm | $\phi M_n$ Mpa | $M_u$ kN-m |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.0 | | 32.16 | 0 | 72.06 | 104 | 7.14 | 253.2 | 32.16 | 6.07 | 5,206 | 0 |
| | 0.1 | | 46.09 | 0 | 72.06 | 104 | 7.27 | 258.1 | 46.09 | 6.18 | 7,833 | 4,009 |
| | 0.2 | | 56.04 | 0 | 72.06 | 104 | 7.33 | 260.1 | 56.04 | 6.23 | 9,717 | 6,820 |
| | 0.3 | | 61.54 | 0 | 72.06 | 104 | 7.35 | 261.0 | 61.54 | 6.25 | 10,759 | 8,469 |
| | 0.4 | | 64.00 | 0 | 72.06 | 104 | 7.36 | 261.3 | 64.00 | 6.26 | 11,226 | 9,012 |
| 1 | 0.5 | 8.47 | 62.29 | 0 | 72.06 | 104 | 7.36 | 261.1 | 62.29 | 6.25 | 10,903 | 8,494 |
| | 0.6 | | 57.20 | 0 | 72.06 | 104 | 7.34 | 260.3 | 57.20 | 6.24 | 9,937 | 6,942 |
| | 0.7 | | 48.71 | 0 | 72.06 | 104 | 7.29 | 258.7 | 48.71 | 6.20 | 8,328 | 4,392 |
| | 0.8 | | 38.20 | 0 | 71.06 | 82.5 | 21.19 | 228.1 | 38.20 | 18.01 | –4,965 | –1.397 |
| | 0.9 | | 53.48 | 0 | 71.06 | 82.5 | 23.36 | 237.0 | 53.48 | 19.86 | –7,822 | –5,906 |
| | 1.0 | | 62.00 | 0 | 71.06 | 104 | 8.13 | 261.0 | 62.00 | 6.25 | –10,848 | –10,716 |

**TABLE 10.22** Factored Shear for an Interior Girder

| Span | Location ($x/L$) | $V_{DC1}$ (kN) Dead Load 1 | $V_{DC2}$ (kN) Dead Load 2 | $V_{DW}$ (kN) Wearing Surface | $V_{LL+IM}$ (kN) Envelopes | $M_{LL+IM}$ (kN-m) Associated | $V_{ps}$ (kN) P/S | $V_u$ (kN) | $M_u$ (kN-m) Associated |
|---|---|---|---|---|---|---|---|---|---|
| | 0.0 | 602.8 | 50.9 | 68.3 | 497.0 | 0.0 | 62.2 | 1762.0 | 0 |
| | 0.1 | 398.7 | 37.1 | 49.8 | 416.1 | 1997.4 | 62.2 | 1342.5 | 7,128 |
| | 0.2 | 249.0 | 23.3 | 31.3 | 340.7 | 3270.3 | 62.2 | 996.5 | 11,838 |
| | 0.3 | 99.3 | 9.5 | 12.8 | 271.9 | 3915.3 | 62.2 | 661.6 | 14,446 |
| | 0.4 | –50.4 | –4.3 | –5.8 | –213.9 | 3228.4 | 62.2 | –366.6 | 13,780 |
| 1 | 0.5 | –200.1 | –18.1 | –24.3 | –277.3 | 3271.7 | 62.2 | –692.6 | 13,270 |
| | 0.6 | –349.8 | –31.9 | –42.8 | –340.5 | 2771.1 | 62.2 | –1018.2 | 10,797 |
| | 0.7 | –499.6 | –45.7 | –61.3 | –404.4 | 1955.7 | 62.2 | –1345.0 | 6,742 |
| | 0.8 | –649.3 | –59.5 | –79.8 | –468.4 | 689.4 | 62.2 | –1671.9 | 879 |
| | 0.9 | –799.0 | –73.3 | –98.3 | –529.5 | –945.3 | 62.2 | –1994.0 | –6,655 |
| | 1.0 | –971.2 | –84.9 | –113.9 | –580.6 | –1849.7 | 62.2 | –2319.5 | –13,110 |

*Note:* $V_v = 0.95[1.25(V_{DC1} + V_{DC2}) + 1.5V_{DW} + 1.75V_{LL+IM}] + V_{ps}$.

**TABLE 10.23** Shear Strength Design for Interior Girder Strength Limit State I

| Span | Location ($x/L$) | $d_v$ (mm) | $\gamma'$ (rad) | $V_p$ (kN) | $v/f_c'$ | $\varepsilon_x$ (1000) | $\theta$ (°) | $\beta$ | $V_c$ (kN) | $S$ (mm) | $\phi V_n$ (kN) | $\lvert V_u \rvert$ (kN) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.0 | 1382 | 0.085 | 606 | 0.133 | –0.256 | 21.0 | 2.68 | 428 | 100 | 1860 | 1762 |
| | 0.1 | 1382 | 0.064 | 459 | 0.101 | –0.382 | 27.0 | 5.60 | 894 | 300 | 1513 | 1342 |
| | 0.2 | 1382 | 0.043 | 309 | 0.078 | –6.241 | 33.0 | 2.37 | 378 | 200 | 1036 | 996 |
| | 0.3 | 1503 | 0.021 | 156 | 0.052 | –6.299 | 38.0 | 2.10 | 365 | 300 | 753 | 662 |
| | 0.4 | 1555 | 0.000 | 0 | 0.036 | –6.357 | 36.0 | 2.23 | 400 | 600 | 511 | 367 |
| 1 | 0.5 | 1503 | 0.021 | 159 | 0.055 | –6.415 | 36.0 | 2.23 | 387 | 400 | 710 | 693 |
| | 0.6 | 1382 | 0.043 | 320 | 0.080 | –6.473 | 30.0 | 2.48 | 396 | 200 | 1076 | 1018 |
| | 0.7 | 1382 | 0.064 | 482 | 0.099 | –0.401 | 27.0 | 5.63 | 899 | 300 | 1538 | 1345 |
| | 0.8 | 1382 | 0.085 | 639 | 0.120 | –0.398 | 23.5 | 6.50 | 1038 | 300 | 1813 | 1672 |
| | 0.9 | 1382 | 0.091 | 670 | 0.152 | –6.372 | 23.5 | 3.49 | 557 | 100 | 2017 | 1994 |
| | 1.0 | 1502 | 0.000 | 0 | 0.233 | –6.280 | 36.0 | 1.00 | 173 | 40 | 2343 | 2319 |

where $\phi$ is shear resistance factor 0.9 and $V_{ps}$ is the secondary shear due to prestress. Factored shear demands $V_u$ for the interior girder are calculated in Table 10.22. To determine the effective web width, assume that the VSL post-tensioning system of 5 to 12 tendon units [VLS, 1994] will be used with a grouted duct diameter of 74 mm. In this example, $b_v = 300 - 74/2 = 263$ mm. Detailed calculations of the shear resistance $\phi V_n$ (using two-leg #15M stirrups $A_v = 400$ mm²) for Span 1 are shown in Table 10.23. The results for Span 2 are similar to Span 1 and the calculations are not repeated for this example.

## References


1. Lin, T. Y. and Burns, N. H., *Design of Prestressed Concrete Structure*, 3rd ed., John Wiley & Sons, New York, 1981.
2. Nawy, E. G., *Prestressed Concrete: A Fundamental Approach*, 2nd ed., Prentice-Hall, Englewood Cliffs, NJ, 1996.
3. Collins, M. P. and Mitchell, D., *Prestressed Concrete Structures*, Prentice-Hall, Englewood Cliffs, NJ, 1991.
4. AASHTO, *AASHTO LRFD Bridge Design Specifications*, 1st ed., American Association of State Highway and Transportation Officials, Washington, D.C., 1994.
5. PCI, *PCI Design Handbook – Precast and Prestressed Concrete*, 3rd ed., Prestressed Concrete Institute, Chicago, IL, 1985.
6. Eubunsky, I. A. and Rubinsky, A., A preliminary investigation of the use of fiberglass for prestressed concrete, *Mag. Concrete Res.*, Sept., 71, 1954.
7. Wines, J. C. and Hoff, G. C., Laboratory Investigation of Plastic — Glass Fiber Reinforcement for Reinforced and Prestressed Concrete, *Report 1,* U.S. Army Corps of Engineers, Waterway Experimental Station, Vicksburg, MI, 1966.
8. Wines, J. C., Dietz, R. J., and Hawly, J. L., Laboratory Investigation of Plastic — Glass Fiber Reinforcement for Reinforced and Prestressed Concrete, *Report 2,* U.S. Army Corps of Engineers, Waterway Experimental Station, Vicksburg, MI, 1966.
9. Iyer, S.I. and Anigol, M., Testing and evaluating fiberglass, graphite, and steel prestressing cables for pretensioned beams, in *Advanced Composite Materials in Civil Engineering Structures*, Iyer, S. I. and Sen, R., Eds., ASCE, New York, 1991, 44.
10. Miesseler, H. J. and Wolff, R., Experience with fiber composite materials and monitoring with optical fiber sensors, in *Advanced Composite Materials in Civil Engineering Structures*, Iyer, S. I. and Sen, R., Eds., ASCE, New York, 1991, 167–182.
11. Kim, P. and Meier, U., CFRP cables for large structures, in *Advanced Composite Materials in Civil Engineering Structures*, Iyer, S. I. and Sen, R., Eds., ASCE, New York, 1991, 233–244.
12. PTI, *Post-Tensioning Manual*, 3rd ed., Post-Tensioning Institute, Phoenix, AZ, 1981.
13. FHWA, *Standard Plans for Highway Bridges, Vol. I, Concrete Superstructures*, U.S. Department of Transportation, FHWA, Washington, D.C., 1990.
14. ACI, *Building Code Requirements for Structural Concrete (ACI318-95) and Commentary* (ACI318R-95), American Concrete Institute, Farmington Hills, MI, 1995.
15. CEB-FIP, *Model Code for concrete structures.* (MC-90). Comité Euro-international du Béton (CEB)-Fédération Internationale de la précontrainte (FIP) (1990). Thomas Telford, London, U.K. 1993.

Sauvageot, G. "Segmental Concrete Bridges."
*Bridge Engineering Handbook.*
Ed. Wai-Fah Chen and Lian Duan
Boca Raton: CRC Press, 2000

# 11

# Segmental Concrete Bridges




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## 11.1 Introduction

Before the advent of segmental construction, concrete bridges would often be made of several precast girders placed side by side, with joints between girders being parallel to the longitudinal axis of the bridge. With the modern segmental concept, the segments are slices of a structural element between joints which are perpendicular to the longitudinal axis of the structure.

When segmental construction first appeared in the early 1950s, it was either cast in place as used in Germany by Finsterwalder et al., or precast as used in France by Eugène Freyssinet and Jean Muller. The development of modern segmental construction is intertwined with the development of balanced cantilever construction.

By the use of the term *balanced cantilever construction,* we are describing a phased construction of a bridge superstructure. The construction starts from the piers cantilevering out to both sides in such a way that each phase is tied to the previous ones by post-tensioning tendons, incorporated into the permanent structure, so that each phase serves as a construction base for the following one.

The first attempts to use balanced cantilever construction, in its pure form, were made by Baumgart, who in 1929 built the Río Peixe Bridge in Brazil in reinforced concrete, casting the 68-m-long main span in free cantilevering. The method did not really prosper, however, until the post-tensioning technique had been sufficiently developed and generally recognized to allow crack-free concrete cantilever construction.

From 1950, several large bridges were built in Germany with the use of balanced cantilever construction with a hinge at midspan, using cast-in-place segments, such as

- Moselbrücke Koblenz, 1954: Road bridge, 20 m wide, with three spans of 101, 114, 123 m plus short ballasted end spans hidden in large abutments; the cross section is made up of twin boxes of variable depth, connected by the top slab.
- Rheinbrücke Bendorf, 1964: Twin motorway bridges, 1,031 m long, with three main river spans of 71, 208, 71 m, built-in free cantilever construction with variable depth box sections.

In France, the cantilever construction took a different direction, emphasizing the use of precast segments.

Precast segments were used by Eugène Freyssinet for construction of the well-known six bridges over the Marne River in France (1946 to 1950). The longitudinal frames were assembled from precast segments, which were prestressed vertically and connected by dry-packed joints and longitudinal post-tensioning tendons. Precast segments were also used by Jean Muller for the execution of a girder bridge in upstate New York, where longitudinal girders were precast in three segments each, which were assembled by dry-packed joints and longitudinal post-tensioning tendons.

From 1960, Jean Muller systematically applied precast segments to cantilever construction of bridges. It is characteristic for precast segmental construction, in its purest form, that segments are match cast, which means that each segment is cast against the previous one so that the end face of one segment will be an imprint of the neighbor segment, ensuring a perfect fit at the erection. The early milestones were as follows:

- Bridge over the Seine at Choisy-le-Roi in France, 1962: Length 37+55+37 = 130 m; the bridge is continuous at midspan, with glued joints between segments (first precast segmental bridge).
- Viaduc d'Oleron in France, 1964 to 1966: Total length 2862 m, span lengths generally 79 m, with hinges in the quarterpoint of every fourth span; the segments were cast on a long bench (long-line method); erection was by self-launching overhead gantry (first large-scale, industrialized precast bridge construction).

In the same period, precast segmental construction was adopted by other designers for bridge construction with cast-in-place joints. Some outstanding structures deserve mention:

- Ager Brücke in Austria, 1959 to 1962: Precast segments placed on scaffold, cast-in-place joints.
- Río Caroni in Venezuela, 1962 to 1964: Bridge with multiple spans of 96-m each. Precast segments 9.2 m long, were connected by 0.40-m-wide cast-in-place joints to constitute the 480-m-long bridge deck weighing 8400 tons, which was placed by incremental launching with temporary intermediate supports.
- Oosterschelde Bridge in The Netherlands, 1962 to 1965: Precast segmental bridge with a total length of 5 km and span lengths of 95 m; the precast segments are connected by cast-in-place, 0.4-m-wide joints and longitudinal post-tensioning.

Since the 1960s, the construction method has undergone refinements, and it has been developed further to cover many special cases, such as progressive construction of cantilever bridges, span-by-span construction of simply supported or continuous spans, and precast-segmental construction of frames, arches, and cable-stayed bridge decks.

In 1980, precast segmental construction was applied to the Long Key and Seven Mile Bridges in the Florida Keys in the United States. The Long Key Bridge has 100 spans of 36 m each, with continuity in groups of eight spans. The Seven Mile Bridge has 270 spans of 42 m each with continuity in groups of seven spans. The spans were assembled from 5.6-m-long precast segments placed on erection girders and made self-supporting by the stressing of longitudinal post-tensioning tendons. The construction method became what is now known as span-by-span construction.

Comparing cast-in-place segmental construction with precast segmental construction, the following features come to mind:

- Cast-in-place segmental construction is a relatively slow construction method. The work is performed *in situ,* i.e., exposed to weather conditions. The time-dependent deformations of the concrete become very important as a result of early loading of the young concrete. This method requires a relatively low degree of investment (travelers).
- Precast segmental construction is a fast construction method determined by the time required for the erection. The major part of the work is performed in the precasting yard, where it can be protected against inclement weather. Precasting can start simultaneously with the foundation work. The time-dependent deformations of the concrete become less important, as the concrete may have reached a higher age by the time the segments are placed in the structure. This method requires relatively important investments in precasting yard, molds, lifting gear, transportation, and erection equipment. Therefore, this method requires a certain volume of work to become economically viable. Typically, the industrialized execution of the structure leads to higher quality of the finished product.

Since the 1960s, the precast segmental construction method has won widespread recognition and is used extensively throughout the world. Currently, very comprehensive bridge schemes, with more than 20,000 segments in one scheme, are being built as large urban and suburban viaducts for road or rail. It is reasonable to expect that the precast segmental construction method, as introduced by Jean Muller, will contribute extensively to meet the infrastructure needs of humankind well into the next millennium.

## 11.2 Balanced Cantilever Girder Bridges

### 11.2.1 Overview

Balanced cantilever segmental construction for concrete box-girder bridges has long been recognized as one of the most efficient methods of building bridges without the need for falsework. This method has great advantages over other forms of construction in urban areas where temporary shoring

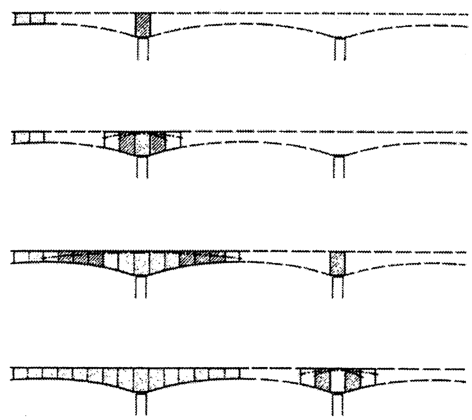


**FIGURE 11.1** Balanced cantilever construction.

would disrupt traffic and services below, in deep gorges, and over waterways where falsework would not only be expensive but also a hazard. Construction commences from the permanent piers and proceeds in a "balanced" manner to midspan (see Figure 11.1). A final closure joint connects cantilevers from adjacent piers. The structure is hence self-supporting at all stages. Nominal out-of-balance forces due to loads on the cantilever can be resisted by several methods where any temporary equipment is reusable from pier to pier.

The most common methods are as follows:

- Monolithic connection to the pier if one is present for the final structure;
- Permanent, if present, or temporary double bearings and vertical temporary post-tensioning;
- A simple prop/tie down to the permanent pile cap;
- A prop against an overhead gantry if one is mobilized for placing segments or supporting formwork.

The cantilevers are usually constructed in 3- to 6-m-long segments. These segments may be cast in place or precast in a nearby purpose-built yard, transported to the specific piers by land, water, or on the completed viaduct, and erected into place. Both methods have merit depending on the specific application.

It is usually difficult to justify the capital outlay for the molds, casting yard, and erection equipment required for precast segmental construction in a project with a deck area of less than 5000 $m^2$. The precasting technique may be viable for smaller projects provided existing casting yard and molds can be mobilized and the segments could be erected by a crane.

## 11.2.2 Span Arrangement and Typical Cross Sections

Typical internal span-to-depth ratios for constant-depth girders are between 18 and 22. However, box girders shallower than 2 m in depth introduce practical difficulties for stressing operations inside the box and girders shallower than 1.5 m become very difficult to form. This sets a minimum economical span for this type of construction of 25 to 30 m. Constant-depth girders deeper than 2.5 to 3.0 m are unusual and therefore for spans greater than 50 m consideration should be given to varying-depth girders through providing a curved soffit or haunches. For haunch lengths of 20 to 25% of the span from the pier, internal span-to-depth ratios of 18 at the pier and as little as 30 at midspan are normally used.

Single-cell box girders provide the most efficient section for casting – these days multicell boxes are rarely used in this method of construction. Inclined webs improve aesthetics but introduce added difficulties in formwork when used in combination with varying-depth girders. The area of

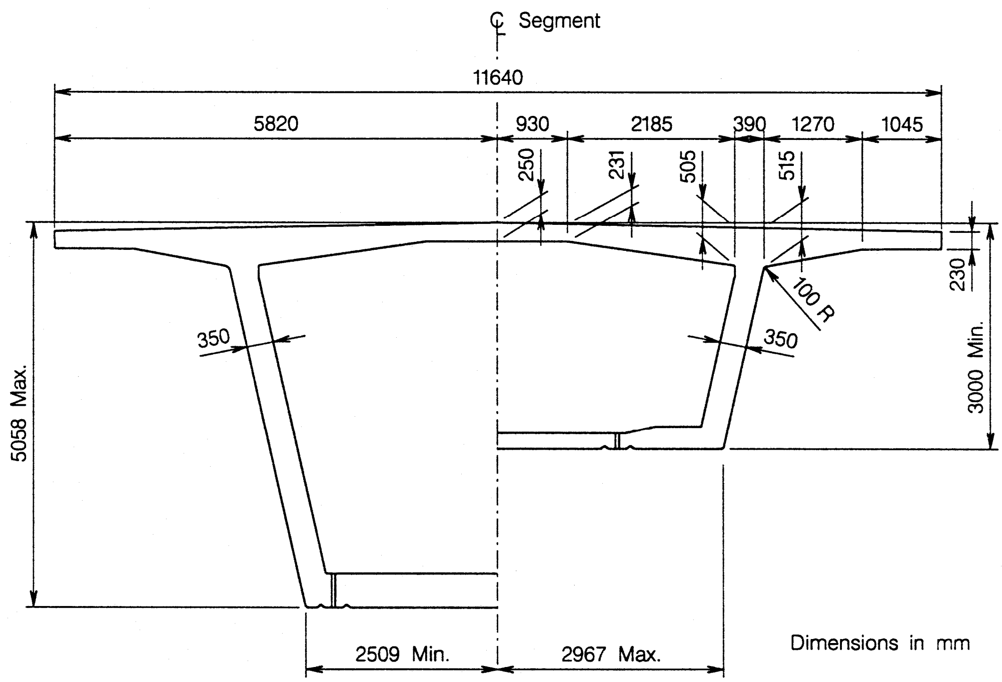



**FIGURE 11.2** Typical cross section of a varying-depth girder for a 93-m span.

the bottom slab at the pier is determined by the modulus required to keep bottom fiber compressions below the allowable maximum at this location. In the case of internal tendons local haunches are used at the intersection of the bottom slab and the webs to provide sufficient space for accommodating the required number of tendon ducts at midspan. The distance between the webs at their intersection with the top slab is determined by achieving a reasonable balance between the moments at this node. Web thicknesses are determined largely by shear considerations with a minimum of 250 mm when no tendon ducts internal to the concrete are present and 300 mm in other cases. Figure 11.2 shows the typical dimensions of a varying-depth box girder.

## 11.2.3 Cast-in-Place Balanced Cantilever Bridges

The cast-in-place technique is preferred for long and irregular span lengths with few repetitions. Bridge structures with one long span and two to four smaller spans usually have a varying-depth girder to carry the longer span, hence making the investment in a mold which accommodates varying-depth segments even more uneconomical. A prime example of application of balanced cantilevering in an urban environment to avoid disruption to existing road services below is the structure of the Bangkok Light Rail Transit System, where it crosses the Rama IV Flyover (see Figure 11.3). The majority of the 26-km viaduct structure is precast, but at this intersection a 60-m span was required to negotiate the existing road at a third level with the flyover in service below. A three-span, 30-, 60-, 30-m structure was utilized with a box-girder depth of 3.5 m at the pier and 2.0 m at midspan and a parabolic curved soffit. The flyover was only disrupted a few nights during concrete placement of the segments directly above as a precaution.

In the above example, the side spans were constructed by balanced cantilevering; however, ideal arrangement of spans normally provides end spans which are greater than half the internal spans. These, therefore, cannot be completed by balanced cantilevering, and various techniques are used to reach the abutments. The most economical and common method is the use of falsework; however,

**FIGURE 11.3** Construction of the Bangkok Transit System over Rama IV Flyover, Thailand.

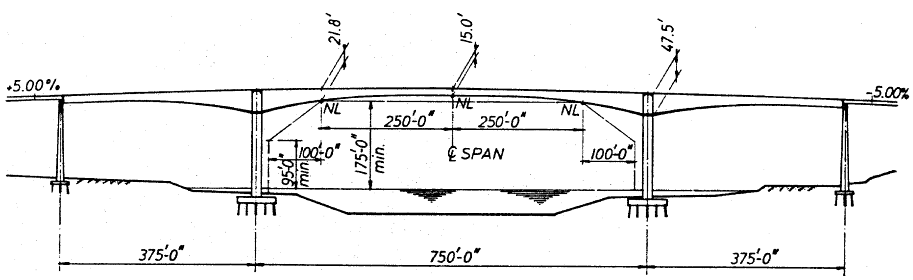



**FIGURE 11.4** Houston Ship Channel Bridge, United States.

should the scale of the project justify use of an auxiliary truss to support the formwork during balanced cantilevering, then this could also be used for completing the end spans.

Another example of a cast-in-place balanced cantilever bridge is the Houston Ship Channel Bridge where a three-span, 114-, 229-, 114-m structure was used over the navigation channel (see Figure 11.4). A three-web box girder carrying four lanes of traffic is fixed to the main piers to make the structure a three-span rigid frame. Unusual span-to-depth ratios were dictated by the maximum allowable grade of the approach viaducts and the clearance required for the ship channel. The soffit was given a third-degree parabolic profile to increase the structural depth near the piers in order to compensate for the very limited height of the center portion of the main span. Maximum depth at the pier is 14.6 m, a span-to-depth ratio of 15.3 to enable a minimum depth at midspan of 4.6 m, and a span-to-depth ratio of 49. The box girder is post-tensioned in three dimensions: four 12.7-mm strands at 600-mm centers transversely in the top slab as well as longitudinal and vertical post-tensioning in the webs.

### 11.2.4 Precast Balanced Cantilever Bridges

Extending segmental construction to balanced cantilevering, and hence eliminating the need for falsework as well as substantial increases in the rate of construction, requires a huge leap in the technology of precasting: match casting. The very first bridge that benefited from match-casting technology was the Choisy-le-Roi Bridge near Paris, designed by Jean Muller and completed in 1964. This method has since grown in popularity and sophistication and is used throughout the world today. The essential feature of match casting is that successive segments are cast against the adjoining segment in the correct relative orientation with each other starting from the first segment away from the pier. The segments are subsequently erected on the pier in the same order, and hence no adjustments are necessary between segments during assembly. The joints are either left dry or made of a very thin layer of epoxy resin, which does not alter the match-cast geometry. Post-tensioning may proceed as early as practicable since there is no need for joints to cure.

The features of this method that provide significant advantages over the cast-in-place method, provided the initial investment in the required equipment is justified by the scale of the project, are immediately obvious and may be listed as follows:

- Casting the superstructure segments may be started at the beginning of the project and at the same time as the construction of the substructure. In fact, this is usually required since the speed of erection is much faster than production output of the casting yard and a stockpile of segments is necessary before erection begins.
- Rate of erection is usually 10 to 15 times the production achieved by the cast-in-place method. The time required for placing reinforcement and tendons and, most importantly, the waiting time for curing of the concrete is eliminated from the critical path.
- Segments are produced in an assembly-line factory environment, providing consistent rates of production and allowing superior quality control. The concrete of the segments is matured, and hence the effects of shrinkage and creep are minimized.

The success of this method relies heavily on accurate geometry control during match casting as the methods available for adjustments during erection offer small and uncertain results. The required levels of accuracy in surveying the segments match-cast against each other are higher than in other areas of civil engineering in order to assure acceptable tolerances at the tip of the cantilevers.

The size and weight of precast segments are limited by the capacity of transportation and placing equipment. For most applications segment weights of 40 to 80 tons are the norm, and segments above 250 tons are seldom economical. An exception to the above is the recent example of the main spans of the Confederation Bridge where complete 192.5-m-long balanced cantilevers weighing 7500 tons were lifted into place using specialized equipment (see Figure 11.5). The 250-m main spans of this fixed link in Atlantic Canada, connecting Cape Tormentine, New Brunswick, and Borden, Prince Edward Island, were constructed by a novel precasting method. The scale of the project was sufficiently large to justify precast segmental construction; however, adverse weather and site conditions provided grounds for constructing the balanced cantilevers, 14 m deep at the piers, in a similar method to cast-in-place construction but in a nearby casting yard. The completed balanced cantilevers were then positioned atop completed pier shafts in a single operation. A light template match-cast against the base of the pier segment allowed fast and accurate alignment control on the spans.

### 11.2.5 Loads on Substructure

The methods for supporting the nominal out-of-balance forces during balanced cantilevering were described earlier. The following forces should be considered in calculating the possible out-of-balance forces:

- In precast construction, one segment out of balance and the loss of a segment on the balancing cantilever as an ultimate condition;
- In precast construction, presence of a stressing platform (5 to 10 tons) on one cantilever only or the loss of the form traveler in the case of cast-in-place construction;

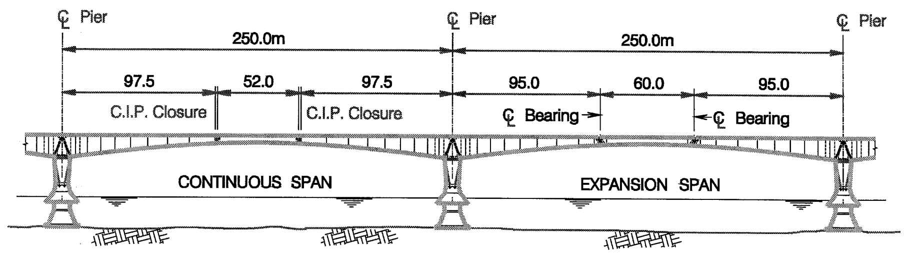



**FIGURE 11.5** Main spans of the Confederation Bridge, Canada.

- Live loading on one side of 1.5 kN/m$^2$;
- Wind loading during construction;
- The possibility of one cantilever having a 2.5% higher dead weight than the other.

The loads on the substructure do not usually govern the design of these elements provided balanced cantilever construction is considered at the onset of the design stage. The out-of-balance forces may provide higher temporary longitudinal moments than for the completed structure; however, in the case of a piled foundation, this usually governs the arrangement and not the number of the piles.

## 11.2.6 Typical Post-Tensioning Layout

Post-tensioning tendons may be internal or external to the concrete section, but inside the box girder, housed in steel pipes, or both. External post-tensioning greatly simplifies the casting process and the reduced eccentricities available compared with internal tendons are normally compensated by lower frictional losses along the tendons and hence higher forces.

The choice of the size of the tendons must be made in relation to the dimensions of the box-girder elements. A minimum number of tendons would be required for the balanced cantilevering process, and these may be anchored on the face of the segments, on internal blisters, or a combination of both. After continuity of opposing cantilevers is achieved, the required number of midspan tendons may be installed across the closure joint and anchored on internal bottom blisters. Depending on the arrangement and length of the spans, economies may be made by arranging some of the tendons to cross two or more piers, deviating from the top at the piers to the bottom at midspan, thereby reducing the number of anchorages and stressing operations. External post-tensioning is best used for these continuity tendons which would allow longer tendon runs due to the reduced frictional losses. Where the tendons are external to the concrete elements, deviators at piers, quarterspan, and midspan are used to achieve the required profile. An example of a typical internal post-tensioning layout is shown in Figure 11.6.

## 11.2.7 Articulation and Hinges

The movements of the structure under the effects of cyclic temperature changes, creep, and shrinkage are traditionally accommodated by provision of halving joint-type hinges at the center of various spans. This practice is now discontinued due to the unacceptable creep deformations that occur at these locations. If such hinges are used, these are placed at contraflexure points to minimize the effects of long-term deflections. A development on simple halving joints is a moment-resisting joint, which allows longitudinal movements only. All types of permanent hinges that are more easily exposed to the elements of water and salt from the roadway provide maintenance difficulties and should be eliminated or reduced wherever possible.

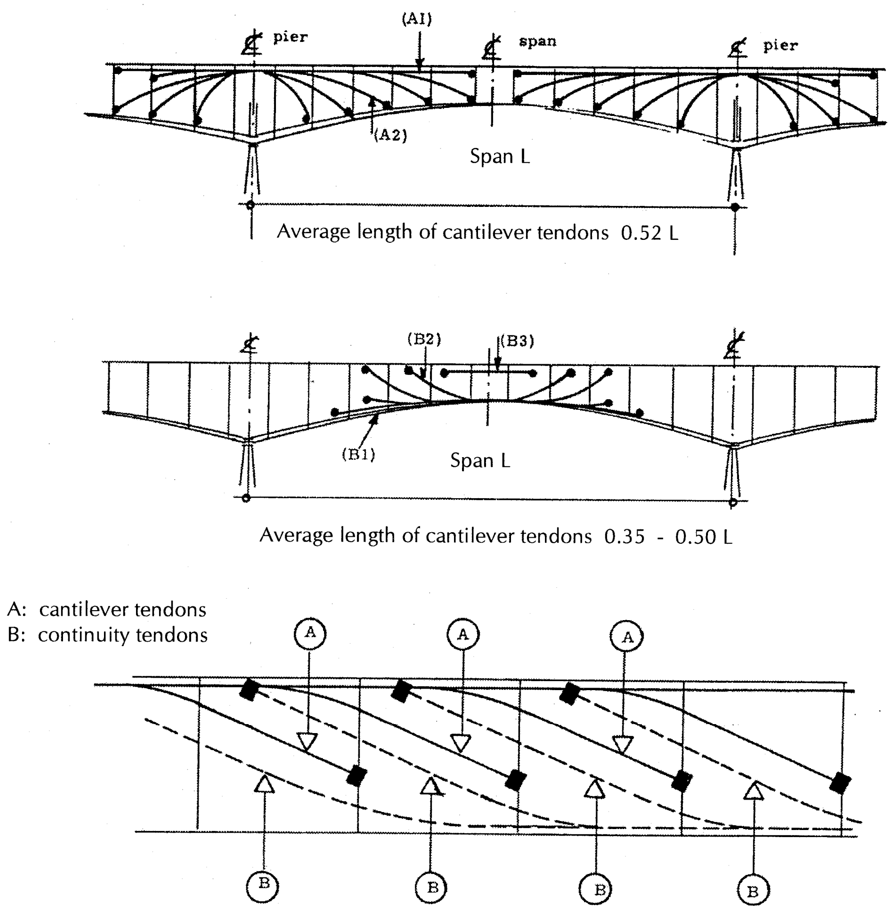



**FIGURE 11.6** Typical post-tensioning layout for internal tendons.

If the piers are sufficiently flexible, then a fully continuous bridge may be realized with joints at abutments only. When seismic considerations are not a dominant design feature and a monolithic connection with the pier is not essential, bearings atop of the piers are preferred as they reduce maintenance and replacement cost. In addition, it will allow free longitudinal movements of the deck. A monolithic connection or a hinged bearing at one or more piers would provide a path for transmitting loads to suitable foundation locations.

## 11.3 Progressive and Span-by-Span Constructed Bridges

### 11.3.1 Overview

In progressive or span-by-span construction methods, construction starts at one end and proceeds continuously to the other end. Generally, progressive construction is used where access to the ground level is restricted either by physical constraints or by environmental concerns. Deck variable cross sections and span lengths up to 60 m are easily accommodated. In contrast, span-by-span precast segmental construction is used typically where speed of construction is of major concern. Span lengths up to 50 m are most economical as it minimizes the size of the erection equipment.

**FIGURE 11.7** Fréburge Viaduct, France—erection with movable stay tower.

### 11.3.2 Progressive Construction

The progressive method step-by-step erection process is derived from cantilever construction, where segments are placed in a successive cantilever fashion. The method is valid for both precast and cast-in-place segments. Due to the excessively high bending moments the cantilever deck has to resist over the permanent pier during construction, either a temporary bent or a temporary movable tower–stay assembly would have to be used. As shown in Figure 11.7, for precast construction using a temporary tower and stay system, segments are transported over the erected portion of the bridge to the end of the completed portion. Using some type of lifting equipment, e.g., a swivel crane, the segment is placed in position and supported temporarily either by post-tensioning to the previous segment or by stays from a tower.

The advantages of this methods are

- Operations are conducted at deck level.
- Reactions on piers are vertical.
- The method can easily accommodate variable horizontal curves.

The disadvantages are

- The first span is erected on falsework.
- Forces in the superstructure during erection are different from those in the completed structure.
- The piers are temporarily subjected to higher reactions from dead load than in the final structure because of the length of the cantilever erected. However, considering the other loads in the final structure, this case is not generally controlling the pier design.

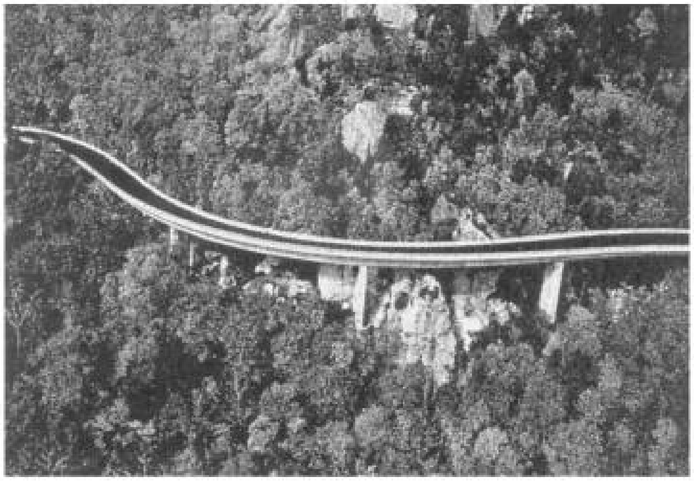


**FIGURE 11.8** Completed Linn Cove Viaduct, United States.

**FIGURE 11.9** Linn Cove Viaduct — pier being constructed from the deck level.

The Linn Cove Viaduct (1983) on the Blue Ridge Parkway in North Carolina shown in Figure 11.8, demonstrated the potential progressive placement when one is forced to overcome extreme environmental and physical constraints. Because access at the ground level was limited, the piers were constructed from the deck level, at the tip of an extended cantilever span. Temporary cable stays could not be used due to the extreme horizontal curvature in the bridge. Instead, temporary bent supports were erected between permanent piers. Figure 11.9 shows one temporary support in the background while a permanent precast pier is being erected from the deck level.

**FIGURE 11.10** Lifting completed span of the Seven Mile Bridge, Florida, using an *overhead truss.*

### 11.3.3 Span-by-Span Construction

As with balanced cantilever and progressive placement, span-by-span construction activity is performed primarily at the deck level and typically implemented for long viaducts having numerous, but relatively short spans, e.g., <50 m. It was initially developed as a cast-in-place method of construction, on formwork, with construction joints at joint of contraflexure. The form traveler is supported either on the bridge piers, on the edge of the previously erected span and the next pier or, at times, even at the ground level. With the precast segmental method, segments are placed and adjusted on a steel erection girder spanning from pier to pier, then post-tensioned together in one operation. Although both the cast-in-place and the precast span-by-span construction methods continue to be used, precast segmental has become the method of choice for most applications.

*Long Key and Seven Mile Bridges, United States*: Two early applications of the precast span-by-span method are the Long Key Bridge (1977) and the Seven Mile Bridge (1978), both located in the Florida Keys. The shorter, 3000-m, 100-span Long Key Bridge is the first application of precast span-by-span construction with dry segment joints and external post-tensioning in the United States.

Essentially the same bridge design concept as Long Key — only much longer — the 10,931-m, 270-span Seven Mile Bridge utilized rectangular precast piers and an overhead truss, as shown in Figure 11.10. The overhead truss allowed easier repositioning from one span to the next one and thus improved overall erection speed.

*Bang Na–Bang Pli–Bang Pakong Expressway,* Thailand: A number of span-by-span highway and rail mega projects have been either completed recently or currently are under construction in Southeast Asia. Probably, the most innovative of these recent applications is the 54,000-m, 1300-span, Bang Na–Bang Pli–Bang Pakong Expressway. The girder supports segment assembly and span installation activities. This erection process can be regarded as "assembly-line" in that there is no requirement for disassembly and reassembly of the erection girder as it travels from pier to pier. The piers, although designed structurally for the construction process, can also be seen to provide an aesthetically pleasing, somewhat "floating," appearance to the six-lane, 27-m-wide box girder. Figure 11.11 shows one of the erection girders as it lifts a segment. With five erection girders erecting a span every 2 days or 780 m of superstructure per week, construction of the viaduct is expected to last approximately 2 years and be completed in 1999, without interruption of traffic below.

*Roize, France*: Another innovative example of span-by-span construction is the 112-m, three-span, prestressed composite truss Roize Bridge (1991) in the French Alps, shown in Figure 11.12. The deck is made of prestressed concrete and steel. Each factory-built tetrahedron module and

**FIGURE 11.11** Bang Na Expressway, Thailand — launching of girder erection.

**FIGURE 11.12** View of the Roize Bridge, France — space truss spans using tetrahedron modules.

precast pretensioned slab is placed on erection beams and adjusted into position. After welding the bottom member joints and casting the closure strips, the modules are post-tensioned together as a completed span. Due to the modular basis, this two-lane bridge represents a new class of super-lightweight, factory-built segments.

*Channel Bridge, United States*: The first precast, prestressed channel bridge in the United States was built in 1974 in San Diego, California, as a pedestrian crossing at San Diego State University. This concept was reused 18 years later as an experimental study for new bridge standards, initially

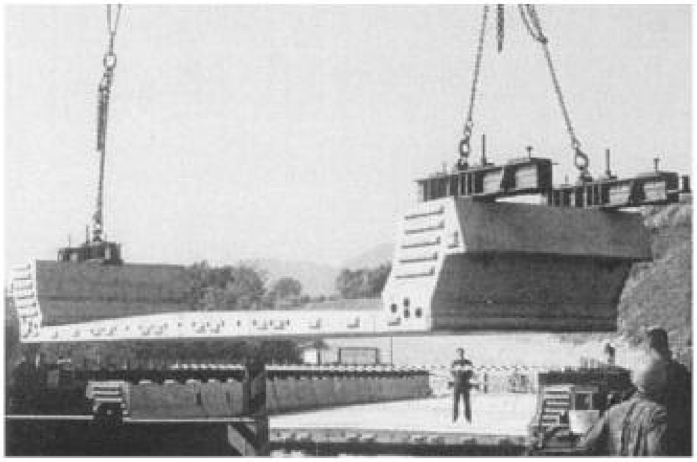


**FIGURE 11.13** Channel Bridge, France, under construction.

by the French Highway Administration. Figure 11.13 shows the 54-m, two-span Champfeuillet Bridge (1992), under construction along the Rhône Alpine Motorway near Grenoble, France. The most innovative aspect of the concept is the use of the concrete parapets as part of the structure. With the primary longitudinal post-tensioning passing through the barriers, an extremely lightweight, shallow section is possible.

Research and implementation of the Channel Bridge, although continuing in Europe, also has begun recently in the United States. Initiated by the Federal Highway Administration (FHWA) and the Highway Innovative Technology Evaluation Center (HITEC), a branch of the Civil Engineering Research Foundation (CERF), at least two applications of the Channel Bridge concept have been completed recently in the United States for the New York State Department of Transportation (NYSDOT).

The primary benefits of the concept are as follows:

- Lightweight, easily placed segments.
- Fast erection times with small investment in erection equipment.
- Increased vertical clearance beneath the superstructure, because the load-carrying members are above the roadway slab, not below.
- A reduction in the number of bridge overpass piers required, which increases safety levels for traffic lanes below.

Span by span, as used today, utilizes post-tensioning tendons outside the concrete, but inside the box girder for ease of precasting and speed of installation together with dry joints, no epoxy, between segments. The post-tensioning tendons are continuous from pier segment to pier segment.

## 11.4 Incrementally Launched Bridges

### 11.4.1 Overview

The incremental launching technique has been used on bridges numbering in the hundreds since its introduction by Professor Fritz Leonhardt in 1961 for the Río Caroni Bridge in Venezuela. It is an effective alternative for the bridge designer to consider when the site meets its particular alignment requirements. The method entails casting the superstructure, or a portion thereof, at a stationary location behind one of the abutments. The completed or partially completed structure is then jacked into place horizontally, i.e., pushed along the bridge alignment. Subsequent segments can then be cast onto the already completed portion and in turn pushed onto the piers. Because all of the casting operations are concentrated at a location easily accessible from the ground, concrete quality of the same level expected from a precasting yard can be achieved. The procedure has the advantage that, like the balanced cantilever technique, it obviates the need for falsework to cast the girder. Moreover, heavy erection equipment, cranes, gantries, and the like, are not necessary, nor is the use of epoxy at segment joints. Usually, the only special equipment required is light steel truss work for a launching nose to reduce the cantilever moments during launching.

### 11.4.2 Special Requirements

There are two peculiarities associated with the technique, which must be appreciated by the designer. The first is that the alignment must be straight or, if it involves curves, the curvature must be constant. The second is that during launching, every section of the girder will be subjected to both the maximum and minimum moments of the span; and the leading cantilever portion will be subjected to slightly higher moments. This second constraint usually leads to slightly deeper sections, on the order of $\frac{1}{15}$ the span, than would otherwise be considered. The girders must also be of constant depth as each section will at sometime be supported on the temporary bearings. Other considerations include the necessity for a large area behind the abutment for the casting operations, the requirement to lift the bridge off of the temporary bearings, and place it on the permanent ones when launching is complete and the need for very careful control of geometry during casting.

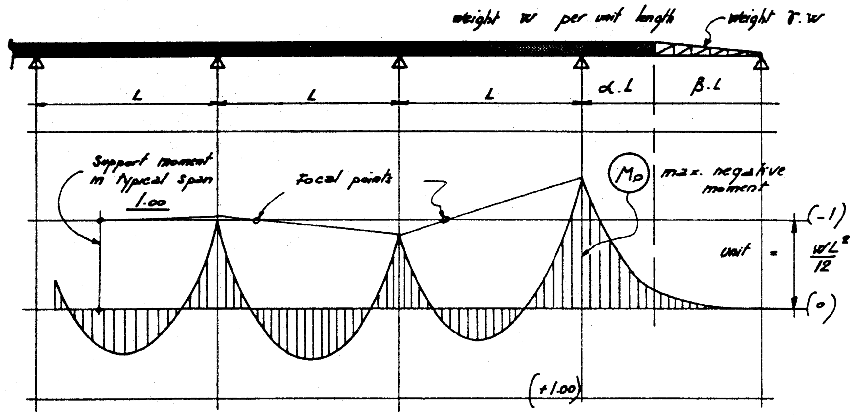
Incremental launching is generally considered for long viaducts with many spans of the same length. Spans up to 100 m can be considered; the requirement for constant-depth girders makes longer spans uneconomical. A single long span in the center of a project can be achieved by launching from both abutments and finishing at the long span with two converging cantilevers. The practical length limit for launching in about 1000 m. Bridges of twice this length can be considered by launching from both abutments.

### 11.4.3 Typical Post-Tensioning Layout

During superstructure launching each section of the girder is subjected to constantly reversing bending moments as it proceeds from temporary support to midspan. Because of the sign change in the applied moments, the efficient use of draped tendons for launching load effects is impossible. The general procedure has therefore been to apply axial prestressing for the launching operation. These tendons are usually straight, being contained in the top and bottom slabs of the girder. The tendons for successive segments must be spliced to these with couplers or stressed in buttresses in an overlapping fashion. This prestressing is subsequently augmented with either draped tendons or short top- and bottom-slab tendons for respective negative and positive moment regions in the completed structure to meet service state requirements. In some instances, permanent draped prestressing has been placed in the configuration required for the final condition, and temporary tendons with an opposing drape are provided to counteract their bending effects during launching. These temporary tendons are then removed when launching is complete.

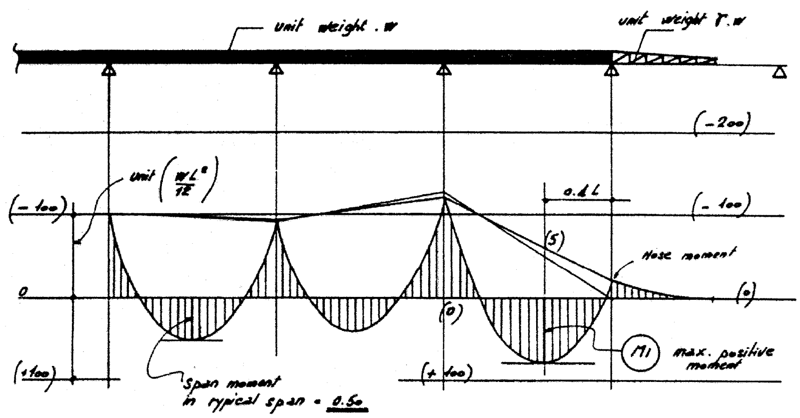
### 11.4.4 Techniques for Reducing Launching Moments

As suggested above, the launching moments in the leading spans, especially the first cantilever span, will be greater than those in the following interior spans. If the girder is simply launched to the first pier with no special provision to reduce these moments, they will in fact be on the order of six times the typical negative moment over a pier. The method used most frequently to overcome this problem has been a light structural-steel launching nose attached to the leading cantilever (see Figures 11.14 and 11.15). This nose supports the girder without the weight penalty of the heavier concrete section. In order to be effective, the nose must be both as light and as stiff as possible.




| α | β | $M_0$ |
|---|---|---|
| 0.20 | 0.80 | 0.82 |
| 0.30 | 0.70 | 1.09 |
| 0.40 | 0.60 | 1.46 |
| 0.50 | 0.50 | 1.95 |
| 1.00 | 0.00 | 6.00 |

**FIGURE 11.14** Critical negative moment during launching with nose. $M_1 = [WL^2/12)\ (6\alpha^3 + 6y)\ (1 - \alpha^3)$. Multiplier $= WL^2/12$. For $y = 0.11$.




| α | β | $M_1$ |
|---|---|---|
| 0.20 | 0.80 | 0.74 |
| 0.30 | 0.70 | 0.79 |
| 0.40 | 0.60 | 0.83 |
| 0.50 | 0.50 | 0.86 |
| 1.00 | 0.00 | 0.93 |

**FIGURE 11.15** Critical positive moment during launching with nose. $M_1 = [(WL^2/12)\ (0.933 - 2.96y\beta^2)$. Multiplier $= WL^2/12$. For $y = 0.11$.

For longer spans, the steel nose is not as effective, and other methods have been employed to reduce launching moments. Temporary piers are a viable solution when ground conditions are such that the foundation costs are relatively modest and the pier height is not too great. If either of these conditions is not found, the cost can escalate rapidly as a temporary pier will be required in every span.

One last method that has been employed successfully is a temporary pylon attached to the deck at the trailing end of the first span which supports stays connected to the leading end. This device is very efficient in reducing the cantilever moment in the leading span; however, it produces an undesirable positive moment when the pylon is at midspan. For this reason, the stays must be equipped with a jack to adjust the stay force as needed during the various stages of the launching operations.

### 11.4.5 Casting Bed and Launching Methods

Segment lengths for incrementally launched bridges are generally greater than for other types of segmental bridges. Typical segment lengths range from 15 to 40 m. Usually, a casting area twice the length of the segment is required for actual casting and the ancillary operations that must be conducted there. The casting bed is generally a significant structure itself, as the strict geometry-control requirements of the technique make settlement of the formwork unacceptable.

Launching has been accomplished in the past either by tendons attached to the girder and horizontal jacks bearing on the abutment or by a horizontal jack bearing on the abutment face connected to a vertical jack which slides on a bearing. The upper surface of the vertical jack is fitted with a friction device to bear on the soffit of the box girder. The vertical jack is inflated to provide the normal force required for transferring the launching force by friction.

## 11.5 Arches, Rigid Frames, and Truss Bridges

### 11.5.1 Arch Bridges

The first step toward the segmental construction of arches was taken shortly after World War I by Eugéne Freyssinet. He employed hydraulic jacks to lift the completed Villeneuve arch from its falsework by applying an internal thrust at its crown. This departure from the classical method of striking the centering to develop the thrust in the arch opened the door to modern arch construction techniques that do not rely on falsework. It also presented the opportunity to reduce the bending moments in the arch by eliminating the dead load bending associated with axial shortening of the ribs.

#### 11.5.1.1 Arches Erected without Falsework

The development of stay-cable and form-traveler technology has made possible the erection of arches in cantilever fashion without a centering supported from below. One early example of this technique was the suite of viaducts built in Caracas, Venezuela, in 1952 (see Figure 11.16). The first quarter of the arch span was supported by light forms which were in turn supported by stay cables attached to a pilaster at the springing of the arch. The crown portion of the arch was then completed with a light centering supported on the already-completed portion of the arch so that no falsework was required in the valley below.

Several variations on this theme were subsequently developed. The methods employed varied, depending on site conditions, from the use of very high pylons with a single group of stays allowing construction of the arch all the way to the crown to those which used the permanent spandrel columns in conjunction with temporary stay diagonals to form a truss. These methods are summarized in Figure 11.17.

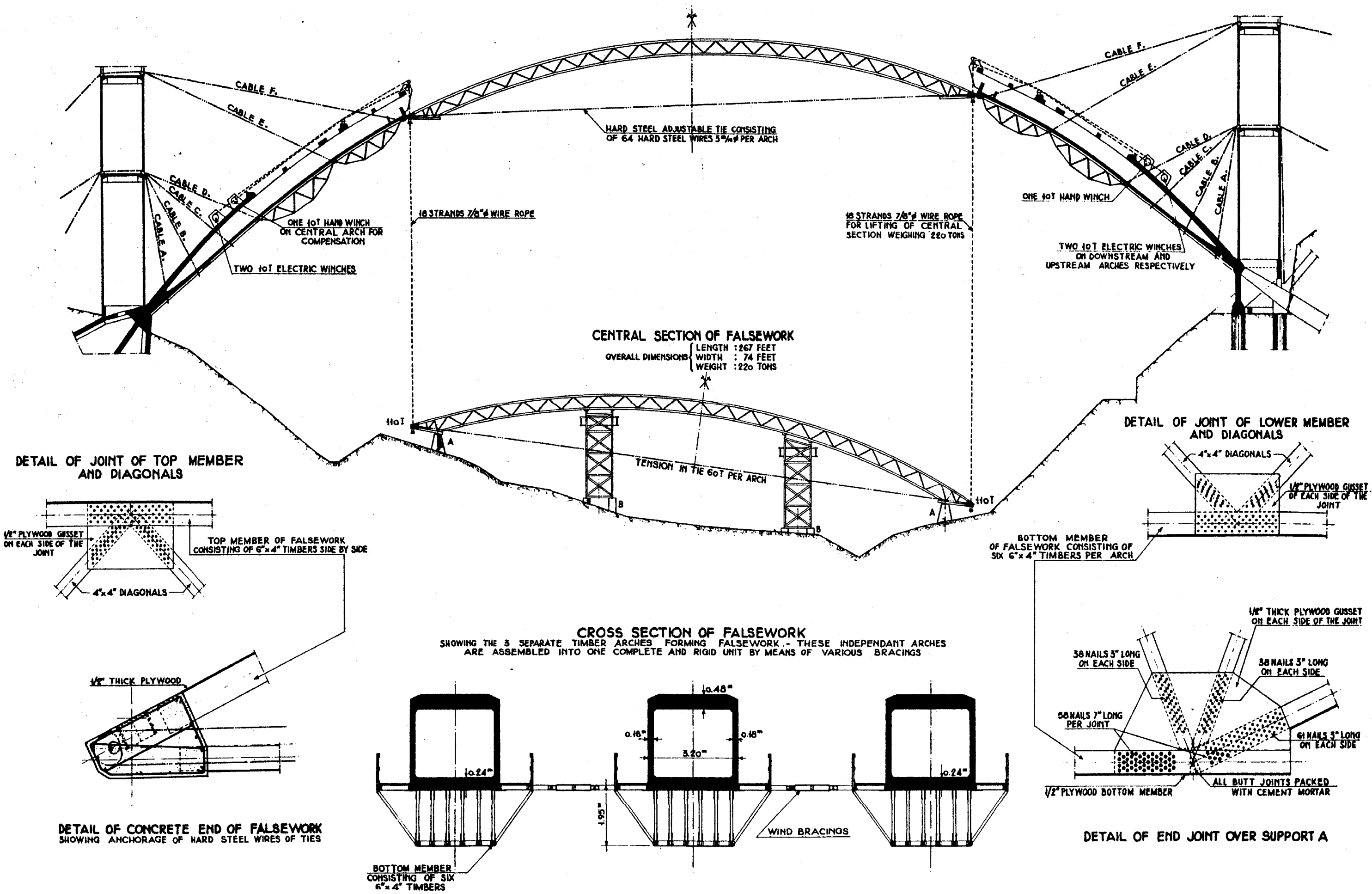



**FIGURE 11.16** Caracas viaducts-erection of center portion of arch falsework.

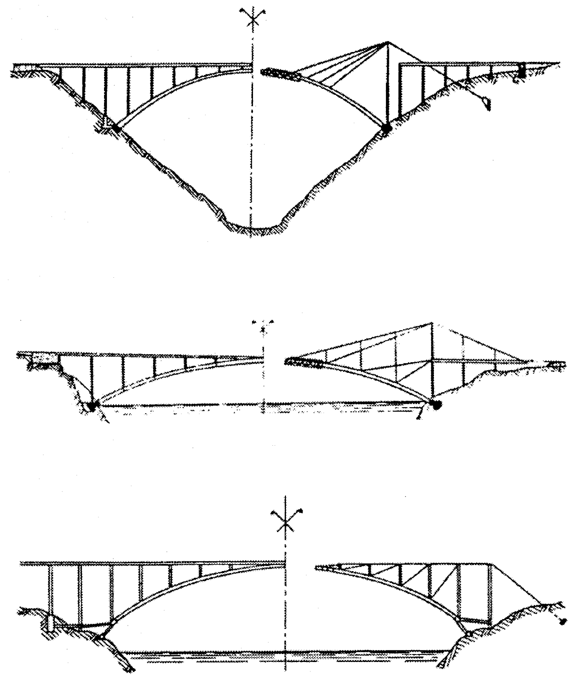


**FIGURE 11.17** Various cantilevers—erection techniques for arches.

#### 11.5.1.2 Precast Arches

The first precast segmental arch bridge was built in France in 1948. This bridge at Luzancy over the Marne River is composed of three box-section arches built up from 2.44-m-long precast segments. The finished span length is 55 m. Because of the severe clearance requirements, the arch has a very unusual span-to-rise ratio of 23 to 1. The segments were connected via 20-mm dry-packed joints and prestressing on the approach behind an abutment, with the resulting rib being moved to its final position by an aerial cableway.

Construction of concrete arches without falsework was employed almost exclusively in conjunction with the cast-in-place cantilever technique until the construction of the Natches Trace Bridge in Tennessee in 1993. This precast arch, which originally was designed for erection on a moveable falsework, was the first precast arch to be erected on stays. The unusual design, which omits spandrel columns, results in a slender appearance. There are two arches: one with a span of 177 m and a rise of 44 m, the other with a span of 141 m and a rise of 31 m. The arch segments are 4.9 m wide and vary in depth from 4 m at the springing to 3 m at the crown (see Figure 11.18).

### 11.5.2 Rigid Frames

Frame bridges can be considered a hybrid of arch and girder forms. They are an appropriate alternative to either of those types for intermediate span lengths. Rigid frame bridges are well suited to segmental construction techniques.

Rigid frame bridges often have some of the same site requirements as arch bridges. They are well suited to valleys and generally will require foundations capable of resisting large horizontal actions. Generally, some form of temporary support will be required until the frame is complete, meaning that construction techniques that eliminate falsework may need slight modification for these structures. One of the most aesthetically convincing applications of the rigid frame is the Bonhomme Bridge in Brittany, France (see Figure 11.19). This slant-leg frame was built using the cast-in-place balanced cantilever technique. Temporary piers were installed below the slant legs to support them

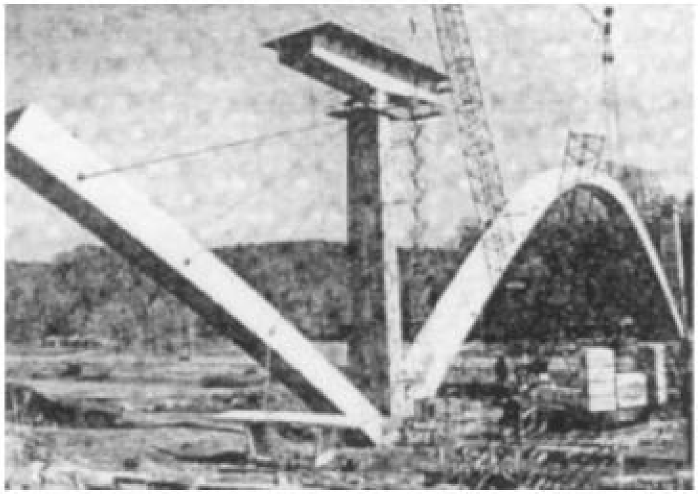


FIGURE 11.18 Natches truss arch—cantilever erection of ribs.

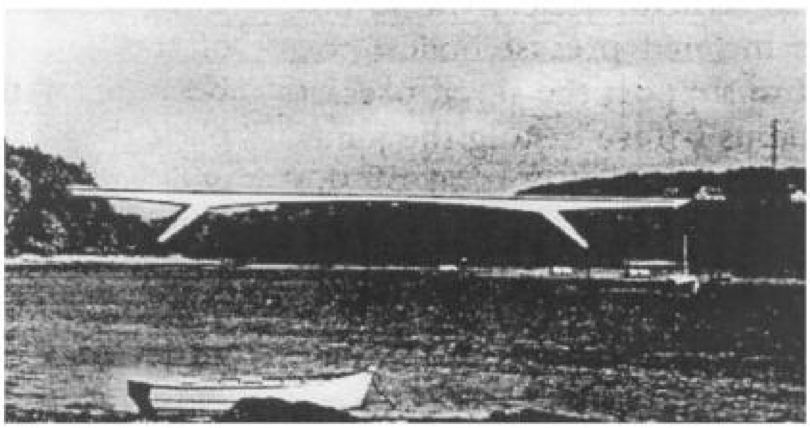


FIGURE 11.19 Bonhomme Bridge in Brittany, France.

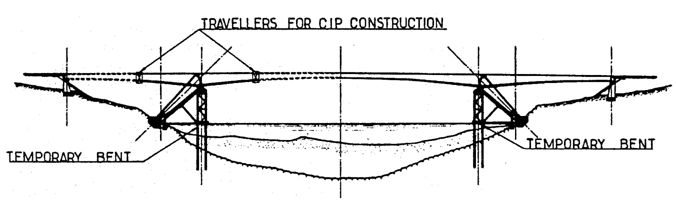



FIGURE 11.20 Temporary support for the Bonhomme Bridge.

before the thrust was developed in the frame (see Figure 11.20). Jacks under the temporary-support piers and at the midspan closure were used to adjust the geometry before closing the span.

### 11.5.3 Segmental Trusses

Although relatively few examples have been built, segmental trusses are interesting, especially for long spans, in that they offer very efficient use of materials. This economy translates directly into lighter elements and smaller loads to be dealt with during construction, as well as reduced material cost.

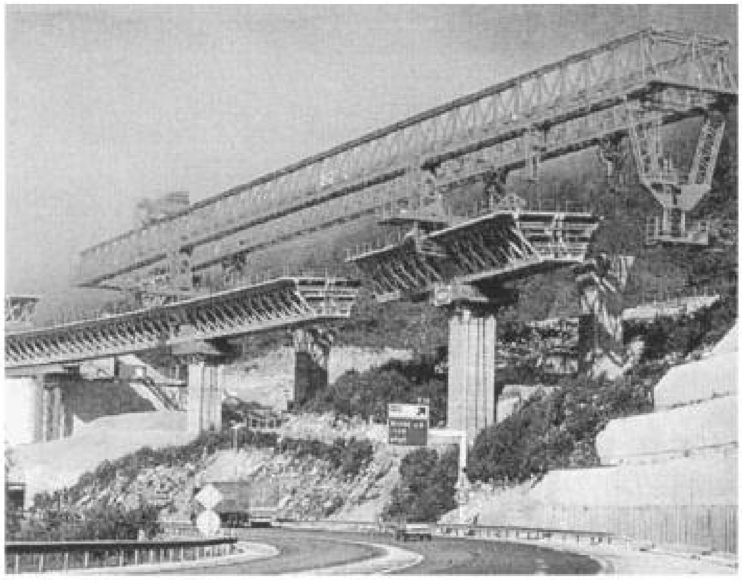


**FIGURE 11.21** Cantilever erection of the Viaduct des Glacièrs, France.

One of the earliest segmental trusses was the Mangfallbrücke in Austria, which was constructed in 1959. It had a total length of 288 m with a maximum span of 108 m and was constructed by the cast-in-place segmental technique in conjunction with temporary piers.

Later examples were developed in precast segmental, of which the Viaduc de Sylans and the Viaduc des Glacieres are the most notable. These sister structures were constructed in balanced cantilever with a self-launching overhead truss (see Figure 11.21). The segments were prestressed in three directions with a combination of external and internal tendons. "X" members for the open webs were precast and subsequently placed in the molds prior to segment casting.

The most recent development in segmental trusses is the composite truss. This concept employs concrete for the top and bottom chords and steel sections for the open webs. In some cases, however, steel is used for the tension chord as well. An excellent example of this type of construction is the bridge over the Roize in France, built with the span-by-span method. This structure was conceived as a truss work of factory-produced steel truss work and precast slabs. These two elements were joined at the site by cast-in-place joints and external tendons (see Figure 11.22). The precast slabs served as the top (compression) chord while a hexagonal steel tube served as the bottom chord. The resulting structure is equally viable as the deck for short-span viaducts and stiffening girder for long-span cable-supported bridges.

## 11.6 Segmental Cable-Stayed Bridges

### 11.6.1 Overview

Theories on cable-stayed bridges are presented in another chapter. We shall address here cable-stayed bridges only as they relate to segmental construction. In the majority of segmental cable-stayed bridges, the methods of construction fall in the three following categories, by order of importance:

- Cantilever construction
- In-stage construction
- Push-out construction

**FIGURE 11.22** The Roize Bridge, France — erection of steel bottom chord and webs.

The choice of material depends upon many factors and load conditions; it should be remembered that concrete is an excellent material for cable-stayed structures, because of its properties in resisting compression and its mass and damping characteristics in resisting aerodynamic vibrations. For the proposed Ceremonial Bridge in Malaysia, with a main span of 1000 m and a single plane of stays, concrete deck in the pylon area is associated with a composite cross section toward the center of the span. Comparative studies show that the replacement of the composite section with its concrete slab by an orthotropic slab would adversely affect the project because of its lack of mass.

## 11.6.2 Cantilever Construction

### 11.6.2.1 Design

It is important to keep the project simple and pay attention to details to achieve economy and efficiency during construction.

The length of segments must be equal and, depending upon the spacing of stays, the segment joints must be such that a stay always falls in the same location within a segment. If the segments are long, the stay should be located toward the free end of the segment. Cross sections must be kept constant as much as possible, the variations being limited to the web and bottom slab thickness. The post-tensioning layout must be repetitive from segment to segment (see Figure 11.23). Erection phases are critical in terms of stability and stresses. Wind effects on the partially built structure must be investigated for static and dynamic effects. A shorter return period is usually used during construction (10 years). Seismic effects must also be investigated in areas prone to earthquakes. To increase stability, temporary cables can be installed at a certain stage of completion.

Stresses in the main elements of the structure often reach a maximum during construction, and the final state of stresses in the finished structure depends greatly on the accuracy of construction. Figure 11.24 shows a typical erection cycle. It is important that all erection phases be reviewed to ensure that the stresses are within allowable limits at each stage.

Stay forces are large and applied on very localized areas of the deck, and their local effects must be analyzed in detail. For instance, the stays apply high, concentrated forces on the section, at the middle, in the case of a single plane of stays, or at the edges with two planes of stays. These forces are not immediately available in the whole cross section, but are spread out at approximately 45°. This shear lag effect is more critical during construction than in service. Construction phases should be checked, assuming a 45° distribution of the horizontal component of the stay force while the vertical component is effectively applied at the stay anchorage (a finite-element computer program will generate the exact cross section stress distribution).

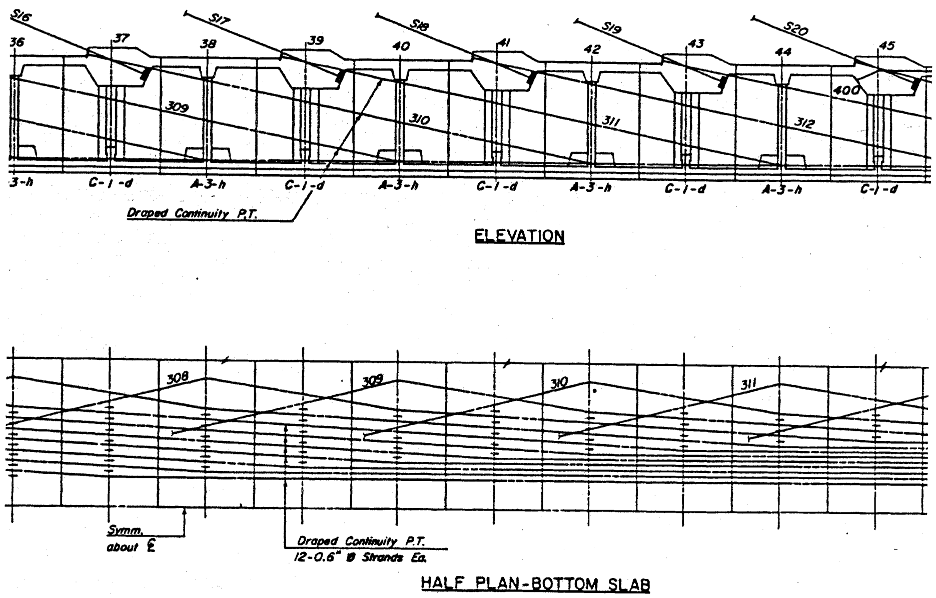



**FIGURE 11.23** Sunshine Skyway, Florida — stay cables and post-tensioning layout.

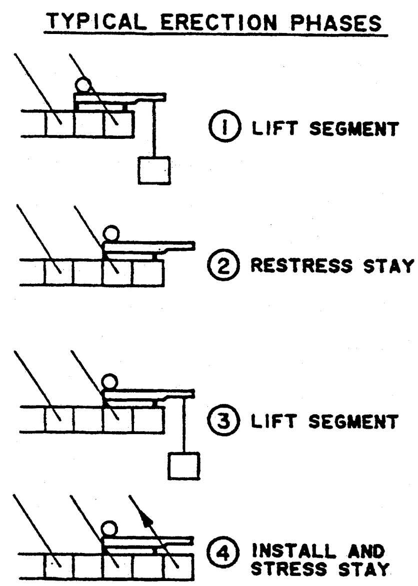



**FIGURE 11.24** Typical erection phases.

This analysis usually shows the necessity of adding a temporary post-tensioning system toward the end of the cantilever, in the area outside the stay centerline (see Figure 11.25).

When a stay is anchored in an already constructed deck, such as a backstay anchored in the side span, the horizontal component of the stay force is distributed half in compression in front of the anchor and half in tension in the back of the anchor. This is called the entrainment effect; care must be taken to have enough tension capacity behind the anchor, either rebars or available compression, to prevent cracking or opening of the joint in case of precast construction.

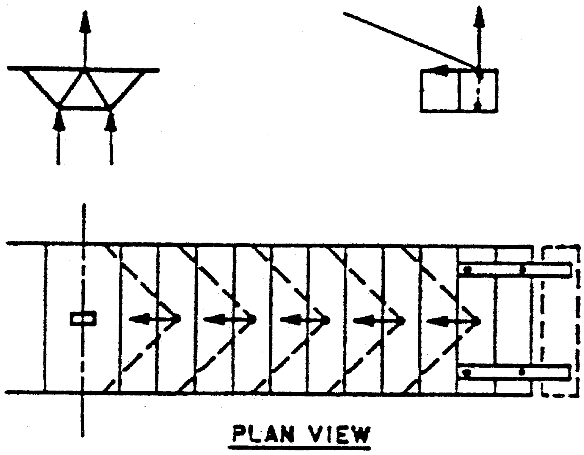



**FIGURE 11.25** Shear lag during construction.

### 11.6.2.2 Cantilever Cast-in-Place Construction

Cast-in-place stayed bridges are built according to the same general principle as a typical box-girder bridge. After the pylon has been built up to the first pylon stay anchorage points and the starting deck segment at the pylon cast, travelers can be installed and cantilever construction started. Temporary stays are sometimes necessary to carry the weight of the traveler plus the newly cast segment before the permanent stay pertaining to that segment can be installed and stressed especially for thin, small inertia decks. A more elegant way is to use the permanent stay, which can be anchored in a precast anchorage block secured to the traveler. The horizontal component of the stay force is carried either by the traveler (see Figure 11.26) or by a precast member, which becomes part of the future segment. The permanent stay can also be anchored in the final deck if the stay anchor structure is staggered ahead of the whole section. This was the case at the Isère Bridge shown in Figures 11.27 and 11.28, with the center spine where the stays are anchored was cast in a first phase and the remainder of the section in a second phase. The phases are as follows:

- Launching of traveler;
- Concreting of the center spine (8 m) and stressing stay to 35% of its final force;
- Launching side forms, connecting bottom slab, and stressing stay to 70% of its final force;
- Concreting top slab and stressing stay to 80% of its final force.

### 11.6.2.3 Cantilever Precast Construction

Precast segmental bridges become economically feasible for relatively large bridges where the cost associated with setting up a casting yard can be offset by the speed of casting segments and the speed of erection. It is very interesting if the approaches to the main span are also precast segmentally, because then the cost of equipment is written off on an even larger volume.

A great example is the Sunshine Skyway Bridge in Florida, with a main span of 366 m for a total length of 1220 m, where the same cross section is used throughout the high-level bridge (see Figures 11.29 and 11.30). The 120-ton segments were precast in a yard close to the site and delivered by barge. They were lifted into place by beam-and-winch assemblies mounted on the previously completed portion of the deck. The same lifting equipment was used for the high approaches to the main spans. The low-level approaches were made of two parallel box girders.

For the James River Bridge in Virginia, the same twin parallel precast box girders were used from one end of the bridge to the other. For the main span, a single plan of stays was used and the two boxes were connected by a transverse frame at each stay anchor location (see Figures 11.31 and

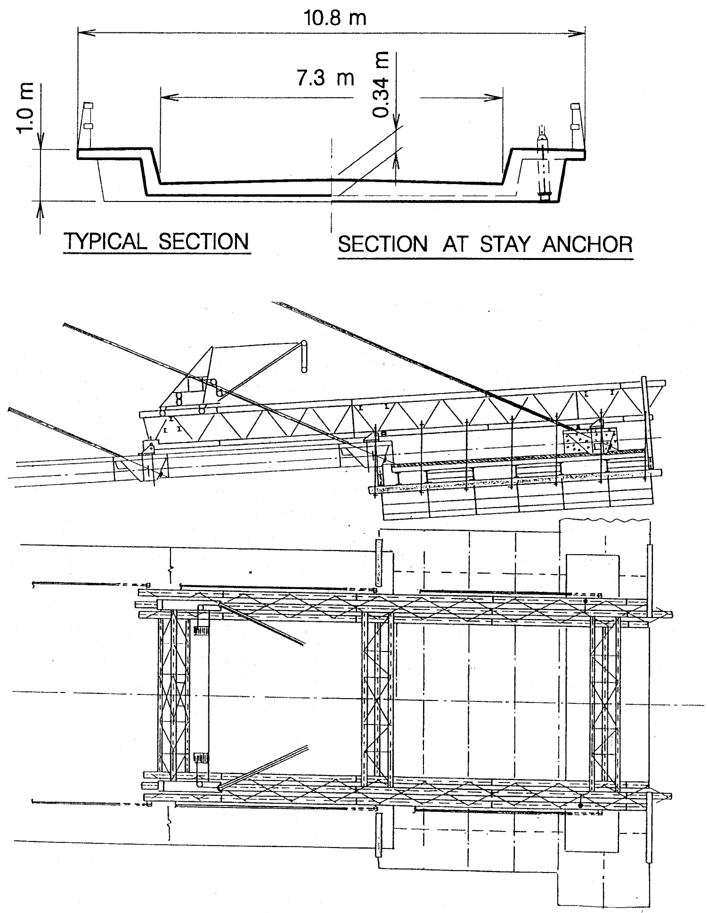



**FIGURE 11.26** Santa Rosa Bridge, Bolivia – general view, cross section, and elevation.

11.32). With this scheme, construction can be carried out at deck level, with the segments erected by the span-by-span method. The main span can be built with crane-type lifting equipment mounted on the completed portion of the deck or, if desired, with cranes at ground level or on barges in the river.

### 11.6.2.4 Structural Steel Segmental Cantilever Construction

Cantilever construction can be applied to steel structures as well, the most recent example being the Normandie Bridge with an 856-m main span and 43.5-m approach spans. Concrete box girders are used for the approaches and part of the main span. The approaches were constructed by incremental launching and the first 116 m out from the pylon by segmental cast-in-place balanced cantilever techniques. Steel segmental construction is used for the remaining 640 m of the main span because of its light weight. The 19.65-m steel segments are barged to the site, lifted in place, secured against the previous segment, and then welded (see Figure 11.33).

## 11.6.3 In-Stage Construction

With this method, the deck is cast on a fixed soffit, with the side forms moving as the segments are cast. Stays can be installed during the casting, then stressed afterward. The advantage is that the bridge does not go through high-stress-level stages during erection and is practically built in its final stage. This method is only a variation of the cast-in-place scheme.

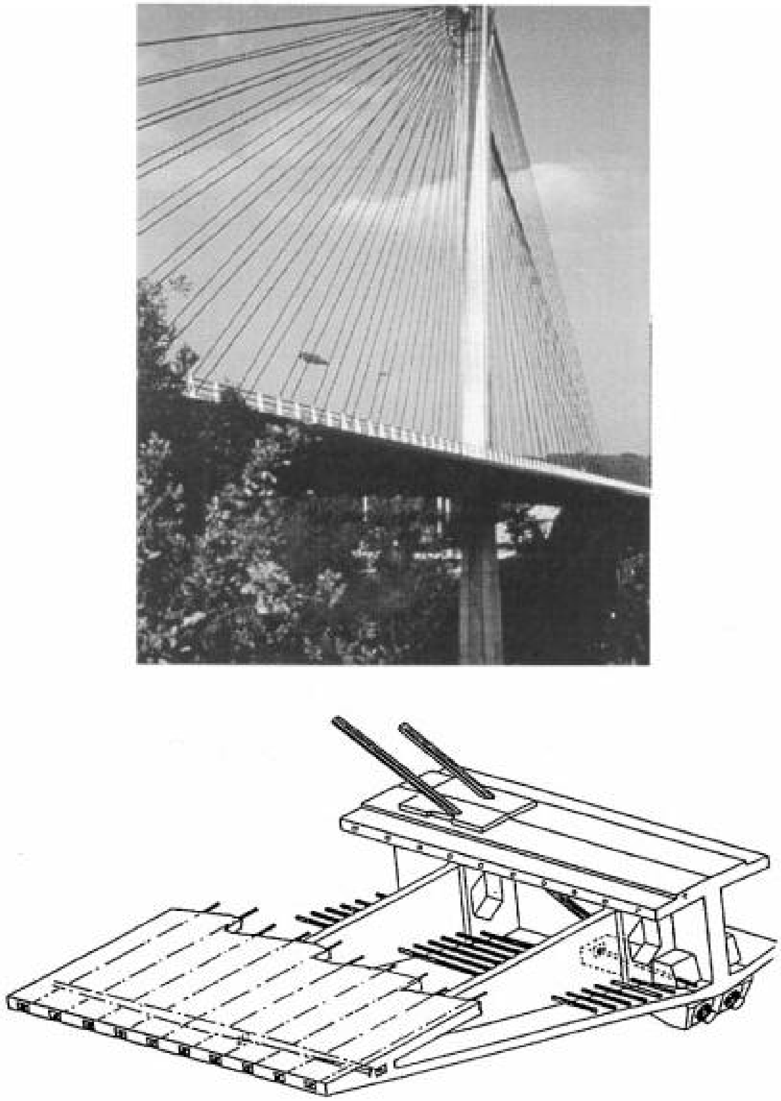


FIGURE 11.27 Isère Bridge, France — general and isometric views.

### 11.6.4 Push-Out Construction

This method is rarely used and not well adapted to cable-stayed bridges. Its use is restricted to sites where temporary supports can be installed. During pushing, the deck is subjected to large moment variations so steel decks are more suitable.

## 11.7 Design Considerations

### 11.7.1 Overview

The intent of this section is to present conditions that the designer should be aware of to produce a satisfactory design. The segmental technique is closely related to the method of construction and the structural system employed. It is usually identified with cantilever construction, but special attention must also be exercised with other methods, such as span-by-span, incremental launching, or progressive placement.

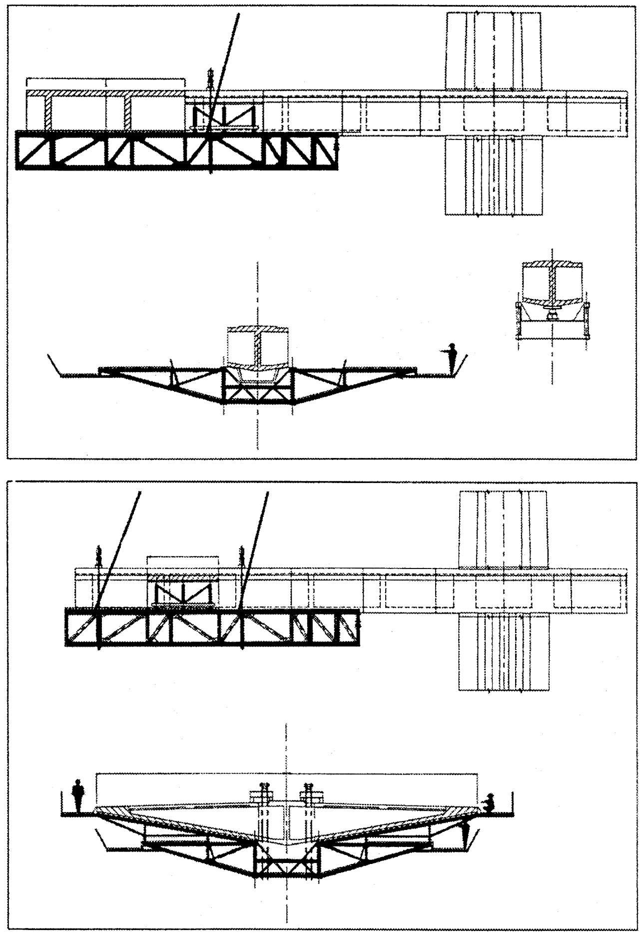


FIGURE 11.28 Isére Bridge — casting sequence.

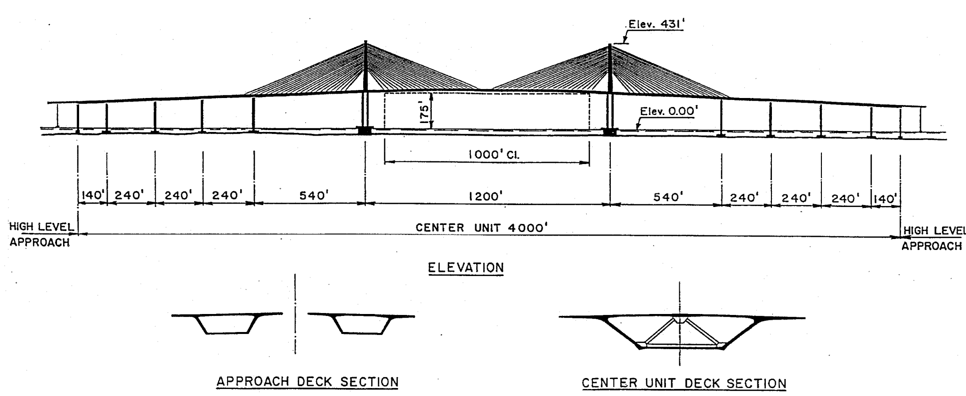



FIGURE 11.29 Sunshine Skyway Bridge, Florida — elevation.

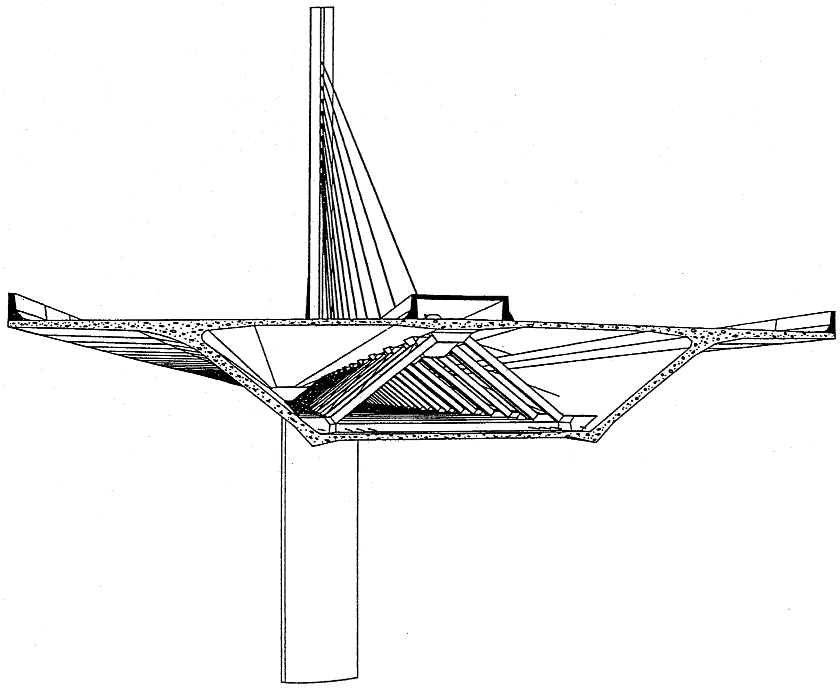


FIGURE 11.30 Sunshine Skyway Bridge — isometric view.

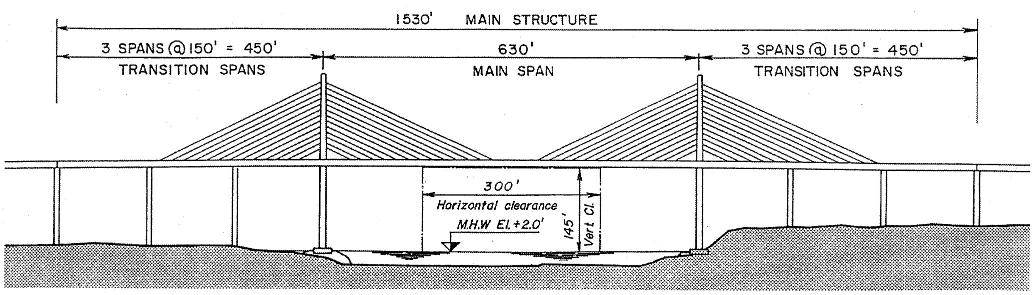



FIGURE 11.31 James River Bridge, Massachusetts – elevation.

## 11.7.2 Span Arrangement

### 11.7.1.1 Balanced Cantilever Construction

The span arrangement should avoid spans of significantly different lengths, if possible. This takes best advantage of the construction method by using cantilevers which are balanced about the column. The abutment spans of bridges built with this method are typically 60 to 65% of the central span length. These shorter end spans minimize the length of the bridge adjacent to the abutment, which must be built by using a different method, typically one employing falsework. Spans shorter than this may require a detail to resist uplift at the abutment resulting in live loading on the adjacent span (see Figure 11.34).

### 11.7.1.2 Span-by-Span Construction

For span-by-span construction the averaging of adjacent span lengths is not required, although it is advantageous to maintain similar span lengths adjacent to one another. The length of the abutment

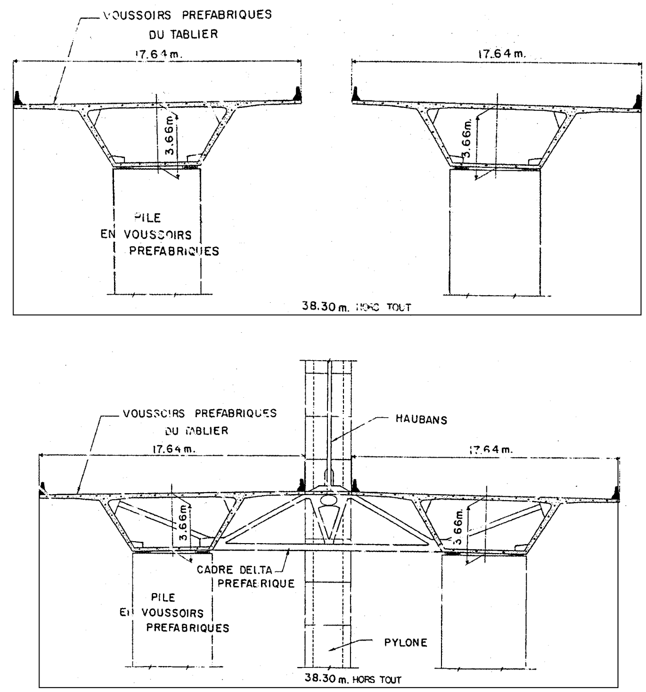



**FIGURE 11.32** James River Bridge — cross sections.

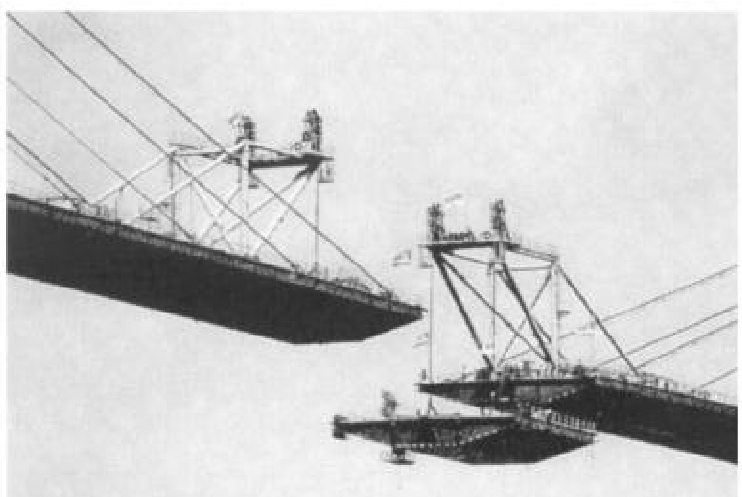


**FIGURE 11.33** Normandie Bridge, France — lifting of a a steel segment.

or end span is typically kept the same as the interior spans. This is reasonable for this type of construction, since the secondary moments due to post-tensioning in the end spans are less than for the interior spans, and the post-tensioning requirement is therefore similar.

### 11.7.1.3 Location of Expansion Joints

Concrete bridge decks have been built with a length up to 1220 m between expansion joints and have had acceptable performance. The placement of expansion joints within a longer viaduct may

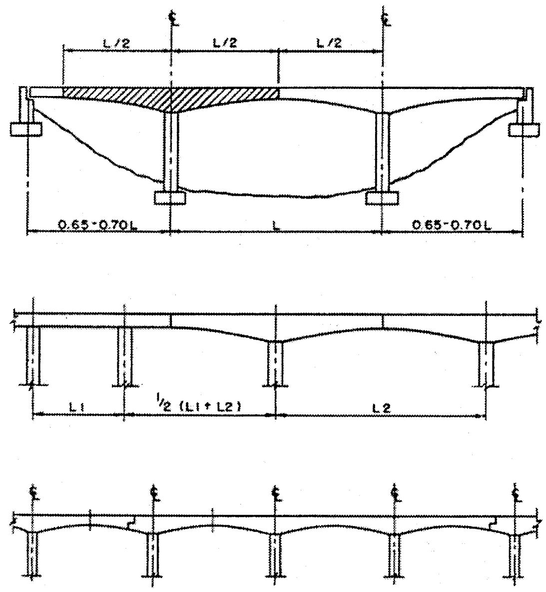



**FIGURE 11.34** Balanced cantilever span arrangement.

be necessary to accommodate the change in length of structure due to creep, shrinkage, and thermal changes. The location of the expansion joint within a span will vary, depending on the method of construction.

For balanced cantilever construction, the expansion joints were initially located at the tip of the cantilevers, which is the middle of the span on the completed structure, for ease of construction. Creep effect under dead load plus post-tensioning drives the tip of the cantilever down, resulting in unacceptable angle break at midspan. This disposition is no longer used. An alternative solution is to place the joint at the point of contraflexure of the equivalent continuous span, thus very effectively reducing the angle of break under creep and live load. However, this technique requires expansion segments at midlength of the cantilever, making construction more difficult. The latest technique goes back to the joint at midspan, but with the addition of a stiffening steel beam across the joint, turning the hinged span into a continuous span with expansion capability. Further refinements are introduced such as the capability of controlling the deflection of the span by vertical jacking on the steel beam during the life of the bridge. This technique has been successfully used as it does not interfere with the cantilever erection process (see Figure 11.35).

For spans built with the use of the span-by-span method the expansion joints are typically located at the centerline of a column. The adjacent box-girder spans are both supported by the column with movement allowed between the spans. With this method, the angle break at the expansion joint is minimized, and there is no requirement for temporary moment restraint between the adjacent sections.

## 11.7.2 Cross-Section Dimensions

### 11.7.2.1 Overall Box-Girder Dimensions

The overall width of a concrete segmental box-girder bridge is quite adaptable to any requirement. Box-girder spans have been built with widths as low as 3.6 m and as great as 27.50 m, with the configuration of the box girder varying significantly.

The depth of precast segmental box girders is generally somewhat greater than that of similar spans with cast-in-place construction. This increased depth is necessary to offset more stringent requirements for extreme fiber axial stresses and restrictions on the locations of post-tensioning

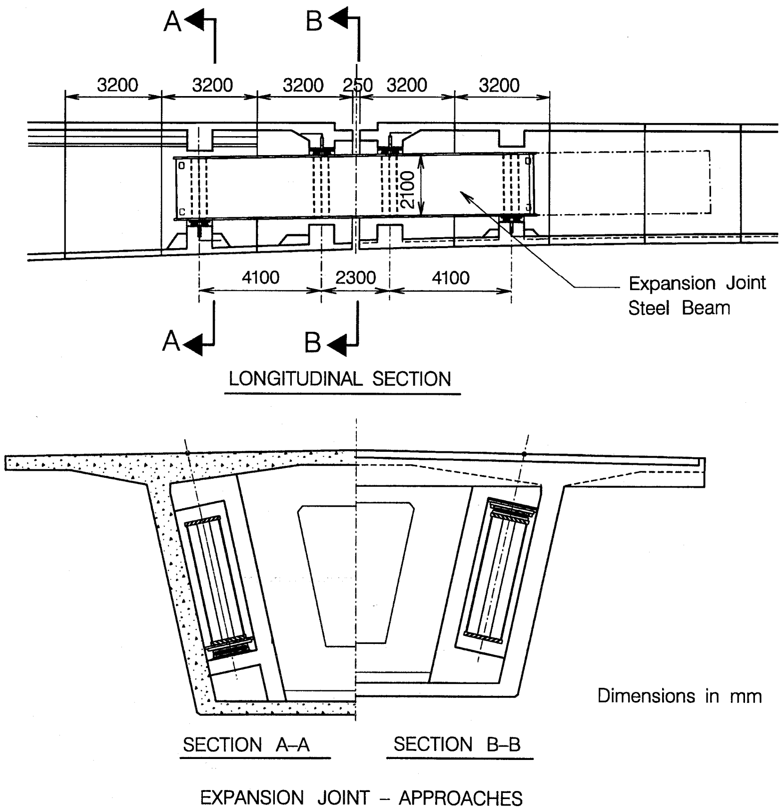



**FIGURE 11.35** Expansion joint — approaches.

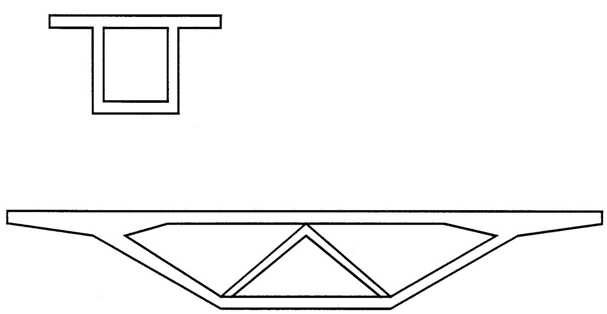


**FIGURE 11.36** Various cross sections.

tendons. Multiple-cell, box-girder bridges will also have more webs to place tendons than a comparable width, single-cell segmental box girder. For span-by-span construction, the span-to-depth ratio should not exceed 25 to 1, and is more comfortable at 20 to 1. For balanced cantilever construction, the span-to-depth ratio at the support should not exceed 18 to 1. However, variable-depth box girders built in balanced cantilever fashion are quite common with straight haunched sections and parabolic extrados. The span-to-depth ratio at midspan of a variable-depth balanced cantilever bridge should not exceed 40 to 1 (see Figure 11.36).

### 11.7.2.2 Web Thickness

The thickness of the web is generally determined such that the required post-tensioning tendons may be placed without interfering with concrete placement or risking cracking during stressing of tendons. Principal stress values at service limit state for no cracking in concrete should be checked in the webs at the neutral axis and at the intersection with top and bottom flanges. This will give a good indication whether the thickness of the web is sufficient. Most design codes also place a limit on the ultimate shear capacity for a box girder to ensure that the web does not fail in diagonal compression prior to the yielding of stirrup reinforcing.

#### 11.7.2.3 Slab Thickness

Slab thicknesses are generally determined to limit deflection under live loading and to provide the necessary flexural capacity. These limits are similar to those of slab thickness for bridge structures built with the use of more traditional construction methods. Span-to-thickness ratios should be in the range of 30 to 1. Since most segmental box girders have transversely post-tensioned top slabs, the minimum thickness of a top slab should be 200 mm, with possibly thicker values at the tendon anchorages. Bottom slab thickness may be less, down to 180 mm, if there is no longitudinal or transverse post-tensioning embedded in the slab.

### 11.7.3 Temperature Gradients

#### 11.7.3.1 Linear Temperature Gradients

Temperature gradients are caused by the top or bottom surface of the structure being warmer than the other. The shape of the temperature distribution along the depth of the section is beyond the scope of this text. However, this distribution may be assumed to be linear or nonlinear with magnitudes given in relevant texts [3]. Due to its high thermal mass, concrete structures are more adversely affected by the thermal gradient than steel structures.

Effects of a linear temperature gradient can be easily evaluated using hand-calculation methods. Once the magnitude of the temperature gradient has been determined, the unrestrained curvature at any point along the span can be determined by

$$R = \frac{\Delta T \cdot \alpha \cdot E_c}{h} \tag{11.1}$$

where

$R$ = radius of curvature
$\Delta T$ = linear temperature differential between top and bottom fibers of cross section
$E_c$ = Modulus of elasticity of concrete
$\alpha$ = thermal expansion coefficient
$h$ = depth of cross section

Once the unrestrained curvature along the structure is known, the final force distribution can be determined by evaluating the redundant support reactions. It is noted that for a statically determinate structure the linear temperature gradient results in zero effect on the structure.

#### 11.7.3.2 Nonlinear Temperature Gradients

Nonlinear temperature gradients are more difficult to evaluate and are best handled by a well-suited computer program. The general theory is presented here; for a more detailed elaboration see Reference [1]. The nonlinear temperature distribution is determined by field measurements and thermodynamic principles. The general shape may be as shown in Figure 11.37. Assuming that the material has linear stress–strain properties, that plane sections will remain plane (Navier–Bernoulli hypothesis), and that temperature varies only with depth (two-dimensional problem), one can make the following theoretical derivation of the problem: the free thermally induced strain is proportional to the temperature distribution; however, this strain distribution violates the second assumptions above, namely, that plane sections remain plane. In order for the section to remain plane under the

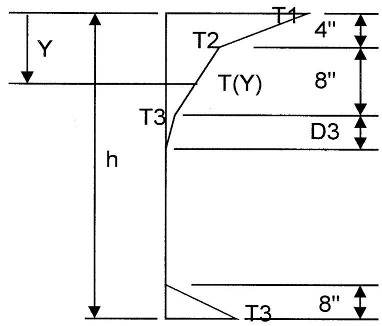



**FIGURE 11.37** Nonlinear gradient.

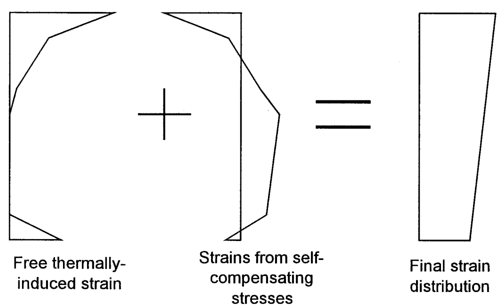



**FIGURE 11.38** Self-compensating stresses.

effects of the applied temperature gradient, there must be some induced stress on said section. This is termed self-compensating stress. The final strain distribution on the section is, therefore, linear and is the sum of the free thermally induced strain and the strain induced by the self-compensating stresses (see Figure 11.38).

The self-compensating stresses can be derived as

$$\sigma(Y) = E_c \cdot \sigma \cdot T(Y) - \frac{P}{A} - \frac{M \cdot Y}{I} \tag{11.2}$$

where

$Y$ = variable along depth of cross section
$T(Y)$ = temperature at abscissa $Y$
$P$ = $\int_Y E \cdot a \cdot T(Y) \cdot b(Y) dY$
$M$ = $\int_Y E \cdot a \cdot T(Y) \cdot b(Y) \cdot Y \cdot dY$
$b(Y)$ = width of section at abscissa $Y$

Similar to the linear gradient, there is now a free unrestrained curvature of the structure along its length. If the structure is continuous, this will result in reactions due to the restraint of the system. The unrestrained curvature at any point along the structure is

$$R = \frac{M}{E_c \cdot I} \tag{11.3}$$

Once the unrestrained curvature along the structure is known, the continuity force distribution can be determined by evaluating the redundant support reactions. The total stress on a section is, therefore, the summation of the self-compensating stresses and the continuity stresses. For a statically determinate structure, the stress on a section is not zero as for a linear temperature gradient; the continuity stresses are zero, but the self-compensating stresses may be significant.

## 11.7.4 Deflection

### 11.7.4.1 Dead Load and Creep Deflection

Global vertical deflections of segmental box-girder bridges due to the effects of dead load and post-tensioning as well as the long-term effect of creep are normally predicted during the design process by the use of a computer analysis program. The deflections are dependent, to a large extent, on the method of construction of the structure, the age of the segments when post-tensioned, and the age of the structure when other loads are applied. It can be expected, therefore, that the actual deflections of the structure would be different from that predicted during design due to changed assumptions. The deflections are usually recalculated by the contractor's engineer, based on the actual construction sequence.

### 11.7.4.2 Camber Requirements

The permanent deflection of the structure after all creep deflections have occurred, normally 10 to 15 years after construction, may be objectionable from the perspective of riding comfort for the users or for the confidence of the general public. Even if there is no structural problem with a span with noticeable sag, it will not inspire public confidence. For these reasons, a camber will normally be cast into the structure so that the permanent deflection of the bridge is nearly zero. It may be preferable to ignore the camber, if it is otherwise necessary to cast a sag in the structure during construction.

### 11.7.4.3 Global Deflection Due to Live Load

Most design codes have a limit on the allowable global deflection of a bridge span due to the effects of live load. The purpose of this limit is to avoid the noticeable vibration for the user and minimize the effects of moving load impact. When structures are used by pedestrians as well as motorists, the limits are further tightened.

### 11.7.4.4 Local Deflection Due to Live Load

Similar to the limits of global deflection of bridge spans, there are also limitations on the deflection of the local elements of the box-girder cross section. For example, the AASHTO Specifications limit the deflection of cantilever arms due to service live load plus impact to $1/300$ of the cantilever length, except where there is pedestrian use [1].

## 11.7.5 Post-Tensioning Layout

### 11.7.5.1 External Post-Tensioning

While most concrete bridges cast on falsework or precast beam bridges have utilized post-tensioning in ducts which are fully encased in the concrete section, other innovations have been made in precast segmental construction. Especially prevalent in structures constructed using the span-by-span method, post-tensioning has been placed inside the hollow cell of the box girder but not encased in concrete along its length. This is know as external post-tensioning. External post-tensioning is easily inspected at any time during the life of the structure, eliminates the problems associated with internal tendons, and eliminates the need for using expensive epoxy adhesive between precast segments. The problems associated with internal tendons are (1) misalignment of the tendons at segment joints, which causes spalling; (2) lack of sheathing at segment joints; and (3) tendon pull-through on spans with tight curvature (see Figure 11.39). External prestressing has been used on many projects in Europe, the United States, and Asia and has performed well.

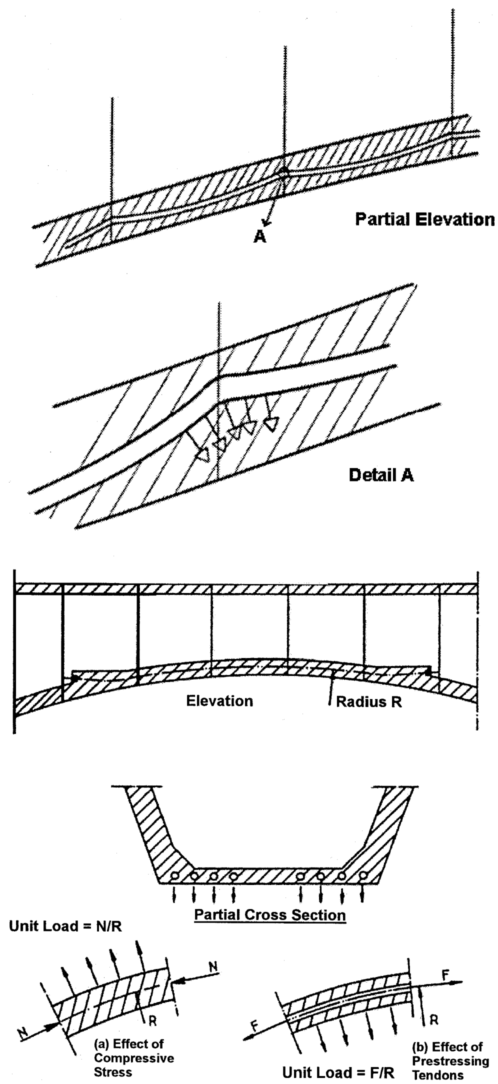



**FIGURE 11.39** Problems with internal tendons.

#### 11.7.5.2 Future Post-Tensioning

The provision for the addition of post-tensioning in the future in order to correct unacceptable creep deflections or to strengthen the structure for additional dead load, i.e., future wearing surface, is now required by many codes. Of the positive and negative moment post-tensioning, 10% is reasonable. Provisions should be made for access, anchorage attachment, and deviation of these additional tendons. External, unbonded tendons are used so that ungrouted ducts in the concrete are not left open.

## 11.8 Seismic Considerations

### 11.8.1 Design Aspects and Design Codes

Due to typical vibration characteristics of bridges, it is generally accepted that under seismic loads, some portion of the structure will be allowed to yield, to dissipate energy, and to increase the period of vibration of the system. This yielding is usually achieved by either allowing the columns to yield plastically (monolithic deck/superstructure connection), or by providing a yielding or a soft bearing system [6].

The same principles also apply to segmental structures, i.e., the segmental superstructure needs to resist the demands imposed by the substructure. Very few implementations of segmental structures are found in seismically active California, where most of the research on earthquake-resistant bridges is conducted in the United States. The Pine Valley Creek Bridge, Parrots Ferry Bridge, and Norwalk/El Segundo Line Overcrossing, all of them being in California, are examples of segmental structures; however, these bridges are all segmentally cast in place, with mild reinforcement crossing the segment joints.

Some guidance for the seismic design of segmental structures is provided in the latest edition of the AASHTO Guide Specifications for Design and Construction of Segmental Concrete Bridges [2], which now contains a chapter dedicated to seismic design. The guide allows precast-segmental construction without reinforcement across the joint, but specifies the following additional requirements for these structures:

- For Seismic Zones C and D [1], either cast-in-place or epoxied joints are required.
- At least 50% of the prestress force should be provided by internal tendons.
- The internal tendons alone should be able to carry 130% of the dead load.

For other seismic design and detailing issues, the reader is referred to the design literature provided by the California Department of Transportation, Caltrans, for cast-in-place structures [5-8].

## 11.8.2 Deck/Superstructure Connection

Regardless of the design approach adopted (ductility through plastic hinging of the column or through bearings), the deck/superstructure connection is a critical element in the seismic resistant system. A brief description of the different possibilities follows.

### 11.8.2.1 Monolithic Deck/Superstructure Connection

For the longitudinal direction, plastic hinging will form at the top and bottom of the columns. Since most of the testing has been conducted on cast-in-place joints, this continues to be the preferred option for these cases. For short columns and for solid columns, the detailing in this area can be readily adapted from standard Caltrans practice for cast-in-place structures, as shown on Figure 11.40. The joint area is then essentially detailed so it is no different from that of a fully cast-in-place bridge. In particular, a Caltrans requirement for positive moment reinforcement over the pier can be detailed with prestressing strand, as shown below. For large spans and tall columns, hollow column sections would be more appropriate. In these cases, care should be taken to confine the main column bars with closely spaced ties, and joint shear reinforcement should be provided according to Reference [3 or 7].

The use of fully precast pier segments in segmental superstructures would probably require special approval of the regulating government agency, since such a solution has not yet been tested for bridges and is not codified. Nevertheless, based upon first principles, and with the help of strut–tie models, it is possible to design systems that would work in practice [6]. The segmental superstructure should be designed to resist at least 130% of the column nominal moment using the strength reduction factors prescribed in Ref. [2].

Of further interest may be a combination of precast and cast-in-place joint as shown in Figure 11.41, which was adapted from Ref. [8]. Here, the precast segment serves as a form for the cast-in-place portion that fills up the remainder of the solid pier cap. Other ideas can also be derived from the building industry where some model testing has been performed. Of particular interest for bridges could be a system that works by leaving dowels in the columns and supplying the precast segment with matching formed holes, which are grouted after the segment is slipped over the reinforcement [9].

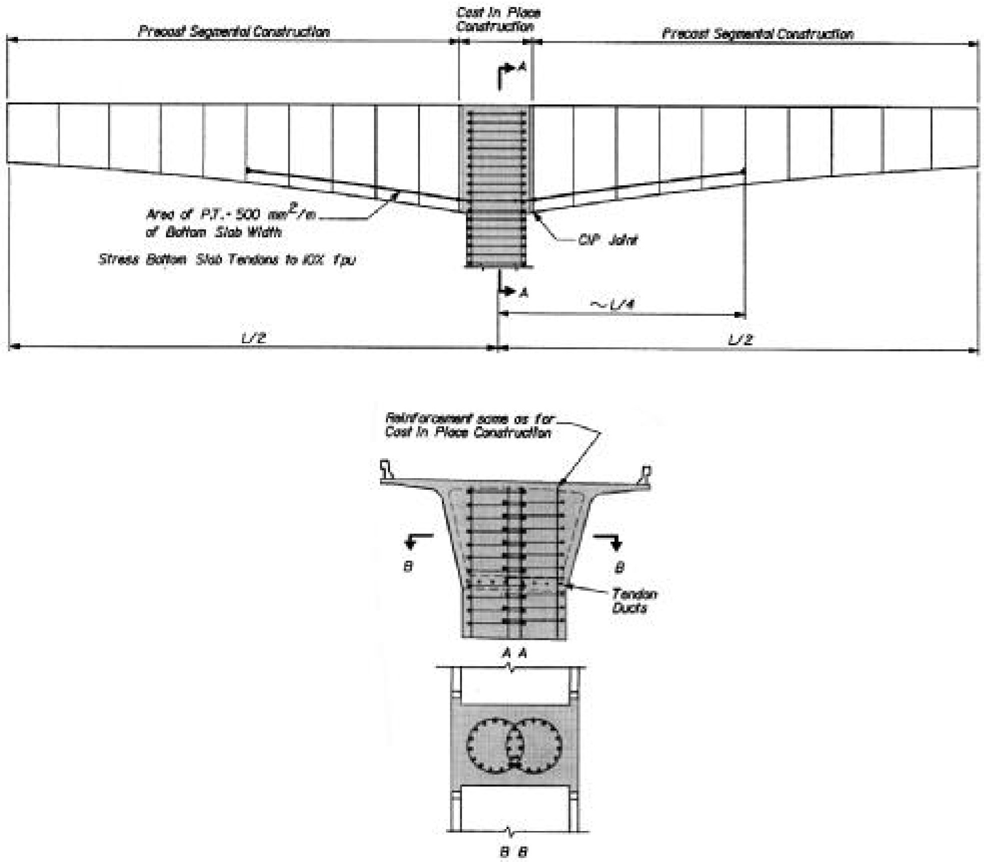



**FIGURE 11.40** Deck/pier connection with cast-in-place joint.

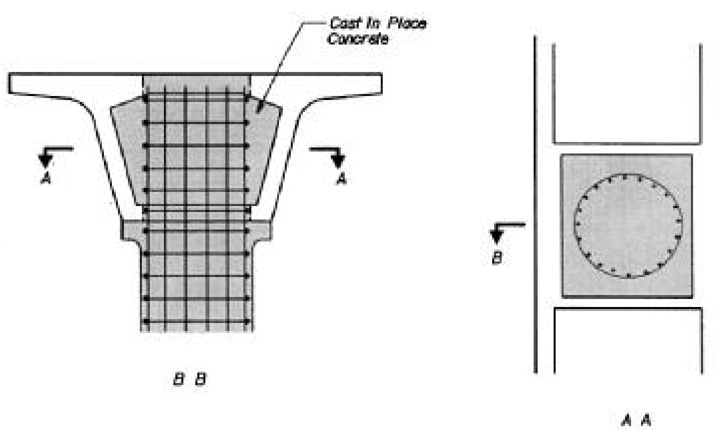



**FIGURE 11.41** Combination of precast and cast-in-place joint.

### 11.8.2.2 Deck/Superstructure Connection via Bearings

Typically, for spans up to 45 m erected with the span-by-span method, the superstructure will be supported on bearings. For action in the longitudinal direction, elastomeric or isolation bearings are preferred to a fixed-end/expansion-end arrangement, since these better distribute the load

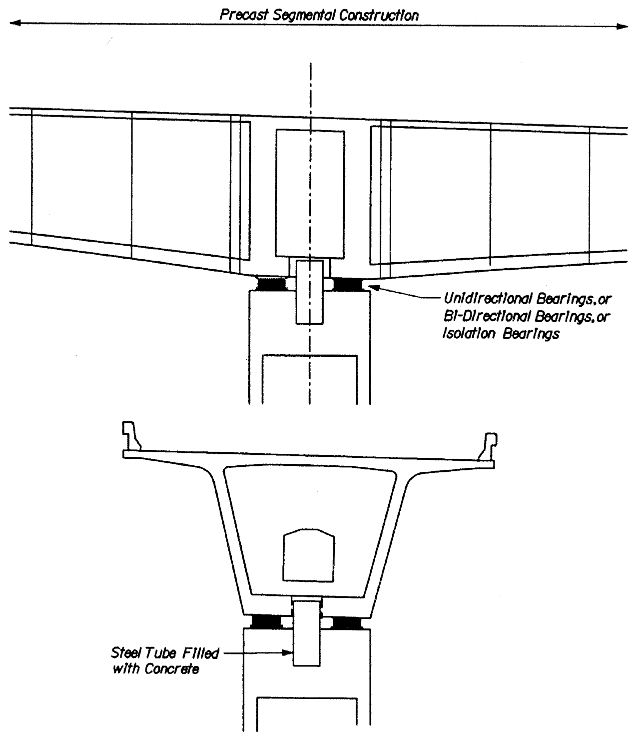



**FIGURE 11.42** Deck/pier connection with bearings.

between the bearings. Furthermore, these bearings will increase the period of the structure, which results in an overall lower induced force level (beneficial for higher-frequency structures), and isolation bearings will provide some structural damping as well.

In the transverse direction, the bearings may be able to transfer load between super- and substructure by shear deformation; however, for the cases where this is not possible, shear keys can be provided as is shown in Figure 11.42. It should be noted that in regions of high seismicity, for structures with tall piers or soft substructures, the bearing demands may become excessive and a monolithic deck–superstructure connection may become necessary.

For the structure-on-bearings approach, the force level for the superstructure can be readily determined, since once the bearing demands are obtained from the analysis, they can be applied to the superstructure and substructure. The superstructure should resist the resulting forces at ultimate (using the applicable code force-reduction factors), whereas the substructure can be allowed to yield plastically if necessary.

### 11.8.2.3 Expansion Hinges

From the seismic point of view, it is desirable to reduce the number of expansion hinges (EH) to a minimum. If EHs are needed, the most beneficial location from the seismic point of view is at midspan. This can be explained by observing Figure 11.43, where the superstructure bending moments, resulting from column plastic hinging ($M_p$), have been plotted for the case of an EH at midspan and for an EH at quarterspan. For the latter, it can be seen that the moment at the face of the column varies within the range of $\pm\frac{3}{4}\,M_p$, whereas with the hinge at midspan, the values are only between $\pm\frac{1}{2}\,M_p$.

The location of expansion hinges within a span, and its characteristics, depends also on the stiffness of the substructure and the type of connection of the superstructure to the piers. Table 11.1 presents general guidelines intended to assist in the selection of location of expansion hinges.

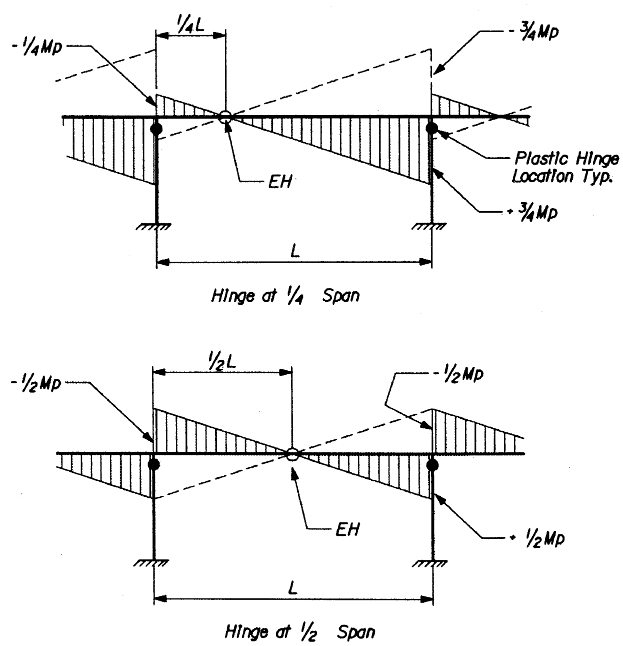



**FIGURE 11.43** Longitudinal superstructure seismic moments with hinges at quarterspan and at midspan.

**TABLE 11.1** Location of Expansion Hinges in Segmental Bridges

| Span Support System | Location of EH | | |
|---|---|---|---|
| | Over Pier | Intermediate Point | Midspan |
| On bearings | • Standard solution for simple spans<br>• For continuous spans generates moderate superstructure moments at adjacent piers | • Complicated erection for cantilever construction<br>• Generates moderate superstructure moments at adjacent piers<br>• Moderate EH openings requiring restrainers and moderate seat widths, or lock-up devices | • Simplest location for cantilever construction<br>• Will require continuity beam inside cross section<br>• Minimizes superstructure seismic moments<br>• Moderate EH openings requiring adequate gap between the end segments |
| Monolithic with pier | Not applicable | • Complicated erection for cantilever construction<br>• Generates very large superstructure moments at adjacent piers<br>• If substructure is stiff, expect relatively small EH movements; otherwise expect very large movements, requiring restrainers and large seat widths, or lock-up devices | • Simplest location for cantilever construction<br>• Will require continuity beam inside cross section<br>• Minimizes superstructure seismic moments<br>• If substructure is stiff, expect relatively small EH movements; otherwise expect very large movements, requiring lock-up devices |

### 11.8.2.4 Precast Segmental Piers

Precast segmental piers are usually hollow cross section to save weight. From research in other areas it can be extrapolated that the precast segments of the pier would be joined by means of unbonded prestressing tendons anchored in the footing. The advantage of unbonded over bonded tendons is that for the former, the prestress force would not increase significantly under high column displacement demands, and would therefore not cause inelastic yielding of the strand, which would otherwise lead to a loss of prestress.

The detail of the connection to the superstructure and foundation would require some insight into the dynamic characteristics of such a connection, which entails joint opening and closing — providing that dry joints are used between segments. This effect is similar to footing rocking, which is well known to be beneficial to the response of a structure in an earthquake. This is due to the period shift and the damping of the soil. The latter effect is clearly not available to the precast columns, but the period shift is. Details need to be developed for the bearing areas at the end of the columns, as well as the provision for clearance of the tendons to move relative to the pier during the event.

If the upper column segment is designed to be connected monolithically to the superstructure, yielding of the reinforcement should be expected. In this case, the expected plastic hinge length should be detailed ductile, using closely spaced ties [3,5].

## 11.9 Casting and Erection

### 11.9.1 Casting

There are obvious major differences in casting and erection when working with cast-in-place cantilever in travelers or in handling precast segments. There are also common features, which must be kept in mind in the design stages to keep the projects simple and thereby economic and efficient, such as

- Keeping the length of segments equal and segments straight, even in curved bridges;
- Maintaining constant cross section dimensions as much as possible;
- Minimizing the number of diaphragms and stiffeners, and avoiding dowels through formwork.

#### 11.9.1.1 Cast-in-Place Cantilevers

Conventional Travelers

The conventional form traveler supports the weight of the fresh concrete of the new segment by means of longitudinal beams or frames extending out in cantilever from the last segment. These beams are tied down to the previous segment. A counterweight is used when launching the traveler forward. The main beams are subjected to some deflections, which may produce cracks in the joint between the old and new segments. Jacking of the form during casting is sometimes needed to avoid these cracks. The weight of a traveler is about 60% of the weight of the segment. The rate of construction is typically one segment per traveler per week. Precast concrete anchor blocks are used to speed up post-tensioning operations. In cold climates, curing can be accelerated by various heating processes.

Construction Camber Control

The most critical practical problem of cast-in-place construction is deflection control. There are five categories of deflections during and after construction:

- Deflection of traveler frame under the weight of the concrete segment;
- Deflection of the concrete cantilever arm during construction under the weight of segment plus post-tensioning;
- Deflection of cantilever arms after construction and before continuity;
- Short- and long-term deflections of the continuous structure;
- Short- and long-term pier shortenings and foundation settlements.

The sum of the various deflection values for the successive sections of the deck allows the construction of a camber diagram to be added to the theoretical profile of the bridge. A construction camber for setting the elevation of the traveler at each joint must also be developed.

#### 11.9.1.2 Precast Segments

Opposite to the precast girder concept where the bridge is cut longitudinally in the precast segmental methods, the bridge is cut transversally, each slice being a segment. Segments are cast in a casting yard one at a time. Furthermore, the new segment is cast against the previously cast segment so that the faces in contact match perfectly. This is the match-cast principle. When the segments are reassembled at the bridge site, they will take the same relative position with regard to the adjacent segments that they had when they were cast. Accuracy of segment geometry is an absolute priority, and adequate surveying methods must be used to ensure follow-up of the geometry.

Match casting of the segments is a prerequisite for the application of glued joints, achieved by covering the end face of one or both of the meeting segments with epoxy at the erection. The epoxy serves as a lubricant during the assembly of the segments, and it ensures a watertight joint in the finished structure. Full watertightness is needed for corrosion protection of internal tendons (tendons inside the concrete). The tensile strength of the epoxy material is higher than that of the concrete, but, even so, the strength of the epoxy is not considered in the structural behavior of the joint. The required shear capacity is generally provided by shear keys, single or multiple, in combination with longitudinal post-tensioning.

With the introduction of external post-tensioning, where the tendons are installed in PE ducts, outside the concrete but inside the box girder, the joints are relieved of the traditional requirement of watertightness and are left dry. The introduction of external tendons in connection with dry joints greatly enhanced the efficiency of precasting.

#### 11.9.1.3 Casting Methods

There are two methods for casting segments. The first one is the long-line method, where all the segments are cast in their correct position on a casting bed that reproduces the span. The second method, used most of the time, is the short-line method, where all segments are cast in the same place in a stationary form, and against the previously cast segment. After casting and initial curing, the previously cast segment is removed for storage, and the freshly cast segment is moved into place (see Figure 11.44).

#### 11.9.1.4 Geometry Control

A pure translation of each segment between cast and match-cast position results in a straight bridge (Figure 11.45). To obtain a bridge with a vertical curve, the match-cast segment must first be translated and given a rotation $\alpha$ in the vertical plane (Figure 11.46). Practically, the bulkhead is left fixed and the mold bottom under the conjugate unit adjusted. To obtain a horizontal curvature, the conjugate unit is given a rotation $\beta$ in the horizontal plane (see Figure 11.47). To obtain a variable superelevation, the conjugate unit is rotated around a horizontal axis located in the middle of the top slab (Figure 11.48).

All these adjustments of the conjugate unit can be combined to obtain the desired geometry of the bridge.

### 11.9.2 Erection

The type of erection equipment depends upon the erection scheme contemplated during the design process; the local conditions, either over water or land; the speed of erection and overall construction schedule. It falls into three categories, independent lifting equipment such as cranes, deck-mounted lifting equipment such as beam and winch or swivel crane, and launching girder equipment.

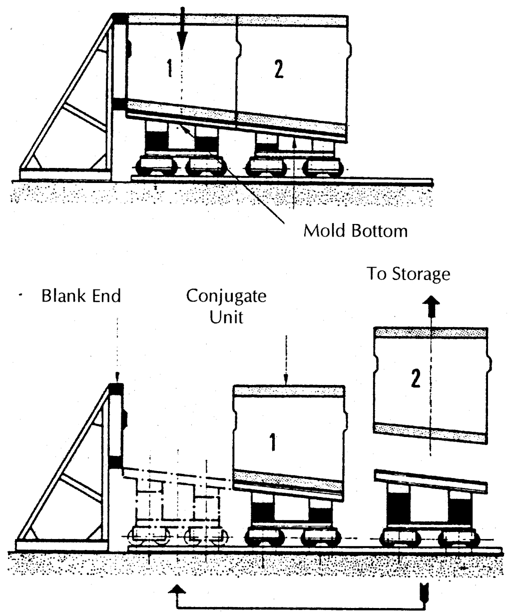



**FIGURE 11.44** Typical short-line precasting operation.

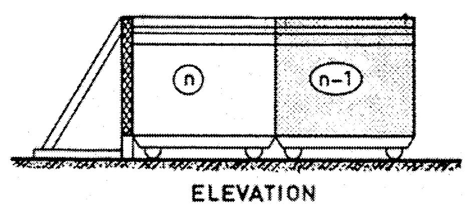



**FIGURE 11.45.** Straight bridge.

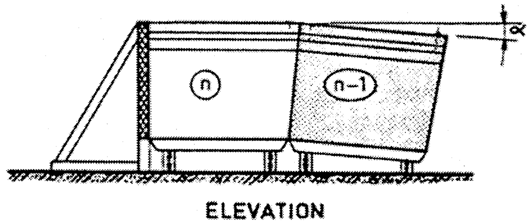



**FIGURE 11.46** Bridge with vertical curve.

#### 11.9.2.1 Balanced Cantilever Method

The principle of the method is to erect or cast the pier segment first, then to place typical segments one by one from each side of the pier, or in pairs simultaneously from both sides. Each newly placed precast segment is fixed to the previous one with temporary PT bars, until the cantilever tendons are installed and stressed. The closure joint between cantilever tips is poured in place and continuity tendons installed and stressed.

In order to carry out this erection scheme, segments must be lifted and installed at the proper location. The simplest way is to use a crane, either on land or barge mounted. Many bridges have

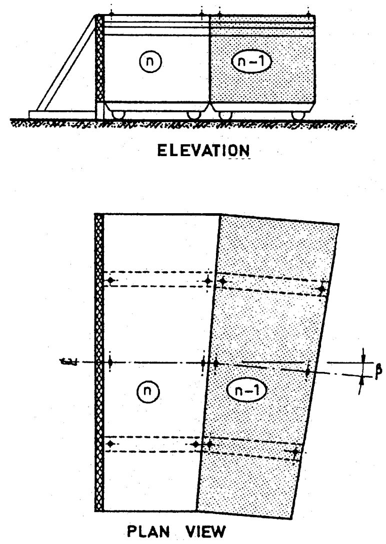



**FIGURE 11.47** Bridge with horizontal curve.

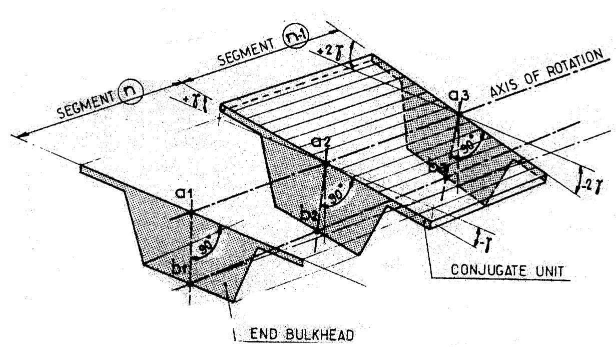



**FIGURE 11.48** Bridge with superelevation.

been erected with cranes as they do not require an investment in special lifting equipment. This method is slow. Typically, two to four segments per day are placed. It is used on relatively short bridges. An alternative is to have a winch on the last segment erected. The winch is mounted on a beam fixed to the segment. It picks up segments from below, directly from truck or barge. After placing the segment, the beam and winch system is moved forward to pick up the next segment and so on. Usually, a beam-and-winch system is placed on each cantilever tip. This method is also slow; however, it does not require a heavy crane on the site, which is always very expensive, especially if the segments are heavy.

When bridges are long and the erection schedule short, the best method is the use of launching girders, which then take full advantage of the precast segmental concept for speed of erection.

There are two essential types of self-launching gantries developed for this erection method. The first type is a gantry with a length slightly longer than the typical span (see Figure 11.49). During erection of the cantilever, the center leg rests on the pier while the rear leg rests on the cantilever tip of the previously erected span, which must resist the corresponding reaction. Prior to launching, the back spans must be made continuous. Then, the center leg is moved to the forward cantilever

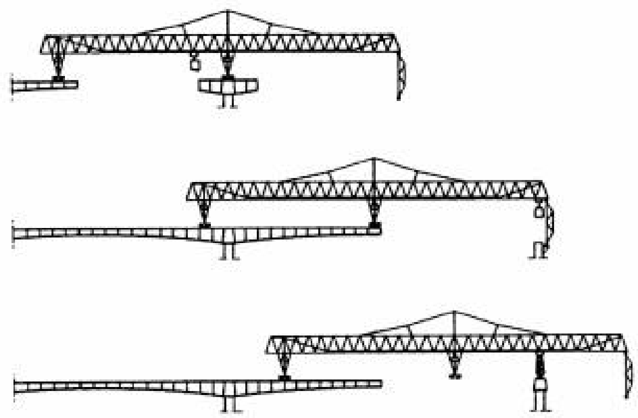


**FIGURE 11.49** French Creek Viaduct, U.S.— single erection truss with portal legs.

tip, which must resist the weight of the gantry plus the weight of the pier segment. This stage controls the design of the gantry, which must be made as light as possible, and of the cantilever.

The second type of gantry has a length that is twice that of the typical span (see Figure 11.50). The reaction from the legs during the erection and launching of the next span is always applied on the piers, so there is no concentrated erection load on the cantilever tip. Each erection cycle consists of the erection of all typical segments of the cantilever and then the placement of the pier segment for the next cantilever, without changing the position of the truss.

The gantries can be categorized by their cross section: single truss, with portal-type legs, and two launching trusses with a gantry across. The twin box girders of the bridge in Hawaii were built with two parallel, but independent trusses (see Figure 11.51), with a typical span of 100.0 m, segment weights of 70 tons; the two bridge structures are 27.5 m apart with different elevations and longitudinal slopes. This system is a refinement of the first type of gantry applied to twin decks with variable geometry.

Normally, the balanced cantilever method is used for spans from 60 to 110 m, with a launching girder. One full, typical cycle of erection is placing segments, installing and stressing post-tensioning tendons, and launching the truss to its next position. It takes about 7 to 10 days, but may vary greatly according to the specifics of a project and the sophistication of the launching girder. With proper equipment and planning, erection of 16 segments per day has been achieved.

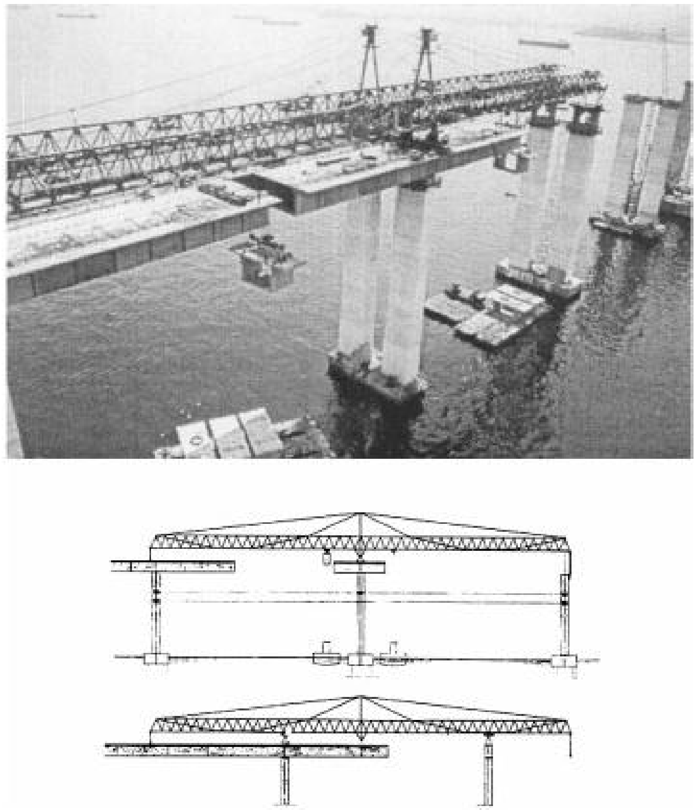


**FIGURE 11.50** Río Niteroi, Brazil — two segments being erected simultaneously from one erection truss.

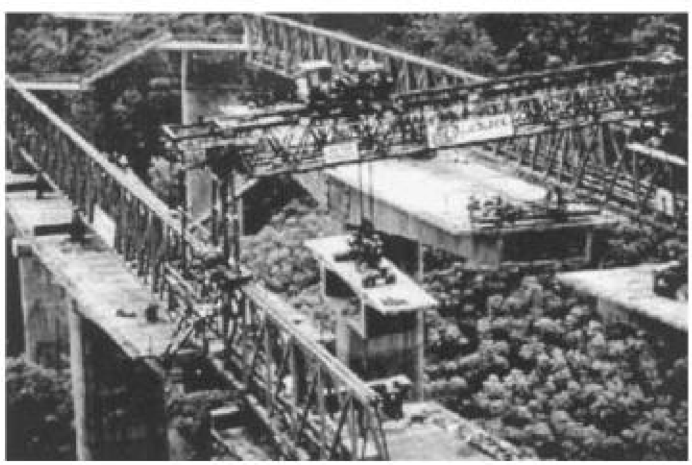


**FIGURE 11.51** H-3 Windward Viaduct, Hawaii.

A modification of this method was used to build the 13-km-long Confederation Bridge (typical span 250 m long), linking Prince Edward Island with New Brunswick in Atlantic Canada. The main girder was constructed in a precasting yard as a precast, balanced cantilever. Then the 197-m-long main girder, with a self-weight of 7500 tons, was placed on the pier with a floating crane (see Figure 11.52).

**FIGURE 11.52** Confederation Bridge, Canada — floating crane.

**FIGURE 11.53** Bang Na–Bang Pli–Bang Pakong Expressway, Thailand — D6 segment erection.

### 11.9.2.2 Span-by-Span Construction

In the first stage, all precast segments for each span are assembled on an erection girder. The second stage is installing and stressing the tendons, and as a result the span becomes self-supported. In comparison with the balanced cantilever method, those girders have to be designed to carry the load of the entire span. Normally, the duration of one erection cycle is 2 to 3 days per span.

The 56-km-long Bang Na–Bang Pli–Bang Pakong Expressway in Bangkok, carrying six lanes of traffic, totaling 27 m in width, is assembled on erection girders. The girder is placed in the middle of a Y-shaped column. The segments, with self-weight up to 100 tons, are placed on the chassis with a swivel crane and then transported to their final position. Two schemes of erection were developed: a swivel crane mounted to the front of the girder picking up segments from trucks on the highway below and a swivel crane placed on the previously erected span with segments delivered over the deck already built (see Figure 11.53).

Another principle used in erection equipment for the span-by-span method, is the so-called overhang girder. In this case, the girder is above the superstructure, and the precast segments are hung from it.

#### 11.9.2.3 Safety

Due to the inherent character of temporary structures, erection equipment is usually designed to take full advantage of the materials and care must be taken to analyze in-depth all construction stages, anticipating mistakes or shortcuts made on site that always occur, and stay within reasonable safety limits. Overall stability when resting on temporary supports or during launching, reversal of forces, bucking, etc., are the most common problems encountered in these structures. Lifting bars or tie-downs must always be designed with failure or mishandling of one of those elements in mind, and appropriate ultimate resisting paths incorporated in the concept.

## 11.10 Future of Segmental Bridges

Since their appearance in 1962, precast segmental technologies have been used worldwide in the design and construction of practically all types of bridges. Nevertheless, in the last 5 years, an important further development of these technologies has taken place in Southeast Asia which will have a decisive impact on the way bridges will be built in the next century.

### 11.10.1 The Challenge

The explosive development of the Southeast Asian economies has been forcing local governments to find new solutions for building infrastructures (build, operate, transfer), which can, when properly managed, become success stories for both the government agencies who organize the projects and for the private groups who develop them.

Privatization of such projects is now accepted by numerous countries as a viable solution for the challenges they face. Large infrastructure projects, worth billions of dollars, are at present being designed, built, financed, and operated by private companies in the region. This trend is expected to extend progressively to the global construction markets. The two key factors of such projects are the amount of toll to be paid by the users and the duration of the concession until the project is transferred to the government agency. For the roadway projects, the tolls vary anywhere from between $1 to $10. The duration of the concessions varies in general from 25 to 35 years.

While basically simple, the scheme presents very complex problems for its implementation. Multidisciplinary skills and a new vision for the design, construction, and operation of the roads and bridges are necessary in order to avoid technical or financial failures. The challenges that private organizations must be able to face can be summarized in just a few words: they need to design and build very competitive projects in the shortest possible time, ensuring the longest possible service life. In light of the experiences gained in the new markets, it is becoming more evident each day that precast segmental technologies often bring the right solutions to these challenges.

### 11.10.2 Concepts

When Jean Muller invented the precast concrete segmental technology in 1962, his vision was to create an industrialized construction system to build any type of bridge with standard modules, assembled with post-tensioning, without any cast-in-place concrete.

To achieve this objective, he developed the concept of match-cast joints, which allows the transverse slicing of concrete box girders and the assembly of such slices — the segments — in the same order as they were produced, without any need for additional *in situ* concrete to complete the bridge deck [10, 11].

In addition to epoxy-glued joints, the use of dry joints became widespread. In 1978, through the design of the Long Key Bridge in Florida, internal post-tensioning was replaced by external post-tensioning [12]. A number of other concepts invented by Jean Muller allowed further development of the modular construction concept: span-by-span assembly method (Long Key Bridge), progressive

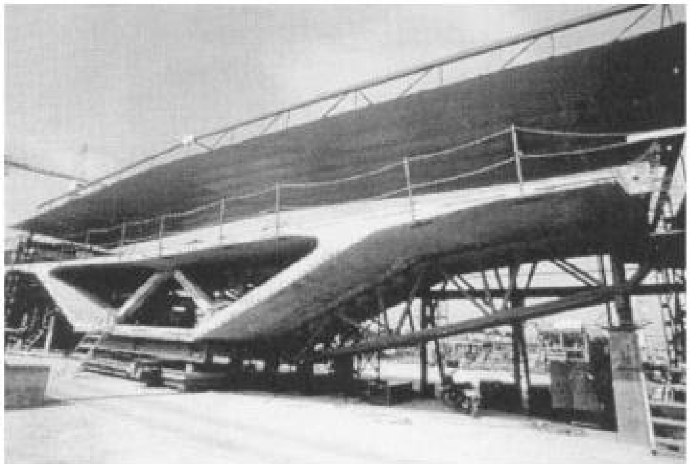


**FIGURE 11.54** Bang Na Expressway, Thailand — D6, six-traffic-lane segment at precast yard.

**TABLE 11.2** Standard Segments

| Segment | Lanes | Widths (m) Min-Max |
|---|---|---|
| D2 | 2 | 08–12 |
| D3 | 3 | 10–15 |
| D4 | 4 | 14–20 |
| D6 | 6 | 18–30 |

placing (Linn Cove Bridge), precast segmental construction of the piers, D6 cable-stayed segments (Sunshine Skyway Bridge), delta frames (James River Bridge, C&D Canal Bridge), etc.

These concepts and others developed more recently for the large projects, which are being built in Canada and Thailand, allow the prefabrication of bridge structures with precast modules ranging from 20 tons, in channel-shaped overpasses, to 7500 tons for the main girders of the Confederation Bridge in Canada. In this project, one of the largest bridges ever built, no cast-in-place structural concrete was used in the construction of the main spans [13]. The construction modules are all manufactured in sophisticated and industrialized precasting plants that ensure an unequaled construction rate and quality (see Figure 11.54).

By further standardizing the segments with the number of traffic lanes that they carry, we have developed the modules in Table 11.2. These modules allow the construction of viaducts of any width, ranging from 7 to 8 m up to 30 m. Concurrently, with the effort to standardize the cross sections for precast segmental bridges, there has been significant development of design, shop drawing software, and geometry control systems. This gives us the capability to produce drawings by the thousands for viaducts, interchanges, and merging sections that give, for each segment, the detailed geometry and dimensions of concrete and rebar and the layout of the post-tensioning. Such shop drawings are an essential part of the system and must be integrated into the structural design; no standardization and, hence, no industrialization is possible without them (Figure 11.55).

### 11.10.3 New Developments

The dynamic business environment of Southeast Asian markets is quite favorable for the introduction of innovative concepts. In recent years, technologies that took over 20 years to develop in Europe and some 10 years to spread throughout the United States were absorbed by countries such

**FIGURE 11.55** The Confederation Bridge, Canada — Prince Edward Island precasting yard.

as Thailand, which had limited prior experience in the field of bridge engineering, and already concepts never used before are being developed for new projects, thus giving these countries a leading position in construction innovation. By using the most innovative technologies, the developers involved in the private roadway projects are dramatically changing the very nature of the construction business that, until very recently, was considered one of the most conservative sectors of the industry. The innovative concepts that the industrialization of bridges is introducing cover different areas:

- Reduction of construction time and construction cost;
- Durability of the structures (25 to 50 to 100 years);
- Replaceability of components such as bearings, post-tensioning, stays;
- Earthquake resistance of the structures;
- Staged construction;
- Integrated inspection and surveillance systems;
- Users' comfort and safety.

It is evident that the multiplication of such private projects, where cost, time, and durability are the decisive factors, will open the way to innovation in the bridge business as never before.

## 11.10.4 Environmental Impact

To prevent private projects from turning into environmental nightmares, private developers need to comply with strict obligations with respect to aesthetics, rights of way, and maintenance of the structures during the duration of the concessions. Government agencies have been developing design, construction, and operation criteria that will progressively become the rules of the BOT projects. As an example, such rules may force structural engineers to conceive structures that can be built in or over crowded areas of cities, with a minimal impact on existing conditions. Or they may impose specific constraints on aesthetics, shapes, or dimensions of structural elements. Further, they may require maintenance costs to be budgeted.

The involvement of the communities in such projects, even if sometimes it may be difficult to manage and may require a profound knowledge of the interests and aspirations of those concerned by the project, is essential for the smooth development of the work. In general, projects that are not well integrated into the context of the local environment or not consistent with the users' expectations run the risk of finishing in disarray or remaining incomplete.

### 11.10.5 Industrial Production of Structures

The experience acquired in large- or medium-size projects demonstrates that the industrialization of the production of structural elements always brings clear advantages in terms of quality and construction time. What frequently has been less noticed is the advantage that such industrialization can offer as far as the cost of a specific project.

The major change in the contractual conditions of the BOT projects is that the cost of "design + construction time" can now be estimated very precisely. If the completion of a project is delayed by 1 month, for instance, in a project worth U.S.$800,000,000, the cost to the developer is approximately equal to the interest that must be paid on that amount. If the interest is 5%, this represents U.S.$40,000,000/year; thus, every month gained in the duration of the design + construction period represents U.S.$3,300,000, or roughly, U.S.$100,000/day.

In these large projects the industrialization of production and the use of sophisticated systems to transport, erect, and assemble the prefabricated modules is reducing the duration of cycles, which usually may take 6 years when managed by the government agencies, to some 3 years, when managed by private organizations in a fully integrated way.

The introduction of the "assembly-line" approach to bridge building was taken to its limits during the construction of the Northumberland Strait Crossing (Confederation Bridge), Canada, a major bridge project which extends over 13 km of icy strait, with extreme weather conditions. The actual assembly of the components that constitute the 43 spans, 250-m each, took place in only 12 months, whereas to build just a single cast-in-place span of 250 m by traditional means is a difficult venture that takes at least 2 years (see Figure 11.55).

### 11.10.6 The Assembly of Structures

The production of the structural modules for precast segmental projects represents half of the process. The other half relates to the transport of these modules to the site, to their erection, and to the assembly methods to constitute the structural integrity of the bridge.

Generally, for the transport of current segments weighing from 30 to 100 tons, equipment already available in the market has been used. The transport is commonly by road, using convoys of "low boys," or by water, using barges. In some projects currently being built, 30 to 40 segments weighing between 50 and 60 tons are transported every night from the casting yards situated some 100 km from the large metropolis, to the site in the center of the city. The segments are picked up directly from the trucks by the assembly gantries, between midnight and five o'clock in the morning, to avoid interfering with heavy city traffic during the day.

The erection and assembly of such segments are also performed in a highly industrialized environment. With the span-by-span construction method, spans of 40 m can be assembled in 2 days, with crews working after hours. The cycle is almost independent of the type of segment, from D2 to D6, and therefore, the method is ideal for spans from 30 to 45 m. For cantilever construction, special gantries have been developed to assemble two parallel viaducts mimicking the procedure used in Hawaii, achieving speeds of construction of 3 weeks to complete two double cantilevers of 100 m [14]. This method very competitively covers spans of 80 to 120 m. Progressive placing of segments, using a swivel crane, has also been improved for this type of construction, which allows construction of spans from 45 to 65 m. Finally, for large cable-stayed spans, the use of precast segmental technologies successfully tested in milestone projects, like the Sunshine Skyway and the James River Bridges, is now being developed to cover different cross sections for the segments and to combine space trusses and composite sections [15].

The use of gigantic floating cranes, such as the *Svanen,* to place units as large as 190 m and with weights of 7500 tons, opens new prospects for the construction of bridges over rivers and straits (see Figure 11.52). Bridges that previously were almost impossible to build competitively and within the common constraints of construction schedules can now be conceived, designed, and built in short periods of time, by intensive use of precast technologies.

**FIGURE 11.56** View of the completed Second Expressway System Project, Thailand.

Clearly, this evolution is going to accelerate and will become global. Equipment designed to be used anywhere in the world will allow for the reduction of costs charged on a specific project. Furthermore, we can expect improvements in the performance and reliability of equipment specifically conceived to perform heavy lifting and assembly of bridge modules. Bridges will be designed which take into consideration the availability and the characteristics of these machines. Construction methods will then become, more than ever, a decisive factor in the design of structures.

## 11.10.7 Prospective

Design-and-build projects that were common in the 1960s provided some of the most innovative contributions to bridge engineering. Engineers and contractors working together produced competitive structures that paved the way for the development that has taken place all over the world during the last quarter century (Figure 11.56).

A new wave of innovative bridge concepts is already being generated by the privatization of roadway and bridge projects. This wave, which began in the vibrant business environment of Southeast Asia, will eventually reach the United States and the European markets. This time, engineers and contractors will be seconded by developers, finance specialists, and industrialists to shape the structures that will be built during the next century. The construction industry will also join other key industries in adopting high-technology and innovation as essential ingredients of its renewal [16].

## References


1. AASHTO, *Standard Specifications for Highway Bridges*, 16th ed., American Association of State Highway and Transportation Officials, Washington, D.C., 1996.
2. AASHTO, *Guide Specifications for Design and Construction of Segmental Concrete Bridges*, Draft 2nd ed., American Association of State Highway and Transportation Officials, Washington, D.C., August 1997.

3. Imbsen, R. A. et al., *Thermal Effects in Concrete Bridge Superstructures*, National Cooperative Highway Research Program Report 276, Transportation Research Board, Washington, D.C., 1985.
4. Priestley, M. J. N. et al., *Seismic Design and Retrofit of Bridges*, John Wiley & Sons, New York, 1996.
5. California Department of Transportation, *Bridge Design Specifications*, Sacramento.
6. California Department of Transportation, *Bridge Memos to Designers*, Sacramento.
7. California Department of Transportation, *Seismic Design Memo*, Sacramento.
8. Riobóo Martin, J. M., A new dimension in precast prestressed concrete bridges for congested urban areas in high seismic zones, *PCI J.*, 37, (2), 1992.
9. Restrepo, J. I. et al., Design of connections of earthquake resisting precast reinforced concrete perimeter frames, *PCI J.*, 40, (5), 1995.
10. Muller, J., Ten years of experience in precast segmental construction, *J. Precast/Prestressed Concrete Insti.*, 20, (1), 28–61, 1975.
11. Podolny, W., et al., *Construction and Design of Prestressed Concrete Segmental Bridges*, John Wiley & Sons, New York, 1982.
12. Muller, J., *Evolution dans la Construction de Grands Ponts: Montage et Entretien*, IABSE, 11th Congress, Vienna, 1980.
13. Sauvageot, G., Northumberland Strait Crossing, Canada, *4th International Bridge Engineering Conference; Proceedings*, 7, Vol. 1, August, 1995, 238–248.
14. Dodson, B., *Bangkok Second Stage Expressway System Segmental Structures*, in *4th International Bridge Engineering Conference, Proceedings* 7, Vol. 2, August, 1995, 199–204.
15. Sauvageot, G., *Hawaii H-3 Precast Segmental Windward Viaduct*, FIP Congress, Washington, D.C., 1994.
16. Muller, J., *Reflections on cable-stayed bridges*, Rev. Gen. Routes Aérodromes, Paris, October, 1994.