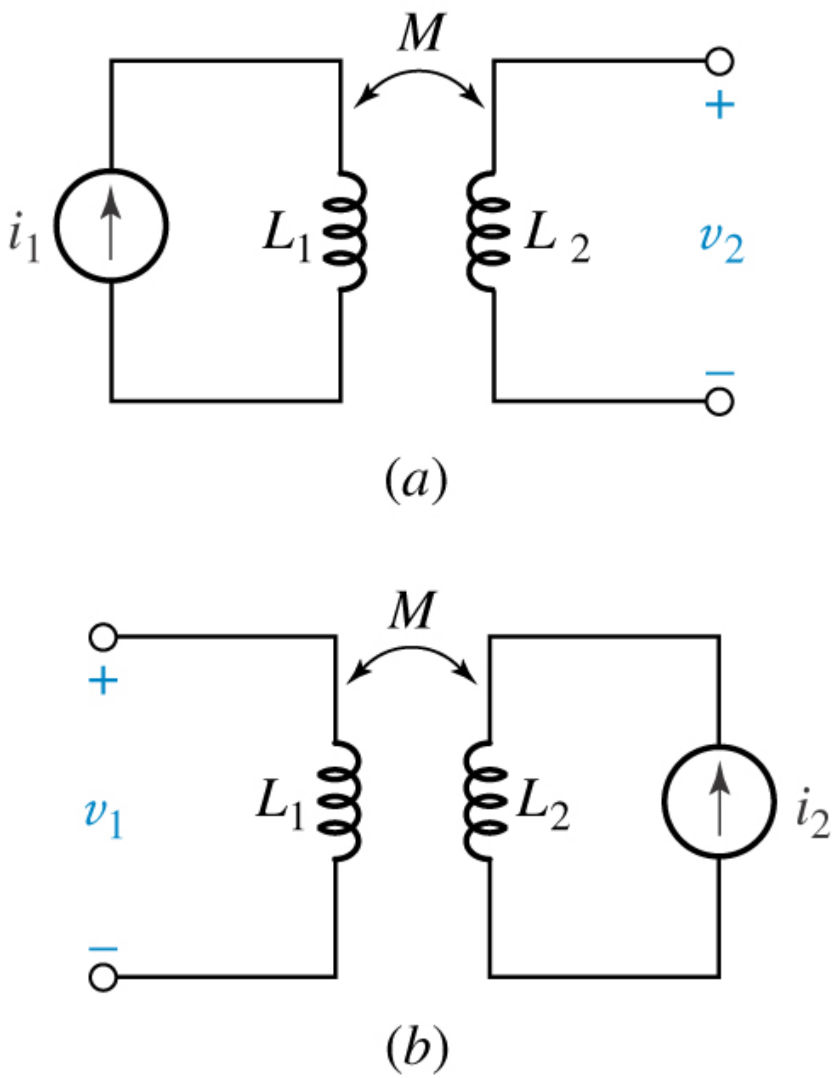
Magnetically-Coupled Circuits

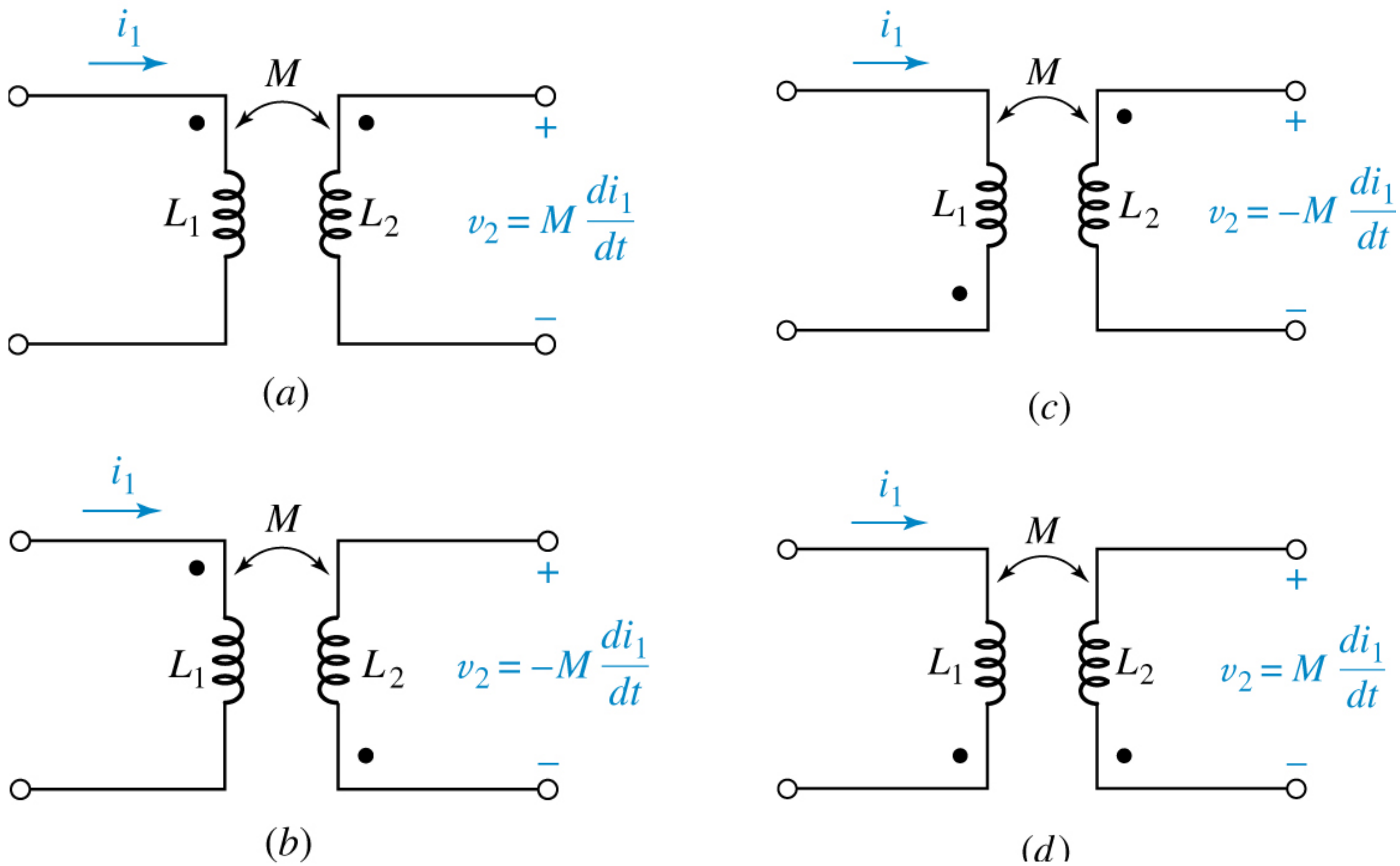
Magnetically-Coupled Circuits



(a) A current i_1 at L_1 produces an open-circuit voltage v_2 at L_2.

(b) A current i_2 at L_2 produces an open-circuit voltage v_1 at L_1.

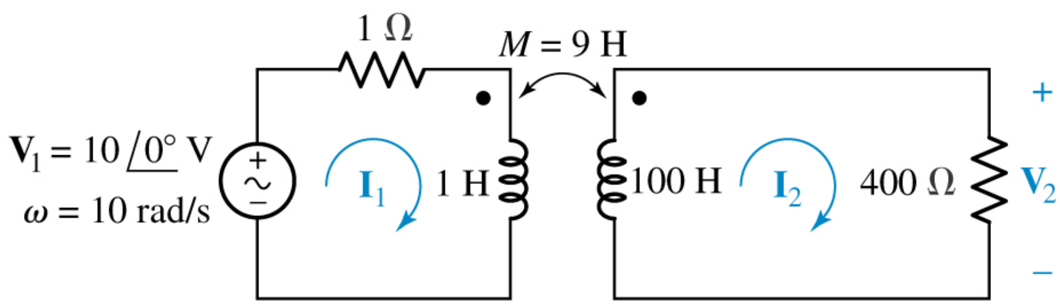
Magnetically-Coupled Circuits



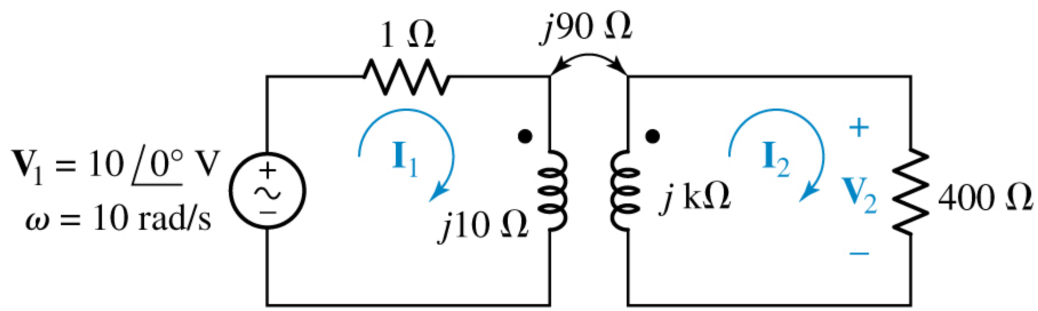
Current entering the dotted terminal of one coil produces a voltage that is sensed positively at the dotted terminal of the second coil. Current entering the undotted terminal of one coil produces a voltage that is sensed positively at the undotted terminal of the second coil.

Magnetically-Coupled Circuits

(a) A circuit containing mutual inductance in which the voltage ratio $\mathbf{V}_2/\ \mathbf{V}_1$ is desired. (b) Self and mutual inductances are replaced by the corresponding impedances.

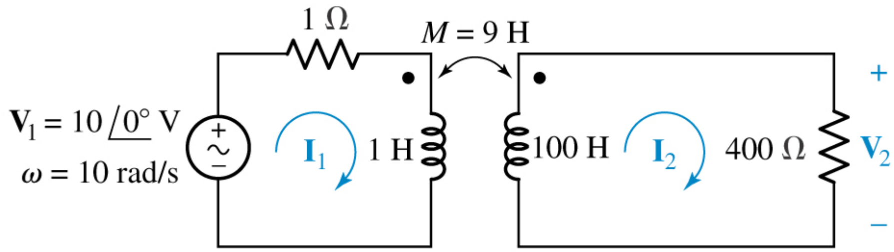


(a)

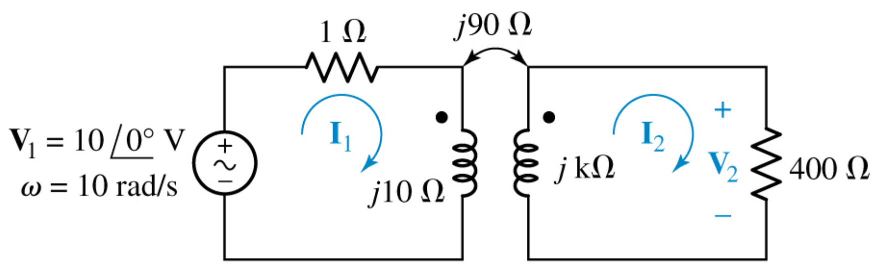


(b)

Magnetically-Coupled Circuits



(a)

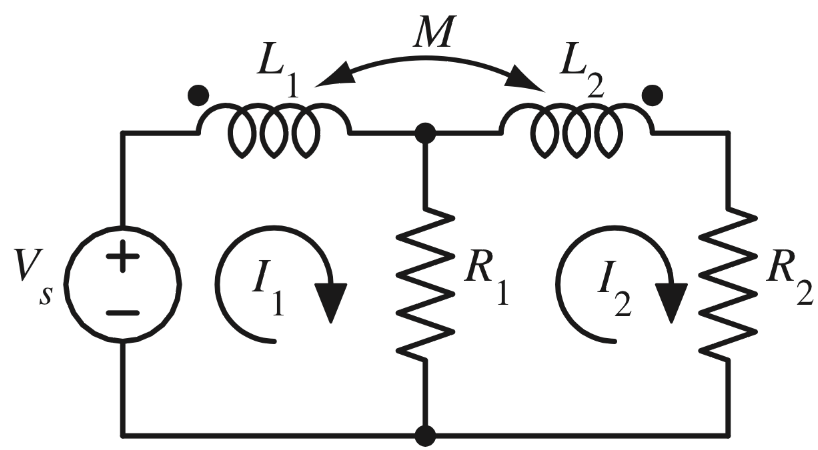


(b)

$$\begin{Bmatrix} -10 + I_1 + j10 I_1 - j90 I_2 = 0 \\ j1000 I_2 - j90 I_1 + 400 I_2 = 0 \end{Bmatrix} \Rightarrow \begin{matrix} I_1 = 2.063\angle -38.5° \\ I_2 = 0.172\angle -16.7° \end{matrix}$$

$V_2 = 400 I_2 = 68.8\ \text{V} \ > \ V_1 \Rightarrow$ Voltage gain is possible with a transformer, but not actual power gain. Current gain is also possible.

Magnetically-Coupled Circuits



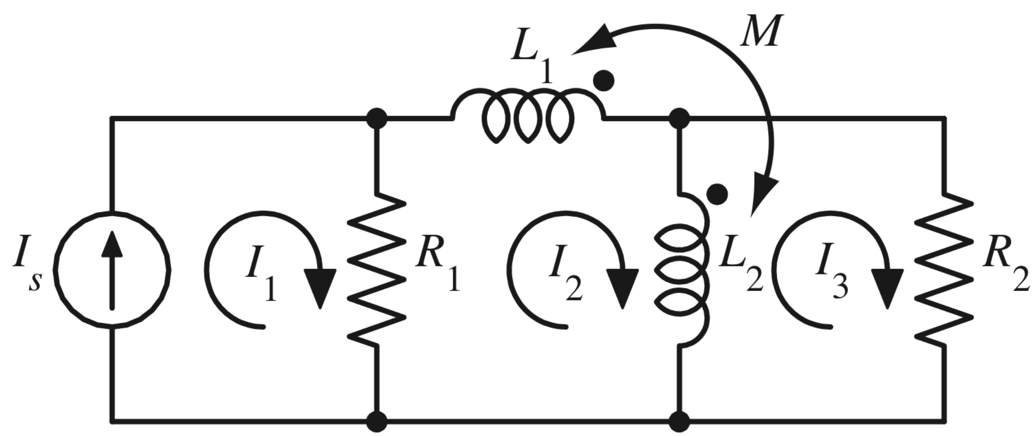
$$v_s(t) = 2\cos(100\pi t)\ \text{V} \ ,\ R_1 = 3\Omega \ ,\ R_2 = 8\Omega$$

$$L_1 = 50\text{mH} \ ,\ L_2 = 100\text{mH} \ ,\ M = 60\text{mH}$$

$$-2 + j100\pi(0.05)I_1 - j100\pi(0.06)I_2 + 3(I_1 - I_2) = 0$$

$$3(I_2 - I_1) + j100\pi(0.1)I_2 - j100\pi(0.06)I_1 + 8I_2 = 0$$

Magnetically-Coupled Circuits



$$i_s(t) = 5\cos(120\pi t)\ \text{A} \ ,\ R_1 = 50\Omega \ ,\ R_2 = 100\Omega$$

$$L_1 = 1\text{H} \ ,\ L_2 = 3\text{H} \ ,\ M = 1.5\text{mH}$$

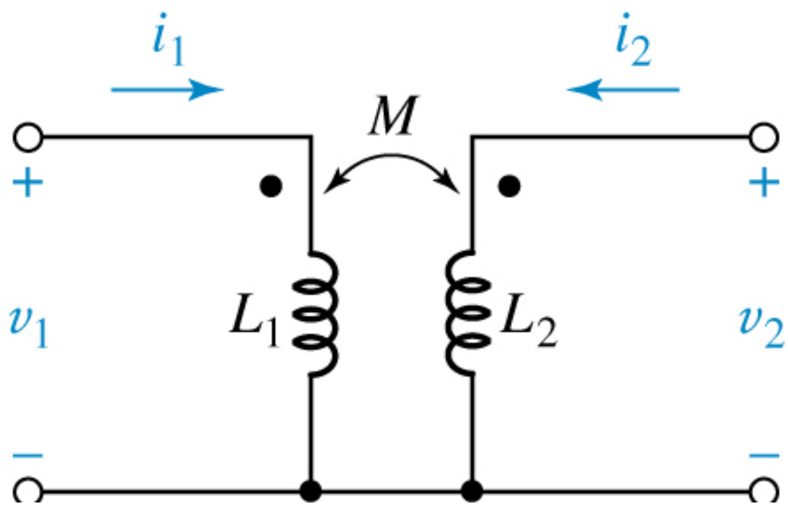
$$50(I_2 - I_1) + j120\pi(1)I_2 - j120\pi(1.5)(I_2 - I_3)$$

$$+ j120\pi(3)(I_2 - I_3) - j120\pi(1.5)I_2 = 0$$

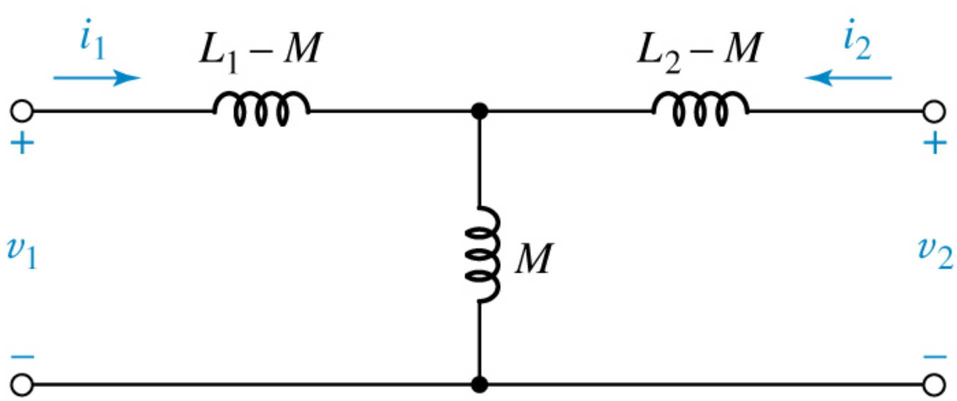
$$j120\pi(3)(I_3 - I_2) + j120\pi(1.5)I_2 + 100I_3 = 0$$

$$I_1 = 5$$

Transformer T Equivalent



(a)



(b)

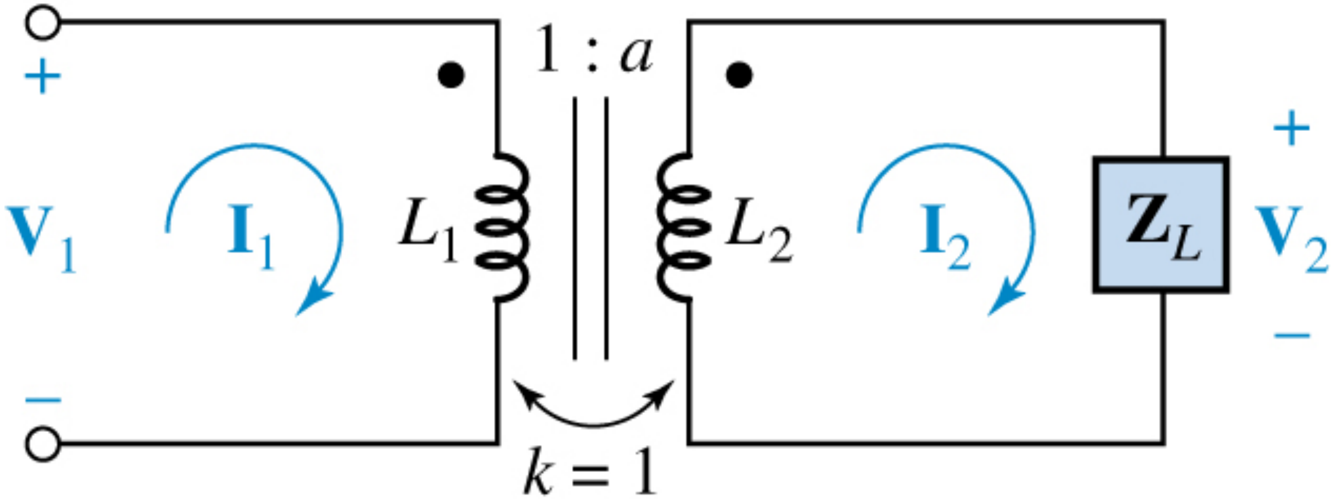
(a) A given transformer which is to be replaced by an equivalent network.

(b) The T equivalent.

$$M \le \sqrt{L_1 L_2}$$

The coupling coefficient k is $\dfrac{M}{\sqrt{L_1 L_2}}$

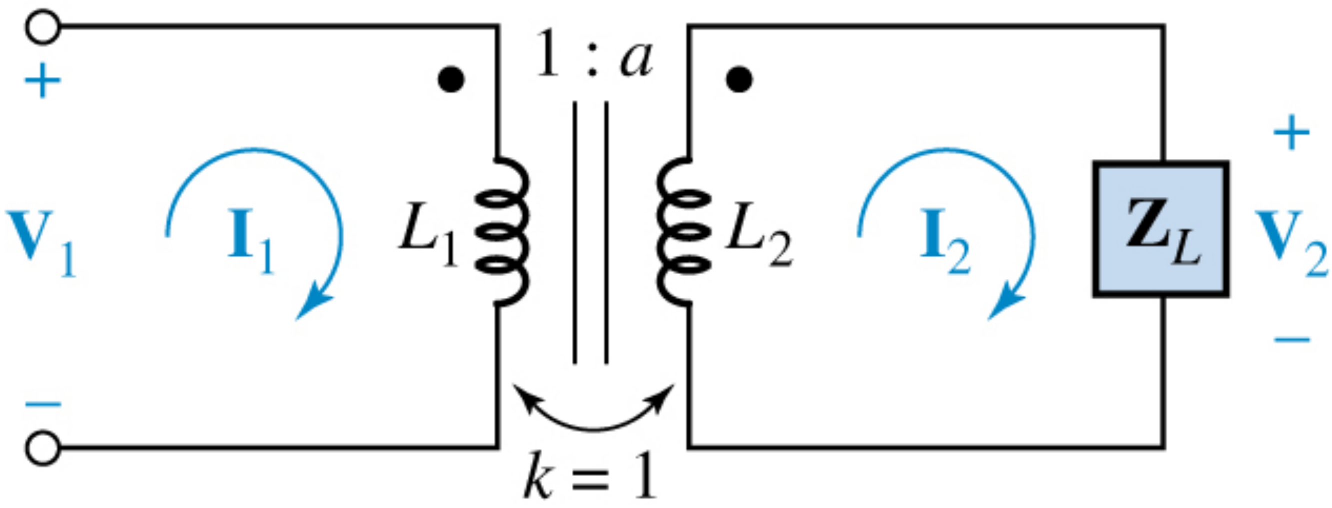
The Ideal Transformer



An ideal transformer is connected to a general load impedance.

The ideal transformer is a transformer with unity coupling coefficient k and with L_1 and L_2 very large (approaching infinity).

The Ideal Transformer

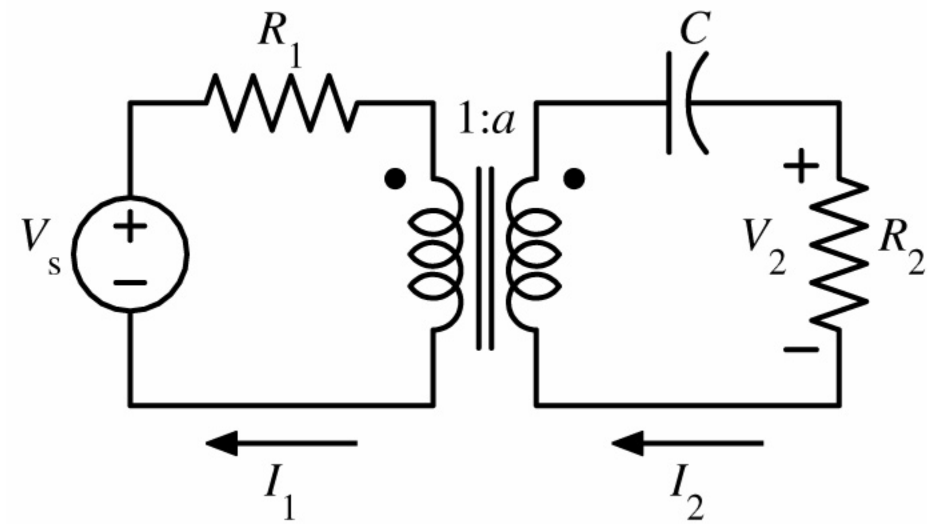


An ideal transformer is connected to a general load impedance.

$$a = \frac{N_2}{N_1} \quad , \quad V_2 = aV_1 \quad , \quad I_1 = aI_2 \quad , \quad \frac{L_2}{L_1} = \frac{N_2^2}{N_1^2} = a^2$$

$Z_{in} = \dfrac{Z_L}{a^2}$, where N_1 and N_2 are the numbers of turns in L_1 and L_2.

The Ideal Transformer



$V_s = 120\angle 0^\circ$, $R_1 = 50\Omega$, $C = 3\mu\text{F}$, $R_2 = 200\Omega$, $a = 5$

$f = 60\text{Hz} \Rightarrow \omega = 377 \text{ rad/s}$

$$-120 + I_1\left(50 + Z_L / 5^2\right) = 0$$

where $Z_L = \dfrac{1}{j377 \times 3 \times 10^{-6}} + 200 = 906.5\angle -77.25^\circ$

$I_1 = 1.7655\angle 31.374^\circ$, $I_2 = I_1 / 5 = 0.3533\angle 31.374^\circ$

$$V_2 = R_2 I_2 = 70.66\angle 31.374^\circ$$