mind that similar progressive steps were occurring in the areas of the telegraph, the telephone, power generation, the phonograph, appliances, and so on.

There is a tendency when reading about the great scientists, inventors, and innovators to believe that their contribution was a totally individual effort. In many instances, this was not the case. In fact, many of the great contributors were friends or associates who provided support and encouragement in their efforts to investigate various theories. At the very least, they were aware of one another's efforts to the degree possible in the days when a letter was often the best form of communication. In particular, note the closeness of the dates during periods of rapid development. One contributor seemed to spur on the efforts of the others or possibly provided the key needed to continue with the area of interest.

In the early stages, the contributors were not electrical, electronic, or computer engineers as we know them today. In most cases, they were physicists, chemists, mathematicians, or even philosophers. In addition, they were not from one or two communities of the Old World. The home country of many of the major contributors introduced in the paragraphs to follow is provided to show that almost every established community had some impact on the development of the fundamental laws of electrical circuits.

As you proceed through the remaining chapters of the text, you will find that a number of the units of measurement bear the name of major contributors in those areas—*volt* after Count Alessandro Volta, *ampere* after André Ampère, *ohm* after Georg Ohm, and so forth—fitting recognition for their important contributions to the birth of a major field of study.

Time charts indicating a limited number of major developments are provided in Fig. 1.2, primarily to identify specific periods of rapid development and to reveal how far we have come in the last few decades. In essence, the current state of the art is a result of efforts that
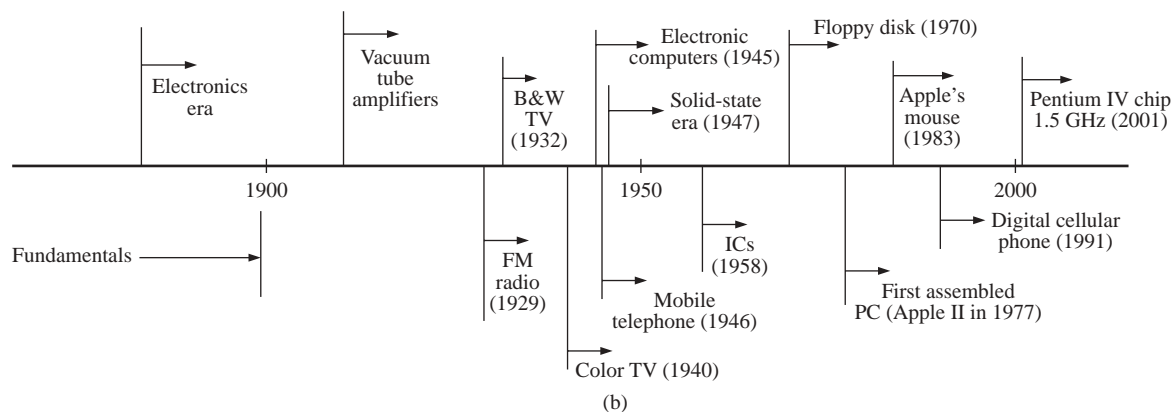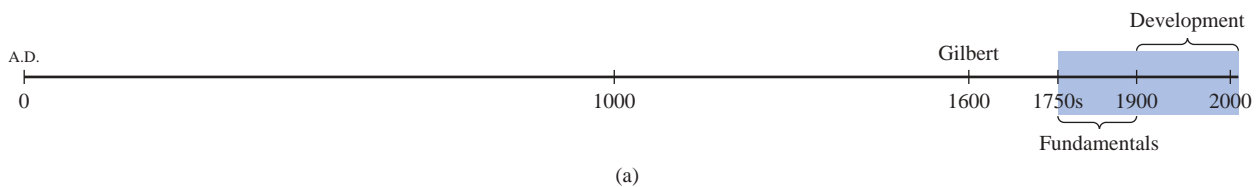

FIG. 1.2
Time charts: (a) long-range; (b) expanded.

began in earnest some 250 years ago, with progress in the last 100 years almost exponential.

As you read through the following brief review, try to sense the growing interest in the field and the enthusiasm and excitement that must have accompanied each new revelation. Although you may find some of the terms used in the review new and essentially meaningless, the remaining chapters will explain them thoroughly.

The Beginning

The phenomenon of **static electricity** has been toyed with since antiquity. The Greeks called the fossil resin substance so often used to demonstrate the effects of static electricity *elektron,* but no extensive study was made of the subject until William Gilbert researched the event in 1600. In the years to follow, there was a continuing investigation of electrostatic charge by many individuals such as Otto von Guericke, who developed the first machine to generate large amounts of charge, and Stephen Gray, who was able to transmit electrical charge over long distances on silk threads. Charles DuFay demonstrated that charges either attract or repel each other, leading him to believe that there were two types of charge—a theory we subscribe to today with our defined positive and negative charges.

There are many who believe that the true beginnings of the electrical era lie with the efforts of Pieter van Musschenbroek and Benjamin Franklin. In 1745, van Musschenbroek introduced the **Leyden jar** for the storage of electrical charge (the first capacitor) and demonstrated electrical shock (and therefore the power of this new form of energy). Franklin used the Leyden jar some seven years later to establish that lightning is simply an electrical discharge, and he expanded on a number of other important theories including the definition of the two types of charge as *positive* and *negative.* From this point on, new discoveries and theories seemed to occur at an increasing rate as the number of individuals performing research in the area grew.

In 1784, Charles Coulomb demonstrated in Paris that the force between charges is inversely related to the square of the distance between the charges. In 1791, Luigi Galvani, professor of anatomy at the University of Bologna, Italy, performed experiments on the effects of electricity on animal nerves and muscles. The first **voltaic cell,** with its ability to produce electricity through the chemical action of a metal dissolving in an acid, was developed by another Italian, Alessandro Volta, in 1799.

The fever pitch continued into the early 1800s with Hans Christian Oersted, a Swedish professor of physics, announcing in 1820 a relationship between magnetism and electricity that serves as the foundation for the theory of **electromagnetism** as we know it today. In the same year, a French physicist, André Ampère, demonstrated that there are magnetic effects around every current-carrying conductor and that current-carrying conductors can attract and repel each other just like magnets. In the period 1826 to 1827, a German physicist, Georg Ohm, introduced an important relationship between potential, current, and resistance which we now refer to as *Ohm's law.* In 1831, an English physicist, Michael Faraday, demonstrated his theory of *electromagnetic induction,* whereby a changing current in one coil can induce a changing current in another coil, even though the two coils are not directly connected. Professor Faraday also did extensive work on a storage device he called the con-

denser, which we refer to today as a *capacitor*. He introduced the idea of adding a dielectric between the plates of a capacitor to increase the storage capacity (Chapter 10). James Clerk Maxwell, a Scottish professor of natural philosophy, performed extensive mathematical analyses to develop what are currently called *Maxwell's equations*, which support the efforts of Faraday linking electric and magnetic effects. Maxwell also developed the *electromagnetic theory of light* in 1862, which, among other things, revealed that electromagnetic waves travel through air at the velocity of light (186,000 miles per second or 3×10^8 meters per second). In 1888, a German physicist, Heinrich Rudolph Hertz, through experimentation with lower-frequency electromagnetic waves (microwaves), substantiated Maxwell's predictions and equations. In the mid 1800s, Professor Gustav Robert Kirchhoff introduced a series of laws of voltages and currents that find application at every level and area of this field (Chapters 5 and 6). In 1895, another German physicist, Wilhelm Röntgen, discovered electromagnetic waves of high frequency, commonly called *X rays* today.

By the end of the 1800s, a significant number of the fundamental equations, laws, and relationships had been established, and various fields of study, including electronics, power generation, and calculating equipment, started to develop in earnest.

The Age of Electronics

Radio   The true beginning of the electronics era is open to debate and is sometimes attributed to efforts by early scientists in applying potentials across evacuated glass envelopes. However, many trace the beginning to Thomas Edison, who added a metallic electrode to the vacuum of the tube and discovered that a current was established between the metal electrode and the filament when a positive voltage was applied to the metal electrode. The phenomenon, demonstrated in 1883, was referred to as the **Edison effect.** In the period to follow, the transmission of radio waves and the development of the radio received widespread attention. In 1887, Heinrich Hertz, in his efforts to verify Maxwell's equations, transmitted radio waves for the first time in his laboratory. In 1896, an Italian scientist, Guglielmo Marconi (often called the father of the radio), demonstrated that telegraph signals could be sent through the air over long distances (2.5 kilometers) using a grounded antenna. In the same year, Aleksandr Popov sent what might have been the first radio message some 300 yards. The message was the name *"Heinrich Hertz"* in respect for Hertz's earlier contributions. In 1901, Marconi established radio communication across the Atlantic.

In 1904, John Ambrose Fleming expanded on the efforts of Edison to develop the first diode, commonly called **Fleming's valve**—actually the first of the *electronic devices*. The device had a profound impact on the design of detectors in the receiving section of radios. In 1906, Lee De Forest added a third element to the vacuum structure and created the first amplifier, the triode. Shortly thereafter, in 1912, Edwin Armstrong built the first regenerative circuit to improve receiver capabilities and then used the same contribution to develop the first nonmechanical oscillator. By 1915 radio signals were being transmitted across the United States, and in 1918 Armstrong applied for a patent for the superheterodyne circuit employed in virtually every television and radio to permit amplification at one frequency rather than at the full range of

incoming signals. The major components of the modern-day radio were now in place, and sales in radios grew from a few million dollars in the early 1920s to over $1 billion by the 1930s. The 1930s were truly the golden years of radio, with a wide range of productions for the listening audience.

Television   The 1930s were also the true beginnings of the television era, although development on the picture tube began in earlier years with Paul Nipkow and his *electrical telescope* in 1884 and John Baird and his long list of successes, including the transmission of television pictures over telephone lines in 1927 and over radio waves in 1928, and simultaneous transmission of pictures and sound in 1930. In 1932, NBC installed the first commercial television antenna on top of the Empire State Building in New York City, and RCA began regular broadcasting in 1939. The war slowed development and sales, but in the mid 1940s the number of sets grew from a few thousand to a few million. Color television became popular in the early 1960s.

Computers   The earliest computer system can be traced back to Blaise Pascal in 1642 with his mechanical machine for adding and subtracting numbers. In 1673 Gottfried Wilhelm von Leibniz used the *Leibniz wheel* to add multiplication and division to the range of operations, and in 1823 Charles Babbage developed the **difference engine** to add the mathematical operations of sine, cosine, logs, and several others. In the years to follow, improvements were made, but the system remained primarily mechanical until the 1930s when electromechanical systems using components such as relays were introduced. It was not until the 1940s that totally electronic systems became the new wave. It is interesting to note that, even though IBM was formed in 1924, it did not enter the computer industry until 1937. An entirely electronic system known as **ENIAC** was dedicated at the University of Pennsylvania in 1946. It contained 18,000 tubes and weighed 30 tons but was several times faster than most electromechanical systems. Although other vacuum tube systems were built, it was not until the birth of the solid-state era that computer systems experienced a major change in size, speed, and capability.
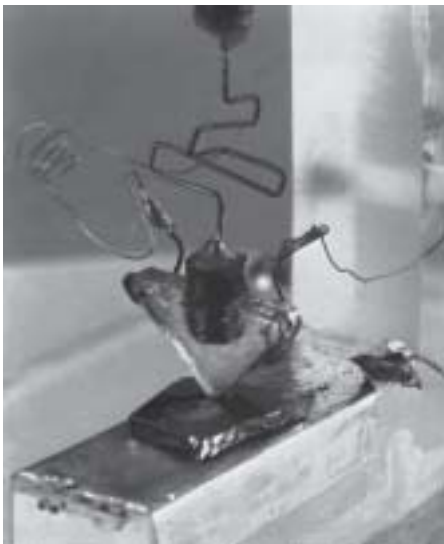
The Solid-State Era

In 1947, physicists William Shockley, John Bardeen, and Walter H. Brattain of Bell Telephone Laboratories demonstrated the point-contact **transistor** (Fig. 1.3), an amplifier constructed entirely of solid-state materials with no requirement for a vacuum, glass envelope, or heater voltage for the filament. Although reluctant at first due to the vast amount of material available on the design, analysis, and synthesis of tube networks, the industry eventually accepted this new technology as the wave of the future. In 1958 the first **integrated circuit (IC)** was developed at Texas Instruments, and in 1961 the first commercial integrated circuit was manufactured by the Fairchild Corporation.

It is impossible to review properly the entire history of the electrical/electronics field in a few pages. The effort here, both through the discussion and the time graphs of Fig. 1.2, was to reveal the amazing progress of this field in the last 50 years. The growth appears to be truly exponential since the early 1900s, raising the interesting question, Where do we go from here? The time chart suggests that the next few

FIG. 1.3

The first transistor. (Courtesy of AT&T, Bell Laboratories.)

decades will probably contain many important innovative contributions that may cause an even faster growth curve than we are now experiencing.

1.3 UNITS OF MEASUREMENT

In any technical field it is naturally important to understand the basic concepts and the impact they will have on certain parameters. However, the application of these rules and laws will be successful only if the mathematical operations involved are applied correctly. In particular, it is vital that the importance of applying the proper unit of measurement to a quantity is understood and appreciated. Students often generate a numerical solution but decide not to apply a unit of measurement to the result because they are somewhat unsure of which unit should be applied. Consider, for example, the following very fundamental physics equation:

$$v = \frac{d}{t} \qquad \begin{aligned} v &= \text{velocity} \\ d &= \text{distance} \\ t &= \text{time} \end{aligned} \qquad \textbf{(1.1)}$$

Assume, for the moment, that the following data are obtained for a moving object:

$$d = 4000 \text{ ft}$$

$$t = 1 \text{ min}$$

and v is desired in miles per hour. Often, without a second thought or consideration, the numerical values are simply substituted into the equation, with the result here that

$$v = \frac{d}{t} = \frac{4000 \text{ ft}}{1 \text{ min}} = \cancel{4000 \text{ mi/h}}$$

As indicated above, the solution is totally incorrect. If the result is desired in *miles per hour,* the unit of measurement for distance must be *miles,* and that for time, *hours.* In a moment, when the problem is analyzed properly, the extent of the error will demonstrate the importance of ensuring that

the numerical value substituted into an equation must have the unit of measurement specified by the equation.

The next question is normally, How do I convert the distance and time to the proper unit of measurement? A method will be presented in a later section of this chapter, but for now it is given that

$$1 \text{ mi} = 5280 \text{ ft}$$

$$4000 \text{ ft} = 0.7576 \text{ mi}$$

$$1 \text{ min} = \tfrac{1}{60} \text{ h} = 0.0167 \text{ h}$$

Substituting into Eq. (1.1), we have

$$v = \frac{d}{t} = \frac{0.7576 \text{ mi}}{0.0167 \text{ h}} = 45.37 \text{ mi/h}$$

which is significantly different from the result obtained before.

To complicate the matter further, suppose the distance is given in kilometers, as is now the case on many road signs. First, we must realize that the prefix *kilo* stands for a multiplier of 1000 (to be introduced

in Section 1.5), and then we must find the conversion factor between kilometers and miles. If this conversion factor is not readily available, we must be able to make the conversion between units using the conversion factors between meters and feet or inches, as described in Section 1.6.

Before substituting numerical values into an equation, try to mentally establish a reasonable range of solutions for comparison purposes. For instance, if a car travels 4000 ft in 1 min, does it seem reasonable that the speed would be 4000 mi/h? Obviously not! This self-checking procedure is particularly important in this day of the hand-held calculator, when ridiculous results may be accepted simply because they appear on the digital display of the instrument.

Finally,

if a unit of measurement is applicable to a result or piece of data, then it must be applied to the numerical value.

To state that $v = 45.37$ without including the unit of measurement *mi/h* is meaningless.

Equation (1.1) is not a difficult one. A simple algebraic manipulation will result in the solution for any one of the three variables. However, in light of the number of questions arising from this equation, the reader may wonder if the difficulty associated with an equation will increase at the same rate as the number of terms in the equation. In the broad sense, this will not be the case. There is, of course, more room for a mathematical error with a more complex equation, but once the proper system of units is chosen and each term properly found in that system, there should be very little added difficulty associated with an equation requiring an increased number of mathematical calculations.

In review, before substituting numerical values into an equation, be absolutely sure of the following:

1. *Each quantity has the proper unit of measurement as defined by the equation.*
2. *The proper magnitude of each quantity as determined by the defining equation is substituted.*
3. *Each quantity is in the same system of units (or as defined by the equation).*
4. *The magnitude of the result is of a reasonable nature when compared to the level of the substituted quantities.*
5. *The proper unit of measurement is applied to the result.*

1.4 SYSTEMS OF UNITS

In the past, the *systems of units* most commonly used were the English and metric, as outlined in Table 1.1. Note that while the English system is based on a single standard, the metric is subdivided into two interrelated standards: the **MKS** and the **CGS.** Fundamental quantities of these systems are compared in Table 1.1 along with their abbreviations. The MKS and CGS systems draw their names from the units of measurement used with each system; the MKS system uses *M*eters, *K*ilograms, and *S*econds, while the CGS system uses *C*entimeters, *G*rams, and *S*econds.

Understandably, the use of more than one system of units in a world that finds itself continually shrinking in size, due to advanced technical developments in communications and transportation, would introduce

TABLE 1.1

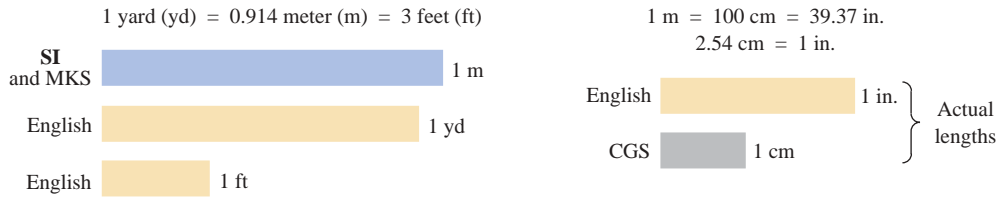*Comparison of the English and metric systems of units.*

English	Metric		
	MKS	**CGS**	**SI**
Length: Yard (yd) (0.914 m)	Meter (m) (39.37 in.) (100 cm)	Centimeter (cm) (2.54 cm = 1 in.)	**Meter (m)**
Mass: **Slug** (14.6 kg)	Kilogram (kg) (1000 g)	Gram (g)	**Kilogram (kg)**
Force: **Pound** (lb) (4.45 N)	Newton (N) (100,000 dynes)	Dyne	**Newton (N)**
Temperature: Fahrenheit (°F) $\left(= \dfrac{9}{5}°C + 32\right)$	Celsius or Centigrade (°C) $\left(= \dfrac{5}{9}(°F - 32)\right)$	Centigrade (°C)	**Kelvin (K)** K = 273.15 + °C
Energy: Foot-pound (ft-lb) (1.356 joules)	Newton-meter (N•m) or joule (J) (0.7376 ft-lb)	Dyne-centimeter or erg (1 joule = 10^7 ergs)	**Joule (J)**
Time: Second (s)	Second (s)	Second (s)	**Second (s)**

unnecessary complications to the basic understanding of any technical data. The need for a standard set of units to be adopted by all nations has become increasingly obvious. The International Bureau of Weights and Measures located at Sèvres, France, has been the host for the General Conference of Weights and Measures, attended by representatives from all nations of the world. In 1960, the General Conference adopted a system called Le Système International d'Unités (International System of Units), which has the international abbreviation **SI.** Since then, it has been adopted by the Institute of Electrical and Electronic Engineers, Inc. (IEEE) in 1965 and by the United States of America Standards Institute in 1967 as a standard for all scientific and engineering literature.

For comparison, the SI units of measurement and their abbreviations appear in Table 1.1. These abbreviations are those usually applied to each unit of measurement, and they were carefully chosen to be the most effective. Therefore, it is important that they be used whenever applicable to ensure universal understanding. Note the similarities of the SI system to the MKS system. This text will employ, whenever possible and practical, all of the major units and abbreviations of the SI system in an effort to support the need for a universal system. Those readers requiring additional information on the SI system should contact the information office of the American Society for Engineering Education (ASEE).*

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*American Society for Engineering Education (ASEE), 1818 N Street N.W., Suite 600, Washington, D.C. 20036-2479; (202) 331-3500; http://www.asee.org/.
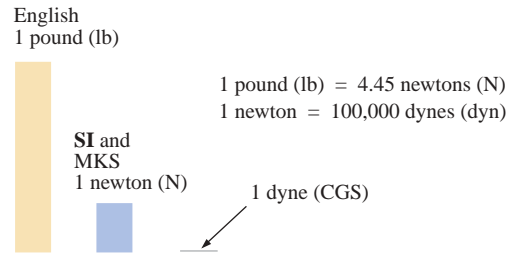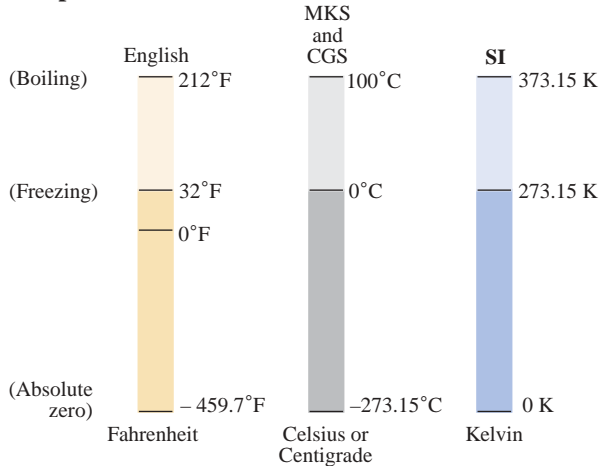
$$\sum_{I}^{S}$$

Length:

1 yard (yd) = 0.914 meter (m) = 3 feet (ft)

SI and MKS   1 m

English   1 yd

English   1 ft

1 m = 100 cm = 39.37 in.
2.54 cm = 1 in.

English   1 in.

CGS   1 cm

Actual lengths

Mass:

1 slug = 14.6 kilograms

1 slug English

1 kilogram = 1000 g

1 kg **SI** and MKS

1 g CGS

Force:

English
1 pound (lb)

1 pound (lb) = 4.45 newtons (N)
1 newton = 100,000 dynes (dyn)

SI and MKS
1 newton (N)

1 dyne (CGS)

Temperature:

MKS and CGS

English

SI

(Boiling)   212°F   100°C   373.15 K

(Freezing)   32°F   0°C   273.15 K

0°F

(Absolute zero)   −459.7°F   −273.15°C   0 K

Fahrenheit   Celsius or Centigrade   Kelvin

$$°F = \frac{9}{5}°C + 32°$$

$$°C = \frac{5}{9}(°F - 32°)$$

$$K = 273.15 + °C$$

Energy:

English
1 ft-lb

SI and MKS
1 joule (J)

1 ft-lb = 1.356 joules
1 joule = 10^7 ergs

1 erg (CGS)

FIG. 1.4

Comparison of units of the various systems of units.

Figure 1.4 should help the reader develop some feeling for the relative magnitudes of the units of measurement of each system of units. Note in the figure the relatively small magnitude of the units of measurement for the CGS system.

A standard exists for each unit of measurement of each system. The standards of some units are quite interesting.

The **meter** was originally defined in 1790 to be 1/10,000,000 the distance between the equator and either pole at sea level, a length preserved on a platinum-iridium bar at the International Bureau of Weights and Measures at Sèvres, France.

The meter is now defined with reference to the speed of light in a vacuum, which is 299,792,458 m/s.

The kilogram is defined as a mass equal to 1000 times the mass of one cubic centimeter of pure water at 4°C.

This standard is preserved in the form of a platinum-iridium cylinder in Sèvres.

The **second** was originally defined as 1/86,400 of the mean solar day. However, since Earth's rotation is slowing down by almost 1 second every 10 years,

the second was redefined in 1967 as 9,192,631,770 periods of the electromagnetic radiation emitted by a particular transition of cesium atom.

1.5 SIGNIFICANT FIGURES, ACCURACY, AND ROUNDING OFF

This section will emphasize the importance of being aware of the source of a piece of data, how a number appears, and how it should be treated. Too often we write numbers in various forms with little concern for the format used, the number of digits that should be included, and the unit of measurement to be applied.

For instance, measurements of 22.1″ and 22.10″ imply different levels of accuracy. The first suggests that the measurement was made by an instrument accurate only to the tenths place; the latter was obtained with instrumentation capable of reading to the hundredths place. The use of zeros in a number, therefore, must be treated with care and the implications must be understood.

In general, there are two types of numbers, *exact* and *approximate.* Exact numbers are precise to the exact number of digits presented, just as we know that there are 12 apples in a dozen and not 12.1. Throughout the text the numbers that appear in the descriptions, diagrams, and examples are considered *exact,* so that a battery of 100 V can be written as 100.0 V, 100.00 V, and so on, since it is 100 V at any level of precision. The additional zeros were not included for purposes of clarity. However, in the laboratory environment, where measurements are continually being taken and the level of accuracy can vary from one instrument to another, it is important to understand how to work with the results. Any reading obtained in the laboratory should be considered *approximate.* The analog scales with their pointers may be difficult to read, and even though the digital meter provides only specific digits on its display, it is limited to the number of digits it can provide, leaving us to wonder about the less significant digits not appearing on the display.

The precision of a reading can be determined by the number of *significant figures (digits)* present. Significant digits are those integers (0 to 9) that can be assumed to be accurate for the measurement being made. The result is that all nonzero numbers are considered significant, with zeros being significant in only some cases. For instance, the zeros in 1005 are considered significant because they define the size of the number and are surrounded by nonzero digits. However, for a number such as 0.064, the two zeros are not considered significant because they are used only to define the location of the decimal point and not the accuracy of the reading. For the number 0.4020, the zero to the left of the decimal point is not significant, but the other two are because they define the magnitude of the number and the fourth-place accuracy of the reading.

When adding approximate numbers, it is important to be sure that the accuracy of the readings is consistent throughout. To add a quantity accurate only to the tenths place to a number accurate to the thousandths

place will result in a total having accuracy only to the tenths place. One cannot expect the reading with the higher level of accuracy to improve the reading with only tenths-place accuracy.

In the addition or subtraction of approximate numbers, the entry with the lowest level of accuracy determines the format of the solution.

For the multiplication and division of approximate numbers, the result has the same number of significant figures as the number with the least number of significant figures.

For approximate numbers (and exact, for that matter) there is often a need to *round off* the result; that is, you must decide on the appropriate level of accuracy and alter the result accordingly. The accepted procedure is simply to note the digit following the last to appear in the rounded-off form, and add a 1 to the last digit if it is greater than or equal to 5, and leave it alone if it is less than 5. For example, $3.186 \cong 3.19 \cong 3.2$, depending on the level of precision desired. The symbol $\cong$ appearing means *approximately equal to.*

EXAMPLE 1.1 Perform the indicated operations with the following approximate numbers and round off to the appropriate level of accuracy.

a. $532.6 + 4.02 + 0.036 = 536.656 \cong$ **536.7** (as determined by 532.6)
b. $0.04 + 0.003 + 0.0064 = 0.0494 \cong$ **0.05** (as determined by 0.04)
c. $4.632 \times 2.4 = 11.1168 \cong$ **11** (as determined by the two significant digits of 2.4)
d. $3.051 \times 802 = 2446.902 \cong$ **2450** (as determined by the three significant digits of 802)
e. $1402/6.4 = 219.0625 \cong$ **220** (as determined by the two significant digits of 6.4)
f. $0.0046/0.05 = 0.0920 \cong$ **0.09** (as determined by the one significant digit of 0.05)

1.6 POWERS OF TEN

It should be apparent from the relative magnitude of the various units of measurement that very large and very small numbers will frequently be encountered in the sciences. To ease the difficulty of mathematical operations with numbers of such varying size, *powers of ten* are usually employed. This notation takes full advantage of the mathematical properties of powers of ten. The notation used to represent numbers that are integer powers of ten is as follows:

$$1 = 10^0 \qquad 1/10 = \quad 0.1 = 10^{-1}$$
$$10 = 10^1 \qquad 1/100 = \quad 0.01 = 10^{-2}$$
$$100 = 10^2 \qquad 1/1000 = \quad 0.001 = 10^{-3}$$
$$1000 = 10^3 \qquad 1/10{,}000 = 0.0001 = 10^{-4}$$

In particular, note that $10^0 = 1$, and, in fact, any quantity to the zero power is 1 ($x^0 = 1$, $1000^0 = 1$, and so on). Also, note that the numbers in the list that are greater than 1 are associated with positive powers of ten, and numbers in the list that are less than 1 are associated with negative powers of ten.

A quick method of determining the proper power of ten is to place a caret mark to the right of the numeral 1 wherever it may occur; then count from this point to the number of places to the right or left before arriving at the decimal point. Moving to the right indicates a positive power of ten, whereas moving to the left indicates a negative power. For example,

$$10,000.0 = 1\underset{1 \quad 2 \ 3 \ 4}{0,000} = 10^{+4}$$

$$0.00001 = 0\underset{5 \ 4 \ 3 \ 2 \ 1}{.00001} = 10^{-5}$$

Some important mathematical equations and relationships pertaining to powers of ten are listed below, along with a few examples. In each case, n and m can be any positive or negative real number.

$$\frac{1}{10^n} = 10^{-n} \qquad \frac{1}{10^{-n}} = 10^n \qquad (1.2)$$

Equation (1.2) clearly reveals that shifting a power of ten from the denominator to the numerator, or the reverse, requires simply changing the sign of the power.

EXAMPLE 1.2

a. $\dfrac{1}{1000} = \dfrac{1}{10^{+3}} = 10^{-3}$

b. $\dfrac{1}{0.00001} = \dfrac{1}{10^{-5}} = 10^{+5}$

The product of powers of ten:

$$(10^n)(10^m) = 10^{(n+m)} \qquad (1.3)$$

EXAMPLE 1.3

a. $(1000)(10,000) = (10^3)(10^4) = 10^{(3+4)} = 10^7$
b. $(0.00001)(100) = (10^{-5})(10^2) = 10^{(-5+2)} = 10^{-3}$

The division of powers of ten:

$$\frac{10^n}{10^m} = 10^{(n-m)} \qquad (1.4)$$

EXAMPLE 1.4

a. $\dfrac{100,000}{100} = \dfrac{10^5}{10^2} = 10^{(5-2)} = 10^3$

b. $\dfrac{1000}{0.0001} = \dfrac{10^3}{10^{-4}} = 10^{(3-(-4))} = 10^{(3+4)} = 10^7$

Note the use of parentheses in part (b) to ensure that the proper sign is established between operators.

The power of powers of ten:

$$(10^n)^m = 10^{(nm)} \qquad \textbf{(1.5)}$$

EXAMPLE 1.5

a. $(100)^4 = (10^2)^4 = 10^{(2)(4)} = 10^8$

b. $(1000)^{-2} = (10^3)^{-2} = 10^{(3)(-2)} = 10^{-6}$

c. $(0.01)^{-3} = (10^{-2})^{-3} = 10^{(-2)(-3)} = 10^6$

Basic Arithmetic Operations

Let us now examine the use of powers of ten to perform some basic arithmetic operations using numbers that are not just powers of ten. The number 5000 can be written as $5 \times 1000 = 5 \times 10^3$, and the number 0.0004 can be written as $4 \times 0.0001 = 4 \times 10^{-4}$. Of course, 10^5 can also be written as 1×10^5 if it clarifies the operation to be performed.

Addition and Subtraction   To perform addition or subtraction using powers of ten, the power of ten *must be the same for each term;* that is,

$$A \times 10^n \pm B \times 10^n = (A \pm B) \times 10^n \qquad \textbf{(1.6)}$$

Equation (1.6) covers all possibilities, but students often prefer to remember a verbal description of how to perform the operation.

Equation (1.6) states

when adding or subtracting numbers in a powers-of-ten format, be sure that the power of ten is the same for each number. Then separate the multipliers, perform the required operation, and apply the same power of ten to the result.

EXAMPLE 1.6

a. $6300 + 75{,}000 = (6.3)(1000) + (75)(1000)$
$= 6.3 \times 10^3 + 75 \times 10^3$
$= (6.3 + 75) \times 10^3$
$= \textbf{81.3} \times \textbf{10}^{\textbf{3}}$

b. $0.00096 - 0.000086 = (96)(0.00001) - (8.6)(0.00001)$
$= 96 \times 10^{-5} - 8.6 \times 10^{-5}$
$= (96 - 8.6) \times 10^{-5}$
$= \textbf{87.4} \times \textbf{10}^{\textbf{-5}}$

Multiplication   In general,

$$(A \times 10^n)(B \times 10^m) = (A)(B) \times 10^{n+m} \qquad \textbf{(1.7)}$$

revealing that the *operations with the powers of ten can be separated from the operation with the multipliers.*

Equation (1.7) states

when multiplying numbers in the powers-of-ten format, first find the product of the multipliers and then determine the power of ten for the result by adding the power-of-ten exponents.

EXAMPLE 1.7

a. $(0.0002)(0.000007) = [(2)(0.0001)][(7)(0.000001)]$
$$= (2 \times 10^{-4})(7 \times 10^{-6})$$
$$= (2)(7) \times (10^{-4})(10^{-6})$$
$$= \mathbf{14 \times 10^{-10}}$$

b. $(340{,}000)(0.00061) = (3.4 \times 10^5)(61 \times 10^{-5})$
$$= (3.4)(61) \times (10^5)(10^{-5})$$
$$= 207.4 \times 10^0$$
$$= \mathbf{207.4}$$

Division   In general,

$$\boxed{\frac{A \times 10^n}{B \times 10^m} = \frac{A}{B} \times 10^{n-m}} \qquad \textbf{(1.8)}$$

revealing again that the *operations with the powers of ten can be separated from the same operation with the multipliers.*

Equation (1.8) states

when dividing numbers in the powers-of-ten format, first find the result of dividing the multipliers. Then determine the associated power for the result by subtracting the power of ten of the denominator from the power of ten of the numerator.

EXAMPLE 1.8

a. $\dfrac{0.00047}{0.002} = \dfrac{47 \times 10^{-5}}{2 \times 10^{-3}} = \left(\dfrac{47}{2}\right) \times \left(\dfrac{10^{-5}}{10^{-3}}\right)$
$$= \mathbf{23.5 \times 10^{-2}}$$

b. $\dfrac{690{,}000}{0.00000013} = \dfrac{69 \times 10^4}{13 \times 10^{-8}} = \left(\dfrac{69}{13}\right) \times \left(\dfrac{10^4}{10^{-8}}\right)$
$$= \mathbf{5.31 \times 10^{12}}$$

Powers   In general,

$$\boxed{(A \times 10^n)^m = A^m \times 10^{nm}} \qquad \textbf{(1.9)}$$

which again permits the separation of the *operation with the powers of ten from the multipliers.*

Equation (1.9) states

when finding the power of a number in the power-of-ten format, first separate the multiplier from the power of ten and determine each separately. Determine the power-of-ten component by multiplying the power of ten by the power to be determined.

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$\sum_{I}^{S}$

EXAMPLE 1.9

a. $(0.00003)^3 = (3 \times 10^{-5})^3 = (3)^3 \times (10^{-5})^3$
$= \mathbf{27 \times 10^{-15}}$

b. $(90,800,000)^2 = (9.08 \times 10^7)^2 = (9.08)^2 \times (10^7)^2$
$= \mathbf{82.4464 \times 10^{14}}$

In particular, remember that the following operations are not the same. One is the product of two numbers in the powers-of-ten format, while the other is a number in the powers-of-ten format taken to a power. As noted below, the results of each are quite different:

$$(10^3)(10^3) \neq (10^3)^3$$
$$(10^3)(10^3) = 10^6 = 1,000,000$$
$$(10^3)^3 = (10^3)(10^3)(10^3) = 10^9 = 1,000,000,000$$

Fixed-Point, Floating-Point, Scientific, and Engineering Notation

There are, in general, four ways in which numbers appear when using a computer or calculator. If powers of ten are not employed, they are written in the **fixed-point** or **floating-point notation.** The fixed-point format requires that the decimal point appear in the same place each time. In the floating-point format, the decimal point will appear in a location defined by the number to be displayed. Most computers and calculators permit a choice of fixed- or floating-point notation. In the fixed format, the user can choose the level of precision for the output as tenths place, hundredths place, thousandths place, and so on. Every output will then fix the decimal point to one location, such as the following examples using thousandths place accuracy:

$$\frac{1}{3} = \mathbf{0.333} \qquad \frac{1}{16} = \mathbf{0.063} \qquad \frac{2300}{2} = \mathbf{1150.000}$$

If left in the floating-point format, the results will appear as follows for the above operations:

$$\frac{1}{3} = \mathbf{0.333333333333} \qquad \frac{1}{16} = \mathbf{0.0625} \qquad \frac{2300}{2} = \mathbf{1150}$$

Powers of ten will creep into the fixed- or floating-point notation if the number is too small or too large to be displayed properly.

Scientific (also called *standard*) **notation** and **engineering notation** make use of powers of ten with restrictions on the mantissa (multiplier) or scale factor (power of the power of ten). Scientific notation requires that the decimal point appear directly after the first digit greater than or equal to 1 but less than 10. A power of ten will then appear with the number (usually following the power notation E), even if it has to be to the zero power. A few examples:

$$\frac{1}{3} = \mathbf{3.33333333333E{-}1} \qquad \frac{1}{16} = \mathbf{6.25E{-}2} \qquad \frac{2300}{2} = \mathbf{1.15E3}$$

Within the scientific notation, the fixed- or floating-point format can be chosen. In the above examples, floating was employed. If fixed is chosen and set at the thousandths-point accuracy, the following will result for the above operations:

$$\frac{1}{3} = \textbf{3.333E}-\textbf{1} \qquad \frac{1}{16} = \textbf{6.250E}-\textbf{2} \qquad \frac{2300}{2} = \textbf{1.150E3}$$

The last format to be introduced is **engineering notation,** which specifies that all powers of ten must be multiples of 3, and the mantissa must be greater than or equal to 1 but less than 1000. This restriction on the powers of ten is due to the fact that specific powers of ten have been assigned prefixes that will be introduced in the next few paragraphs. Using engineering notation in the floating-point mode will result in the following for the above operations:

$$\frac{1}{3} = \textbf{333.333333333E}-\textbf{3} \qquad \frac{1}{16} = \textbf{62.5E}-\textbf{3} \qquad \frac{2300}{2} = \textbf{1.15E3}$$

Using engineering notation with three-place accuracy will result in the following:

$$\frac{1}{3} = \textbf{333.333E}-\textbf{3} \qquad \frac{1}{16} = \textbf{62.500E}-\textbf{3} \qquad \frac{2300}{2} = \textbf{1.150E3}$$

Prefixes

Specific powers of ten in engineering notation have been assigned prefixes and symbols, as appearing in Table 1.2. They permit easy recognition of the power of ten and an improved channel of communication between technologists.

TABLE 1.2

Multiplication Factors	SI Prefix	SI Symbol
$1\ 000\ 000\ 000\ 000 = 10^{12}$	tera	T
$1\ 000\ 000\ 000 = 10^{9}$	giga	G
$1\ 000\ 000 = 10^{6}$	mega	M
$1\ 000 = 10^{3}$	kilo	k
$0.001 = 10^{-3}$	milli	m
$0.000\ 001 = 10^{-6}$	micro	μ
$0.000\ 000\ 001 = 10^{-9}$	nano	n
$0.000\ 000\ 000\ 001 = 10^{-12}$	pico	p

EXAMPLE 1.10

a. 1,000,000 ohms $= 1 \times 10^{6}$ ohms
$\qquad\qquad\qquad = 1$ megohm (MΩ)

b. 100,000 meters $= 100 \times 10^{3}$ meters
$\qquad\qquad\qquad = 100$ kilometers (km)

c. 0.0001 second $= 0.1 \times 10^{-3}$ second
$\qquad\qquad\qquad = 0.1$ millisecond (ms)

d. 0.000001 farad $= 1 \times 10^{-6}$ farad
$\qquad\qquad\qquad = 1$ microfarad (μF)

Here are a few examples with numbers that are not strictly powers of ten.

EXAMPLE 1.11

a. 41,200 m is equivalent to 41.2×10^3 m = 41.2 kilometers = **41.2 km.**
b. 0.00956 J is equivalent to 9.56×10^{-3} J = 9.56 millijoules = **9.56 mJ.**
c. 0.000768 s is equivalent to 768×10^{-6} s = 768 microseconds = **768 μs.**

d. $\dfrac{8400 \text{ m}}{0.06} = \dfrac{8.4 \times 10^3 \text{ m}}{6 \times 10^{-2}} = \left(\dfrac{8.4}{6}\right) \times \left(\dfrac{10^3}{10^{-2}}\right)$ m

$\qquad = 1.4 \times 10^5 \text{ m} = 140 \times 10^3 \text{ m} = 140 \text{ kilometers} = \textbf{140 km}$

e. $(0.0003)^4 \text{ s} = (3 \times 10^{-4})^4 \text{ s} = 81 \times 10^{-16} \text{ s}$

$\qquad = 0.0081 \times 10^{-12} \text{ s} = 0.008 \text{ picosecond} = \textbf{0.0081 ps}$

1.7 CONVERSION BETWEEN LEVELS OF POWERS OF TEN

It is often necessary to convert from one power of ten to another. For instance, if a meter measures kilohertz (kHz), it may be necessary to find the corresponding level in megahertz (MHz), or if time is measured in milliseconds (ms), it may be necessary to find the corresponding time in microseconds (μs) for a graphical plot. The process is not a difficult one if we simply keep in mind that an increase or a decrease in the power of ten must be associated with the opposite effect on the multiplying factor. The procedure is best described by a few examples.

EXAMPLE 1.12

a. Convert 20 kHz to megahertz.
b. Convert 0.01 ms to microseconds.
c. Convert 0.002 km to millimeters.

Solutions:

a. In the power-of-ten format:

$$20 \text{ kHz} = 20 \times 10^3 \text{ Hz}$$

The conversion requires that we find the multiplying factor to appear in the space below:

<div align="center">
Increase by 3

$20 \times 10^3 \text{ Hz} \quad \Rightarrow \quad \underline{\quad} \times 10^6 \text{ Hz}$

Decrease by 3
</div>

Since the power of ten will be *increased* by a factor of *three,* the multiplying factor must be *decreased* by moving the decimal point *three* places to the left, as shown below:

$$\underset{3}{020.} = 0.02$$

and $\qquad 20 \times 10^3 \text{ Hz} = 0.02 \times 10^6 \text{ Hz} = \textbf{0.02 MHz}$

b. In the power-of-ten format:

$$0.01 \text{ ms} = 0.01 \times 10^{-3} \text{ s}$$

<div align="center">
Reduce by 3

and $\qquad 0.01 \times 10^{-3} \text{ s} = \underline{\quad} \times 10^{-6} \text{ s}$

Increase by 3
</div>

Since the power of ten will be *reduced* by a factor of three, the multiplying factor must be *increased* by moving the decimal point three places to the right, as follows:

$$0.010 = 10$$

and $\quad 0.01 \times 10^{-3}$ s $= 10 \times 10^{-6}$ s $= \textbf{10 } \pmb{\mu}\textbf{s}$

There is a tendency when comparing -3 to -6 to think that the power of ten has increased, but keep in mind when making your judgment about increasing or decreasing the magnitude of the multiplier that 10^{-6} is a great deal smaller than 10^{-3}.

c.

Reduce by 6

$$0.002 \times 10^{3} \text{ m} \quad \Rightarrow \quad \underline{\quad} \times 10^{-3} \text{ m}$$

Increase by 6

In this example we have to be very careful because the difference between $+3$ and -3 is a factor of 6, requiring that the multiplying factor be modified as follows:

$$0.002000 = 2000$$

and $\quad 0.002 \times 10^{3}$ m $= 2000 \times 10^{-3}$ m $= \textbf{2000 mm}$

1.8 CONVERSION WITHIN AND BETWEEN SYSTEMS OF UNITS

The conversion within and between systems of units is a process that cannot be avoided in the study of any technical field. It is an operation, however, that is performed incorrectly so often that this section was included to provide one approach that, if applied properly, will lead to the correct result.

There is more than one method of performing the conversion process. In fact, some people prefer to determine mentally whether the conversion factor is multiplied or divided. This approach is acceptable for some elementary conversions, but it is risky with more complex operations.

The procedure to be described here is best introduced by examining a relatively simple problem such as converting inches to meters. Specifically, let us convert 48 in. (4 ft) to meters.

If we multiply the 48 in. by a factor of 1, the magnitude of the quantity remains the same:

$$48 \text{ in.} = 48 \text{ in.}(1) \qquad \textbf{(1.10)}$$

Let us now look at the conversion factor, which is the following for this example:

$$1 \text{ m} = 39.37 \text{ in.}$$

Dividing both sides of the conversion factor by 39.37 in. will result in the following format:

$$\frac{1 \text{ m}}{39.37 \text{ in.}} = (1)$$

Note that the end result is that the ratio 1 m/39.37 in. equals 1, as it should since they are equal quantities. If we now substitute this factor (1) into Eq. (1.10), we obtain

$$48 \text{ in.}(1) = 48 \cancel{\text{in.}}\left(\frac{1 \text{ m}}{39.37 \cancel{\text{in.}}}\right)$$

which results in the cancellation of inches as a unit of measure and leaves meters as the unit of measure. In addition, since the 39.37 is in the denominator, it must be divided into the 48 to complete the operation:

$$\frac{48}{39.37} \text{ m} = \textbf{1.219 m}$$

Let us now review the method, which has the following steps:

1. *Set up the conversion factor to form a numerical value of (1) with the unit of measurement to be removed from the original quantity in the denominator.*
2. *Perform the required mathematics to obtain the proper magnitude for the remaining unit of measurement.*

EXAMPLE 1.13

a. Convert 6.8 min to seconds.
b. Convert 0.24 m to centimeters.

Solutions:

a. The conversion factor is

$$1 \text{ min} = 60 \text{ s}$$

Since the minute is to be removed as the unit of measurement, it must appear in the denominator of the (1) factor, as follows:

Step 1: $\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = (1)$

Step 2: $6.8 \text{ min}(1) = 6.8 \cancel{\text{min}}\left(\frac{60 \text{ s}}{1 \cancel{\text{min}}}\right) = (6.8)(60) \text{ s}$

$$= \textbf{408 s}$$

b. The conversion factor is

$$1 \text{ m} = 100 \text{ cm}$$

Since the meter is to be removed as the unit of measurement, it must appear in the denominator of the (1) factor as follows:

Step 1: $\left(\frac{100 \text{ cm}}{1 \text{ m}}\right) = 1$

Step 2: $0.24 \text{ m}(1) = 0.24 \cancel{\text{m}}\left(\frac{100 \text{ cm}}{1 \cancel{\text{m}}}\right) = (0.24)(100) \text{ cm}$

$$= \textbf{24 cm}$$

The products (1)(1) and (1)(1)(1) are still 1. Using this fact, we can perform a series of conversions in the same operation.

EXAMPLE 1.14

a. Determine the number of minutes in half a day.
b. Convert 2.2 yards to meters.

Solutions:

a. Working our way through from days to hours to minutes, always ensuring that the unit of measurement to be removed is in the denominator, will result in the following sequence:

$$0.5 \, day\left(\frac{24 \, h}{1 \, day}\right)\left(\frac{60 \, min}{1 \, h}\right) = (0.5)(24)(60) \, min$$

$$= \textbf{720 min}$$

b. Working our way through from yards to feet to inches to meters will result in the following:

$$2.2 \, yards\left(\frac{3 \, ft}{1 \, yard}\right)\left(\frac{12 \, in.}{1 \, ft}\right)\left(\frac{1 \, m}{39.37 \, in.}\right) = \frac{(2.2)(3)(12)}{39.37} \, m$$

$$= \textbf{2.012 m}$$

The following examples are variations of the above in practical situations.

EXAMPLE 1.15

a. In Europe and Canada, and many other locations throughout the world, the speed limit is posted in kilometers per hour. How fast in miles per hour is 100 km/h?
b. Determine the speed in miles per hour of a competitor who can run a 4-min mile.

Solutions:

a. $\left(\dfrac{100 \, km}{h}\right)(1)(1)(1)(1)$

$$= \left(\frac{100 \, km}{h}\right)\left(\frac{1000 \, m}{1 \, km}\right)\left(\frac{39.37 \, in.}{1 \, m}\right)\left(\frac{1 \, ft}{12 \, in.}\right)\left(\frac{1 \, mi}{5280 \, ft}\right)$$

$$= \frac{(100)(1000)(39.37)}{(12)(5280)} \, \frac{mi}{h}$$

$$= \textbf{62.14 mi/h}$$

Many travelers use 0.6 as a conversion factor to simplify the math involved; that is,

$$(100 \, km/h)(0.6) \cong 60 \, mi/h$$

and $\qquad\qquad (60 \, km/h)(0.6) \cong 36 \, mi/h$

b. Inverting the factor 4 min/1 mi to 1 mi/4 min, we can proceed as follows:

$$\left(\frac{1 \, mi}{4 \, min}\right)\left(\frac{60 \, min}{h}\right) = \frac{60}{4} \, mi/h = \textbf{15 mi/h}$$

1.9 SYMBOLS

Throughout the text, various symbols will be employed that the reader may not have had occasion to use. Some are defined in Table 1.3, and others will be defined in the text as the need arises.

TABLE 1.3

Symbol	Meaning
$\neq$	Not equal to $6.12 \neq 6.13$
$>$	Greater than $4.78 > 4.20$
$\gg$	Much greater than $840 \gg 16$
$<$	Less than $430 < 540$
$\ll$	Much less than $0.002 \ll 46$
$\geq$	Greater than or equal to $x \geq y$ is satisfied for $y = 3$ and $x > 3$ or $x = 3$
$\leq$	Less than or equal to $x \leq y$ is satisfied for $y = 3$ and $x < 3$ or $x = 3$
$\cong$	Approximately equal to $3.14159 \cong 3.14$
Σ	Sum of $\Sigma (4 + 6 + 8) = 18$
$\| \|$	Absolute magnitude of $|a| = 4$, where $a = -4$ or $+4$
$\therefore$	Therefore $x = \sqrt{4} \quad \therefore x = \pm 2$
$\equiv$	By definition Establishes a relationship between two or more quantities

1.10 CONVERSION TABLES

Conversion tables such as those appearing in Appendix B can be very useful when time does not permit the application of methods described in this chapter. However, even though such tables appear easy to use, frequent errors occur because the operations appearing at the head of the table are not performed properly. In any case, when using such tables, try to establish mentally some order of magnitude for the quantity to be determined compared to the magnitude of the quantity in its original set of units. This simple operation should prevent several impossible results that may occur if the conversion operation is improperly applied.

For example, consider the following from such a conversion table:

To convert from	To	Multiply by
Miles	Meters	1.609×10^3

A conversion of 2.5 mi to meters would require that we multiply 2.5 by the conversion factor; that is,

$$2.5 \text{ mi}(1.609 \times 10^3) = 4.0225 \times 10^3 \text{ m}$$

A conversion from 4000 m to miles would require a division process:

$$\frac{4000 \text{ m}}{1.609 \times 10^3} = 2486.02 \times 10^{-3} = 2.48602 \text{ mi}$$

In each of the above, there should have been little difficulty realizing that 2.5 mi would convert to a few thousand meters and 4000 m would be only a few miles. As indicated above, this kind of anticipatory thinking will eliminate the possibility of ridiculous conversion results.

1.11 CALCULATORS

In some texts, the calculator is not discussed in detail. Instead, students are left with the general exercise of choosing an appropriate calculator and learning to use it properly on their own. However, some discussion about the use of the calculator must be included to eliminate some of the impossible results obtained (and often strongly defended by the user—because the calculator says so) through a correct understanding of the process by which a calculator performs the various tasks. Time and space do not permit a detailed explanation of all the possible operations, but it is assumed that the following discussion will enlighten the user to the fact that it is important to understand the manner in which a calculator proceeds with a calculation and not to expect the unit to accept data in any form and always generate the correct answer.

When choosing a calculator (scientific for our use), be absolutely sure that it has the ability to operate on complex numbers (polar and rectangular) which will be described in detail in Chapter 13. For now simply look up the terms in the index of the operator's manual, and be sure that the terms appear and that the basic operations with them are discussed. Next, be aware that some calculators perform the operations with a minimum number of steps while others can require a downright lengthy or complex series of steps. Speak to your instructor if unsure about your purchase. For this text, the TI-86 of Fig. 1.5 was chosen because of its treatment of complex numbers.

FIG. 1.5
Texas Instruments TI-86 calculator. (Courtesy of Texas Instruments, Inc.)

Initial Settings

Format and accuracy are the first two settings that must be made on any scientific calculator. For most calculators the choices of formats are *Normal, Scientific,* and *Engineering.* For the TI-86 calculator, pressing the 2nd function (yellow) key followed by the $\boxed{\text{MODE}}$ key will provide a list of options for the initial settings of the calculator. For calculators without a $\boxed{\text{MODE}}$ choice, consult the operator's manual for the manner in which the format and accuracy level are set.

Examples of each are shown below:

Normal: 1/3 = 0.33
Scientific: 1/3 = 3.33E−1
Engineering: 1/3 = 333.33E−3

Note that the Normal format simply places the decimal point in the most logical location. The Scientific ensures that the number preceding the decimal point is a single digit followed by the required power of ten. The Engineering format will always ensure that the power of ten is a multiple of 3 (whether it be positive, negative, or zero).

In the above examples the accuracy was hundredths place. To set this accuracy for the TI-86 calculator, return to the $\boxed{\text{MODE}}$ selection and choose 2 to represent two-place accuracy or hundredths place.

Initially you will probably be most comfortable with the Normal mode with hundredths-place accuracy. However, as you begin to analyze networks, you may find the Engineering mode more appropriate since you will be working with component levels and results that have powers of ten that have been assigned abbreviations and names. Then again, the Scientific mode may the best choice for a particular analysis. In any event, take the time now to become familiar with the differences between the various modes, and learn how to set them on your calculator.

Order of Operations

Although being able to set the format and accuracy is important, these features are not the source of the impossible results that often arise because of improper use of the calculator. Improper results occur primarily because users fail to realize that no matter how simple or complex an equation, the calculator will perform the required operations in a specific order.

For instance, the operation

$$\frac{8}{3+1}$$

is often entered as

$$\boxed{8}\;\boxed{\div}\;\boxed{3}\;\boxed{+}\;\boxed{1}\; = \frac{8}{3} + 1 = 2.67 + 1 = 3.67$$

which is totally incorrect (2 is the answer).

The user must be aware that the calculator **will not** perform the addition first and then the division. In fact, addition and subtraction are the last operations to be performed in any equation. It is therefore very important that the reader carefully study and thoroughly understand the next few paragraphs in order to use the calculator properly.

1. The first operations to be performed by a calculator can be set using **parentheses ( )**. It does not matter which operations are within

the parentheses. The parentheses simply dictate that this part of the equation is to be determined first. There is no limit to the number of parentheses in each equation—all operations within parentheses will be performed first. For instance, for the example above, if parentheses are added as shown below, the addition will be performed first and the correct answer obtained:

$$\frac{8}{(3+1)} = \boxed{8}\boxed{\div}\boxed{(}\boxed{3}\boxed{+}\boxed{1}\boxed{)} = \frac{8}{4} = \mathbf{2}$$

2. Next, ***powers and roots*** are performed, such as x^2, $\sqrt{x}$, and so on.

3. ***Negation*** (applying a negative sign to a quantity) and ***single-key operations*** such as sin, $\tan^{-1}$, and so on, are performed.

4. ***Multiplication and division*** are then performed.

5. ***Addition and subtraction*** are performed last.

It may take a few moments and some repetition to remember the order, but at least you are now aware that there is an order to the operations and are aware that ignoring them can result in meaningless results.

EXAMPLE 1.16

a. Determine

$$\sqrt{\frac{9}{3}}$$

b. Find

$$\frac{3+9}{4}$$

c. Determine

$$\frac{1}{4} + \frac{1}{6} + \frac{2}{3}$$

Solutions:

a. The following calculator operations will result in an incorrect answer of 1 because the square-root operation will be performed before the division.

$$\boxed{\sqrt{}}\boxed{9}\boxed{\div}\boxed{3} = \frac{\sqrt{9}}{3} = \frac{3}{3} = 1$$

However, recognizing that we must first divide 9 by 3, we can use parentheses as follows to define this operation as the first to be performed, and the correct answer will be obtained:

$$\boxed{\sqrt{}}\boxed{(}\boxed{9}\boxed{\div}\boxed{3}\boxed{)} = \sqrt{\left(\frac{9}{3}\right)} = \sqrt{3} = \mathbf{1.67}$$

b. If the problem is entered as it appears, the incorrect answer of 5.25 will result.

$$\boxed{3}\boxed{+}\boxed{9}\boxed{\div}\boxed{4} = 3 + \frac{9}{4} = 5.25$$

Using brackets to ensure that the addition takes place before the division will result in the correct answer as shown below:

$$\boxed{(}\boxed{3}\boxed{+}\boxed{9}\boxed{)}\boxed{\div}\boxed{4} = \frac{(3+9)}{4} = \frac{12}{4} = \mathbf{3}$$

respect to length, mass, force, and temperature? If so, explain.

7. Which of the four systems of units appearing in Table 1.1 has the smallest units for length, mass, and force? When would this system be used most effectively?

*8. Which system of Table 1.1 is closest in definition to the SI system? How are the two systems different? Why do you think the units of measurement for the SI system were chosen as listed in Table 1.1? Give the best reasons you can without referencing additional literature.

9. What is room temperature (68°F) in the MKS, CGS, and SI systems?

10. How many foot-pounds of energy are associated with 1000 J?

11. How many centimeters are there in $\frac{1}{2}$ yd?

SECTION 1.6   Powers of Ten

12. Express the following numbers as powers of ten:
    a. 10,000
    b. 0.0001
    c. 1000
    d. 1,000,000
    e. 0.0000001
    f. 0.00001

13. Using only those powers of ten listed in Table 1.2, express the following numbers in what seems to you the most logical form for future calculations:
    a. 15,000
    b. 0.03000
    c. 7,400,000
    d. 0.0000068
    e. 0.00040200
    f. 0.0000000002

14. Perform the following operations and express your answer as a power of ten:
    a. $4200 + 6,800,000$
    b. $9 \times 10^4 + 3.6 \times 10^3$
    c. $0.5 \times 10^{-3} - 6 \times 10^{-5}$
    d. $1.2 \times 10^3 + 50,000 \times 10^{-3} - 0.006 \times 10^5$

15. Perform the following operations and express your answer as a power of ten:
    a. $(100)(100)$
    b. $(0.01)(1000)$
    c. $(10^3)(10^6)$
    d. $(1000)(0.00001)$
    e. $(10^{-6})(10,000,000)$
    f. $(10,000)(10^{-8})(10^{35})$

16. Perform the following operations and express your answer as a power of ten:
    a. $(50,000)(0.0003)$
    b. 2200×0.08
    c. $(0.000082)(0.00007)$
    d. $(30 \times 10^{-4})(0.0002)(7 \times 10^8)$

17. Perform the following operations and express your answer as a power of ten:
    a. $\dfrac{100}{1000}$
    b. $\dfrac{0.01}{100}$
    c. $\dfrac{10,000}{0.00001}$
    d. $\dfrac{0.0000001}{100}$
    e. $\dfrac{10^{38}}{0.000100}$
    f. $\dfrac{(100)^{1/2}}{0.01}$

18. Perform the following operations and express your answer as a power of ten:
    a. $\dfrac{2000}{0.00008}$
    b. $\dfrac{0.00408}{60,000}$
    c. $\dfrac{0.000215}{0.00005}$
    d. $\dfrac{78 \times 10^9}{4 \times 10^{-6}}$

19. Perform the following operations and express your answer as a power of ten:
    a. $(100)^3$
    b. $(0.0001)^{1/2}$
    c. $(10,000)^8$
    d. $(0.00000010)^9$

20. Perform the following operations and express your answer as a power of ten:
    a. $(2.2 \times 10^3)^3$
    b. $(0.0006 \times 10^2)^4$
    c. $(0.004)(6 \times 10^2)^2$
    d. $((2 \times 10^{-3})(0.8 \times 10^4)(0.003 \times 10^5))^3$

21. Perform the following operations and express your answer in scientific notation:
    a. $(-0.001)^2$
    b. $\dfrac{(100)(10^{-4})}{10}$
    c. $\dfrac{(0.001)^2(100)}{10,000}$
    d. $\dfrac{(10^2)(10,000)}{0.001}$
    e. $\dfrac{(0.0001)^3(100)}{1,000,000}$
    *f. $\dfrac{[(100)(0.01)]^{-3}}{[(100)^2][0.001]}$

*22. Perform the following operations and express your answer in engineering notation:
    a. $\dfrac{(300)^2(100)}{10^4}$
    b. $[(40,000)^2][(20)^{-3}]$
    c. $\dfrac{(60,000)^2}{(0.02)^2}$
    d. $\dfrac{(0.000027)^{1/3}}{210,000}$
    e. $\dfrac{[(4000)^2][300]}{0.02}$
    f. $[(0.000016)^{1/2}][(100,000)^5][0.02]$
    g. $\dfrac{[(0.003)^3][(0.00007)^2][(800)^2]}{[(100)(0.0009)]^{1/2}}$ (a challenge)

SECTION 1.7   Conversion between Levels of Powers of Ten

23. Fill in the blanks of the following conversions:
    a. $6 \times 10^3 = $ _____ $\times 10^6$
    b. $4 \times 10^{-4} = $ _____ $\times 10^{-6}$
    c. $50 \times 10^5 = $ _____ $\times 10^3 = $ _____ $\times 10^6$
       $ = $ _____ $\times 10^9$
    d. $30 \times 10^{-8} = $ _____ $\times 10^{-3} = $ _____ $\times 10^{-6}$
       $ = $ _____ $\times 10^{-9}$

24. Perform the following conversions:
    a. 2000 μs to milliseconds
    b. 0.04 ms to microseconds
    c. 0.06 μF to nanofarads
    d. 8400 ps to microseconds
    e. 0.006 km to millimeters
    f. 260×10^3 mm to kilometers

SECTION 1.8   Conversion within and between Systems of Units

For Problems 25 to 27, convert the following:

25. **a.** 1.5 min to seconds
    b. 0.04 h to seconds
    c. 0.05 s to microseconds
    d. 0.16 m to millimeters
    e. 0.00000012 s to nanoseconds
    f. 3,620,000 s to days
    g. 1020 mm to meters

26. **a.** 0.1 μF (microfarad) to picofarads
    b. 0.467 km to meters
    c. 63.9 mm to centimeters
    d. 69 cm to kilometers
    e. 3.2 h to milliseconds
    f. 0.016 mm to micrometers
    g. 60 sq cm (cm^2) to square meters (m^2)

*27. **a.** 100 in. to meters
    b. 4 ft to meters
    c. 6 lb to newtons
    d. 60,000 dyn to pounds
    e. 150,000 cm to feet
    f. 0.002 mi to meters (5280 ft = 1 mi)
    g. 7800 m to yards

28. What is a mile in feet, yards, meters, and kilometers?

29. Calculate the speed of light in miles per hour using the defined speed of Section 1.4.

30. Find the velocity in miles per hour of a mass that travels 50 ft in 20 s.

31. How long in seconds will it take a car traveling at 100 mi/h to travel the length of a football field (100 yd)?

32. Convert 6 mi/h to meters per second.

33. If an athlete can row at a rate of 50 m/min, how many days would it take to cross the Atlantic ($\cong$3000 mi)?

34. How long would it take a runner to complete a 10-km race if a pace of 6.5 min/mi were maintained?

35. Quarters are about 1 in. in diameter. How many would be required to stretch from one end of a football field to the other (100 yd)?

36. Compare the total time in hours to cross the United States ($\cong$3000 mi) at an average speed of 55 mi/h versus an average speed of 65 mi/h. What is your reaction to the total time required versus the safety factor?

*37. Find the distance in meters that a mass traveling at 600 cm/s will cover in 0.016 h.

*38. Each spring there is a race up 86 floors of the 102-story Empire State Building in New York City. If you were able to climb 2 steps/second, how long would it take you to reach the 86th floor if each floor is 14 ft. high and each step is about 9 in.?

*39. The record for the race in Problem 38 is 10 minutes, 47 seconds. What was the racer's speed in min/mi for the race?

*40. If the race of Problem 38 were a horizontal distance, how long would it take a runner who can run 5-minute miles to cover the distance? Compare this with the record speed of Problem 39. Gravity is certainly a factor to be reckoned with!

SECTION 1.10   Conversion Tables

41. Using Appendix B, determine the number of
    a. Btu in 5 J of energy.
    b. cubic meters in 24 oz of a liquid.
    c. seconds in 1.4 days.
    d. pints in 1 m^3 of a liquid.

SECTION 1.11   Calculators

Perform the following operations using a calculator:

42. $6(4 + 8)$

43. $\sqrt{3^2 + 4^2}$

44. $\tan^{-1}\dfrac{4}{3}$

45. $\sqrt{\dfrac{400}{6^2 + 10}}$

SECTION 1.12   Computer Analysis

46. Investigate the availability of computer courses and computer time in your curriculum. Which languages are commonly used, and which software packages are popular?

47. Develop a list of five popular computer languages with a few characteristics of each. Why do you think some languages are better for the analysis of electric circuits than others?

GLOSSARY

C++   A computer language having an efficient communication link between the user and the machine language of the central processing unit (CPU) of a computer.

CGS system   The system of units employing the *C*entimeter, *G*ram, and *S*econd as its fundamental units of measure.

Difference engine   One of the first mechanical calculators.

Edison effect   Establishing a flow of charge between two elements in an evacuated tube.

Electromagnetism   The relationship between magnetic and electrical effects.

Engineering notation   A method of notation that specifies that all powers of ten used to define a number be multiples of 3 with a mantissa greater than or equal to 1 but less than 1000.

ENIAC   The first totally electronic computer.

$\sum_{I}^{S}$

Fixed-point notation Notation using a decimal point in a particular location to define the magnitude of a number.

Fleming's valve The first of the electronic devices, the diode.

Floating-point notation Notation that allows the magnitude of a number to define where the decimal point should be placed.

Integrated circuit (IC) A subminiature structure containing a vast number of electronic devices designed to perform a particular set of functions.

Joule (J) A unit of measurement for energy in the SI or MKS system. Equal to 0.7378 foot-pound in the English system and 10^7 ergs in the CGS system.

Kelvin (K) A unit of measurement for temperature in the SI system. Equal to $273.15 + °C$ in the MKS and CGS systems.

Kilogram (kg) A unit of measure for mass in the SI and MKS systems. Equal to 1000 grams in the CGS system.

Language A communication link between user and computer to define the operations to be performed and the results to be displayed or printed.

Leyden jar One of the first charge-storage devices.

Menu A computer-generated list of choices for the user to determine the next operation to be performed.

Meter (m) A unit of measure for length in the SI and MKS systems. Equal to 1.094 yards in the English system and 100 centimeters in the CGS system.

MKS system The system of units employing the *M*eter, *K*ilogram, and *S*econd as its fundamental units of measure.

Newton (N) A unit of measurement for force in the SI and MKS systems. Equal to 100,000 dynes in the CGS system.

Pound (lb) A unit of measurement for force in the English system. Equal to 4.45 newtons in the SI or MKS system.

Program A sequential list of commands, instructions, etc., to perform a specified task using a computer.

PSpice A software package designed to analyze various dc, ac, and transient electrical and electronic systems.

Scientific notation A method for describing very large and very small numbers through the use of powers of ten, which requires that the multiplier be a number between 1 and 10.

Second (s) A unit of measurement for time in the SI, MKS, English, and CGS systems.

SI system The system of units adopted by the IEEE in 1965 and the USASI in 1967 as the International System of Units (*S*ystème *I*nternational d'Unités).

Slug A unit of measure for mass in the English system. Equal to 14.6 kilograms in the SI or MKS system.

Software package A computer program designed to perform specific analysis and design operations or generate results in a particular format.

Static electricity Stationary charge in a state of equilibrium.

Transistor The first semiconductor amplifier.

Voltaic cell A storage device that converts chemical to electrical energy.

2

Current and Voltage

2.1   ATOMS AND THEIR STRUCTURE

A basic understanding of the fundamental concepts of current and voltage requires a degree of familiarity with the atom and its structure. The simplest of all atoms is the hydrogen atom, made up of two basic particles, the **proton** and the **electron,** in the relative positions shown in Fig. 2.1(a). The **nucleus** of the hydrogen atom is the proton, a positively charged particle. *The orbiting electron carries a negative charge that is equal in magnitude to the positive charge of the proton.* In all other ele-

(a) Hydrogen atom
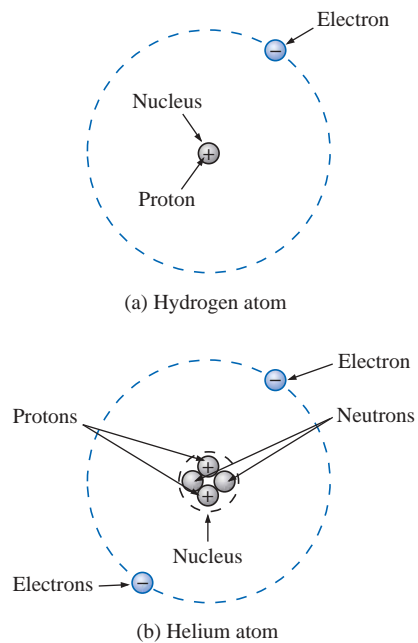
(b) Helium atom

FIG. 2.1
The hydrogen and helium atoms.

ments, the nucleus also contains **neutrons,** which are slightly heavier than protons and have no electrical charge. The helium atom, for example, has two neutrons in addition to two electrons and two protons, as shown in Fig. 2.1(b). *In all neutral atoms the number of electrons is equal to the number of protons.* The mass of the electron is 9.11×10^{-28} g, and that of the proton and neutron is 1.672×10^{-24} g. The mass of the proton (or neutron) is therefore approximately 1836 times that of the electron. The radii of the proton, neutron, and electron are all of the order of magnitude of 2×10^{-15} m.

For the hydrogen atom, the radius of the smallest orbit followed by the electron is about 5×10^{-11} m. The radius of this orbit is approximately 25,000 times that of the radius of the electron, proton, or neutron. This is approximately equivalent to a sphere the size of a dime revolving about another sphere of the same size more than a quarter of a mile away.

Different atoms will have various numbers of electrons in the concentric shells about the nucleus. The first shell, which is closest to the nucleus, can contain only two electrons. If an atom should have three electrons, the third must go to the next shell. The second shell can contain a maximum of eight electrons; the third, 18; and the fourth, 32; as determined by the equation $2n^2$, where n is the shell number. These shells are usually denoted by a number ($n = 1, 2, 3, \ldots$) or letter ($n = k, l, m, \ldots$).

Each shell is then broken down into subshells, where the first subshell can contain a maximum of two electrons; the second subshell, six electrons; the third, 10 electrons; and the fourth, 14; as shown in Fig. 2.2. The subshells are usually denoted by the letters *s, p, d,* and *f,* in that order, outward from the nucleus.
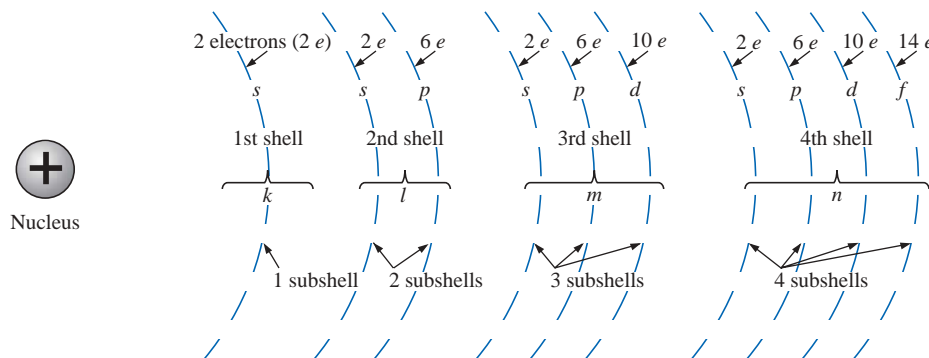
FIG. 2.2
Shells and subshells of the atomic structure.

It has been determined by experimentation that *unlike charges attract, and like charges repel.* The force of attraction or repulsion between two charged bodies Q_1 and Q_2 can be determined by **Coulomb's law:**

$$F \text{ (attraction or repulsion)} = \frac{kQ_1Q_2}{r^2} \qquad \text{(newtons, N)} \quad \textbf{(2.1)}$$

where F is in newtons, $k = $ a constant $= 9.0 \times 10^9$ N·m^2/C^2, Q_1 and Q_2 are the charges in coulombs (to be introduced in Section 2.2), and r is

the distance in meters between the two charges. In particular, note the squared r term in the denominator, resulting in rapidly decreasing levels of F for increasing values of r. (See Fig. 2.3.)

In the atom, therefore, electrons will repel each other, and protons and electrons will attract each other. Since the nucleus consists of many positive charges (protons), a strong attractive force exists for the electrons in orbits close to the nucleus [note the effects of a large charge Q and a small distance r in Eq. (2.1)]. As the distance between the nucleus and the orbital electrons increases, the binding force diminishes until it reaches its lowest level at the outermost subshell (largest r). Due to the weaker binding forces, less energy must be expended to remove an electron from an outer subshell than from an inner subshell. Also, it is generally true that electrons are more readily removed from atoms having outer subshells that are incomplete *and*, in addition, possess few electrons. These properties of the atom that permit the removal of electrons under certain conditions are essential if motion of charge is to be created. Without this motion, this text could venture no further—our basic quantities rely on it.

Copper is the most commonly used metal in the electrical/electronics industry. An examination of its atomic structure will help identify why it has such widespread applications. The copper atom (Fig. 2.4) has one more electron than needed to complete the first three shells. This incomplete outermost subshell, possessing only one electron, and the distance between this electron and the nucleus reveal that the twenty-ninth electron is loosely bound to the copper atom. If this twenty-ninth electron gains sufficient energy from the surrounding medium to leave its parent atom, it is called a **free electron.** In one cubic inch of copper at room temperature, there are approximately $1.4 \times 10^{+24}$ free electrons. Other metals that exhibit the same properties as copper, but to a different degree, are silver, gold, aluminum, and tungsten. Additional discussion of conductors and their characteristics can be found in Section 3.2.
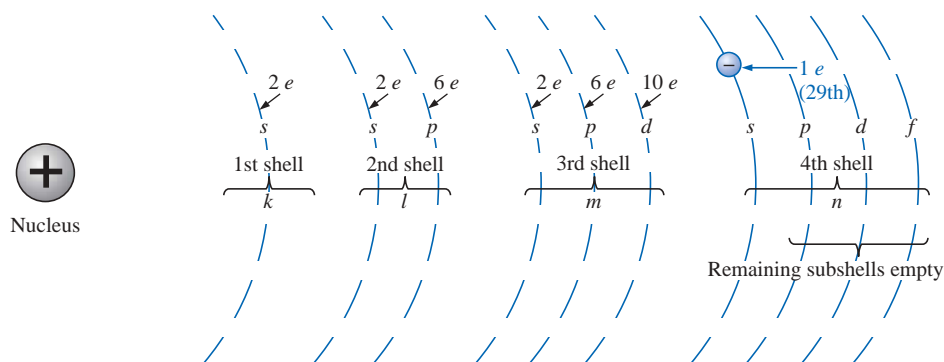
FIG. 2.3
Charles Augustin de Coulomb.

FIG. 2.4
The copper atom.

2.2 CURRENT

Consider a short length of copper wire cut with an imaginary perpendicular plane, producing the circular cross section shown in Fig. 2.5. At room temperature with no external forces applied, there exists within the copper wire the random motion of free electrons created by
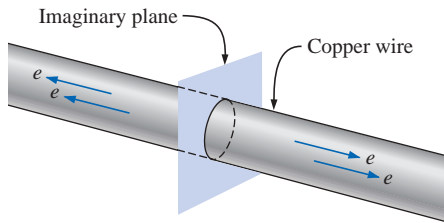
FIG. 2.5

Random motion of electrons in a copper wire with no external "pressure" (voltage) applied.
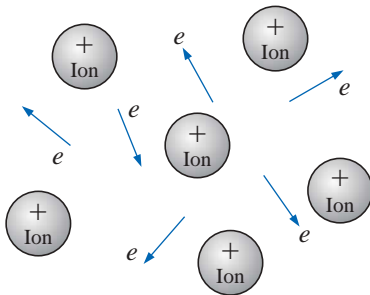
FIG. 2.6

Random motion of free electrons in an atomic structure.

the thermal energy that the electrons gain from the surrounding medium. When atoms lose their free electrons, they acquire a net positive charge and are referred to as **positive ions.** The free electrons are able to move within these positive ions and leave the general area of the parent atom, while the positive ions only oscillate in a mean fixed position. For this reason,

the free electron is the charge carrier in a copper wire or any other solid conductor of electricity.

An array of positive ions and free electrons is depicted in Fig. 2.6. Within this array, the free electrons find themselves continually gaining or losing energy by virtue of their changing direction and velocity. Some of the factors responsible for this random motion include (1) the collisions with positive ions and other electrons, (2) the attractive forces for the positive ions, and (3) the force of repulsion that exists between electrons. This random motion of free electrons is such that over a period of time, the number of electrons moving to the right across the circular cross section of Fig. 2.5 is exactly equal to the number passing over to the left.

With no external forces applied, the net flow of charge in a conductor in any one direction is zero.

Let us now connect copper wire between two battery terminals and a light bulb, as shown in Fig. 2.7, to create the simplest of electric circuits. The battery, at the expense of chemical energy, places a net positive charge at one terminal and a net negative charge on the other. The instant the final connection is made, the free electrons (of negative charge) will drift toward the positive terminal, while the positive ions left behind in the copper wire will simply oscillate in a mean fixed position. The negative terminal is a "supply" of electrons to be drawn from when the electrons of the copper wire drift toward the positive terminal.
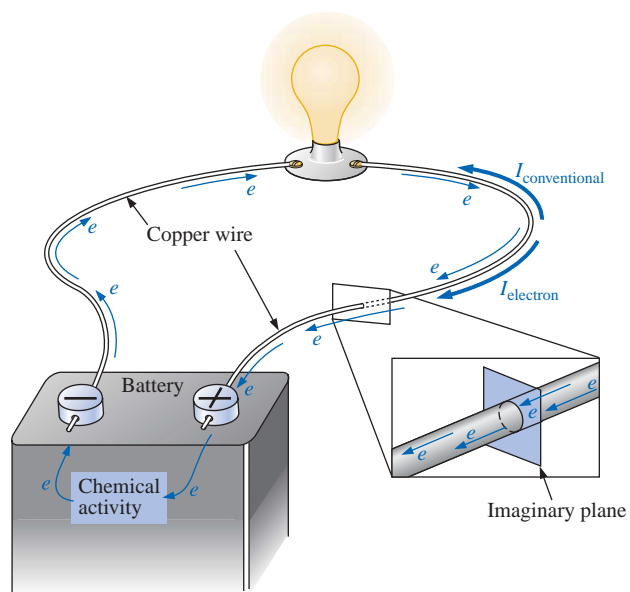
FIG. 2.7

Basic electric circuit.

The chemical activity of the battery will absorb the electrons at the positive terminal and will maintain a steady supply of electrons at the negative terminal. The flow of charge (electrons) through the bulb will heat up the filament of the bulb through friction to the point that it will glow red hot and emit the desired light.

If 6.242×10^{18} *electrons* drift at uniform velocity through the imaginary circular cross section of Fig. 2.7 in 1 *second*, the flow of charge, or *current,* is said to be 1 **ampere** (A) in honor of André Marie Ampère (Fig. 2.8). The discussion of Chapter 1 clearly reveals that this is an enormous number of electrons passing through the surface in 1 second. The current associated with only a few electrons per second would be inconsequential and of little practical value. To establish numerical values that permit immediate comparisons between levels, a **coulomb** (C) of charge was defined as the total charge associated with 6.242×10^{18} electrons. The charge associated with one electron can then be determined from

$$\text{Charge/electron} = Q_e = \frac{1 \text{ C}}{6.242 \times 10^{18}} = 1.6 \times 10^{-19} \text{ C}$$

The current in amperes can now be calculated using the following equation:

$$\boxed{I = \frac{Q}{t}} \qquad \begin{array}{l} I = \text{amperes (A)} \\ Q = \text{coulombs (C)} \\ t = \text{seconds (s)} \end{array} \qquad \textbf{(2.2)}$$

The capital letter I was chosen from the French word for current: *intensité.* The SI abbreviation for each quantity in Eq. (2.2) is provided to the right of the equation. The equation clearly reveals that for equal time intervals, the more charge that flows through the wire, the heavier the current.

Through algebraic manipulations, the other two quantities can be determined as follows:

$$\boxed{Q = It} \qquad \text{(coulombs, C)} \qquad \textbf{(2.3)}$$

and

$$\boxed{t = \frac{Q}{I}} \qquad \text{(seconds, s)} \qquad \textbf{(2.4)}$$

EXAMPLE 2.1 The charge flowing through the imaginary surface of Fig. 2.7 is 0.16 C every 64 ms. Determine the current in amperes.

Solution: Eq. (2.2):

$$I = \frac{Q}{t} = \frac{0.16 \text{ C}}{64 \times 10^{-3} \text{ s}} = \frac{160 \times 10^{-3} \text{ C}}{64 \times 10^{-3} \text{ s}} = \textbf{2.50 A}$$

EXAMPLE 2.2 Determine the time required for 4×10^{16} electrons to pass through the imaginary surface of Fig. 2.7 if the current is 5 mA.

Solution: Determine Q:

$$4 \times 10^{16} \text{ electrons} \left(\frac{1 \text{ C}}{6.242 \times 10^{18} \text{ electrons}} \right) = 0.641 \times 10^{-2} \text{ C}$$

$$= 0.00641 \text{ C} = 6.41 \text{ mC}$$

French (Lyon, Paris)
(1775–1836)
Mathematician and Physicist
Professor of Mathematics, École Polytechnique in Paris

Courtesy of the
Smithsonian Institution
Photo No. 76,524

On September 18, 1820, introduced a new field of study, *electrodynamics,* devoted to the effect of electricity in motion, including the interaction between currents in adjoining conductors and the interplay of the surrounding magnetic fields. Constructed the first *solenoid* and demonstrated how it could behave like a magnet (the first *electromagnet*). Suggested the name *galvanometer* for an instrument designed to measure current levels.

FIG. 2.8
André Marie Ampère.

Calculate t [Eq. (2.4)]:

$$t = \frac{Q}{I} = \frac{6.41 \times 10^{-3}\,\text{C}}{5 \times 10^{-3}\,\text{A}} = \mathbf{1.282\ s}$$

A second glance at Fig. 2.7 will reveal that two directions of charge flow have been indicated. One is called *conventional flow,* and the other is called *electron flow.* This text will deal only with conventional flow for a variety of reasons, including the fact that it is the most widely used at educational institutions and in industry, it is employed in the design of all electronic device symbols, and it is the popular choice for all major computer software packages. The flow controversy is a result of an assumption made at the time electricity was discovered that the positive charge was the moving particle in metallic conductors. Be assured that the choice of conventional flow will not create great difficulty and confusion in the chapters to follow. Once the direction of I is established, the issue is dropped and the analysis can continue without confusion.

Safety Considerations

It is important to realize that even small levels of current through the human body can cause serious, dangerous side effects. Experimental results reveal that the human body begins to react to currents of only a few milliamperes. Although most individuals can withstand currents up to perhaps 10 mA for very short periods of time without serious side effects, any current over 10 mA should be considered dangerous. In fact, currents of 50 mA can cause severe shock, and currents of over 100 mA can be fatal. In most cases the skin resistance of the body when dry is sufficiently high to limit the current through the body to relatively safe levels for voltage levels typically found in the home. However, be aware that when the skin is wet due to perspiration, bathing, etc., or when the skin barrier is broken due to an injury, the skin resistance drops dramatically, and current levels could rise to dangerous levels for the same voltage shock. In general, therefore, simply remember that *water and electricity don't mix.* Granted, there are safety devices in the home today [such as the ground fault current interrupt (GFCI) breaker to be introduced in Chapter 4] that are designed specifically for use in wet areas such as the bathroom and kitchen, but accidents happen. Treat electricity with respect—not fear.

2.3 VOLTAGE

The flow of charge described in the previous section is established by an external "pressure" derived from the energy that a mass has by virtue of its position: **potential energy.**

Energy, by definition, is the *capacity to do work.* If a mass (m) is raised to some height (h) above a reference plane, it has a measure of potential energy expressed in *joules* (J) that is determined by

$$\boxed{W \text{ (potential energy)} = mgh} \qquad \text{(joules, J)} \qquad \mathbf{(2.5)}$$

where g is the gravitational acceleration (9.754 m/s^2). This mass now has the "potential" to do work such as crush an object placed on the ref-

erence plane. If the weight is raised further, it has an increased measure of potential energy and can do additional work. There is an obvious *difference in potential* between the two heights above the reference plane.

In the battery of Fig. 2.7, the internal chemical action will establish (through an expenditure of energy) an accumulation of negative charges (electrons) on one terminal (the negative terminal) and positive charges (positive ions) on the other (the positive terminal). A "positioning" of the charges has been established that will result in a **potential difference** between the terminals. If a conductor is connected between the terminals of the battery, the electrons at the negative terminal have sufficient potential energy to overcome collisions with other particles in the conductor and the repulsion from similar charges to reach the positive terminal to which they are attracted.

Charge can be raised to a higher potential level through the expenditure of energy from an external source, or it can lose potential energy as it travels through an electrical system. In any case, by definition:

A potential difference of 1 volt (V) exists between two points if 1 joule (J) of energy is exchanged in moving 1 coulomb (C) of charge between the two points.

The unit of measurement **volt** was chosen to honor Alessandro Volta (Fig. 2.9).

Pictorially, if one joule of energy (1 J) is required to move the one coulomb (1 C) of charge of Fig. 2.10 from position *x* to position *y*, the potential difference or voltage between the two points is one volt (1 V). If the energy required to move the 1 C of charge increases to 12 J due to additional opposing forces, then the potential difference will increase to 12 V. Voltage is therefore an indication of how much energy is involved in moving a charge between two points in an electrical system. Conversely, the higher the voltage rating of an energy source such as a battery, the more energy will be available to move charge through the system. Note in the above discussion that two points are always involved when talking about voltage or potential difference. In the future, therefore, it is very important to keep in mind that

a potential difference or voltage is always measured between two points in the system. Changing either point may change the potential difference between the two points under investigation.

In general, the potential difference between two points is determined by

$$V = \frac{W}{Q} \qquad \text{(volts)} \qquad \textbf{(2.6)}$$

Through algebraic manipulations, we have

$$W = QV \qquad \text{(joules)} \qquad \textbf{(2.7)}$$

and

$$Q = \frac{W}{V} \qquad \text{(coulombs)} \qquad \textbf{(2.8)}$$

FIG. 2.9
Count Alessandro Volta.

$W = 1$ J
1 C
x
y
$V_{xy} = 1$ volt

FIG. 2.10
Defining the unit of measurement for voltage.

EXAMPLE 2.3 Find the potential difference between two points in an electrical system if 60 J of energy are expended by a charge of 20 C between these two points.

Solution: Eq. (2.6):

$$V = \frac{W}{Q} = \frac{60 \text{ J}}{20 \text{ C}} = \mathbf{3 \text{ V}}$$

EXAMPLE 2.4 Determine the energy expended moving a charge of 50 μC through a potential difference of 6 V.

Solution: Eq. (2.7):

$$W = QV = (50 \times 10^{-6} \text{ C})(6 \text{ V}) = 300 \times 10^{-6} \text{ J} = \mathbf{300 \ \mu J}$$

Notation plays a very important role in the analysis of electrical and electronic systems. To distinguish between sources of voltage (batteries and the like) and losses in potential across dissipative elements, the following notation will be used:

E for voltage sources (volts)
V for voltage drops (volts)

An occasional source of confusion is the terminology applied to this subject matter. Terms commonly encountered include *potential, potential difference, voltage, voltage difference* (*drop* or *rise*), and *electromotive force.* As noted in the description above, some are used interchangeably. The following definitions are provided as an aid in understanding the meaning of each term:

Potential: The voltage at a point with respect to another point in the electrical system. Typically the reference point is ground, which is at zero potential.

Potential difference: The algebraic difference in potential (or voltage) between two points of a network.

Voltage: When isolated, like potential, the voltage at a point with respect to some reference such as ground (0 V).

Voltage difference: The algebraic difference in voltage (or potential) between two points of the system. A voltage drop or rise is as the terminology would suggest.

Electromotive force (emf): The force that establishes the flow of charge (or current) in a system due to the application of a difference in potential. This term is not applied that often in today's literature but is associated primarily with sources of energy.

In summary, the applied **potential difference** (in volts) of a voltage source in an electric circuit is the "pressure" to set the system in motion and "cause" the flow of charge or current through the electrical system. A mechanical analogy of the applied voltage is the pressure applied to the water in a main. The resulting flow of water through the system is likened to the flow of charge through an electric circuit. Without the applied pressure from the spigot, the water will simply sit in the hose, just as the electrons of a copper wire do not have a general direction without an applied voltage.

2.4 FIXED (dc) SUPPLIES

The terminology *dc* employed in the heading of this section is an abbreviation for **direct current,** which encompasses the various electrical systems in which there is a *unidirectional* ("one direction") flow of charge. A great deal more will be said about this terminology in the chapters to follow. For now, we will consider only those supplies that provide a fixed voltage or current.

dc Voltage Sources

Since the dc voltage source is the more familiar of the two types of supplies, it will be examined first. The symbol used for all dc voltage supplies in this text appears in Fig. 2.11. The relative lengths of the bars indicate the terminals they represent.

Dc voltage sources can be divided into three broad categories: (1) batteries (chemical action), (2) generators (electromechanical), and (3) power supplies (rectification).
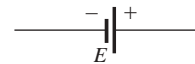
FIG. 2.11
Symbol for a dc voltage source.

Batteries

General Information   For the layperson, the battery is the most common of the dc sources. By definition, a battery (derived from the expression "battery of cells") consists of a combination of two or more similar **cells,** a cell being the fundamental source of electrical energy developed through the conversion of chemical or solar energy. All cells can be divided into the **primary** or **secondary** types. The secondary is rechargeable, whereas the primary is not. That is, the chemical reaction of the secondary cell can be reversed to restore its capacity. The two most common rechargeable batteries are the lead-acid unit (used primarily in automobiles) and the nickel-cadmium battery (used in calculators, tools, photoflash units, shavers, and so on). The obvious advantage of the rechargeable unit is the reduced costs associated with not having to continually replace discharged primary cells.

All the cells appearing in this chapter except the **solar cell,** which absorbs energy from incident light in the form of photons, establish a potential difference at the expense of chemical energy. In addition, each has a positive and a negative *electrode* and an **electrolyte** to complete the circuit between electrodes within the battery. The electrolyte is the contact element and the source of ions for conduction between the terminals.

Alkaline and Lithium-Iodine Primary Cells   The popular alkaline primary battery employs a powdered zinc anode ($+$); a potassium (alkali metal) hydroxide electrolyte; and a manganese dioxide, carbon cathode ($-$) as shown in Fig. 2.12(a). In particular, note in Fig. 2.12(b) that the larger the cylindrical unit, the higher the current capacity. The lantern is designed primarily for long-term use. Figure 2.13 shows two lithium-iodine primary units with an area of application and a rating to be introduced later in this section.

Lead-Acid Secondary Cell   For the secondary lead-acid unit appearing in Fig. 2.14, the electrolyte is sulfuric acid, and the electrodes are spongy lead (Pb) and lead peroxide (PbO_2). When a load is applied to the battery terminals, there is a transfer of electrons from the spongy lead electrode to the lead peroxide electrode through the load. This

(a)

Positive cover — plated steel

Can — steel

Metallized plastic film label

Electrolyte — potassium hydroxide

Anode — powdered zinc

Cathode — manganese dioxide, carbon

Current collector — brass

Inner cell cover — steel

Separator — non-woven fabric

Seal — nylon

Negative cover — plated steel

Metal washer

Metal spur

Rivet — brass

(b)

[7.2 Ah] (0–1 A) [16.0 Ah] (0–1 A) Capacity Continuous Current

1.5 V "AAA" cell

9 V transistor

1.5 V "C" cell 1.5 V "D" cell 1.5 V "AA" cell 6 V lantern

[1.1 Ah] (0–300 mA) [2.5 Ah] (0–500 mA) [520 mAh] (0–250 mA) [22.0 Ah] (0–1.5 A) Capacity Continuous Current

FIG. 2.12

(a) Cutaway of cylindrical Energizer® alkaline cell; (b) Eveready® Energizer primary cells. (Courtesy of Eveready Battery Company, Inc.)
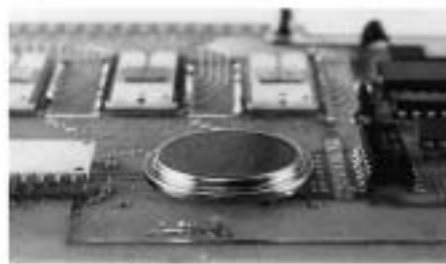
(a) Lithiode™ lithium-iodine cell
2.8 V, 870 mAh
Long-life power sources with printed circuit board mounting capability

(b) Lithium-iodine pacemaker cell
2.8 V, 2.0 Ah

FIG. 2.13

Lithium-iodine primary cells. (Courtesy of Catalyst Research Corp.)

transfer of electrons will continue until the battery is completely discharged. The discharge time is determined by how diluted the acid has become and how heavy the coating of lead sulfate is on each plate. The state of discharge of a lead storage cell can be determined by measuring the **specific gravity** of the electrolyte with a hydrometer. The specific gravity of a substance is defined to be the ratio of the weight of a given volume of the substance to the weight of an equal volume of water at 4°C. For fully charged batteries, the specific gravity should be somewhere between 1.28 and 1.30. When the specific gravity drops to about 1.1, the battery should be recharged.

Since the lead storage cell is a secondary cell, it can be recharged at any point during the discharge phase simply by applying an external **dc current source** across the cell that will pass current through the cell in a direction opposite to that in which the cell supplied current to the
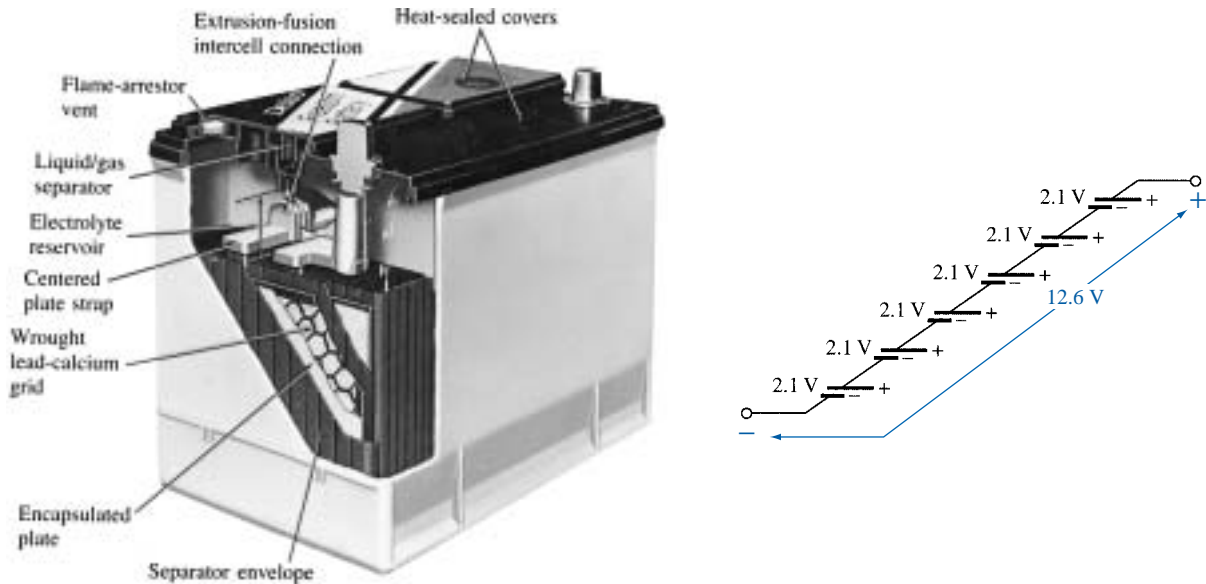
FIG. 2.14

Maintenance-free 12-V (actually 12.6-V) lead-acid battery. (Courtesy of Delco-Remy, a division of General Motors Corp.)

load. This will remove the lead sulfate from the plates and restore the concentration of sulfuric acid.

The output of a lead storage cell over most of the discharge phase is about 2.1 V. In the commercial lead storage batteries used in the automobile, 12.6 V can be produced by six cells in series, as shown in Fig. 2.14. In general, lead-acid storage batteries are used in situations where a high current is required for relatively short periods of time. At one time all lead-acid batteries were vented. Gases created during the discharge cycle could escape, and the vent plugs provided access to replace the water or electrolyte and to check the acid level with a hydrometer. The use of a grid made from a wrought lead–calcium alloy strip rather than the lead-antimony cast grid commonly used has resulted in maintenance-free batteries such as that appearing in Fig. 2.14. The lead-antimony structure was susceptible to corrosion, overcharge, gasing, water usage, and self-discharge. Improved design with the lead-calcium grid has either eliminated or substantially reduced most of these problems.

It would seem that with all the years of technology surrounding batteries, smaller, more powerful units would now be available. However, when it comes to the electric car, which is slowly gaining interest and popularity throughout the world, the lead-acid battery is still the primary source of power. A "station car," manufactured in Norway and used on a test basis in San Francisco for typical commuter runs, has a total weight of 1650 pounds, with 550 pounds (a third of its weight) for the lead-acid rechargeable batteries. Although the station car will travel at speeds of 55 mph, its range is limited to 65 miles on a charge. It would appear that long-distance travel with significantly reduced weight factors for the batteries will depend on a new, innovative approach to battery design.

Nickel-Cadmium Secondary-Cell   The nickel-cadmium battery is a rechargeable battery that has been receiving enormous interest and

1.2 V   1.2 V   7.2 V   1.2 V   1.2 V
4 Ah   1.2   100 mAh   500 mAh   180 mAh

(a)

Eveready® BH 500 cell
1.2 V, 500 mAh
App: Where vertical height is severe
limitation

(b)
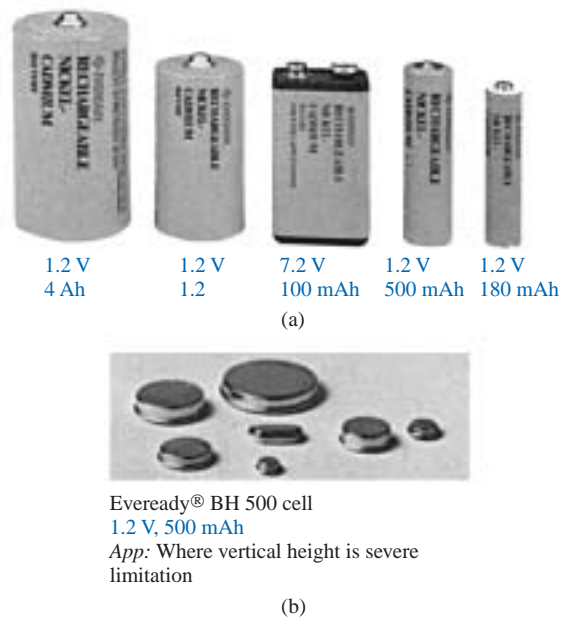
FIG. 2.15

Rechargeable nickel-cadmium batteries. (Courtesy of Eveready Batteries.)

development in recent years. For applications such as flashlights, shavers, portable televisions, power drills, and so on, the nickel-cadmium (Ni-Cad) battery of Fig. 2.15 is the secondary battery of choice because the current levels are lower and the period of continuous drain is usually longer. A typical nickel-cadmium battery can survive over 1000 charge/discharge cycles over a period of time that can last for years.

It is important to recognize that when an appliance or a system calls for a Ni-Cad battery, a primary cell should not be used. The appliance or system may have an internal charging network that would be dysfunctional with a primary cell. In addition, be aware that all Ni-Cad batteries are about 1.2 V per cell, while the most common primary cells are typically 1.5 V per cell. There is some ambiguity about how often a secondary cell should be recharged. For the vast majority of situations, the battery can be used until there is some indication that the energy level is low, such as a dimming light from a flashlight, less power from a drill, or a blinking light if one is provided with the equipment. Keep in mind that secondary cells do have some "memory." If they are recharged continuously after being used for a short period of time, they may begin to believe they are short-term units and actually fail to hold the charge for the rated period of time. In any event, always try to avoid a "hard" discharge, which results when every bit of energy is drained from a cell. Too many hard discharge cycles will reduce the cycle life of the battery. Finally, be aware that the charging mechanism for nickel-cadmium cells is quite different from that for lead-acid batteries. The nickel-cadmium battery is charged by a constant current source, with the terminal voltage staying pretty steady through the entire charging cycle. The lead-acid battery is charged by a constant voltage source, permitting the current to vary as determined by the state of the battery. The capacity of the Ni-Cad battery increases almost linearly throughout most of the charging cycle. One may find that Ni-Cad

batteries are relatively warm when charging. The lower the capacity level of the battery when charging, the higher the temperature of the cell. As the battery approaches rated capacity, the temperature of the cell approaches room temperature.

Nickel-Hydrogen and Nickel–Metal Hydride Secondary Cells
Two other types of secondary cell include the nickel-hydrogen and nickel–metal hydride cells. The nickel-hydrogen cell is currently limited primarily to space vehicle applications where high-energy-density batteries are required that are rugged and reliable and can withstand a high number of charge/discharge cycles over a relatively long period of time. The nickel–metal hydride cell is actually a hybrid of the nickel-cadmium and nickel-hydrogen cells, combining the positive characteristics of each to create a product with a high power level in a small package that has a long cycle life. Although relatively expensive, this hybrid is a valid option for applications such as portable computers, as shown in Fig. 2.16.

10.8 V, 2.9 Ah,
600 mA (monochrome display),
900 mA (color display)

FIG. 2.16
Nickel–metal hydride (Ni-MH) battery for the IBM lap-top computer.

Solar Cell   A high-density, 40-W **solar cell** appears in Fig. 2.17 with some of its associated data and areas of application. Since the maximum available wattage in an average bright sunlit day is 100 mW/cm^2, and since conversion efficiencies are currently between 10% and 14%, the maximum available power per square centimeter from most commercial units is between 10 mW and 14 mW. For a square meter, however, the return would be 100 W to 140 W. A more detailed description of the solar cell will appear in your electronics courses. For now it is important to realize that a fixed illumination of the solar cell will provide a fairly steady dc voltage for driving various loads, from watches to automobiles.

Ampere-Hour Rating   Batteries have a capacity rating given in ampere-hours (Ah) or milliampere-hours (mAh). Some of these ratings are included in the above figures. A battery with an **ampere-hour rating** of 100 will theoretically provide a steady current of 1 A for 100 h, 2 A for 50 h, 10 A for 10 h, and so on, as determined by the following equation:

$$\text{Life (hours)} = \frac{\text{ampere-hour rating (Ah)}}{\text{amperes drawn (A)}} \quad \textbf{(2.9)}$$

Two factors that affect this rating, however, are the temperature and the rate of discharge. The disc-type Eveready® BH 500 cell appearing in Fig. 2.15 has the terminal characteristics appearing in Fig. 2.18. Figure 2.18 reveals that

the capacity of a dc battery decreases with an increase in the current demand

and

the capacity of a dc battery decreases at relatively (compared to room temperature) low and high temperatures.

For the 1-V unit of Fig. 2.18(a), the rating is just above 500 mAh at a discharge current of 100 mA, but it drops to about 300 mAh at about 1 A. For a unit that is less than 1½ in. in diameter and less than ½ in. in thickness, however, these are excellent terminal characteristics. Figure
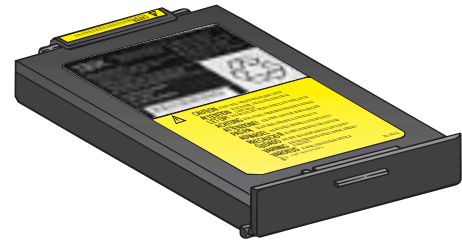
40-W, high-density solar module 100-mm × 100-mm (4″ × 4″) square cells are used to provide maximum power in a minimum of space. The 33 series cell module provides a strong 12-V battery charging current for a wide range of temperatures (−40°C to 60°C)

FIG. 2.17
Solar module. (Courtesy of Motorola Semiconductor Products.)
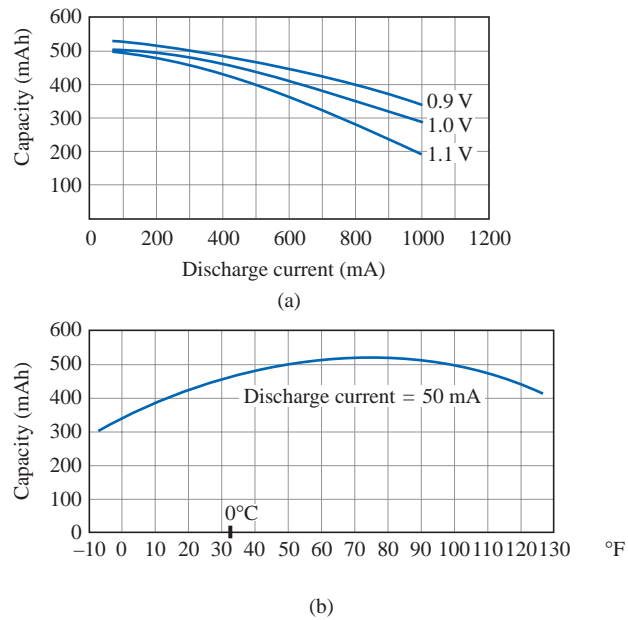
FIG. 2.18

Eveready® BH 500 cell characteristics: (a) capacity versus discharge current; (b) capacity versus temperature. (Courtesy of Eveready Batteries.)

2.18(b) reveals that the maximum mAh rating (at a current drain of 50 mA) occurs at about 75°F ($\cong$24°C), or just above average room temperature. Note that the curve drops to the right and left of this maximum value. We are all aware of the reduced "strength" of a battery at low temperatures. Note that it has dropped to almost 300 mAh at about −8°F.

Another curve of interest appears in Fig. 2.19. It provides the expected cell voltage at a particular drain over a period of hours of use. It is noteworthy that the loss in hours between 50 mA and 100 mA is much greater than between 100 mA and 150 mA, even though the increase in current is the same between levels. In general,

the terminal voltage of a dc battery decreases with the length of the discharge time at a particular drain current.
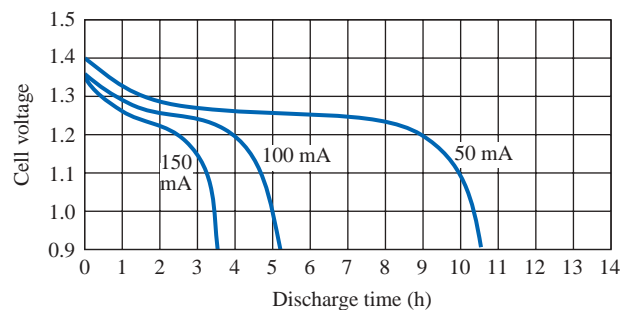
FIG. 2.19

Eveready® BH 500 cell discharge curves. (Courtesy of Eveready Batteries.)

EXAMPLE 2.5

a. Determine the capacity in milliampere-hours and life in minutes for the 0.9-V BH 500 cell of Fig. 2.18(a) if the discharge current is 600 mA.
b. At what temperature will the mAh rating of the cell of Fig. 2.18(b) be 90% of its maximum value if the discharge current is 50 mA?

Solutions:

a. From Fig. 2.18(a), the capacity at 600 mA is about 450 mAh. Thus, from Eq. (2.9),

$$\text{Life} = \frac{450 \text{ mAh}}{600 \text{ mA}} = 0.75 \text{ h} = \textbf{45 min}$$

b. From Fig. 2.18(b), the maximum is approximately 520 mAh. The 90% level is therefore 468 mAh, which occurs just above freezing, or **1°C,** and at the higher temperature of **45°C.**

Generators   The **dc generator** is quite different, both in construction (Fig. 2.20) and in mode of operation, from the battery. When the shaft of the generator is rotating at the nameplate speed due to the applied torque of some external source of mechanical power, a voltage of rated value will appear across the external terminals. The terminal voltage and power-handling capabilities of the dc generator are typically higher than those of most batteries, and its lifetime is determined only by its construction. Commercially used dc generators are typically of the 120-V or 240-V variety. As pointed out earlier in this section, for the purposes of this text, no distinction will be made between the symbols for a battery and a generator.
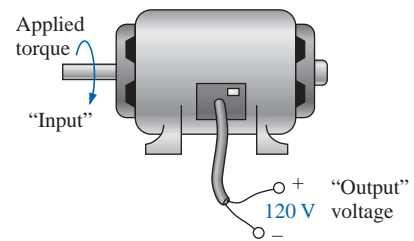
FIG. 2.20
dc generator.

Power Supplies   The dc supply encountered most frequently in the laboratory employs the **rectification** and *filtering* processes as its means toward obtaining a steady dc voltage. Both processes will be covered in detail in your basic electronics courses. In total, a time-varying voltage (such as ac voltage available from a home outlet) is converted to one of a fixed magnitude. A dc laboratory supply of this type appears in Fig. 2.21.

Most dc laboratory supplies have a regulated, adjustable voltage output with three available terminals, as indicated in Figs. 2.21 and 2.22(a). The symbol for ground or zero potential (the reference) is also shown in Fig. 2.22(a). If 10 V above ground potential are required, then the connections are made as shown in Fig. 2.22(b). If 15 V below ground potential are required, then the connections are made as shown in Fig. 2.22(c). If connections are as shown in Fig. 2.22(d), we say we have a "floating" voltage of 5 V since the reference level is not included. Seldom is the configuration of Fig. 2.22(d) employed since it fails to protect the operator by providing a direct low-resistance path to ground and to establish a common ground for the system. In any case, the positive and negative terminals must be part of any circuit configuration.

dc Current Sources

The wide variety of types of, and applications for, the dc voltage source has resulted in its becoming a rather familiar device, the characteristics of which are understood, at least basically, by the layperson. For example, it is common knowledge that a 12-V car battery has a terminal volt-

FIG. 2.21
dc laboratory supply. (Courtesy of Leader Instruments Corporation.)
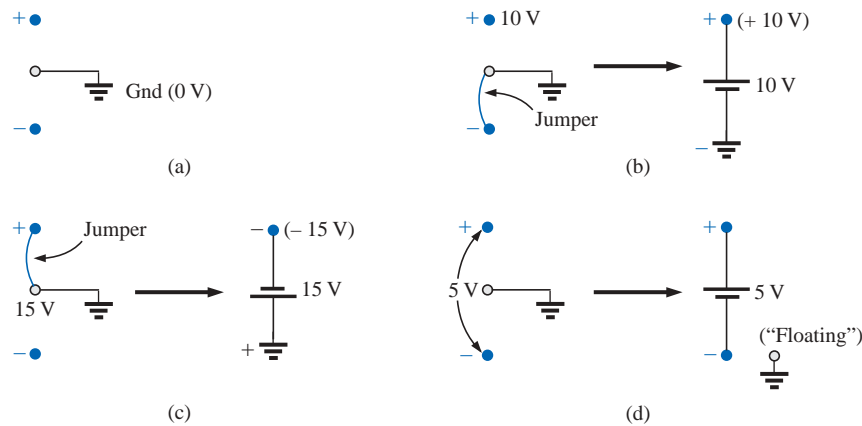
FIG. 2.22

dc laboratory supply: (a) available terminals; (b) positive voltage with respect to (w.r.t.) ground; (c) negative voltage w.r.t. ground; (d) floating supply.
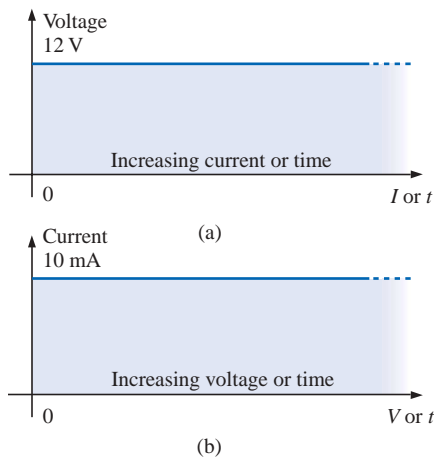
FIG. 2.23

Terminal characteristics: (a) ideal voltage source; (b) ideal current source.

age (at least approximately) of 12 V, even though the current drain by the automobile may vary under different operating conditions. In other words, *a dc voltage source will provide ideally a fixed terminal voltage, even though the current demand from the electrical/electronic system may vary,* as depicted in Fig. 2.23(a). A **dc current source** is the dual of the voltage source; that is,

the current source will supply, ideally, a fixed current to an electrical/electronic system, even though there may be variations in the terminal voltage as determined by the system,

as depicted in Fig. 2.23(b). Do not become alarmed if the concept of a current source is strange and somewhat confusing at this point. It will be covered in great detail in later chapters. Also, additional exposure will be provided in basic electronics courses.

2.5 CONDUCTORS AND INSULATORS

Different wires placed across the same two battery terminals will allow different amounts of charge to flow between the terminals. Many factors, such as the density, mobility, and stability characteristics of a material, account for these variations in charge flow. In general, however,

conductors *are those materials that permit a generous flow of electrons with very little external force (voltage) applied.*

In addition,

good conductors typically have only one electron in the valence (most distant from the nucleus) ring.

Since **copper** is used most frequently, it serves as the standard of comparison for the relative conductivity in Table 2.1. Note that aluminum, which has seen some commercial use, has only 61% of the conductivity level of copper, but keep in mind that this must be weighed against the cost and weight factors.

Insulators *are those materials that have very few free electrons and require a large applied potential (voltage) to establish a measurable current level.*

TABLE 2.1
Relative conductivity of various materials.

Metal	Relative Conductivity (%)
Silver	105
Copper	**100**
Gold	70.5
Aluminum	61
Tungsten	31.2
Nickel	22.1
Iron	14
Constantan	3.52
Nichrome	1.73
Calorite	1.44

A common use of insulating material is for covering current-carrying wire, which, if uninsulated, could cause dangerous side effects. Power-line repair people wear rubber gloves and stand on rubber mats as safety measures when working on high-voltage transmission lines. A number of different types of insulators and their applications appear in Fig. 2.24.

(a) (b) (c)
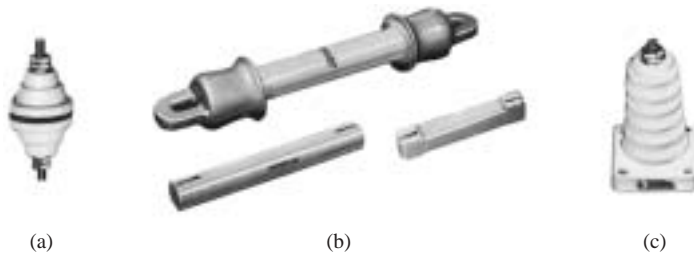
FIG. 2.24
Insulators: (a) insulated thru-panel bushings; (b) antenna strain insulators; (c) porcelain stand-off insulators. (Courtesy of Herman H. Smith, Inc.)

It must be pointed out, however, that even the best insulator will break down (permit charge to flow through it) if a sufficiently large potential is applied across it. The breakdown strengths of some common insulators are listed in Table 2.2. According to this table, for insu-

TABLE 2.2
Breakdown strength of some common insulators.

Material	Average Breakdown Strength (kV/cm)
Air	30
Porcelain	70
Oils	140
Bakelite	150
Rubber	270
Paper (paraffin-coated)	500
Teflon	600
Glass	900
Mica	2000

PROBLEMS

SECTION 2.1   Atoms and Their Structure

1. The numbers of orbiting electrons in aluminum and silver are 13 and 47, respectively. Draw the electronic configuration, including all the shells and subshells, and discuss briefly why each is a good conductor.

2. Find the force of attraction between a proton and an electron separated by a distance equal to the radius of the smallest orbit followed by an electron (5×10^{-11} m) in a hydrogen atom.

3. Find the force of attraction in newtons between the charges Q_1 and Q_2 in Fig. 2.34 when
   a. $r = 1$ m   **b.** $r = 3$ m
   c. $r = 10$ m
   (Note how quickly the force drops with an increase in r.)
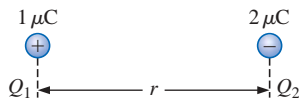
FIG. 2.34
Problem 3.

*4. Find the force of repulsion in newtons between Q_1 and Q_2 in Fig. 2.35 when
   a. $r = 1$ mi   **b.** $r = 0.01$ m
   c. $r = 1/16$ in.
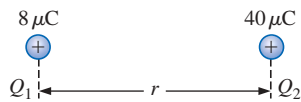
FIG. 2.35
Problem 4.

*5. Plot the force of attraction (in newtons) versus separation (in meters) for two charges of 2 mC and -4 μC. Set r to 0.5 m and 1 m, followed by 1-m intervals to 10 m. Comment on the shape of the curve. Is it linear or nonlinear? What does it tell you about the force of attraction between charges as they are separated? What does it tell you about any function plotted against a squared term in the denominator?

6. Determine the distance between two charges of 20 μC if the force between the two charges is 3.6×10^4 N.

*7. Two charged bodies, Q_1 and Q_2, when separated by a distance of 2 m, experience a force of repulsion equal to 1.8 N.
   a. What will the force of repulsion be when they are 10 m apart?
   b. If the ratio $Q_1/Q_2 = 1/2$, find Q_1 and Q_2 ($r = 10$ m).

SECTION 2.2   Current

8. Find the current in amperes if 650 C of charge pass through a wire in 50 s.

9. If 465 C of charge pass through a wire in 2.5 min, find the current in amperes.

10. If a current of 40 A exists for 1 min, how many coulombs of charge have passed through the wire?

11. How many coulombs of charge pass through a lamp in 2 min if the current is constant at 750 mA?

12. If the current in a conductor is constant at 2 mA, how much time is required for 4600×10^{-6} C to pass through the conductor?

13. If $21.847 \times 10^{+18}$ electrons pass through a wire in 7 s, find the current.

14. How many electrons pass through a conductor in 1 min if the current is 1 A?

15. Will a fuse rated at 1 A "blow" if 86 C pass through it in 1.2 min?

*16. If $0.784 \times 10^{+18}$ electrons pass through a wire in 643 ms, find the current.

*17. Which would you prefer?
   a. A penny for every electron that passes through a wire in 0.01 μs at a current of 2mA, or
   b. A dollar for every electron that passes through a wire in 1.5 ns if the current is 100 μA.

SECTION 2.3   Voltage

18. What is the voltage between two points if 96 mJ of energy are required to move 50×10^{18} electrons between the two points?

19. If the potential difference between two points is 42 V, how much work is required to bring 6 C from one point to the other?

20. Find the charge Q that requires 96 J of energy to be moved through a potential difference of 16 V.

21. How much charge passes through a battery of 22.5 V if the energy expended is 90 J?

22. If a conductor with a current of 200 mA passing through it converts 40 J of electrical energy into heat in 30 s, what is the potential drop across the conductor?

*23. Charge is flowing through a conductor at the rate of 420 C/min. If 742 J of electrical energy are converted to heat in 30 s, what is the potential drop across the conductor?

*24. The potential difference between two points in an electric circuit is 24 V. If 0.4 J of energy were dissipated in a period of 5 ms, what would the current be between the two points?

SECTION 2.4   Fixed (dc) Supplies

25. What current will a battery with an Ah rating of 200 theoretically provide for 40 h?

26. What is the Ah rating of a battery that can provide 0.8 A for 76 h?

27. For how many hours will a battery with an Ah rating of 32 theoretically provide a current of 1.28 A?

28. Find the mAh rating of the Eveready® BH 500 battery at 100°F and 0°C at a discharge current of 50 mA using Fig. 2.18(b).

29. Find the mAh rating of the 1.0-V Eveready® BH 500 battery if the current drain is 550 mA using Fig. 2.18(a). How long will it supply this current?

30. For how long can 50 mA be drawn from the battery of Fig. 2.19 before its terminal voltage drops below 1 V? Determine the number of hours at a drain current of 150 mA, and compare the ratio of drain current to the resulting ratio of hours of availability.

31. A standard 12-V car battery has an ampere-hour rating of 40 Ah, whereas a heavy-duty battery has a rating of 60 Ah. How would you compare the energy levels of each and the available current for starting purposes?

*32. Using the relevant equations of the past few sections, determine the available energy (in joules) from the Eveready battery of Fig. 2.15(b).

*33. A portable television using a 12-V, 3-Ah rechargeable battery can operate for a period of about 5.5 h. What is the average current drawn during this period? What is the energy expended by the battery in joules?

34. Discuss briefly the difference among the three types of dc voltage supplies (batteries, rectification, and generators).

35. Compare the characteristics of a dc current source with those of a dc voltage source. How are they similar and how are they different?

SECTION 2.5   Conductors and Insulators

36. Discuss two properties of the atomic structure of copper that make it a good conductor.

37. Name two materials not listed in Table 2.1 that are good conductors of electricity.

38. Explain the terms *insulator* and *breakdown strength.*

39. List three uses of insulators not mentioned in Section 2.5.

SECTION 2.6   Semiconductors

40. What is a semiconductor? How does it compare with a conductor and an insulator?

41. Consult a semiconductor electronics text and note the extensive use of germanium and silicon semiconductor materials. Review the characteristics of each material.

SECTION 2.7   Ammeters and Voltmeters

42. What are the significant differences in the way ammeters and voltmeters are connected?

43. If an ammeter reads 2.5 A for a period of 4 min, determine the charge that has passed through the meter.

44. Between two points in an electric circuit, a voltmeter reads 12.5 V for a period of 20 s. If the current measured by an ammeter is 10 mA, determine the energy expended and the charge that flowed between the two points.

GLOSSARY

Ammeter   An instrument designed to read the current through elements in series with the meter.

Ampere (A)   The SI unit of measurement applied to the flow of charge through a conductor.

Ampere-hour rating (Ah)   The rating applied to a source of energy that will reveal how long a particular level of current can be drawn from that source.

Cell   A fundamental source of electrical energy developed through the conversion of chemical or solar energy.

Conductors   Materials that permit a generous flow of electrons with very little voltage applied.

Copper   A material possessing physical properties that make it particularly useful as a conductor of electricity.

Coulomb (C)   The fundamental SI unit of measure for charge. It is equal to the charge carried by 6.242×10^{18} electrons.

Coulomb's law   An equation defining the force of attraction or repulsion between two charges.

dc current source   A source that will provide a fixed current level even though the load to which it is applied may cause its terminal voltage to change.

dc generator   A source of dc voltage available through the turning of the shaft of the device by some external means.

Direct current   Current having a single direction (unidirectional) and a fixed magnitude over time.

Electrolytes   The contact element and the source of ions between the electrodes of the battery.

Electron   The particle with negative polarity that orbits the nucleus of an atom.

Free electron   An electron unassociated with any particular atom, relatively free to move through a crystal lattice structure under the influence of external forces.

Insulators   Materials in which a very high voltage must be applied to produce any measurable current flow.

Neutron   The particle having no electrical charge, found in the nucleus of the atom.

Nucleus   The structural center of an atom that contains both protons and neutrons.

Positive ion   An atom having a net positive charge due to the loss of one of its negatively charged electrons.

Potential difference   The algebraic difference in potential (or voltage) between two points in an electrical system.

Potential energy   The energy that a mass possesses by virtue of its position.

Primary cell   Sources of voltage that cannot be recharged.

Proton   The particle of positive polarity found in the nucleus of an atom.

Rectification   The process by which an ac signal is converted to one that has an average dc level.

Secondary cell   Sources of voltage that can be recharged.

Semiconductor A material having a conductance value between that of an insulator and that of a conductor. Of significant importance in the manufacture of semiconductor electronic devices.

Solar cell Sources of voltage available through the conversion of light energy (photons) into electrical energy.

Specific gravity The ratio of the weight of a given volume of a substance to the weight of an equal volume of water at 4°C.

Volt (V) The unit of measurement applied to the difference in potential between two points. If one joule of energy is required to move one coulomb of charge between two points, the difference in potential is said to be one volt.

Voltmeter An instrument designed to read the voltage across an element or between any two points in a network.

3

Resistance
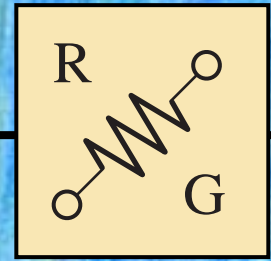
3.1   INTRODUCTION

The flow of charge through any material encounters an opposing force similar in many respects to mechanical friction. This opposition, due to the collisions between electrons and between electrons and other atoms in the material, *which converts electrical energy into another form of energy such as heat,* is called the **resistance** of the material. The unit of measurement of resistance is the **ohm,** for which the symbol is Ω, the capital Greek letter omega. The circuit symbol for resistance appears in Fig. 3.1 with the graphic abbreviation for resistance (R).
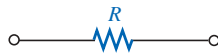
FIG. 3.1

Resistance symbol and notation.

The resistance of any material with a uniform cross-sectional area is determined by the following four factors:

1. *Material*
2. *Length*
3. *Cross-sectional area*
4. *Temperature*

The chosen material, with its unique molecular structure, will react differentially to pressures to establish current through its core. Conductors that permit a generous flow of charge with little external pressure will have low resistance levels, while insulators will have high resistance characteristics.

As one might expect, the longer the path the charge must pass through, the higher the resistance level, whereas the larger the area (and therefore available room), the lower the resistance. Resistance is thus directly proportional to length and inversely proportional to area.

R
G

As the temperature of most conductors increases, the increased motion of the particles within the molecular structure makes it increasingly difficult for the "free" carriers to pass through, and the resistance level increases.

At a fixed temperature of 20°C (room temperature), the resistance is related to the other three factors by

$$R = \rho\frac{l}{A} \qquad \text{(ohms, } \Omega) \qquad\qquad \textbf{(3.1)}$$

where ρ (Greek letter rho) is a characteristic of the material called the **resistivity,** l is the length of the sample, and A is the cross-sectional area of the sample.

The units of measurement substituted into Eq. (3.1) are related to the application. For circular wires, units of measurement are usually defined as in Section 3.2. For most other applications involving important areas such as integrated circuits, the units are as defined in Section 3.4.

3.2 RESISTANCE: CIRCULAR WIRES

For a circular wire, the quantities appearing in Eq. (3.1) are defined by Fig. 3.2. For two wires of the same physical size at the same temperature, as shown in Fig. 3.3(a),

the higher the resistivity, the more the resistance.

As indicated in Fig. 3.3(b),

the longer the length of a conductor, the more the resistance.

Figure 3.3(c) reveals for remaining similar determining variables that

the smaller the area of a conductor, the more the resistance.

Finally, Figure 3.3(d) states that for metallic wires of identical construction and material,

the higher the temperature of a conductor, the more the resistance.

For circular wires, the quantities of Eq. (3.1) have the following units:

ρ: CM-ohms/ft at $T = 20°C$
l: feet
A: circular mils (CM)

FIG. 3.2

Factors affecting the resistance of a conductor.

$\rho_2 > \rho_1$

(a)

$l_2 > l_1$

(b)

$A_2 < A_1$

(c)

$T_2 > T_1$

(d)
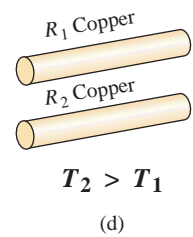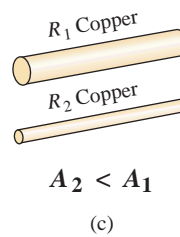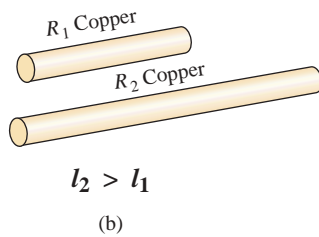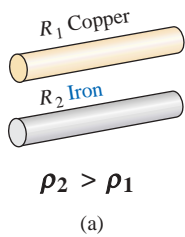
FIG. 3.3

Cases in which $R_2 > R_1$. For each case, all remaining parameters that control the resistance level are the same.

Note that the area of the conductor is measured in **circular mils** (CM) and *not* in *square meters, inches,* and so on, as determined by the equation

$$\text{Area (circle)} = \pi r^2 = \frac{\pi d^2}{4} \qquad \begin{array}{l} r = \text{radius} \\ d = \text{diameter} \end{array} \qquad \textbf{(3.2)}$$

The *mil* is a unit of measurement for length and is related to the inch by

$$1 \text{ mil} = \frac{1}{1000} \text{ in.}$$

or $$1000 \text{ mils} = 1 \text{ in.}$$

By definition,

a wire with a diameter of 1 mil has an area of 1 circular mil (CM), as shown in Fig. 3.4.

One square mil was superimposed on the 1-CM area of Fig. 3.4 to clearly show that the square mil has a larger surface area than the circular mil.

Applying the above definition to a wire having a diameter of 1 mil, and applying Eq. (3.2), we have

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4} (1 \text{ mil})^2 = \frac{\pi}{4} \text{ sq mils} \overset{\text{by definition}}{\equiv} 1 \text{ CM}$$

Therefore,

$$1 \text{ CM} = \frac{\pi}{4} \text{ sq mils} \qquad \textbf{(3.3a)}$$

or $$1 \text{ sq mil} = \frac{4}{\pi} \text{ CM} \qquad \textbf{(3.3b)}$$

Dividing Eq. (3.3b) through will result in

$$1 \text{ sq mil} = \frac{4}{\pi} \text{ CM} = 1.273 \text{ CM}$$

which certainly agrees with the pictorial representation of Fig. 3.4. For a wire with a diameter of N mils (where N can be any positive number),

$$A = \frac{\pi d^2}{4} = \frac{\pi N^2}{4} \text{ sq mils}$$

Substituting the fact that $4/\pi$ CM $= 1$ sq mil, we have

$$A = \frac{\pi N^2}{4} \text{ (sq mils)} = \left(\frac{\pi N^2}{4}\right)\left(\frac{4}{\pi} \text{ CM}\right) = N^2 \text{ CM}$$

Since $d = N,$ the area in circular mils is simply equal to the diameter in mils square; that is,

$$A_{CM} = (d_{mils})^2 \qquad \textbf{(3.4)}$$

Verification that an area can simply be the diameter squared is provided in part by Fig. 3.5 for diameters of 2 and 3 mils. Although some areas are not circular, they have the same area as 1 circular mil.
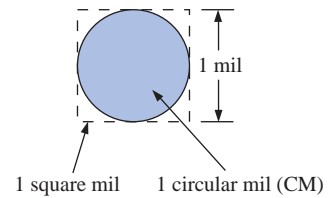
FIG. 3.4
Defining the circular mil (CM).
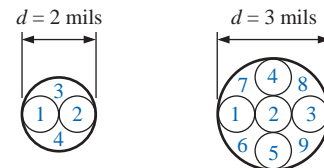
$A = (2 \text{ mils})^2 = 4 \text{ CM} \qquad A = (3 \text{ mils})^2 = 9 \text{ CM}$

FIG. 3.5
Verification of Eq. (3.4): $A_{CM} = (d_{mils})^2$.

In the future, therefore, to find the area in circular mils, the diameter must first be converted to mils. Since 1 mil = 0.001 in., if the diameter is given in inches, simply move the decimal point three places to the right. For example,

$$0.02 \text{ in.} = 0.020 \text{ mils} = 20 \text{ mils}$$

If the diameter is in fractional form, first convert it to decimal form and then proceed as above. For example,

$$\frac{1}{8} \text{ in.} = 0.125 \text{ in.} = 125 \text{ mils}$$

The constant ρ (resistivity) is different for every material. Its value is the resistance of a length of wire 1 ft by 1 mil in diameter, measured at 20°C (Fig. 3.6). The unit of measurement for ρ can be determined from Eq. (3.1) by first solving for ρ and then substituting the units of the other quantities. That is,

$$\rho = \frac{AR}{l}$$

and

$$\text{Units of } \rho = \frac{\text{CM} \cdot \Omega}{\text{ft}}$$

The resistivity ρ is also measured in ohms per mil-foot, as determined by Fig. 3.6, or *ohm-meters* in the SI system of units. Some typical values of ρ are provided in Table 3.1.
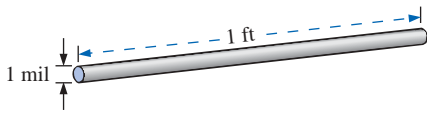
FIG. 3.6
Defining the constant ρ (resistivity).

TABLE 3.1
Resistivity (ρ) of various materials.

Material	ρ @ 20°C
Silver	9.9
Copper	**10.37**
Gold	14.7
Aluminum	17.0
Tungsten	33.0
Nickel	47.0
Iron	74.0
Constantan	295.0
Nichrome	600.0
Calorite	720.0
Carbon	21,000.0

EXAMPLE 3.1 What is the resistance of a 100-ft length of copper wire with a diameter of 0.020 in. at 20°C?

Solution:

$$\rho = 10.37 \frac{\text{CM} \cdot \Omega}{\text{ft}} \qquad 0.020 \text{ in.} = 20 \text{ mils}$$

$$A_{\text{CM}} = (d_{\text{mils}})^2 = (20 \text{ mils})^2 = 400 \text{ CM}$$

$$R = \rho \frac{l}{A} = \frac{(10.37 \text{ CM} \cdot \Omega/\text{ft})(100 \text{ ft})}{400 \text{ CM}}$$

$$R = \mathbf{2.59 \ \Omega}$$

EXAMPLE 3.2 An undetermined number of feet of wire have been used from the carton of Fig. 3.7. Find the length of the remaining copper wire if it has a diameter of 1/16 in. and a resistance of 0.5 Ω.

Solution:
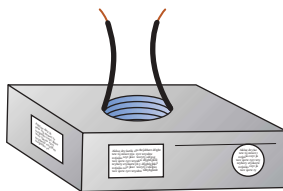
$$\rho = 10.37 \text{ CM} \cdot \Omega/\text{ft} \qquad \frac{1}{16} \text{ in.} = 0.0625 \text{ in.} = 62.5 \text{ mils}$$

$$A_{\text{CM}} = (d_{\text{mils}})^2 = (62.5 \text{ mils})^2 = 3906.25 \text{ CM}$$

$$R = \rho \frac{l}{A} \Rightarrow l = \frac{RA}{\rho} = \frac{(0.5 \ \Omega)(3906.25 \text{ CM})}{10.37 \frac{\text{CM} \cdot \Omega}{\text{ft}}} = \frac{1953.125}{10.37}$$

$$l = \mathbf{188.34 \text{ ft}}$$

FIG. 3.7
Example 3.2.

EXAMPLE 3.3 What is the resistance of a copper bus-bar, as used in the power distribution panel of a high-rise office building, with the dimensions indicated in Fig. 3.8?

Solution:

$$A_{CM} \begin{cases} 5.0 \text{ in.} = 5000 \text{ mils} \\[6pt] \dfrac{1}{2} \text{ in.} = 500 \text{ mils} \\[6pt] A = (5000 \text{ mils})(500 \text{ mils}) = 2.5 \times 10^6 \text{ sq mils} \\[6pt] = 2.5 \times 10^6 \text{ sq mils} \left(\dfrac{4/\pi \text{ CM}}{1 \text{ sq mil}} \right) \\[6pt] A = 3.185 \times 10^6 \text{ CM} \end{cases}$$

$$R = \rho \frac{l}{A} = \frac{(10.37 \text{ CM} \cdot \Omega / \text{ft})(3 \text{ ft})}{3.185 \times 10^6 \text{ CM}} = \frac{31.110}{3.185 \times 10^6}$$

$$R = \mathbf{9.768 \times 10^{-6} \ \Omega}$$
$$\text{(quite small, } 0.000009768 \ \Omega)$$
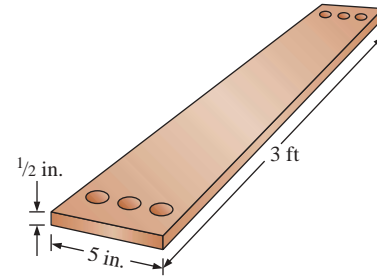
FIG. 3.8
Example 3.3.

We will find in the chapters to follow that the less the resistance of a conductor, the lower the losses in conduction from the source to the load. Similarly, since resistivity is a major factor in determining the resistance of a conductor, the lower the resistivity, the lower the resistance for the same size conductor. Table 3.1 would suggest therefore that silver, copper, gold, and aluminum would be the best conductors and the most common. In general, there are other factors, however, such as **malleability** (ability of a material to be shaped), **ductility** (ability of a material to be drawn into long, thin wires), temperature sensitivity, resistance to abuse, and, of course, cost, that must all be weighed when choosing a conductor for a particular application.

In general, copper is the most widely used material because it is quite malleable, ductile, and available; has good thermal characteristics; and is less expensive than silver or gold. It is certainly not cheap, however. Wiring is removed quickly from buildings to be torn down, for example, to extract the copper. At one time aluminum was introduced for general wiring because it is cheaper than copper, but its thermal characteristics created some difficulties. It was found that the heating due to current flow and the cooling that occurred when the circuit was turned off resulted in expansion and contraction of the aluminum wire to the point where connections could eventually work themselves loose and dangerous side effects could result. Aluminum is still used today, however, in areas such as integrated circuit manufacturing and in situations where the connections can be made secure. Silver and gold are, of course, much more expensive than copper or aluminum, but there are places where the cost is justified. Silver has excellent plating characteristics for surface preparations, and gold is used quite extensively in integrated circuits. Tungsten has a resistivity three times that of copper, but there are occasions when its physical characteristics (durability, hardness) are the overriding considerations.

3.3 WIRE TABLES

The wire table was designed primarily to standardize the size of wire produced by manufacturers throughout the United States. As a result,

TABLE 3.2
American Wire Gage (AWG) sizes.

	AWG #	Area (CM)	Ω/1000 ft at 20°C	Maximum Allowable Current for RHW Insulation (A)*
(4/0)	**0000**	211,600	0.0490	**230**
(3/0)	**000**	167,810	0.0618	**200**
(2/0)	**00**	133,080	0.0780	**175**
(1/0)	**0**	105,530	0.0983	**150**
	1	83,694	0.1240	**130**
	2	66,373	0.1563	**115**
	3	52,634	0.1970	**100**
	4	41,742	0.2485	**85**
	5	33,102	0.3133	—
	6	26,250	0.3951	**65**
	7	20,816	0.4982	—
	8	16,509	0.6282	**50**
	9	13,094	0.7921	—
	10	10,381	0.9989	**30**
	11	8,234.0	1.260	—
	12	6,529.0	1.588	**20**
	13	5,178.4	2.003	—
	14	4,106.8	2.525	**15**
	15	3,256.7	3.184	
	16	2,582.9	4.016	
	17	2,048.2	5.064	
	18	1,624.3	6.385	
	19	1,288.1	8.051	
	20	1,021.5	10.15	
	21	810.10	12.80	
	22	642.40	16.14	
	23	509.45	20.36	
	24	404.01	25.67	
	25	320.40	32.37	
	26	254.10	40.81	
	27	201.50	51.47	
	28	159.79	64.90	
	29	126.72	81.83	
	30	100.50	103.2	
	31	79.70	130.1	
	32	63.21	164.1	
	33	50.13	206.9	
	34	39.75	260.9	
	35	31.52	329.0	
	36	25.00	414.8	
	37	19.83	523.1	
	38	15.72	659.6	
	39	12.47	831.8	
	40	9.89	1049.0	

*Not more than three conductors in raceway, cable, or direct burial.

the manufacturer has a larger market and the consumer knows that standard wire sizes will always be available. The table was designed to assist the user in every way possible; it usually includes data such as the cross-sectional area in circular mils, diameter in mils, ohms per 1000 feet at 20°C, and weight per 1000 feet.

The American Wire Gage (AWG) sizes are given in Table 3.2 for solid round copper wire. A column indicating the maximum allowable current in amperes, as determined by the National Fire Protection Association, has also been included.

The chosen sizes have an interesting relationship: For every drop in 3 gage numbers, the area is doubled; and for every drop in 10 gage numbers, the area increases by a factor of 10.

Examining Eq. (3.1), we note also that *doubling the area cuts the resistance in half, and increasing the area by a factor of 10 decreases the resistance of 1/10 the original,* everything else kept constant.

The actual sizes of the gage wires listed in Table 3.2 are shown in Fig. 3.9 with a few of their areas of application. A few examples using Table 3.2 follow.

EXAMPLE 3.4 Find the resistance of 650 ft of #8 copper wire ($T = 20°C$).

Solution: For #8 copper wire (solid), Ω/1000 ft at 20°C = 0.6282 Ω, and

$$650 \; \text{ft} \left(\frac{0.6282 \; \Omega}{1000 \; \text{ft}} \right) = \textbf{0.408} \; \boldsymbol{\Omega}$$

EXAMPLE 3.5 What is the diameter, in inches, of a #12 copper wire?

Solution: For #12 copper wire (solid), $A = 6529.9$ CM, and

$$d_{\text{mils}} = \sqrt{A_{\text{CM}}} = \sqrt{6529.9 \; \text{CM}} \cong 80.81 \; \text{mils}$$

$$d = \textbf{0.0808 in.} \; \text{(or close to 1/12 in.)}$$

EXAMPLE 3.6 For the system of Fig. 3.10, the total resistance of *each* power line cannot exceed 0.025 Ω, and the maximum current to be drawn by the load is 95 A. What gage wire should be used?

Solution:

$$R = \rho \frac{l}{A} \Rightarrow A = \rho \frac{l}{R} = \frac{(10.37 \; \text{CM} \cdot \Omega/\text{ft})(100 \; \text{ft})}{0.025 \; \Omega} = 41{,}480 \; \text{CM}$$

Using the wire table, we choose the wire with the next largest area, which is #4, to satisfy the resistance requirement. We note, however, that 95 A must flow through the line. This specification requires that #3 wire be used since the #4 wire can carry a maximum current of only 85 A.

$D = 0.365$ in. $\cong$ $^1/_3$ in.

Stranded for increased flexibility

00

Power distribution

$D = 0.081$ in. $\cong$ $^1/_{12}$ in. $D = 0.064$ in. $\cong$ $^1/_{16}$ in.

12   14

Lighting, outlets, general home use

$D = 0.032$ in. $\cong$ $^1/_{32}$ in. $D = 0.025$ in. $=$ $^1/_{40}$ in.

20   22

Radio, television

$D = 0.013$ in. $\cong$ $^1/_{75}$ in.

28

Telephone, instruments

FIG. 3.9
Popular wire sizes and some of their areas of application.

Solid round copper wire

Input   Load

100 ft

FIG. 3.10
Example 3.6.

3.4 RESISTANCE: METRIC UNITS

The design of resistive elements for various areas of application, including thin-film resistors and integrated circuits, uses metric units for the quantities of Eq. (3.1). In SI units, the resistivity would be measured in ohm-meters, the area in square meters, and the length in
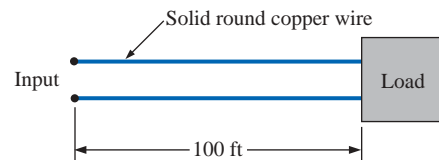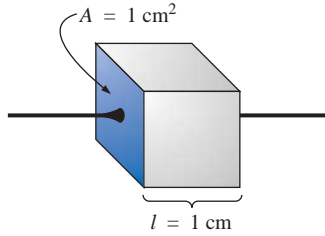
FIG. 3.11

Defining ρ in ohm-centimeters.

TABLE 3.3

Resistivity (ρ) of various materials in ohm-centimeters.

Silver	1.645×10^{-6}
Copper	$\mathbf{1.723 \times 10^{-6}}$
Gold	2.443×10^{-6}
Aluminum	2.825×10^{-6}
Tungsten	5.485×10^{-6}
Nickel	7.811×10^{-6}
Iron	12.299×10^{-6}
Tantalum	15.54×10^{-6}
Nichrome	99.72×10^{-6}
Tin oxide	250×10^{-6}
Carbon	3500×10^{-6}

meters. However, the meter is generally too large a unit of measure for most applications, and so the centimeter is usually employed. The resulting dimensions for Eq. (3.1) are therefore

ρ:	ohm-centimeters
l:	centimeters
A:	square centimeters

The units for ρ can be derived from

$$\rho = \frac{RA}{l} = \frac{\Omega \cdot \text{cm}^2}{\text{cm}} = \Omega \cdot \text{cm}$$

The resistivity of a material is actually the resistance of a sample such as that appearing in Fig. 3.11. Table 3.3 provides a list of values of ρ in ohm-centimeters. Note that the area now is expressed in square centimeters, which can be determined using the basic equation $A = \pi d^2/4$, eliminating the need to work with circular mils, the special unit of measure associated with circular wires.

EXAMPLE 3.7 Determine the resistance of 100 ft of #28 copper telephone wire if the diameter is 0.0126 in.

Solution: Unit conversions:

$$l = 100 \, \text{ft}\left(\frac{12 \, \text{in.}}{1 \, \text{ft}}\right)\left(\frac{2.54 \, \text{cm}}{1 \, \text{in.}}\right) = 3048 \, \text{cm}$$

$$d = 0.0126 \, \text{in.}\left(\frac{2.54 \, \text{cm}}{1 \, \text{in.}}\right) = 0.032 \, \text{cm}$$

Therefore,

$$A = \frac{\pi d^2}{4} = \frac{(3.1416)(0.032 \, \text{cm})^2}{4} = 8.04 \times 10^{-4} \, \text{cm}^2$$

$$R = \rho\frac{l}{A} = \frac{(1.723 \times 10^{-6} \, \Omega \cdot \text{cm})(3048 \, \text{cm})}{8.04 \times 10^{-4} \, \text{cm}^2} \cong \mathbf{6.5 \, \Omega}$$

Using the units for circular wires and Table 3.2 for the area of a #28 wire, we find

$$R = \rho\frac{l}{A} = \frac{(10.37 \, \text{CM} \cdot \Omega/\text{ft})(100 \, \text{ft})}{159.79 \, \text{CM}} \cong \mathbf{6.5 \, \Omega}$$

EXAMPLE 3.8 Determine the resistance of the thin-film resistor of Fig. 3.12 if the **sheet resistance** R_s (defined by $R_s = \rho/d$) is 100 Ω.

Solution: For deposited materials of the same thickness, the sheet resistance factor is usually employed in the design of thin-film resistors. Equation (3.1) can be written

$$R = \rho\frac{l}{A} = \rho\frac{l}{dw} = \left(\frac{\rho}{d}\right)\left(\frac{l}{w}\right) = R_s\frac{l}{w}$$

where l is the length of the sample and w is the width. Substituting into the above equation yields

$$R = R_s\frac{l}{w} = \frac{(100 \, \Omega)(0.6 \, \text{cm})}{0.3 \, \text{cm}} = \mathbf{200 \, \Omega}$$

as one might expect since $l = 2w$.
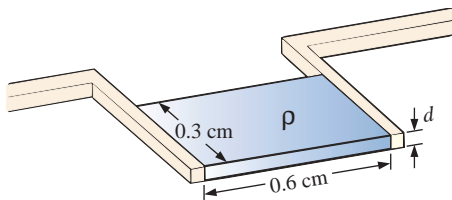
FIG. 3.12

Thin-film resistor (note Fig. 3.22).

The conversion factor between resistivity in circular mil-ohms per foot and ohm-centimeters is the following:

$$\rho \ (\Omega \cdot \text{cm}) = (1.662 \times 10^{-7}) \times (\text{value in CM} \cdot \Omega/\text{ft})$$

For example, for copper, $\rho = 10.37 \ \text{CM} \cdot \Omega/\text{ft}$:

$$\rho \ (\Omega \cdot \text{cm}) = 1.662 \times 10^{-7}(10.37 \ \text{CM} \cdot \Omega/\text{ft})$$
$$= 1.723 \times 10^{-6} \ \Omega \cdot \text{cm}$$

as indicated in Table 3.3.

The resistivity in IC design is typically in ohm-centimeter units, although tables often provide ρ in ohm-meters or microhm-centimeters. Using the conversion technique of Chapter 1, we find that the conversion factor between ohm-centimeters and ohm-meters is the following:

$$1.723 \times 10^{-6} \ \Omega \cdot \cancel{\text{cm}} \left[\frac{1 \text{ m}}{100 \ \cancel{\text{cm}}} \right] = \frac{1}{100}[1.723 \times 10^{-6}] \ \Omega \cdot \text{m}$$

or the value in ohm-meters is 1/100 the value in ohm-centimeters, and

$$\rho \ (\Omega \cdot \text{m}) = \left(\frac{1}{100} \right) \times (\text{value in } \Omega \cdot \text{cm})$$

Similarly:

$$\rho \ (\mu\Omega \cdot \text{cm}) = (10^6) \times (\text{value in } \Omega \cdot \text{cm})$$

For comparison purposes, typical values of ρ in ohm-centimeters for conductors, semiconductors, and insulators are provided in Table 3.4.

TABLE 3.4
Comparing levels of ρ in $\Omega \cdot cm$.

Conductor		Semiconductor		Insulator	
Copper	1.723×10^{-6}	Ge	50	In general:	10^{15}
		Si	200×10^3		
		GaAs	70×10^6		

In particular, note the power-of-ten difference between conductors and insulators (10^{21})—a difference of huge proportions. There is a significant difference in levels of ρ for the list of semiconductors, but the power-of-ten difference between the conductor and insulator levels is at least 10^6 for each of the semiconductors listed.

3.5 TEMPERATURE EFFECTS

Temperature has a significant effect on the resistance of conductors, semiconductors, and insulators.

Conductors

Conductors have a generous number of free electrons, and any introduction of thermal energy will have little impact on the total number of

free carriers. In fact, the thermal energy will only increase the intensity of the random motion of the particles within the material and make it increasingly difficult for a general drift of electrons in any one direction to be established. The result is that

for good conductors, an increase in temperature will result in an increase in the resistance level. Consequently, conductors have a positive temperature coefficient.

The plot of Fig. 3.13(a) has a positive temperature coefficient.
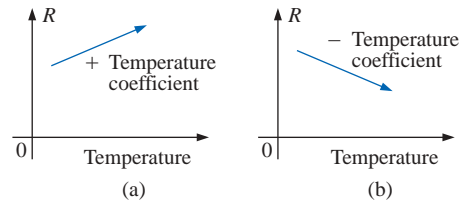
FIG. 3.13
(a) Positive temperature coefficient—conductors; (b) negative temperature coefficient—semiconductors.

Semiconductors

In semiconductors an increase in temperature will impart a measure of thermal energy to the system that will result in an increase in the number of free carriers in the material for conduction. The result is that

for semiconductor materials, an increase in temperature will result in a decrease in the resistance level. Consequently, semiconductors have negative temperature coefficients.

The thermistor and photoconductive cell of Sections 3.10 and 3.11 of this chapter are excellent examples of semiconductor devices with negative temperature coefficients. The plot of Fig. 3.13(b) has a negative temperature coefficient.

Insulators

As with semiconductors, an increase in temperature will result in a decrease in the resistance of an insulator. The result is a negative temperature coefficient.

Inferred Absolute Temperature

Figure 3.14 reveals that for copper (and most other metallic conductors), the resistance increases almost linearly (in a straight-line relationship) with an increase in temperature. Since temperature can have such a pronounced effect on the resistance of a conductor, it is important that we have some method of determining the resistance at any temperature within operating limits. An equation for this purpose can be obtained by approximating the curve of Fig. 3.14 by the straight dashed line that intersects the temperature scale at $-234.5°C$. Although the actual curve extends to **absolute zero** ($-273.15°C$, or 0 K), the straight-line approximation is quite accurate for the normal operating temperature range. At two different temperatures, T_1 and T_2, the resistance of copper is R_1 and R_2, as indicated on the curve. Using a property of similar triangles, we may develop a mathematical relationship between these values of resis-
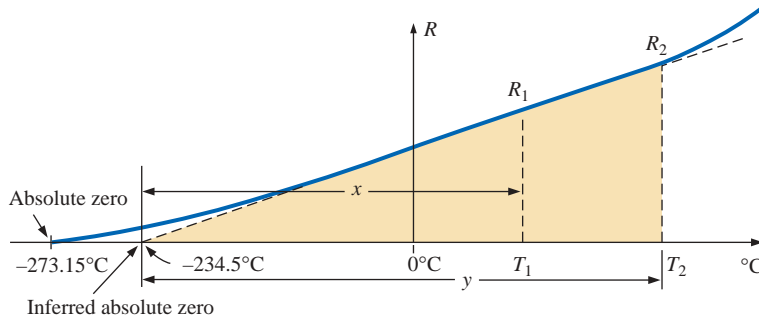
FIG. 3.14
Effect of temperature on the resistance of copper.

tances at different temperatures. Let x equal the distance from $-234.5°C$ to T_1 and y the distance from $-234.5°C$ to T_2, as shown in Fig. 3.14. From similar triangles,

$$\frac{x}{R_1} = \frac{y}{R_2}$$

or

$$\frac{234.5 + T_1}{R_1} = \frac{234.5 + T_2}{R_2} \qquad \textbf{(3.5)}$$

The temperature of $-234.5°C$ is called the **inferred absolute temperature** of copper. For different conducting materials, the intersection of the straight-line approximation will occur at different temperatures. A few typical values are listed in Table 3.5.

The minus sign does not appear with the inferred absolute temperature on either side of Eq. (3.5) because x and y are the *distances* from $-234.5°C$ to T_1 and T_2, respectively, and therefore are simply magnitudes. For T_1 and T_2 less than zero, x and y are less than $-234.5°C$, and the distances are the differences between the inferred absolute temperature and the temperature of interest.

Equation (3.5) can easily be adapted to any material by inserting the proper inferred absolute temperature. It may therefore be written as follows:

$$\frac{|T_1| + T_1}{R_1} = \frac{|T_1| + T_2}{R_2} \qquad \textbf{(3.6)}$$

where $|T_1|$ indicates that the inferred absolute temperature of the material involved is inserted as a positive value in the equation. In general, therefore, associate the sign only with T_1 and T_2.

TABLE 3.5
Inferred absolute temperatures (T_i).

Material	°C
Silver	−243
Copper	**−234.5**
Gold	−274
Aluminum	−236
Tungsten	−204
Nickel	−147
Iron	−162
Nichrome	−2,250
Constantan	−125,000

EXAMPLE 3.9 If the resistance of a copper wire is 50 Ω at 20°C, what is its resistance at 100°C (boiling point of water)?

Solution: Eq. (3.5):

$$\frac{234.5°C + 20°C}{50\ \Omega} = \frac{234.5°C + 100°C}{R_2}$$

$$R_2 = \frac{(50\ \Omega)(334.5°C)}{254.5°C} = \textbf{65.72 }\boldsymbol{\Omega}$$

EXAMPLE 3.10 If the resistance of a copper wire at freezing (0°C) is 30 Ω, what is its resistance at −40°C?

Solution: Eq. (3.5):

$$\frac{234.5°C + 0}{30\ \Omega} = \frac{234.5°C - 40°C}{R_2}$$

$$R_2 = \frac{(30\ \Omega)(194.5°C)}{234.5°C} = \textbf{24.88 } \boldsymbol{\Omega}$$

EXAMPLE 3.11 If the resistance of an aluminum wire at room temperature (20°C) is 100 mΩ (measured by a milliohmmeter), at what temperature will its resistance increase to 120 mΩ?

Solution: Eq. (3.5):

$$\frac{236°C + 20°C}{100\ m\Omega} = \frac{236°C + T_2}{120\ m\Omega}$$

and

$$T_2 = 120\ m\Omega\left(\frac{256°C}{100\ m\Omega}\right) - 236°C$$

$$T_2 = \textbf{71.2°C}$$

Temperature Coefficient of Resistance

There is a second popular equation for calculating the resistance of a conductor at different temperatures. Defining

$$\alpha_{20} = \frac{1}{|T_1| + 20°C} \qquad (\Omega/°C/\Omega) \qquad \textbf{(3.7)}$$

as the **temperature coefficient of resistance** at a temperature of 20°C, and R_{20} as the resistance of the sample at 20°C, the resistance R_1 at a temperature T_1 is determined by

$$R_1 = R_{20}[1 + \alpha_{20}(T_1 - 20°C)] \qquad \textbf{(3.8)}$$

The values of α_{20} for different materials have been evaluated, and a few are listed in Table 3.6.

Equation (3.8) can be written in the following form:

$$\alpha_{20} = \frac{\left(\dfrac{R_1 - R_{20}}{T_1 - 20°C}\right)}{R_{20}} = \frac{\dfrac{\Delta R}{\Delta T}}{R_{20}}$$

from which the units of Ω/°C/Ω for α_{20} are defined.

Since $\Delta R/\Delta T$ is the slope of the curve of Fig. 3.14, we can conclude that

the higher the temperature coefficient of resistance for a material, the more sensitive the resistance level to changes in temperature.

Referring to Table 3.5, we find that copper is more sensitive to temperature variations than is silver, gold, or aluminum, although the differences are quite small. The slope defined by α_{20} for constantan is so small that the curve is almost horizontal.

TABLE 3.6

Temperature coefficient of resistance for various conductors at 20°C.

Material	Temperature Coefficient (α_{20})
Silver	0.0038
Copper	**0.00393**
Gold	0.0034
Aluminum	0.00391
Tungsten	0.005
Nickel	0.006
Iron	0.0055
Constantan	0.000008
Nichrome	0.00044

Since R_{20} of Eq. (3.8) is the resistance of the conductor at 20°C and $T_1 - 20°C$ is the change in temperature from 20°C, Equation (3.8) can be written in the following form:

$$R = \rho \frac{l}{A}[1 + \alpha_{20}\,\Delta T] \qquad \textbf{(3.9)}$$

providing an equation for resistance in terms of all the controlling parameters.

PPM/°C

For resistors, as for conductors, resistance changes with a change in temperature. The specification is normally provided in parts per million per degree Celsius **(PPM/°C),** providing an immediate indication of the sensitivity level of the resistor to temperature. For resistors, a 5000-PPM level is considered high, whereas 20 PPM is quite low. A 1000-PPM/°C characteristic reveals that a 1° change in temperature will result in a change in resistance equal to 1000 PPM, or $1000/1,000,000 = 1/1000$ of its nameplate value—not a significant change for most applications. However, a 10° change would result in a change equal to 1/100 (1%) of its nameplate value, which is becoming significant. The concern, therefore, lies not only with the PPM level but with the range of expected temperature variation.

In equation form, the change in resistance is given by

$$\Delta R = \frac{R_{\text{nominal}}}{10^6}(\text{PPM})(\Delta T) \qquad \textbf{(3.10)}$$

where R_{nominal} is the nameplate value of the resistor at room temperature and ΔT is the change in temperature from the reference level of 20°C.

EXAMPLE 3.12 For a 1-kΩ carbon composition resistor with a PPM of 2500, determine the resistance at 60°C.

Solution:

$$\Delta R = \frac{1000\ \Omega}{10^6}(2500)(60°C - 20°C)$$

$$= 100\ \Omega$$

and $\qquad R = R_{\text{nominal}} + \Delta R = 1000\ \Omega + 100\ \Omega$

$$= \textbf{1100 }\boldsymbol{\Omega}$$

3.6 SUPERCONDUCTORS

There is no question that the field of electricity/electronics is one of the most exciting of the 20th century. Even though new developments appear almost weekly from extensive research and development activities, every once in a while there is some very special step forward that has the whole field at the edge of its seat waiting to see what might develop in the near future. Such a level of excitement and interest surrounds the research

drive to develop a room-temperature **superconductor**—an advance that will rival the introduction of semiconductor devices such as the transistor (to replace tubes), wireless communication, or the electric light. The implications of such a development are so far-reaching that it is difficult to forecast the vast impact it will have on the entire field.

The intensity of the research effort throughout the world today to develop a room-temperature superconductor is described by some researchers as "unbelievable, contagious, exciting, and demanding" but an adventure in which they treasure the opportunity to be involved. Progress in the field since 1986 suggests that the use of superconductivity in commercial applications will grow quite rapidly in the next few decades. It is indeed an exciting era full of growing anticipation! Why this interest in superconductors? What are they all about? In a nutshell,

superconductors *are conductors of electric charge that, for all practical purposes, have zero resistance.*

In a conventional conductor, electrons travel at average speeds in the neighborhood of 1000 mi/s (they can cross the United States in about 3 seconds), even though Einstein's theory of relativity suggests that the maximum speed of information transmission is the speed of light, or 186,000 mi/s. The relatively slow speed of conventional conduction is due to collisions with other atoms in the material, repulsive forces between electrons (like charges repel), thermal agitation that results in indirect paths due to the increased motion of the neighboring atoms, impurities in the conductor, and so on. In the superconductive state, there is a pairing of electrons, denoted by the **Cooper effect,** in which electrons travel in pairs and help each other maintain a significantly higher velocity through the medium. In some ways this is like "drafting" by competitive cyclists or runners. There is an oscillation of energy between partners or even "new" partners (as the need arises) to ensure passage through the conductor at the highest possible velocity with the least total expenditure of energy.

Even though the concept of superconductivity first surfaced in 1911, it was not until 1986 that the possibility of superconductivity at room temperature became a renewed goal of the research community. For some 74 years superconductivity could be established only at temperatures colder than 23 K. (Kelvin temperature is universally accepted as the unit of measurement for temperature for superconductive effects. Recall that K = 273.15° + °C, so a temperature of 23 K is −250°C, or −418°F.) In 1986, however, physicists Alex Muller and George Bednorz of the IBM Zurich Research Center found a ceramic material, lanthanum barium copper oxide, that exhibited superconductivity at 30 K. Although it would not appear to be a significant step forward, it introduced a new direction to the research effort and spurred others to improve on the new standard. In October 1987 both scientists received the Nobel prize for their contribution to an important area of development.

In just a few short months, Professors Paul Chu of the University of Houston and Man Kven Wu of the University of Alabama raised the temperature to 95 K using a superconductor of yttrium barium copper oxide. The result was a level of excitement in the scientific community that brought research in the area to a new level of effort and investment. The major impact of such a discovery was that liquid nitrogen (boiling point of 77 K) could now be used to bring the material down to the required temperature rather than liquid helium, which boils at 4 K. The

result is a tremendous saving in the cooling expense since liquid helium is at least ten times more expensive than liquid nitrogen. Pursuing the same direction, some success has been achieved at 125 K and 162 K using a thallium compound (unfortunately, however, thallium is a very poisonous substance).

Figure 3.15 clearly reveals that there was little change in the temperature for superconductors until the discovery of 1986. The curve then takes a sharp curve upward, suggesting that room-temperature superconductors may become available in a few short years. However, unless there is a significant breakthrough in the near future, this goal no longer seems feasible. The effort continues and is receiving an increasing level of financing and worldwide attention. Now, increasing numbers of corporations are trying to capitalize on the success already attained, as will be discussed later in this section.
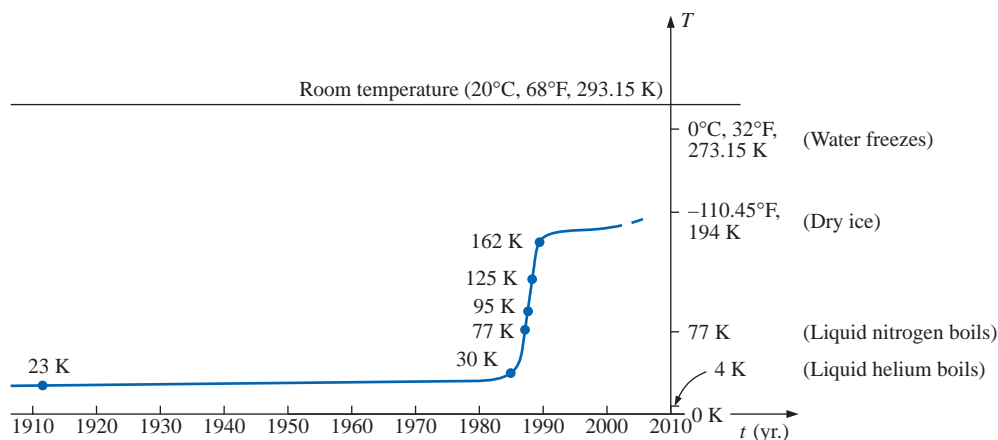
FIG. 3.15
Rising temperatures of superconductors.

The fact that ceramics have provided the recent breakthrough in superconductivity is probably a surprise when you consider that they are also an important class of insulators. However, the ceramics that exhibit the characteristics of superconductivity are compounds that include copper, oxygen, and rare earth elements such as yttrium, lanthanum, and thallium. There are also indicators that the current compounds may be limited to a maximum temperature of 200 K (about 100 K short of room temperature), leaving the door wide open to innovative approaches to compound selection. The temperature at which a superconductor reverts back to the characteristics of a conventional conductor is called the *critical temperature,* denoted by T_c. Note in Fig. 3.16 that the resistivity level changes abruptly at T_c. The sharpness of the transition region is a function of the purity of the sample. Long listings of critical temperatures for a variety of tested compounds can be found in reference materials providing tables of a wide variety to support research in physics, chemistry, geology, and related fields. Two such publications include the CRC (The Chemical Rubber Co.) *Handbook of Tables for Applied Engineering Science* and the CRC *Handbook of Chemistry and Physics.*
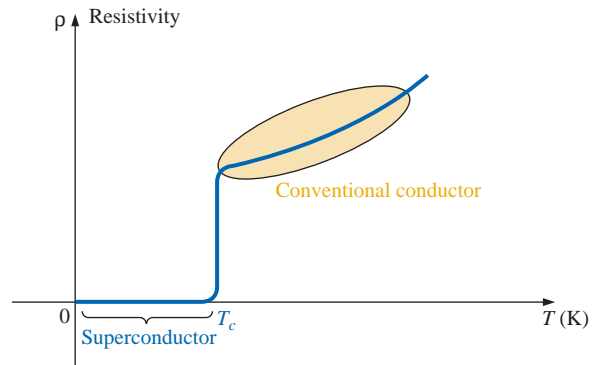
FIG. 3.16

Defining the critical temperature T_c.

Even though ceramic compounds have established higher transition temperatures, there is concern about their brittleness and current density limitations. In the area of integrated circuit manufacturing, current density levels must equal or exceed 1 MA/cm^2, or 1 million amperes through a cross-sectional area about one-half the size of a dime. Recently IBM attained a level of 4 MA/cm^2 at 77 K, permitting the use of superconductors in the design of some new-generation, high-speed computers.

Although room-temperature success has not been attained, there are numerous applications for some of the superconductors developed thus far. It is simply a matter of balancing the additional cost against the results obtained or deciding whether any results at all can be obtained without the use of this zero-resistance state. Some research efforts require high-energy accelerators or strong magnets attainable only with superconductive materials. Superconductivity is currently applied in the design of 300-mi/h Meglev trains (trains that ride on a cushion of air established by opposite magnetic poles), in powerful motors and generators, in nuclear magnetic resonance imaging systems to obtain cross-sectional images of the brain (and other parts of the body), in the design of computers with operating speeds four times that of conventional systems, and in improved power distribution systems.

The range of future uses for superconductors is a function of how much success physicists have in raising the operating temperature and how well they can utilize the successes obtained thus far. However, it would appear that it is only a matter of time (the eternal optimist) before magnetically levitated trains increase in number, improved medical diagnostic equipment is available, computers operate at much higher speeds, high-efficiency power and storage systems are available, and transmission systems operate at very high efficiency levels due to this area of developing interest. Only time will reveal the impact that this new direction will have on the quality of life.

3.7   TYPES OF RESISTORS

Fixed Resistors

Resistors are made in many forms, but all belong in either of two groups: fixed or variable. The most common of the low-wattage, fixed-

type resistors is the molded carbon composition resistor. The basic construction is shown in Fig. 3.17.

The relative sizes of all fixed and variable resistors change with the wattage (power) rating, increasing in size for increased wattage ratings in order to withstand the higher currents and dissipation losses. The relative sizes of the molded composition resistors for different wattage ratings are shown in Fig. 3.18. Resistors of this type are readily available in values ranging from 2.7 Ω to 22 MΩ.

The temperature-versus-resistance curves for a 10,000-Ω and 0.5-MΩ composition-type resistor are shown in Fig. 3.19. Note the small percent resistance change in the normal temperature operating range. Several other types of fixed resistors using high-resistance wire or metal films are shown in Fig. 3.20.

The miniaturization of parts—used quite extensively in computers—requires that resistances of different values be placed in very small packages. Some examples appear in Fig. 3.21.

For use with printed circuit boards, fixed resistor networks in a variety of configurations are available in miniature packages, such as those shown in Fig. 3.22. The figure includes a photograph of three different casings and the internal resistor configuration for the single in-line structure to the right.

FIG. 3.17
Fixed composition resistor.
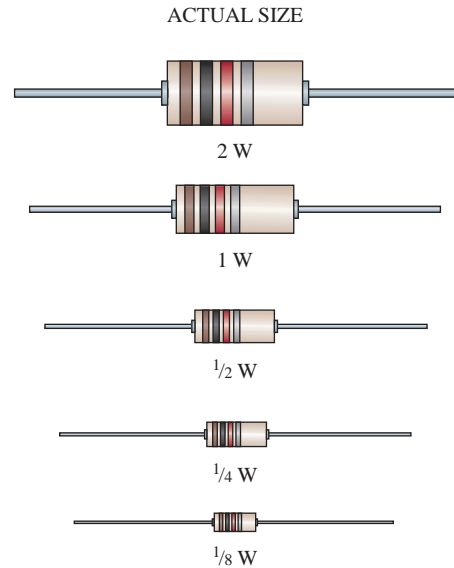
FIG. 3.18
Fixed composition resistors of different wattage ratings.

Variable Resistors

Variable resistors, as the name implies, have a terminal resistance that can be varied by turning a dial, knob, screw, or whatever seems appropriate for the application. They can have two or three terminals, but most have three terminals. If the two- or three-terminal device is used as a variable resistor, it is usually referred to as a **rheostat.** If the three-terminal device is used for controlling potential levels, it is then commonly called a **potentiometer.** Even though a three-terminal device can be used as a rheostat or potentiometer (depending on how it is connected), it is typically called a *potentiometer* when listed in trade magazines or requested for a particular application.

FIG. 3.19
Curves showing percentage temporary resistance changes from +20°C values.
(Courtesy of Allen-Bradley Co.)

(a) Vitreous-enameled wire-wound resistor
    App: All types of equipment

(b) High-voltage cermet film resistors (on a high grade ceramic body).
    App: For high-voltage applications up to 10 kV requiring high levels of stability.

(c) Metal-film precision resistors
    App: Where high stability, low temperature coefficient, and low noise level desired
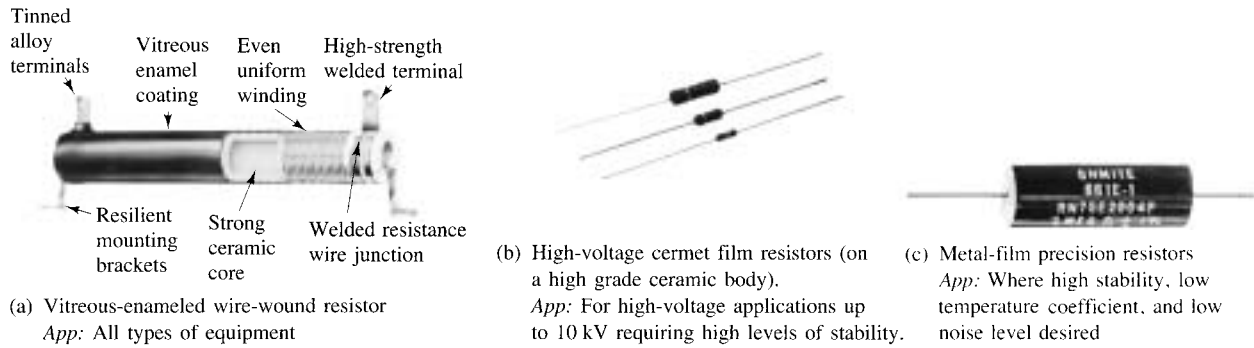
FIG. 3.20
Fixed resistors. [Parts (a) and (c) courtesy of Ohmite Manufacturing Co. Part (b) courtesy of Philips Components Inc.]

(a) Surface mount power resistor ideal for printed circuit boards. Patented J-bends eliminate need for solder connections. (0.8 W to 3 W in wire-wound, film, or power film construction)

(b) Precision power wire-wound resistors with ratings as high as 2 W and tolerances as low as 0.05%. Temperature coefficients as low as 20 ppm/°C are also available.

(c) Thick-film chip resistors for design flexibility with hybrid circuitry. Pre-tinned, gold or silver electrodes available. Operating temperature range −55°C to +150°C.

FIG. 3.21
Miniature fixed resistors. [Part (a) courtesy of Ohmite Manufacturing Co. Parts (b) and (c) courtesy of Dale Electronics, Inc.]

FIG. 3.22
Thick-film resistor networks. (Courtesy of Dale Electronics, Inc.)

The symbol for a three-terminal potentiometer appears in Fig. 3.23(a). When used as a variable resistor (or rheostat), it can be hooked up in one of two ways, as shown in Fig. 3.23(b) and (c). In Fig. 3.23(b), points *a* and *b* are hooked up to the circuit, and the remaining terminal is left hanging. The resistance introduced is determined by that portion of the resistive element between points *a* and *b*. In Fig. 3.23(c), the resistance is again between points *a* and *b,* but now the remaining resistance is "shorted-out" (effect removed) by the connection from *b* to *c.* The universally accepted symbol for a rheostat appears in Fig. 3.23(d).

Most potentiometers have three terminals in the relative positions shown in Fig. 3.24. The knob, dial, or screw in the center of the housing controls the motion of a contact that can move along the resistive element connected between the outer two terminals. The contact is connected to the center terminal, establishing a resistance from movable contact to each outer terminal.
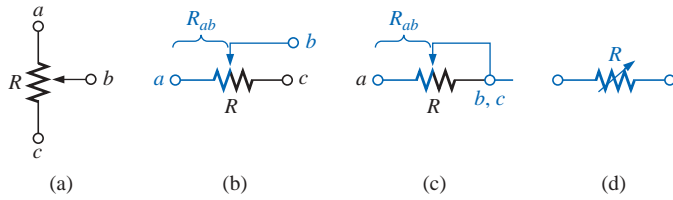
FIG. 3.23

Potentiometer: (a) symbol; (b) and (c) rheostat connections; (d) rheostat symbol.

(a) External view

(b) Internal view
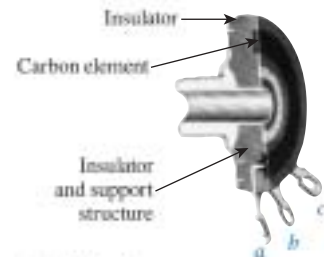
(c) Carbon element

FIG. 3.24

Molded composition–type potentiometer. (Courtesy of Allen-Bradley Co.)

The resistance between the outside terminals a and c of Fig. 3.25(a) (and Fig. 3.24) is always fixed at the full rated value of the potentiometer, regardless of the position of the wiper arm b.

In other words, the resistance between terminals *a* and *c* of Fig. 3.25(a) for a 1-MΩ potentiometer will always be 1 MΩ, no matter how we turn the control element and move the contact. In Fig. 3.25(a) the center contact is not part of the network configuration.

The resistance between the wiper arm and either outside terminal can be varied from a minimum of 0 Ω to a maximum value equal to the full rated value of the potentiometer.

In Fig. 3.25(b) the wiper arm has been placed 1/4 of the way down from point *a* to point *c*. The resulting resistance between points *a* and *b* will therefore be 1/4 of the total, or 250 kΩ (for a 1-MΩ potentiometer), and the resistance between *b* and *c* will be 3/4 of the total, or 750 kΩ.

The sum of the resistances between the wiper arm and each outside terminal will equal the full rated resistance of the potentiometer.

This was demonstrated by Fig. 3.25(b), where 250 kΩ + 750 kΩ = 1 MΩ. Specifically:

$$R_{ac} = R_{ab} + R_{bc}$$ **(3.11)**

FIG. 3.25

Terminal resistance of a potentiometer: (a) between outside terminals; (b) among all three terminals.

Therefore, as the resistance from the wiper arm to one outside contact increases, the resistance between the wiper arm and the other outside terminal must decrease accordingly. For example, if R_{ab} of a 1-kΩ potentiometer is 200 Ω, then the resistance R_{bc} must be 800 Ω. If R_{ab} is further decreased to 50 Ω, then R_{bc} must increase to 950 Ω, and so on.

The molded carbon composition potentiometer is typically applied in networks with smaller power demands, and it ranges in size from 20 Ω to 22 MΩ (maximum values). Other commercially available potentiometers appear in Fig. 3.26.

(a)

(b)
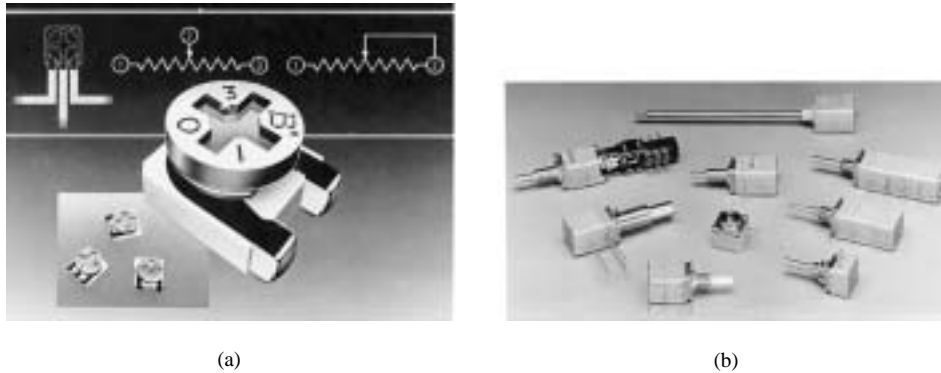
FIG. 3.26

Potentiometers: (a) 4-mm ($\approx$5/32″) trimmer (courtesy of Bourns, Inc.);
(b) conductive plastic and cermet element (courtesy of Clarostat Mfg. Co.).
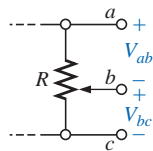
FIG. 3.27

Potentiometer control of voltage levels.

When the device is used as a potentiometer, the connections are as shown in Fig. 3.27. It can be used to control the level of V_{ab}, V_{bc}, or both, depending on the application. Additional discussion of the potentiometer in a loaded situation can be found in the chapters that follow.

3.8 COLOR CODING AND STANDARD RESISTOR VALUES

A wide variety of resistors, fixed or variable, are large enough to have their resistance in ohms printed on the casing. Some, however, are too small to have numbers printed on them, so a system of **color coding** is used. For the fixed molded composition resistor, four or five color bands are printed on one end of the outer casing, as shown in Fig. 3.28. Each color has the numerical value indicated in Table 3.7. The color bands are always read from the end that has the band closest to it, as shown in Fig. 3.28. The first and second bands represent the first and second digits, respectively. The third band determines the power-of-ten multiplier for the first two digits (actually the number of zeros that follow the second digit) or a multiplying factor if gold or silver. The fourth band is the manufacturer's tolerance, which is an indication of the precision by which the resistor was made. If the fourth band is omitted, the tolerance is assumed to be $\pm$20%. The fifth band is a reliability factor, which gives the percentage of failure per 1000 hours of use. For instance,
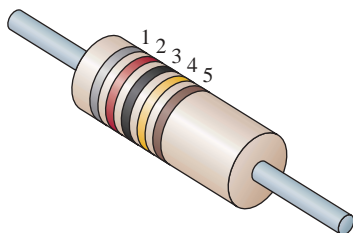
FIG. 3.28

Color coding of fixed molded composition resistor.

TABLE 3.7
Resistor color coding.

Bands 1–3*	Band 3	Band 4	Band 5
0 Black	0.1 Gold ⎱ multiplying	5% Gold	1% Brown
1 Brown	0.01 Silver ⎰ factors	10% Silver	0.1% Red
2 Red		20% No band	0.01% Orange
3 Orange			0.001% Yellow
4 Yellow			
5 Green			
6 Blue			
7 Violet			
8 Gray			
9 White			

*With the exception that black is not a valid color for the first band.

a 1% failure rate would reveal that one out of every 100 (or 10 out of every 1000) will fail to fall within the tolerance range after 1000 hours of use.

EXAMPLE 3.13 Find the range in which a resistor having the following color bands must exist to satisfy the manufacturer's tolerance:

a.

1st band	2nd band	3rd band	4th band	5th band
Gray	Red	Black	Gold	Brown
8	2	0	±5%	1%

b.

1st band	2nd band	3rd band	4th band	5th band
Orange	White	Gold	Silver	No color
3	9	0.1	±10%	

Solutions:

a. **82 Ω ± 5% (1% reliability)**

   Since 5% of 82 = 4.10, the resistor should be within the range 82 Ω ± 4.10 Ω, or *between 77.90 and 86.10 Ω*.

b. **3.9 Ω ± 10% = 3.9 ± 0.39 Ω**

   The resistor should lie somewhere *between 3.51 and 4.29 Ω*.

One might expect that resistors would be available for a full range of values such as 10 Ω, 20 Ω, 30 Ω, 40 Ω, 50 Ω, and so on. However, this is not the case with some typical commercial values, such as 27 Ω, 56 Ω, and 68 Ω. This may seem somewhat strange and out of place. There is a reason for the chosen values, which is best demonstrated by examining the list of standard values of commercially available resistors in Table 3.8. The values in boldface blue are available with 5%, 10%, and 20% tolerances, making them the most common of the commercial variety. The values in boldface black are typically available with 5% and 10% tolerances, and those in normal print are available only in the 5% variety. If we separate the values available into tolerance levels, we have Table 3.9, which clearly reveals how few are available up to 100 Ω with 20% tolerances.

An examination of the impact of the tolerance level will now help explain the choice of numbers for the commercial values. Take the

TABLE 3.8
Standard values of commercially available resistors.

Ohms (Ω)					Kilohms (kΩ)		Megohms (MΩ)	
0.10	1.0	10	100	1000	10	100	1.0	10.0
0.11	1.1	11	110	1100	11	110	1.1	11.0
0.12	1.2	12	120	1200	12	120	1.2	12.0
0.13	1.3	13	130	1300	13	130	1.3	13.0
0.15	1.5	15	150	1500	15	150	1.5	15.0
0.16	1.6	16	160	1600	16	160	1.6	16.0
0.18	1.8	18	180	1800	18	180	1.8	18.0
0.20	2.0	20	200	2000	20	200	2.0	20.0
0.22	2.2	22	220	2200	22	220	2.2	22.0
0.24	2.4	24	240	2400	24	240	2.4	
0.27	2.7	27	270	2700	27	270	2.7	
0.30	3.0	30	300	3000	30	300	3.0	
0.33	3.3	33	330	3300	33	330	3.3	
0.36	3.6	36	360	3600	36	360	3.6	
0.39	3.9	39	390	3900	39	390	3.9	
0.43	4.3	43	430	4300	43	430	4.3	
0.47	4.7	47	470	4700	47	470	4.7	
0.51	5.1	51	510	5100	51	510	5.1	
0.56	5.6	56	560	5600	56	560	5.6	
0.62	6.2	62	620	6200	62	620	6.2	
0.68	6.8	68	680	6800	68	680	6.8	
0.75	7.5	75	750	7500	75	750	7.5	
0.82	8.2	82	820	8200	82	820	8.2	
0.91	9.1	91	910	9100	91	910	9.1	

TABLE 3.9
Standard values and their tolerances.

±5%	±10%	±20%
10	10	10
11		
12	12	
13		
15	15	15
16		
18	18	
20		
22	22	22
24		
27	27	
30		
33	33	33
36		
39	39	
43		
47	47	47
51		
56	56	
62		
68	68	68
75		
82	82	
91		

sequence 47 Ω–68 Ω–100 Ω, which are all available with 20% tolerances. In Fig. 3.29(a), the tolerance band for each has been determined and plotted on a single axis. Take note that, with this tolerance (which is all that the manufacturer will guarantee), the full range of resistor values is available from 37.6 Ω to 120 Ω. In other words, the manufacturer is guaranteeing the full range, using the tolerances to fill in the
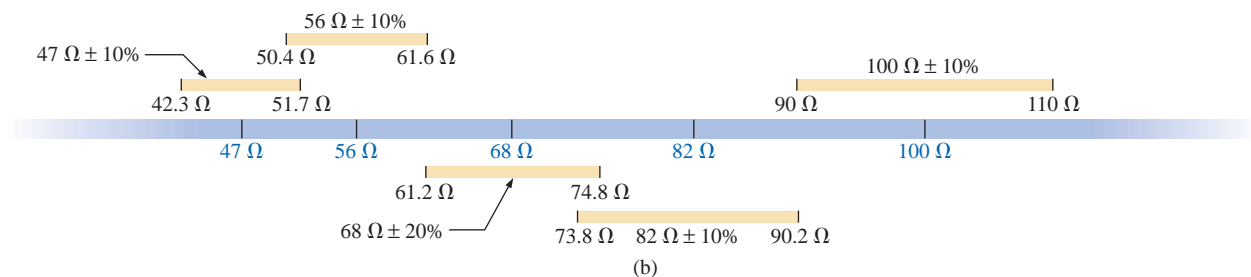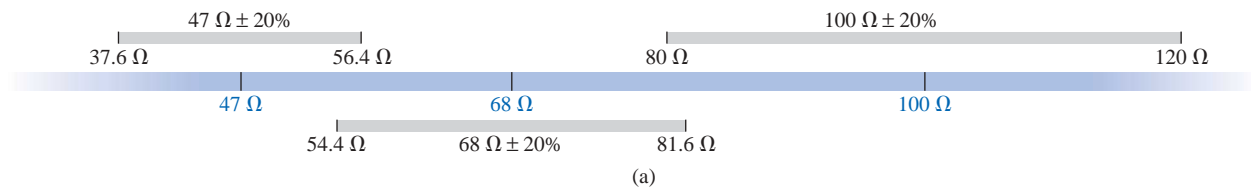
FIG. 3.29
Guaranteeing the full range of resistor values for the given tolerance: (a) 20%; (b) 10%.

gaps. Dropping to the 10% level introduces the 56-Ω and 82-Ω resistors to fill in the gaps, as shown in Fig. 3.29(b). Dropping to the 5% level would require additional resistor values to fill in the gaps. In total, therefore, the resistor values were chosen to ensure that the full range was covered, as determined by the tolerances employed. Of course, if a specific value is desired but is not one of the standard values, combinations of standard values will often result in a total resistance very close to the desired level. If this approach is still not satisfactory, a potentiometer can be set to the exact value and then inserted in the network.

Throughout the text you will find that many of the resistor values are not standard values. This was done to reduce the mathematical complexity, which might deter from or cloud the procedure or analysis technique being introduced. In the problem sections, however, standard values are frequently employed to ensure that the reader starts to become familiar with the commercial values available.

3.9 CONDUCTANCE

By finding the reciprocal of the resistance of a material, we have a measure of how well the material will conduct electricity. The quantity is called **conductance,** has the symbol *G,* and is measured in *siemens* (S) (note Fig. 3.30). In equation form, conductance is

$$G = \frac{1}{R} \qquad \text{(siemens, S)} \qquad \textbf{(3.12)}$$

A resistance of 1 MΩ is equivalent to a conductance of 10^{-6} S, and a resistance of 10 Ω is equivalent to a conductance of 10^{-1} S. The larger the conductance, therefore, the less the resistance and the greater the conductivity.

In equation form, the conductance is determined by

$$G = \frac{A}{\rho l} \qquad \text{(S)} \qquad \textbf{(3.13)}$$

indicating that increasing the area or decreasing either the length or the resistivity will increase the conductance.

EXAMPLE 3.14 What is the relative increase or decrease in conductivity of a conductor if the area is reduced by 30% and the length is increased by 40%? The resistivity is fixed.

Solution: Eq. (3.11):

$$G_i = \frac{A_i}{\rho_i l_i}$$

with the subscript i for the initial value. Using the subscript n for new value:

$$G_n = \frac{A_n}{\rho_n l_n} = \frac{0.70 A_i}{\rho_i(1.4 l_i)} = \frac{0.70}{1.4} \frac{A_i}{\rho_i l_i} = \frac{0.70}{1.4 G_i}$$

and $\qquad G_n = \textbf{0.5}\textbf{G}_i$

German (Lenthe, Berlin)
(1816–92)
Electrical Engineer
Telegraph Manufacturer,
Siemens & Halske AG

Bettman Archives
Photo Number 336.19

Developed an *electroplating process* during a brief stay in prison for acting as a second in a duel between fellow officers of the Prussian army. Inspired by the electronic telegraph invented by Sir Charles Wheatstone in 1817, he improved on the design and proceeded to lay cable with the help of his brother Carl across the Mediterranean and from Europe to India. His inventions included the first *self-excited generator,* which depended on the *residual* magnetism of its electronmagnet rather than an inefficient permanent magnet. In 1888 he was raised to the rank of nobility with the addition of *von* to his name. The current firm of Siemens AG has manufacturing outlets in some 35 countries with sales offices in some 125 countries.

FIG. 3.30
Werner von Siemens.

3.10 OHMMETERS

The **ohmmeter** is an instrument used to perform the following tasks and several other useful functions:

1. Measure the resistance of individual or combined elements
2. Detect open-circuit (high-resistance) and short-circuit (low-resistance) situations
3. Check continuity of network connections and identify wires of a multilead cable
4. Test some semiconductor (electronic) devices
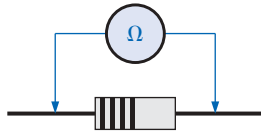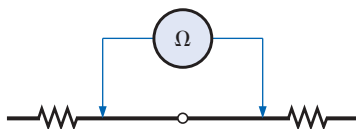
For most applications, the ohmmeters used most frequently are the ohmmeter section of a VOM or DMM. The details of the internal circuitry and the method of using the meter will be left primarily for a laboratory exercise. In general, however, the resistance of a resistor can be measured by simply connecting the two leads of the meter across the resistor, as shown in Fig. 3.31. There is no need to be concerned about which lead goes on which end; the result will be the same in either case since resistors offer the same resistance to the flow of charge (current) in either direction. If the VOM is employed, a switch must be set to the proper resistance range, and a nonlinear scale (usually the top scale of the meter) must be properly read to obtain the resistance value. The DMM also requires choosing the best scale setting for the resistance to be measured, but the result appears as a numerical display, with the proper placement of the decimal point as determined by the chosen scale. When measuring the resistance of a single resistor, it is usually best to remove the resistor from the network before making the measurement. If this is difficult or impossible, at least one end of the resistor must not be connected to the network, or the reading may include the effects of the other elements of the system.

If the two leads of the meter are touching in the ohmmeter mode, the resulting resistance is zero. A connection can be checked as shown in Fig. 3.32 by simply hooking up the meter to either side of the connection. If the resistance is zero, the connection is secure. If it is other than zero, the connection could be weak, and, if it is infinite, there is no connection at all.

If one wire of a harness is known, a second can be found as shown in Fig. 3.33. Simply connect the end of the known lead to the end of any other lead. When the ohmmeter indicates zero ohms (or very low resistance), the second lead has been identified. The above procedure can also be used to determine the first known lead by simply connecting the meter to any wire at one end and then touching all the leads at the other end until a zero-ohm indication is obtained.

FIG. 3.31

Measuring the resistance of a single element.

FIG. 3.32

Checking the continuity of a connection.
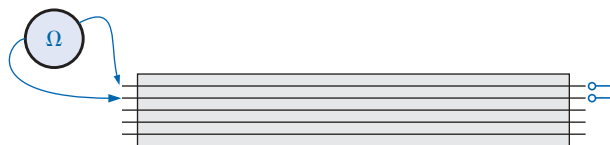
FIG. 3.33

Identifying the leads of a multilead cable.

Preliminary measurements of the condition of some electronic devices such as the diode and transistor can be made using the ohmmeter. The meter can also be used to identify the terminals of such devices.

One important note about the use of any ohmmeter:

Never hook up an ohmmeter to a live circuit!

The reading will be meaningless and you may damage the instrument. The ohmmeter section of any meter is designed to pass a small sensing current through the resistance to be measured. A large external current could damage the movement and would certainly throw off the calibration of the instrument. In addition,

never store a VOM or a DMM in the resistance mode.

The two leads of the meter could touch and the small sensing current could drain the internal battery. VOMs should be stored with the selector switch on the highest voltage range, and the selector switch of DMMs should be in the off position.

3.11   THERMISTORS

The **thermistor** is a two-terminal semiconductor device whose resistance, as the name suggests, is temperature sensitive. A representative characteristic appears in Fig. 3.34 with the graphic symbol for the device. Note the nonlinearity of the curve and the drop in resistance from about 5000 Ω to 100 Ω for an increase in temperature from 20°C to 100°C. The decrease in resistance with an increase in temperature indicates a negative temperature coefficient.

The temperature of the device can be changed internally or externally. An increase in current through the device will raise its temperature, causing a drop in its terminal resistance. Any externally applied heat source will result in an increase in its body temperature and a drop in resistance. This type of action (internal or external) lends itself well to control mechanisms. Many different types of thermistors are shown in Fig. 3.35. Materials employed in the manufacture of thermistors include oxides of cobalt, nickel, strontium, and manganese.
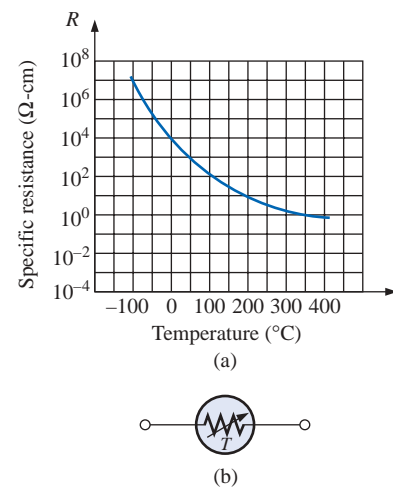
FIG. 3.34
Thermistor: (a) characteristics; (b) symbol.

FIG. 3.35
NTC (negative temperature coefficient) and PTC (positive temperature coefficient) thermistors. (Courtesy of Siemens Components, Inc.)

equation for the resistance R is defined in terms of the variables, and the result is obtained. The true value of developing in the above sequence is the fact that you can place the program in memory and, when the need arises, call it up and change a variable or two—the result will appear immediately. There is no need to reenter all the definitions—just change the numerical value.

In the chapters to follow, Mathcad will appear at every opportunity to demonstrate its ability to perform calculations in a quick, effective manner. You will probably want to learn more about this time-saving and accuracy-checking option.

PROBLEMS

SECTION 3.2   Resistance: Circular Wires

1. Convert the following to mils:
   a. 0.5 in.   **b.** 0.01 in.
   c. 0.004 in.   **d.** 1 in.
   e. 0.02 ft   **f.** 0.01 cm

2. Calculate the area in circular mils (CM) of wires having the following diameters:
   a. 0.050 in.   **b.** 0.016 in.
   c. 0.30 in.   **d.** 0.1 cm
   e. 0.003 ft   **f.** 0.0042 m

3. The area in circular mils is
   a. 1600 CM   **b.** 900 CM
   c. 40,000 CM   **d.** 625 CM
   e. 7.75 CM   **f.** 81 CM
   What is the diameter of each wire in inches?

4. What is the resistance of a copper wire 200 ft long and 0.01 in. in diameter ($T = 20°C$)?

5. Find the resistance of a silver wire 50 yd long and 0.0045 in. in diameter ($T = 20°C$).

6. a. What is the area in circular mils of an aluminum conductor that is 80 ft long with a resistance of 2.5 Ω?
   b. What is its diameter in inches?

7. A 2.2-Ω resistor is to be made of nichrome wire. If the available wire is 1/32 in. in diameter, how much wire is required?

8. a. What is the area in circular mils of a copper wire that has a resistance of 2.5 Ω and is 300 ft long ($T = 20°C$)?
   b. Without working out the numerical solution, determine whether the area of an aluminum wire will be smaller or larger than that of the copper wire. Explain.
   c. Repeat (b) for a silver wire.

9. In Fig. 3.45, three conductors of different materials are presented.
   a. Without working out the numerical solution, determine which section would appear to have the most resistance. Explain.
   b. Find the resistance of each section and compare with the result of (a) ($T = 20°C$).

Silver: $l = 1$ ft, $d = 1$ mil

Copper: $l = 10$ ft, $d = 10$ mils

Aluminum: $l = 50$ ft, $d = 50$ mils
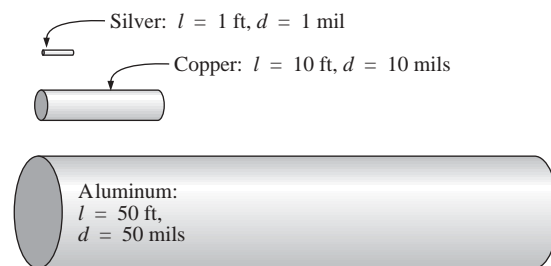
FIG. 3.45
Problem 9.

10. A wire 1000 ft long has a resistance of 0.5 kΩ and an area of 94 CM. Of what material is the wire made ($T = 20°C$)?

*11. a. What is the resistance of a copper bus-bar with the dimensions shown ($T = 20°C$) in Fig. 3.46?
   b. Repeat (a) for aluminum and compare the results.
   c. Without working out the numerical solution, determine whether the resistance of the bar (aluminum or copper) will increase or decrease with an increase in length. Explain your answer.
   d. Repeat (c) for an increase in cross-sectional area.

$^1/_2$ in.

3 in.

4 ft
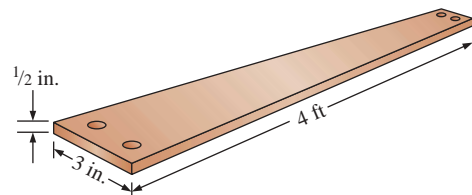
FIG. 3.46
Problem 11.

12. Determine the increase in resistance of a copper conductor if the area is reduced by a factor of 4 and the length is doubled. The original resistance was 0.2 Ω. The temperature remains fixed.

*13. What is the new resistance level of a copper wire if the length is changed from 200 ft to 100 yd, the area is changed from 40,000 CM to 0.04 in.^{2}, and the original resistance was 800 mΩ?

SECTION 3.3   Wire Tables

14. a. Using Table 3.2, find the resistance of 450 ft of #11 and #14 AWG wires.
  b. Compare the resistances of the two wires.
  c. Compare the areas of the two wires.

15. a. Using Table 3.2, find the resistance of 1800 ft of #8 and #18 AWG wires.
  b. Compare the resistances of the two wires.
  c. Compare the areas of the two wires.

16. a. For the system of Fig. 3.47, the resistance of each line cannot exceed 0.006 Ω, and the maximum current drawn by the load is 110 A. What gage wire should be used?
  b. Repeat (a) for a maximum resistance of 0.003 Ω, $d = 30$ ft, and a maximum current of 110 A.
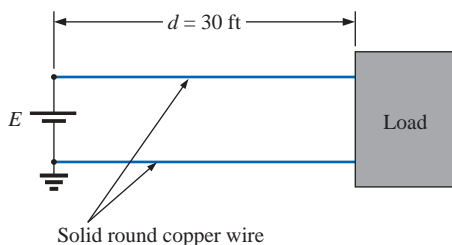
FIG. 3.47
Problem 16.

*17. a. From Table 3.2, determine the maximum permissible current density (A/CM) for an AWG #0000 wire.
  b. Convert the result of (a) to A/in.^{2}.
  c. Using the result of (b), determine the cross-sectional area required to carry a current of 5000 A.

SECTION 3.4   Resistance: Metric Units

18. Using metric units, determine the length of a copper wire that has a resistance of 0.2 Ω and a diameter of 1/10 in.

19. Repeat Problem 11 using metric units; that is, convert the given dimensions to metric units before determining the resistance.

20. If the sheet resistance of a tin oxide sample is 100 Ω, what is the thickness of the oxide layer?

21. Determine the width of a carbon resistor having a sheet resistance of 150 Ω if the length is 1/2 in. and the resistance is 500 Ω.

*22. Derive the conversion factor between ρ (CM·Ω/ft) and ρ (Ω·cm) by
  a. Solving for ρ for the wire of Fig. 3.48 in CM·Ω/ft.
  b. Solving for ρ for the same wire of Fig. 3.48 in Ω·cm by making the necessary conversions.

  c. Use the equation $\rho_2 = k\rho_1$ to determine the conversion factor k if ρ_1 is the solution of part (a) and ρ_2 the solution of part (b).
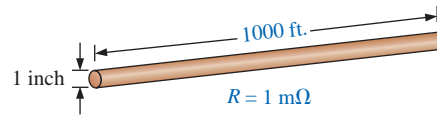
FIG. 3.48
Problem 22.

SECTION 3.5   Temperature Effects

23. The resistance of a copper wire is 2 Ω at 10°C. What is its resistance at 60°C?

24. The resistance of an aluminum bus-bar is 0.02 Ω at 0°C. What is its resistance at 100°C?

25. The resistance of a copper wire is 4 Ω at 70°F. What is its resistance at 32°F?

26. The resistance of a copper wire is 0.76 Ω at 30°C. What is its resistance at −40°C?

27. If the resistance of a silver wire is 0.04 Ω at −30°C, what is its resistance at 0°C?

*28. a. The resistance of a copper wire is 0.002 Ω at room temperature (68°F). What is its resistance at 32°F (freezing) and 212°F (boiling)?
  b. For (a), determine the change in resistance for each 10° change in temperature between room temperature and 212°F.

29. a. The resistance of a copper wire is 0.92 Ω at 4°C. At what temperature (°C) will it be 1.06 Ω?
  b. At what temperature will it be 0.15 Ω?

*30. a. If the resistance of a 1000-ft length of copper wire is 10 Ω at room temperature (20°C), what will its resistance be at 50 K (Kelvin units) using Eq. (3.6)?
  b. Repeat part (a) for a temperature of 38.65 K. Comment on the results obtained by reviewing the curve of Fig. 3.14.
  c. What is the temperature of absolute zero in Fahrenheit units?

31. a. Verify the value of α_{20} for copper in Table 3.6 by substituting the inferred absolute temperature into Eq. (3.7).
  b. Using Eq. (3.8) find the temperature at which the resistance of a copper conductor will increase to 1 Ω from a level of 0.8 Ω at 20°C.

32. Using Eq. (3.8), find the resistance of a copper wire at 16°C if its resistance at 20°C is 0.4 Ω.

*33. Determine the resistance of a 1000-ft coil of #12 copper wire sitting in the desert at a temperature of 115°F.

34. A 22-Ω wire-wound resistor is rated at +200 PPM for a temperature range of −10°C to +75°C. Determine its resistance at 65°C.

35. Determine the PPM rating of the 10-kΩ carbon composition resistor of Fig. 3.19 using the resistance level determined at 90°C.

SECTION 3.6 Superconductors

36. Visit your local library and find a table listing the critical temperatures for a variety of materials. List at least five materials with the critical temperatures that are not mentioned in this text. Choose a few materials that have relatively high critical temperatures.

37. Find at least one article on the application of superconductivity in the commercial sector, and write a short summary, including all interesting facts and figures.

*38. Using the required 1-MA/cm^2 density level for IC manufacturing, determine what the resulting current would be through a #12 house wire. Compare the result obtained with the allowable limit of Table 3.2.

*39. Research the SQUID magnetic field detector and review its basic mode of operation and an application or two.

SECTION 3.7 Types of Resistors

40. **a.** What is the approximate increase in size from a 1-W to a 2-W carbon resistor?
    b. What is the approximate increase in size from a 1/2-W to a 2-W carbon resistor?
    c. In general, can we conclude that for the same type of resistor, an increase in wattage rating requires an increase in size (volume)? Is it almost a linear relationship? That is, does twice the wattage require an increase in size of 2:1?

41. If the 10-kΩ resistor of Fig. 3.19 is exactly 10 kΩ at room temperature, what is its approximate resistance at −30°C and 100°C (boiling)?

42. Repeat Problem 41 at a temperature of 120°F.

43. If the resistance between the outside terminals of a linear potentiometer is 10 kΩ, what is its resistance between the wiper (movable) arm and an outside terminal if the resistance between the wiper arm and the other outside terminal is 3.5 kΩ?

44. If the wiper arm of a linear potentiometer is one-quarter the way around the contact surface, what is the resistance between the wiper arm and each terminal if the total resistance is 25 kΩ?

*45. Show the connections required to establish 4 kΩ between the wiper arm and one outside terminal of a 10-kΩ potentiometer while having only zero ohms between the other outside terminal and the wiper arm.

SECTION 3.8 Color Coding and Standard Resistor Values

46. Find the range in which a resistor having the following color bands must exist to satisfy the manufacturer's tolerance:

	1st band	2nd band	3rd band	4th band
a.	green	blue	orange	gold
b.	red	red	brown	silver
c.	brown	black	black	—

47. Find the color code for the following 10% resistors:
    a. 220 Ω   **b.** 4700 Ω
    c. 68 kΩ   **d.** 9.1 MΩ

48. Is there an overlap in coverage between 20% resistors? That is, determine the tolerance range for a 10-Ω 20% resistor and a 15-Ω 20% resistor, and note whether their tolerance ranges overlap.

49. Repeat Problem 48 for 10% resistors of the same value.

SECTION 3.9 Conductance

50. Find the conductance of each of the following resistances:
    a. 0.086 Ω   **b.** 4 kΩ
    c. 2.2 MΩ
    Compare the three results.

51. Find the conductance of 1000 ft of #18 AWG wire made of
    a. copper
    b. aluminum
    c. iron

*52. The conductance of a wire is 100 S. If the area of the wire is increased by 2/3 and the length is reduced by the same amount, find the new conductance of the wire if the temperature remains fixed.

SECTION 3.10 Ohmmeters

53. How would you check the status of a fuse with an ohmmeter?

54. How would you determine the on and off states of a switch using an ohmmeter?

55. How would you use an ohmmeter to check the status of a light bulb?

SECTION 3.11 Thermistors

*56. **a.** Find the resistance of the thermistor having the characteristics of Fig. 3.34 at −50°C, 50°C, and 200°C. Note that it is a log scale. If necessary, consult a reference with an expanded log scale.
     b. Does the thermistor have a positive or a negative temperature coefficient?
     c. Is the coefficient a fixed value for the range −100°C to 400°C? Why?
     d. What is the approximate rate of change of ρ with temperature at 100°C?

SECTION 3.12 Photoconductive Cell

*57. **a.** Using the characteristics of Fig. 3.36, determine the resistance of the photoconductive cell at 10 and 100 foot-candles of illuminaton. As in Problem 56, note that it is a log scale.
     b. Does the cell have a positive or a negative illumination coefficient?
     c. Is the coefficient a fixed value for the range 0.1 to 1000 foot-candles? Why?
     d. What is the approximate rate of change of R with illumination at 10 foot-candles?

SECTION 3.13 Varistors

58. a. Referring to Fig. 3.38(a), find the terminal voltage of the device at 0.5 mA, 1 mA, 3 mA, and 5 mA.
  b. What is the total change in voltage for the indicated range of current levels?
  c. Compare the ratio of maximum to minimum current levels above to the corresponding ratio of voltage levels.

SECTION 3.15 Mathcad

59. Verify the results of Example 3.3 using Mathcad.

60. Verify the results of Example 3.11 using Mathcad.

GLOSSARY

Absolute zero The temperature at which all molecular motion ceases; $-273.15°C$.

Circular mil (CM) The cross-sectional area of a wire having a diameter of one mil.

Color coding A technique employing bands of color to indicate the resistance levels and tolerance of resistors.

Conductance (G) An indication of the relative ease with which current can be established in a material. It is measured in siemens (S).

Cooper effect The "pairing" of electrons as they travel through a medium.

Ductility The property of a material that allows it to be drawn into long, thin wires.

Inferred absolute temperature The temperature through which a straight-line approximation for the actual resistance-versus-temperature curve will intersect the temperature axis.

Malleability The property of a material that allows it to be worked into many different shapes.

Negative temperature coefficient of resistance The value revealing that the resistance of a material will decrease with an increase in temperature.

Ohm (Ω) The unit of measurement applied to resistance.

Ohmmeter An instrument for measuring resistance levels.

Photoconductive cell A two-terminal semiconductor device whose terminal resistance is determined by the intensity of the incident light on its exposed surface.

Positive temperature coefficient of resistance The value revealing that the resistance of a material will increase with an increase in temperature.

Potentiometer A three-terminal device through which potential levels can be varied in a linear or nonlinear manner.

PPM/°C Temperature sensitivity of a resistor in parts per million per degree Celsius.

Resistance A measure of the opposition to the flow of charge through a material.

Resistivity (ρ) A constant of proportionality between the resistance of a material and its physical dimensions.

Rheostat An element whose terminal resistance can be varied in a linear or nonlinear manner.

Sheet resistance Defined by ρ/d for thin-film and integrated circuit design.

Superconductor Conductors of electric charge that have for all practical purposes zero ohms.

Thermistor A two-terminal semiconductor device whose resistance is temperature sensitive.

Varistor A voltage-dependent, nonlinear resistor used to suppress high-voltage transients.

4
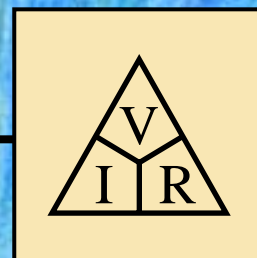
Ohm's Law, Power, and Energy

4.1   OHM'S LAW

Consider the following relationship:

$$\text{Effect} = \frac{\text{cause}}{\text{opposition}} \qquad \textbf{(4.1)}$$

Every conversion of energy from one form to another can be related to this equation. In electric circuits, the *effect* we are trying to establish is the flow of charge, or *current.* The *potential difference,* or voltage, between two points is the *cause* ("pressure"), and the opposition is the *resistance* encountered.

An excellent analogy for the simplest of electrical circuits is the water in a hose connected to a pressure valve. Think of the electrons in the copper wire as the water in the hose, the pressure valve as the applied voltage, and the size of the hose as the factor that determines the resistance. If the pressure valve is closed, the water simply sits in the hose without motion, much like the electrons in a conductor without an applied voltage. When we open the pressure valve, water will flow through the hose much like the electrons in a copper wire when the voltage is applied. In other words, the absence of the "pressure" in one case and the voltage in the other will simply result in a system without motion or reaction. The rate at which the water will flow in the hose is a function of the size of the hose. A hose with a very small diameter will limit the rate at which water can flow through the hose, just as a copper wire with a small diameter will have a high resistance and will limit the current.

In summary, therefore, the absence of an applied "pressure" such as voltage in an electric circuit will result in no reaction in the system and no current in the electric circuit. Current is a reaction to the applied voltage and not the factor that gets the system in motion. To continue with the analogy, the more the pressure at the spigot, the more the rate

German (Erlangen,
  Cologne)
(1789–1854)
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In 1827, developed one of the most important laws of electric circuits: *Ohm's law.* When the law was first introduced, the supporting documentation was considered lacking and foolish, causing him to lose his teaching position and search for a living doing odd jobs and some tutoring. It took some 22 years for his work to be recognized as a major contribution to the field. He was then awarded a chair at the University of Munich and received the Copley Medal of the Royal Society in 1841. His research also extended into the areas of molecular physics, acoustics, and telegraphic communication.

FIG. 4.1
Georg Simon Ohm.
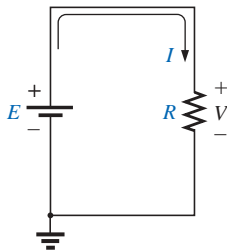
FIG. 4.2
Basic circuit.

of water flow through the hose, just as applying a higher voltage to the same circuit will result in a higher current.

Substituting the terms introduced above into Eq. (4.1) results in

$$\text{Current} = \frac{\text{potential difference}}{\text{resistance}}$$

and
$$I = \frac{E}{R}$$
(amperes, A) **(4.2)**

Equation (4.2) is known as **Ohm's law** in honor of Georg Simon Ohm (Fig. 4.1). The law clearly reveals that for a fixed resistance, the greater the voltage (or pressure) across a resistor, the more the current, and the more the resistance for the same voltage, the less the current. In other words, the current is proportional to the applied voltage and inversely proportional to the resistance.

By simple mathematical manipulations, the voltage and resistance can be found in terms of the other two quantities:

$$E = IR$$
(volts, V) **(4.3)**

and
$$R = \frac{E}{I}$$
(ohms, Ω) **(4.4)**

The three quantities of Eqs. (4.2) through (4.4) are defined by the simple circuit of Fig. 4.2. The current I of Eq. (4.2) results from applying a dc supply of E volts across a network having a resistance R. Equation (4.3) determines the voltage E required to establish a current I through a network with a total resistance R, and Equation (4.4) provides the resistance of a network that results in a current I due to an impressed voltage E.

Note in Fig. 4.2 that the voltage source "pressures" current in a direction that passes from the negative to the positive terminal of the battery. This will always be the case for single-source circuits. The effect of more than one source in the network will be examined in the chapters to follow. The symbol for the voltage of the battery (a source of electrical energy) is the uppercase letter E, whereas the loss in potential energy across the resistor is given the symbol V. The polarity of the voltage drop across the resistor is as defined by the applied source because the two terminals of the battery are connected directly across the resistive element.

EXAMPLE 4.1 Determine the current resulting from the application of a 9-V battery across a network with a resistance of 2.2 Ω.

Solution: Eq. (4.2):

$$I = \frac{E}{R} = \frac{9 \text{ V}}{2.2 \text{ Ω}} = \textbf{4.09 A}$$

EXAMPLE 4.2 Calculate the resistance of a 60-W bulb if a current of 500 mA results from an applied voltage of 120 V.

Solution: Eq. (4.4):

$$R = \frac{E}{I} = \frac{120 \text{ V}}{500 \times 10^{-3} \text{ A}} = \textbf{240 } \boldsymbol{\Omega}$$

For an isolated resistive element, the polarity of the voltage drop is as shown in Fig. 4.3(a) for the indicated current direction. A reversal in current will reverse the polarity, as shown in Fig. 4.3(b). In general, the flow of charge is from a high (+) to a low (−) potential. Polarities as established by current direction will become increasingly important in the analysis to follow.
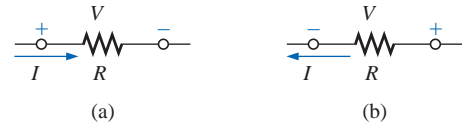
FIG. 4.3
Defining polarities.

EXAMPLE 4.3 Calculate the current through the 2-kΩ resistor of Fig. 4.4 if the voltage drop across it is 16 V.

Solution:

$$I = \frac{V}{R} = \frac{16 \text{ V}}{2 \times 10^3 \text{ } \Omega} = \textbf{8 mA}$$
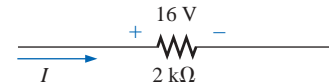
FIG. 4.4
Example 4.3.

EXAMPLE 4.4 Calculate the voltage that must be applied across the soldering iron of Fig. 4.5 to establish a current of 1.5 A through the iron if its internal resistance is 80 Ω.

Solution:

$$E = IR = (1.5 \text{ A})(80 \text{ } \Omega) = \textbf{120 V}$$
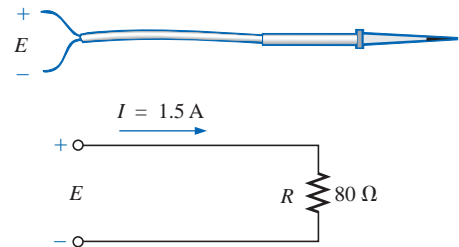
FIG. 4.5
Example 4.4.

In a number of the examples in this chapter, such as Example 4.4 above, the voltage applied is actually that obtained from an ac outlet in the home, office, or laboratory. This approach was used to provide an opportunity for the student to relate to real-world situations as soon as possible and to demonstrate that a number of the equations derived in this chapter are applicable to ac networks also. Chapter 13 will provide a direct relationship between ac and dc voltages that permits the mathematical substitutions used in this chapter. In other words, don't be concerned about the fact that some of the voltages and currents appearing in the examples of this chapter are actually ac voltages, because the equations for dc networks have exactly the same format, and all the solutions will be correct.

4.2 PLOTTING OHM'S LAW

Graphs, characteristics, plots, and the like, play an important role in every technical field as a mode through which the broad picture of the behavior or response of a system can be conveniently displayed. It is therefore critical to develop the skills necessary both to read data and to plot them in such a manner that they can be interpreted easily.

For most sets of characteristics of electronic devices, the current is represented by the vertical axis (ordinate), and the voltage by the horizontal axis (abscissa), as shown in Fig. 4.6. First note that the vertical axis is in
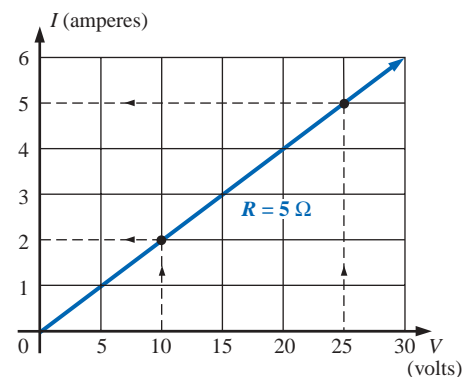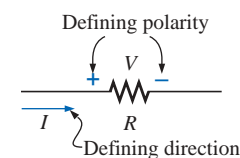
FIG. 4.6
Plotting Ohm's law.

amperes and the horizontal axis is in volts. For some plots, *I* may be in milliamperes (mA), microamperes (μA), or whatever is appropriate for the range of interest. The same is true for the levels of voltage on the horizontal axis. Note also that the chosen parameters require that the spacing between numerical values of the vertical axis be different from that of the horizontal axis. The linear (straight-line) graph reveals that the resistance is not changing with current or voltage level; rather, it is a fixed quantity throughout. The current direction and the voltage polarity appearing at the top of Fig. 4.6 are the defined direction and polarity for the provided plot. If the current direction is opposite to the defined direction, the region below the horizontal axis is the region of interest for the current *I*. If the voltage polarity is opposite to that defined, the region to the left of the current axis is the region of interest. For the standard fixed resistor, the first quadrant, or region, of Fig. 4.6 is the only region of interest. However, you will encounter many devices in your electronics courses that will use the other quadrants of a graph.

Once a graph such as Fig. 4.6 is developed, the current or voltage at any level can be found from the other quantity by simply using the resulting plot. For instance, at *V* = 25 V, if a vertical line is drawn on Fig. 4.6 to the curve as shown, the resulting current can be found by drawing a horizontal line over to the current axis, where a result of 5 A is obtained. Similarly, at *V* = 10 V, a vertical line to the plot and a horizontal line to the current axis will result in a current of 2 A, as determined by Ohm's law.

If the resistance of a plot is unknown, it can be determined at any point on the plot since a straight line indicates a fixed resistance. At any point on the plot, find the resulting current and voltage, and simply substitute into the following equation:

$$R_{dc} = \frac{V}{I} \qquad (4.5)$$

To test Eq. (4.5), consider a point on the plot where *V* = 20 V and *I* = 4 A. The resulting resistance is R_{dc} = *V/I* = 20 V/4 A = 5 Ω. For comparison purposes, a 1-Ω and 10-Ω resistor were plotted on the graph of Fig. 4.7. Note that the less the resistance, the steeper the slope (closer to the vertical axis) of the curve.

If we write Ohm's law in the following manner and relate it to the basic straight-line equation

$$I = \frac{1}{R} \cdot E + 0$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$y = m \cdot x + b$$

we find that the slope is equal to 1 divided by the resistance value, as indicated by the following:

$$m = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta I}{\Delta V} = \frac{1}{R} \qquad (4.6)$$

where Δ signifies a small, finite change in the variable.

Equation (4.6) clearly reveals that the greater the resistance, the less the slope. If written in the following form, Equation (4.6) can be used to determine the resistance from the linear curve:
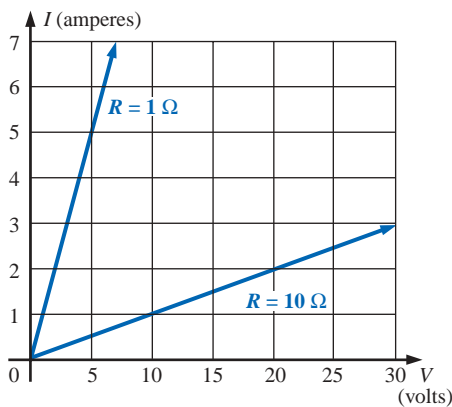
FIG. 4.7

Demonstrating on an I-V plot that the less the resistance, the steeper is the slope.

$$R = \frac{\Delta V}{\Delta I} \qquad \text{(ohms)} \qquad \textbf{(4.7)}$$

The equation states that by choosing a particular ΔV (or ΔI), one can obtain the corresponding ΔI (or ΔV, respectively) from the graph, as shown in Fig. 4.8, and then determine the resistance. It the plot is a straight line, Equation (4.7) will provide the same result no matter where the equation is applied. However, if the plot curves at all, the resistance will change.
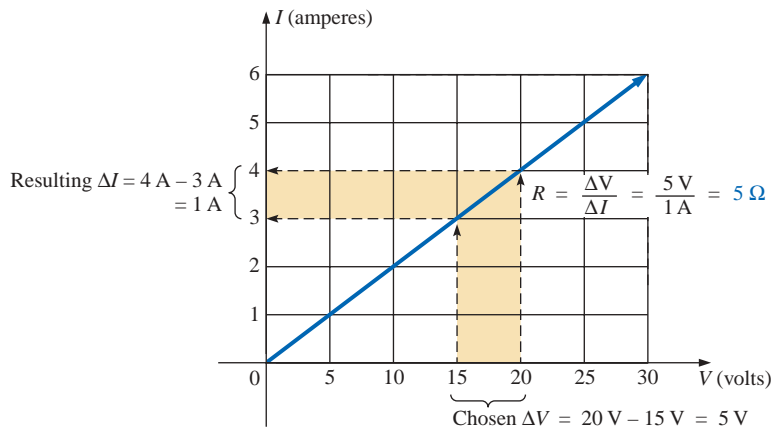
FIG. 4.8
Applying Eq. (4.6).

EXAMPLE 4.5 Determine the resistance associated with the curve of Fig. 4.9 using Eqs. (4.5) and (4.7), and compare results.

Solution: At $V = 6$ V, $I = 3$ mA, and

$$R_{dc} = \frac{V}{I} = \frac{6\text{ V}}{3\text{ mA}} = \textbf{2 k}\boldsymbol{\Omega}$$

For the interval between 6 V and 8 V,

$$R = \frac{\Delta V}{\Delta I} = \frac{2\text{ V}}{1\text{ mA}} = \textbf{2 k}\boldsymbol{\Omega}$$

The results are equivalent.
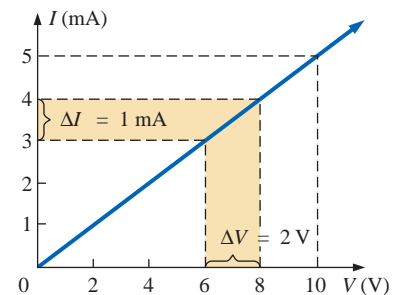
FIG. 4.9
Example 4.5.

Before leaving the subject, let us first investigate the characteristics of a very important semiconductor device called the **diode,** which will be examined in detail in basic electronics courses. This device will ideally act like a low-resistance path to current in one direction and a high-resistance path to current in the reverse direction, much like a switch that will pass current in only one direction. A typical set of characteristics appears in Fig. 4.10. Without any mathematical calculations, the closeness of the characteristic to the voltage axis for negative values of applied voltage indicates that this is the low-conductance (high resistance, switch opened) region. Note that this region extends to approximately 0.7 V positive. However, for values of applied voltage greater than 0.7 V, the vertical rise in the characteristics indicates a high-conductivity (low resistance, switch closed) region. Application of Ohm's law will now verify the above conclusions.
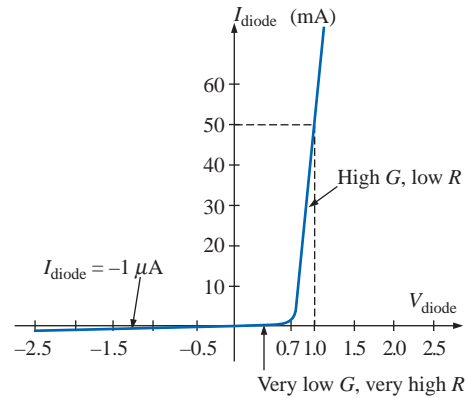
FIG. 4.10
Semiconductor diode characteristic.

At $V = +1$ V,

$$R_{\text{diode}} = \frac{V}{I} = \frac{1 \text{ V}}{50 \text{ mA}} = \frac{1 \text{ V}}{50 \times 10^{-3} \text{ A}}$$

$$= 20 \ \Omega$$

(a relatively low value for most applications)

At $V = -1$ V,

$$R_{\text{diode}} = \frac{V}{I} = \frac{1 \text{ V}}{1 \ \mu\text{A}}$$

$$= 1 \text{ M}\Omega$$

(which is often represented by an open-circuit equivalent)

4.3 POWER

Power is an indication of how much work (the conversion of energy from one form to another) can be done in a specified amount of time, that is, a *rate* of doing work. For instance, a large motor has more power than a small motor because it can convert more electrical energy into mechanical energy in the same period of time. Since converted energy is measured in *joules* (J) and time in seconds (s), power is measured in joules/second (J/s). The electrical unit of measurement for power is the watt (W), defined by

$$1 \text{ watt (W)} = 1 \text{ joule/second (J/s)} \qquad \textbf{(4.8)}$$

In equation form, power is determined by

$$P = \frac{W}{t} \qquad \text{(watts, W, or joules/second, J/s)} \qquad \textbf{(4.9)}$$

with the energy W measured in joules and the time t in seconds.

Throughout the text, the abbreviation for energy (W) can be distinguished from that for the watt (W) by the fact that one is in italics while the other is in roman. In fact, all variables in the dc section appear in italics while the units appear in roman.

The unit of measurement, the watt, is derived from the surname of James Watt (Fig. 4.11), who was instrumental in establishing the standards for power measurements. He introduced the **horsepower** (hp) as a measure of the average power of a strong dray horse over a full working day. It is approximately 50% more than can be expected from the average horse. The horsepower and watt are related in the following manner:

$$1 \text{ horsepower} \cong 746 \text{ watts}$$

The power delivered to, or absorbed by, an electrical device or system can be found in terms of the current and voltage by first substituting Eq. (2.7) into Eq. (4.9):

$$P = \frac{W}{t} = \frac{QV}{t} = V\frac{Q}{t}$$

But

$$I = \frac{Q}{t}$$

so that

$$P = VI \qquad \text{(watts)} \qquad \textbf{(4.10)}$$

By direct substitution of Ohm's law, the equation for power can be obtained in two other forms:

$$P = VI = V\left(\frac{V}{R}\right)$$

and

$$P = \frac{V^2}{R} \qquad \text{(watts)} \qquad \textbf{(4.11)}$$

or

$$P = VI = (IR)I$$

and

$$P = I^2R \qquad \text{(watts)} \qquad \textbf{(4.12)}$$

The result is that the power absorbed by the resistor of Fig. 4.12 can be found directly depending on the information available. In other words, if the current and resistance are known, it pays to use Eq. (4.12) directly, and if V and I are known, use of Eq. (4.10) is appropriate. It saves having to apply Ohm's law before determining the power.

Power can be delivered or absorbed as defined by the polarity of the voltage and the direction of the current. For all dc voltage sources, power is being *delivered* by the source if the current has the direction appearing in Fig. 4.13(a). Note that the current has the same direction as established by the source in a single-source network. If the current direction and polarity are as shown in Fig. 4.13(b) due to a multisource network, the battery is absorbing power much as when a battery is being charged.

For resistive elements, all the power delivered is dissipated in the form of heat because the voltage polarity is defined by the current direction (and vice versa), and current will always enter the terminal of higher potential corresponding with the absorbing state of Fig. 4.13(b). A reversal of the current direction in Fig. 4.12 will also reverse the polarity of the voltage across the resistor and match the conditions of Fig. 4.13(b).

Scottish (Greenock, Birmingham)
(1736–1819)
Instrument Maker and Inventor
Elected Fellow of the Royal Society of London in 1785

Courtesy of the
Smithsonian Institution
Photo No. 30,391

In 1757, at the age of 21, used his innovative talents to design mathematical instruments such as the *quadrant, compass,* and various *scales*. In 1765, introduced the use of a separate *condenser* to increase the efficiency of steam engines. In the years to follow he received a number of important patents on improved engine design, including a rotary motion for the steam engine (versus the reciprocating action) and a double-action engine, in which the piston pulled as well as pushed in its cyclic motion. Introduced the term **horsepower** as the average power of a strong dray (small cart) horse over a full working day.

FIG. 4.11
James Watt.
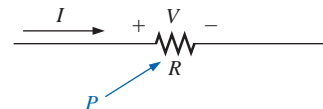
FIG. 4.12
Defining the power to a resistive element.

(a)   (b)
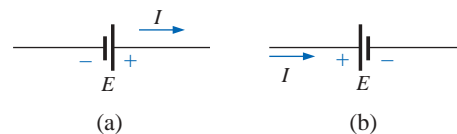
FIG. 4.13
Battery power: (a) supplied; (b) absorbed.

The magnitude of the power delivered or absorbed by a battery is given by

$$P = EI \qquad \text{(watts)} \qquad \textbf{(4.13)}$$

with E the battery terminal voltage and I the current through the source.
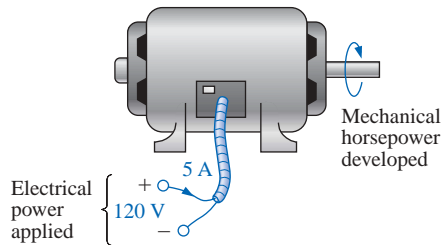
EXAMPLE 4.6 Find the power delivered to the dc motor of Fig. 4.14.

Solution:

$$P = VI = (120\text{ V})(5\text{ A}) = 600\text{ W} = \textbf{0.6 kW}$$

EXAMPLE 4.7 What is the power dissipated by a 5-Ω resistor if the current is 4 A?

Solution:

$$P = I^2R = (4\text{ A})^2(5\ \Omega) = \textbf{80 W}$$

FIG. 4.14
Example 4.6.

FIG. 4.15
The nonlinear I-V characteristics of a 75-W light bulb.

EXAMPLE 4.8 The *I-V* characteristics of a light bulb are provided in Fig. 4.15. Note the nonlinearity of the curve, indicating a wide range in resistance of the bulb with applied voltage as defined by the discussion of Section 4.2. If the rated voltage is 120 V, find the wattage rating of the bulb. Also calculate the resistance of the bulb under rated conditions.

Solution: At 120 V,

$$I = 0.625\text{ A}$$

and $\qquad P = VI = (120\text{ V})(0.625\text{ A}) = \textbf{75 W}$

At 120 V,

$$R = \frac{V}{I} = \frac{120\text{ V}}{0.625\text{ A}} = \textbf{192 }\Omega$$

Sometimes the power is given and the current or voltage must be determined. Through algebraic manipulations, an equation for each variable is derived as follows:

$$P = I^2R \Rightarrow I^2 = \frac{P}{R}$$

and

$$I = \sqrt{\frac{P}{R}} \qquad \text{(amperes)} \qquad \textbf{(4.14)}$$

$$P = \frac{V^2}{R} \Rightarrow V^2 = PR$$

and

$$V = \sqrt{PR} \qquad \text{(volts)} \qquad \textbf{(4.15)}$$

EXAMPLE 4.9 Determine the current through a 5-kΩ resistor when the power dissipated by the element is 20 mW.

Solution: Eq. (4.14):

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{20 \times 10^{-3}\,\text{W}}{5 \times 10^{3}\,\Omega}} = \sqrt{4 \times 10^{-6}} = 2 \times 10^{-3}\,\text{A}$$
$$= \mathbf{2\ mA}$$

4.4 WATTMETERS

As one might expect, there are instruments that can measure the power delivered by a source and to a dissipative element. One such instrument, the **wattmeter,** appears in Fig. 4.16. Since power is a function of both the current and the voltage levels, four terminals must be connected as shown in Fig. 4.17 to measure the power to the resistor R.

FIG. 4.16
Wattmeter. (Courtesy of Electrical Instrument Service, Inc.)

FIG. 4.17
Wattmeter connections.

If the current coils (CC) and potential coils (PC) of the wattmeter are connected as shown in Fig. 4.17, there will be an up-scale reading on the wattmeter. A reversal of either coil will result in a below-zero indication. Three voltage terminals may be available on the voltage side to permit a choice of voltage levels. On most wattmeters, the current terminals are physically larger than the voltage terminals to provide safety and to ensure a solid connection.

4.5 EFFICIENCY

A flowchart for the energy levels associated with any system that converts energy from one form to another is provided in Fig. 4.18. Take particular note of the fact that the output energy level must always be less than the applied energy due to losses and storage within the system. The best one can hope for is that W_o and W_i are relatively close in magnitude.

Conservation of energy requires that

Energy input = energy output + energy lost or stored in the system

Dividing both sides of the relationship by t gives

$$\frac{W_{\text{in}}}{t} = \frac{W_{\text{out}}}{t} + \frac{W_{\text{lost or stored by the system}}}{t}$$

FIG. 4.18
Energy flow through a system.

Since $P = W/t$, we have the following:

$$P_i = P_o + P_{\text{lost or stored}} \qquad \text{(W)} \qquad \textbf{(4.16)}$$

The **efficiency** (η) of the system is then determined by the following equation:

$$\text{Efficiency} = \frac{\text{power output}}{\text{power input}}$$

and
$$\eta = \frac{P_o}{P_i} \qquad \text{(decimal number)} \qquad \textbf{(4.17)}$$

where η (lowercase Greek letter eta) is a decimal number. Expressed as a percentage,

$$\eta\% = \frac{P_o}{P_i} \times 100\% \qquad \text{(percent)} \qquad \textbf{(4.18)}$$

In terms of the input and output energy, the efficiency in percent is given by

$$\eta\% = \frac{W_o}{W_i} \times 100\% \qquad \text{(percent)} \qquad \textbf{(4.19)}$$

The maximum possible efficiency is 100%, which occurs when $P_o = P_i$, or when the power lost or stored in the system is zero. Obviously, the greater the internal losses of the system in generating the necessary output power or energy, the lower the net efficiency.

EXAMPLE 4.10 A 2-hp motor operates at an efficiency of 75%. What is the power input in watts? If the applied voltage is 220 V, what is the input current?

Solution:

$$\eta\% = \frac{P_o}{P_i} \times 100\%$$

$$0.75 = \frac{(2 \text{ hp})(746 \text{ W/hp})}{P_i}$$

and
$$P_i = \frac{1492 \text{ W}}{0.75} = \textbf{1989.33 W}$$

$$P_i = EI \quad \text{or} \quad I = \frac{P_i}{E} = \frac{1989.33 \text{ W}}{220 \text{ V}} = \textbf{9.04 A}$$

EXAMPLE 4.11 What is the output in horsepower of a motor with an efficiency of 80% and an input current of 8 A at 120 V?

Solution:

$$\eta\% = \frac{P_o}{P_i} \times 100\%$$

$$0.80 = \frac{P_o}{(120 \text{ V})(8 \text{ A})}$$

and
$$P_o = (0.80)(120 \text{ V})(8 \text{ A}) = 768 \text{ W}$$

with
$$768 \text{ W}\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = \textbf{1.029 hp}$$

EXAMPLE 4.12 If $\eta = 0.85$, determine the output energy level if the applied energy is 50 J.

Solution:

$$\eta = \frac{W_o}{W_i} \Rightarrow W_o = \eta W_i$$

$$= (0.85)(50 \text{ J})$$
$$= \textbf{42.5 J}$$

The very basic components of a generating (voltage) system are depicted in Fig. 4.19. The source of mechanical power is a structure such as a paddlewheel that is turned by water rushing over the dam. The gear train will then ensure that the rotating member of the generator is turning at rated speed. The output voltage must then be fed through a transmission system to the load. For each component of the system, an
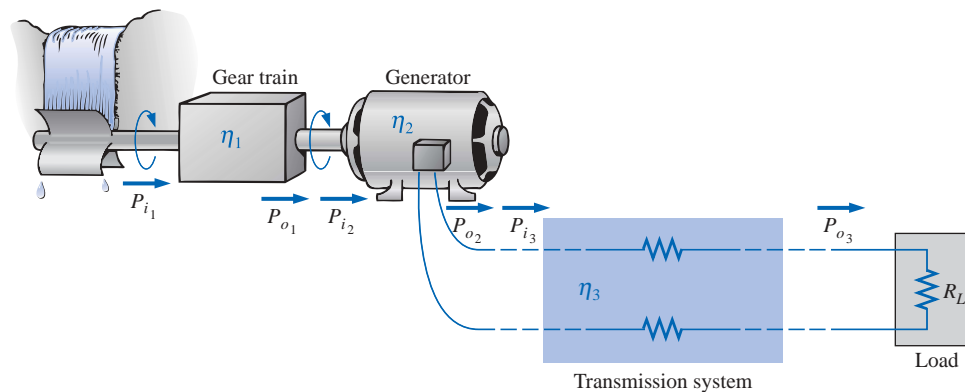
FIG. 4.19
Basic components of a generating system.

input and output power have been indicated. The efficiency of each system is given by

$$\eta_1 = \frac{P_{o_1}}{P_{i_1}} \qquad \eta_2 = \frac{P_{o_2}}{P_{i_2}} \qquad \eta_3 = \frac{P_{o_3}}{P_{i_3}}$$

If we form the product of these three efficiencies,

$$\eta_1 \cdot \eta_2 \cdot \eta_3 = \frac{P_{o_1}}{P_{i_1}} \cdot \frac{P_{o_2}}{P_{i_2}} \cdot \frac{P_{o_3}}{P_{i_3}}$$

and substitute the fact that $P_{i_2} = P_{o_1}$ and $P_{i_3} = P_{o_2}$, we find that the quantities indicated above will cancel, resulting in P_{o_3}/P_{i_1}, which is a measure of the efficiency of the entire system. In general, for the representative cascaded system of Fig. 4.20,

$$\boxed{\eta_{\text{total}} = \eta_1 \cdot \eta_2 \cdot \eta_3 \cdots \eta_n} \qquad \textbf{(4.20)}$$
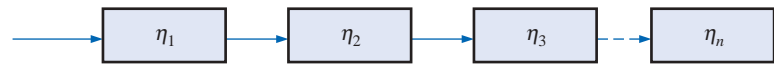
FIG. 4.20
Cascaded system.

EXAMPLE 4.13 Find the overall efficiency of the system of Fig. 4.19 if $\eta_1 = 90\%$, $\eta_2 = 85\%$, and $\eta_3 = 95\%$.

Solution:

$$\eta_T = \eta_1 \cdot \eta_2 \cdot \eta_3 = (0.90)(0.85)(0.95) = 0.727, \text{ or } \textbf{72.7\%}$$

EXAMPLE 4.14 If the efficiency η_1 drops to 40%, find the new overall efficiency and compare the result with that obtained in Example 4.13.

Solution:

$$\eta_T = \eta_1 \cdot \eta_2 \cdot \eta_3 = (0.40)(0.85)(0.95) = 0.323, \text{ or } \textbf{32.3\%}$$

Certainly 32.3% is noticeably less than 72.7%. The total efficiency of a cascaded system is therefore determined primarily by the lowest efficiency (weakest link) and is less than (or equal to if the remaining efficiencies are 100%) the least efficient link of the system.

4.6 ENERGY

For power, which is the rate of doing work, to produce an energy conversion of any form, it must be *used over a period of time.* For example, a motor may have the horsepower to run a heavy load, but unless the motor is *used* over a period of time, there will be no energy conversion. In addition, the longer the motor is used to drive the load, the greater will be the energy expended.

The **energy** (W) lost or gained by any system is therefore determined by

$$\boxed{W = Pt} \qquad \text{(wattseconds, Ws, or joules)} \qquad \textbf{(4.21)}$$

Since power is measured in watts (or joules per second) and time in seconds, the unit of energy is the *wattsecond* or *joule* (note Fig. 4.21) as indicated above. The wattsecond, however, is too small a quantity for most practical purposes, so the *watthour* (Wh) and *kilowatthour* (kWh) were defined, as follows:

$$\text{Energy (Wh)} = \text{power (W)} \times \text{time (h)} \qquad \textbf{(4.22)}$$

$$\text{Energy (kWh)} = \frac{\text{power (W)} \times \text{time (h)}}{1000} \qquad \textbf{(4.23)}$$

Note that the energy in kilowatthours is simply the energy in watthours divided by 1000. To develop some sense for the kilowatthour energy level, consider that 1 kWh is the energy dissipated by a 100-W bulb in 10 h.

The **kilowatthour meter** is an instrument for measuring the energy supplied to the residential or commercial user of electricity. It is normally connected directly to the lines at a point just prior to entering the power distribution panel of the building. A typical set of dials is shown in Fig. 4.22(a) with a photograph of an analog kilowatthour meter. As indicated, each power of ten below a dial is in kilowatthours. The more rapidly the aluminum disc rotates, the greater the energy demand. The dials are connected through a set of gears to the rotation of this disc. A solid-state digital meter with an extended range of capabilities appears in Fig. 4.22(b).

British (Salford, Manchester)
(1818–89)
Physicist
Honorary Doctorates from the Universities of Dublin and Oxford

Bettmann Archive Photo Number 076800P

Contributed to the important fundamental *law of conservation of energy* by establishing that various forms of energy, whether electrical, mechanical, or heat, are in the same family and can be exchanged from one form to another. In 1841 introduced *Joule's law*, which stated that the heat developed by electric current in a wire is proportional to the product of the current squared and the resistance of the wire (I^2R). He further determined that the heat emitted was equivalent to the power absorbed and therefore heat is a form of energy.

FIG. 4.21
James Prescott Joule.

(a)

(b)

FIG. 4.22
Kilowatthour meters: (a) analog; (b) digital. (Courtesy of ABB Electric Metering Systems.)

EXAMPLE 4.15 For the dial positions of Fig. 4.22(a), calculate the electricity bill if the previous reading was 4650 kWh and the average cost is 9¢ per kilowatthour.

Solution:

$$5360 \text{ kWh} - 4650 \text{ kWh} = 710 \text{ kWh used}$$

$$710 \text{ kWh} \left(\frac{9¢}{\text{kWh}} \right) = \textbf{\$63.90}$$

EXAMPLE 4.16 How much energy (in kilowatthours) is required to light a 60-W bulb continuously for 1 year (365 days)?

Solution:

$$W = \frac{Pt}{1000} = \frac{(60 \text{ W})(24 \text{ h/day})(365 \text{ days})}{1000} = \frac{525{,}600 \text{ Wh}}{1000}$$

$$= \textbf{525.60 kWh}$$

EXAMPLE 4.17 How long can a 205-W television set be on before using more than 4 kWh of energy?

Solution:

$$W = \frac{Pt}{1000} \Rightarrow t \text{ (hours)} = \frac{(W)(1000)}{P}$$

$$= \frac{(4 \text{ kWh})(1000)}{205 \text{ W}} = \textbf{19.51 h}$$

EXAMPLE 4.18 What is the cost of using a 5-hp motor for 2 h if the rate is 9¢ per kilowatthour?

Solution:

$$W \text{ (kilowatthours)} = \frac{Pt}{1000} = \frac{(5 \text{ hp} \times 746 \text{ W/hp})(2 \text{ h})}{1000} = 7.46 \text{ kWh}$$

$$\text{Cost} = (7.46 \text{ kWh})(9¢/\text{kWh}) = \textbf{67.14¢}$$

EXAMPLE 4.19 What is the total cost of using all of the following at 9¢ per kilowatthour?

A 1200-W toaster for 30 min
Six 50-W bulbs for 4 h
A 400-W washing machine for 45 min
A 4800-W electric clothes dryer for 20 min

Solution:

$$W$$
$$= \frac{(1200 \text{ W})(\frac{1}{2} \text{ h}) + (6)(50 \text{ W})(4 \text{ h}) + (400 \text{ W})(\frac{3}{4} \text{ h}) + (4800 \text{ W})(\frac{1}{3} \text{ h})}{1000}$$

$$= \frac{600 \text{ Wh} + 1200 \text{ Wh} + 300 \text{ Wh} + 1600 \text{ Wh}}{1000} = \frac{3700 \text{ Wh}}{1000}$$

$$W = 3.7 \text{ kWh}$$

$$\text{Cost} = (3.7 \text{ kWh})(9¢/\text{kWh}) = \textbf{33.3¢}$$

The chart in Fig. 4.23 shows the average cost per kilowatthour compared to the kilowatthours used per customer. Note that the cost today is above the level of 1926 and the average customer uses more than 20 times as much electrical energy in a year. Keep in mind that the chart of Fig. 4.23 is the average cost across the nation. Some states have average rates close to 5¢ per kilowatthour, whereas others approach 12¢ per kilowatthour.

Table 4.1 lists some common household appliances with their typical wattage ratings. It might prove interesting for the reader to calculate the cost of operating some of these appliances over a period of time using the chart in Fig. 4.23 to find the cost per kilowatthour.

RESIDENTIAL SERVICE
Total electric utility industry
(including Alaska and Hawaii since 1960)
Average use per customer
and average revenue per kWh

kWh per customer

Average yearly use,
kWh per customer

Cents/kWh

Average revenue,
cents per kWh
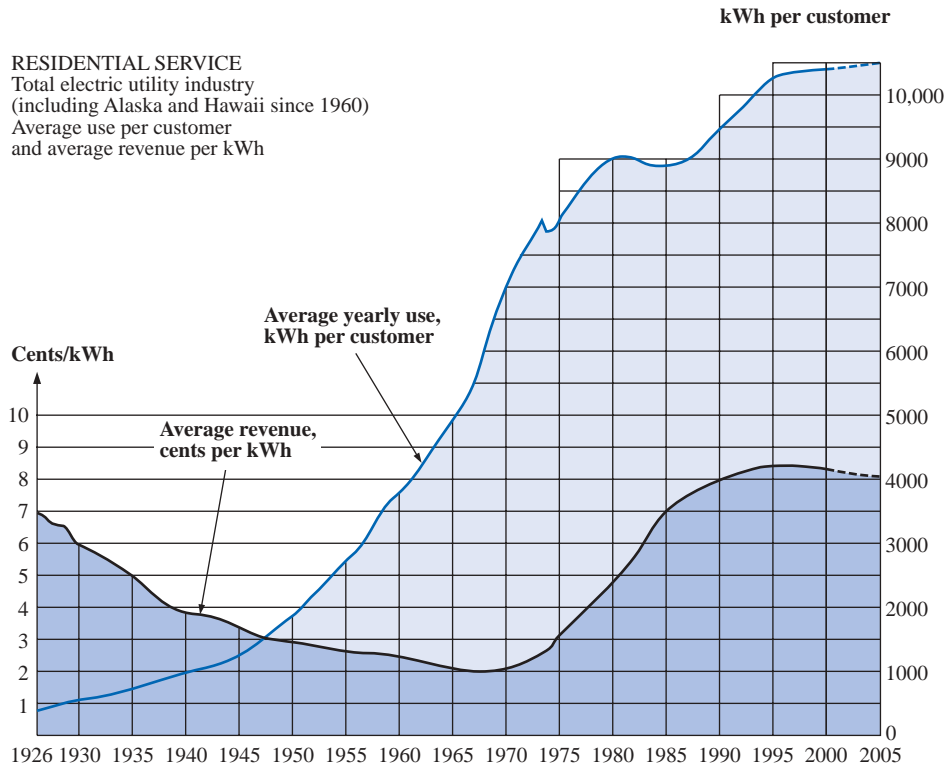
FIG. 4.23

Cost per kWh and average kWh per customer versus time. (Courtesy of Edison Electric Institute.)

TABLE 4.1

Typical wattage ratings of some common household items.

Appliance	Wattage Rating	Appliance	Wattage Rating
Air conditioner	860	Lap-top computer:	
Blow dryer	1,300	Sleep	< 1 W (Typically 0.3 W to 0.5 W)
Cassette player/recorder	5	Normal	10–20 W
Cellular phone:		High	25–35 W
Standby	$\cong$ 35 mW	Microwave oven	1,200
Talk	$\cong$ 4.3 W	Pager	1–2 mW
Clock	2	Phonograph	75
Clothes dryer (electric)	4,800	Projector	1,200
Coffee maker	900	Radio	70
Dishwasher	1,200	Range (self-cleaning)	12,200
Fan:		Refrigerator (automatic defrost)	1,800
Portable	90	Shaver	15
Window	200	Stereo equipment	110
Heater	1,322	Sun lamp	280
Heating equipment:		Toaster	1,200
Furnace fan	320	Trash compactor	400
Oil-burner motor	230	TV (color)	200
Iron, dry or steam	1,100	Videocassette recorder	110
		Washing machine	500
		Water heater	4,500

Courtesy of General Electric Co.

on the meter as shown in Fig. 4.31. Note that both are providing the expected results.

Now for one of the most important things to learn about applying EWB:

Always stop or end the simulation (clicking on 0 or choosing OFF) before making any changes in the network. When the simulation is initiated, it will stay in that mode until turned off.

There was obviously a great deal of material to learn in this first exercise using Electronics Workbench. Be assured, however, that as we continue with more examples, you will find the procedure quite straightforward and actually enjoyable to apply.

PROBLEMS

SECTION 4.1   Ohm's Law

1. What is the potential drop across a 6-Ω resistor if the current through it is 2.5 A?

2. What is the current through a 72-Ω resistor if the voltage drop across it is 12 V?

3. How much resistance is required to limit the current to 1.5 mA if the potential drop across the resistor is 6 V?

4. At starting, what is the current drain on a 12-V car battery if the resistance of the starting motor is 0.056 Ω?

5. If the current through a 0.02-MΩ resistor is 3.6 μA, what is the voltage drop across the resistor?

6. If a voltmeter has an internal resistance of 15 kΩ, find the current through the meter when it reads 62 V.

7. If a refrigerator draws 2.2 A at 120 V, what is its resistance?

8. If a clock has an internal resistance of 7.5 kΩ, find the current through the clock if it is plugged into a 120-V outlet.

9. A washing machine is rated at 4.2 A at 120 V. What is its internal resistance?

10. If a soldering iron draws 0.76 A at 120 V, what is its resistance?

11. The input current to a transistor is 20 μA. If the applied (input) voltage is 24 mV, determine the input resistance of the transistor.

12. The internal resistance of a dc generator is 0.5 Ω. Determine the loss in terminal voltage across this internal resistance if the current is 15 A.

*13. a. If an electric heater draws 9.5 A when connected to a 120-V supply, what is the internal resistance of the heater?
   b. Using the basic relationships of Chapter 2, how much energy is converted in 1 h?

SECTION 4.2   Plotting Ohm's Law

14. Plot the linear curves of a 100-Ω and a 0.5-Ω resistor on the graph of Fig. 4.6. If necessary, reproduce the graph.

15. Sketch the characteristics of a device that has an internal resistance of 20 Ω from 0 to 10 V and an internal resistance of 2 Ω for higher voltages. Use the axes of Fig. 4.6. If necessary, reproduce the graph.

16. Plot the linear curves of a 2-kΩ and a 50-kΩ resistor on the graph of Fig. 4.6. Use a horizontal scale that extends from 0 to 20 V and a vertical axis scaled off in milliamperes. If necessary, reproduce the graph.

17. What is the change in voltage across a 2-kΩ resistor established by a change in current of 400 mA through the resistor?

*18. a. Using the axes of Fig. 4.10, sketch the characteristics of a device that has an internal resistance of 500 Ω from 0 to 1 V and 50 Ω between 1 V and 2 V. Its resistance then changes to −20 Ω for higher voltages. The result is a set of characteristics very similar to those of an electronic device called a *tunnel diode.*
   b. Using the above characteristics, determine the resulting current at voltages of 0.7 V, 1.5 V, and 2.5 V.

SECTION 4.3   Power

19. If 420 J of energy are absorbed by a resistor in 7 min, what is the power to the resistor?

20. The power to a device is 40 joules per second (J/s). How long will it take to deliver 640 J?

21. a. How many joules of energy does a 2-W nightlight dissipate in 8 h?
   b. How many kilowatthours does it dissipate?

22. A resistor of 10 Ω has charge flowing through it at the rate of 300 coulombs per minute (C/min). How much power is dissipated?

23. How long must a steady current of 2 A exist in a resistor that has 3 V across it to dissipate 12 J of energy?

24. What is the power delivered by a 6-V battery if the charge flows at the rate of 48 C/min?

25. The current through a 4-Ω resistor is 7 mA. What is the power delivered to the resistor?

26. The voltage drop across a 3-Ω resistor is 9 mV. What is the power input to the resistor?

27. If the power input to a 4-Ω resistor is 64 W, what is the current through the resistor?

28. A 1/2-W resistor has a resistance of 1000 Ω. What is the maximum current that it can safely handle?

29. A 2.2-kΩ resistor in a stereo system dissipates 42 mW of power. What is the voltage across the resistor?

30. A dc battery can deliver 45 mA at 9 V. What is the power rating?

31. What are the "hot" resistance level and current rating of a 120-V, 100-W bulb?

32. What are the internal resistance and voltage rating of a 450-W automatic washer that draws 3.75 A?

33. A calculator with an internal 3-V battery draws 0.4 mW when fully functional.
   a. What is the current demand from the supply?
   b. If the calculator is rated to operate 500 h on the same battery, what is the ampere-hour rating of the battery?

34. A 20-kΩ resistor has a rating of 100 W. What are the maximum current and the maximum voltage that can be applied to the resistor?

*35. a. Plot power versus current for a 100-Ω resistor. Use a power scale from 0 to 1 W and a current scale from 0 to 100 mA with divisions of 0.1 W and 10 mA, respectively.
   b. Is the curve linear or nonlinear?
   c. Using the resulting plot, determine the current at a power level of 500 mW.

*36. A small, portable black-and-white television draws 0.455 A at 9 V.
   a. What is the power rating of the television?
   b. What is the internal resistance of the television?
   c. What is the energy converted in 6 h of typical battery life?

*37. a. If a home is supplied with a 120-V, 100-A service, find the maximum power capability.
   b. Can the homeowner safely operate the following loads at the same time?
   5-hp motor
   3000-W clothes dryer
   2400-W electric range
   1000-W steam iron

SECTION 4.5 Efficiency

38. What is the efficiency of a motor that has an output of 0.5 hp with an input of 450 W?

39. The motor of a power saw is rated 68.5% efficient. If 1.8 hp are required to cut a particular piece of lumber, what is the current drawn from a 120-V supply?

40. What is the efficiency of a dryer motor that delivers 1 hp when the input current and voltage are 4 A and 220 V, respectively?

41. A stereo system draws 2.4 A at 120 V. The audio output power is 50 W.
   a. How much power is lost in the form of heat in the system?
   b. What is the efficiency of the system?

42. If an electric motor having an efficiency of 87% and operating off a 220-V line delivers 3.6 hp, what input current does the motor draw?

43. A motor is rated to deliver 2 hp.
   a. If it runs on 110 V and is 90% efficient, how many watts does it draw from the power line?
   b. What is the input current?
   c. What is the input current if the motor is only 70% efficient?

44. An electric motor used in an elevator system has an efficiency of 90%. If the input voltage is 220 V, what is the input current when the motor is delivering 15 hp?

45. A 2-hp motor drives a sanding belt. If the efficiency of the motor is 87% and that of the sanding belt 75% due to slippage, what is the overall efficiency of the system?

46. If two systems in cascade each have an efficiency of 80% and the input energy is 60 J, what is the output energy?

47. The overall efficiency of two systems in cascade is 72%. If the efficiency of one is 0.9, what is the efficiency in percent of the other?

*48. If the total input and output power of two systems in cascade are 400 W and 128 W, respectively, what is the efficiency of each system if one has twice the efficiency of the other?

49. a. What is the total efficiency of three systems in cascade with efficiencies of 98%, 87%, and 21%?
   b. If the system with the least efficiency (21%) were removed and replaced by one with an efficiency of 90%, what would be the percentage increase in total efficiency?

50. a. Perform the following conversions:
   1 Wh to joules
   1 kWh to joules
   b. Based on the results of part (a), discuss when it is more appropriate to use one unit versus the other.

SECTION 4.6 Energy

51. A 10-Ω resistor is connected across a 15-V battery.
   a. How many joules of energy will it dissipate in 1 min?
   b. If the resistor is left connected for 2 min instead of 1 min, will the energy used increase? Will the power dissipation level increase?

52. How much energy in kilowatthours is required to keep a 230-W oil-burner motor running 12 h a week for 5 months? (Use 4⅓ weeks = 1 month.)

53. How long can a 1500-W heater be on before using more than 10 kWh of energy?

54. How much does it cost to use a 30-W radio for 3 h at 9¢ per kilowatthour?

55. a. In 10 h an electrical system converts 500 kWh of electrical energy into heat. What is the power level of the system?
   b. If the applied voltage is 208 V, what is the current drawn from the supply?
   c. If the efficiency of the system is 82%, how much energy is lost or stored in 10 h?

56. a. At 9¢ per kilowatthour, how long can one play a 250-W color television for $1?

b. For $1, how long can one use a 4.8-kW dryer?

c. Compare the results of parts (a) and (b), and comment on the effect of the wattage level on the relative cost of using an appliance.

57. What is the total cost of using the following at 9¢ per kilowatthour?
860-W air conditioner for 24 h
4800-W clothes dryer for 30 min
400-W washing machine for 1 h
1200-W dishwasher for 45 min

***58.** What is the total cost of using the following at 9¢ per kilowatthour?
110-W stereo set for 4 h
1200-W projector for 20 min
60-W tape recorder for 1.5 h
150-W color television set for 3 h 45 min

SECTION 4.9   Computer Analysis

PSpice or Electronics Workbench

59. Repeat the analysis of the circuit of Fig. 4.29 with $E = 400$ mV and $R = 0.04$ MΩ.

60. Repeat the analysis of the circuit of Fig. 4.29, but reverse the polarity of the battery and use $E = 0.02$ V and $R = 240$ Ω.

Programming Language (C++, QBASIC, Pascal, etc.)

61. Write a program to calculate the cost of using five different appliances for varying lengths of time if the cost is 9¢ per kilowatthour.

62. Request *I, R,* and *t* and determine *V, P,* and *W.* Print out the results with the proper units.

GLOSSARY

Circuit breaker   A two-terminal device designed to ensure that current levels do not exceed safe levels. If "tripped," it can be reset with a switch or a reset button.

Diode   A semiconductor device whose behavior is much like that of a simple switch; that is, it will pass current ideally in only one direction when operating within specified limits.

Efficiency (η)   A ratio of output to input power that provides immediate information about the energy-converting characteristics of a system.

Energy *(W)*   A quantity whose change in state is determined by the product of the rate of conversion (*P*) and the period involved (*t*). It is measured in joules (J) or wattseconds (Ws).

Fuse   A two-terminal device whose sole purpose is to ensure that current levels in a circuit do not exceed safe levels.

Horsepower (hp)   Equivalent to 746 watts in the electrical system.

Kilowatthour meter   An instrument for measuring kilowatthours of energy supplied to a residential or commercial user of electricity.

Ohm's law   An equation that establishes a relationship among the current, voltage, and resistance of an electrical system.

Power   An indication of how much work can be done in a specified amount of time; a *rate* of doing work. It is measured in joules/second (J/s) or watts (W).

Wattmeter   An instrument capable of measuring the power delivered to an element by sensing both the voltage across the element and the current through the element.

5

Series Circuits

5.1 INTRODUCTION

Two types of current are readily available to the consumer today. One is *direct current* (dc), in which ideally the flow of charge (current) does not change in magnitude (or direction) with time. The other is *sinusoidal alternating current* (ac), in which the flow of charge is continually changing in magnitude (and direction) with time. The next few chapters are an introduction to circuit analysis purely from a dc approach. The methods and concepts will be discussed in detail for direct current; when possible, a short discussion will suffice to cover any variations we might encounter when we consider ac in the later chapters.

The battery of Fig. 5.1, by virtue of the potential difference between its terminals, has the ability to cause (or "pressure") charge to flow through the simple circuit. The positive terminal attracts the electrons through the wire at the same rate at which electrons are supplied by the negative terminal. As long as the battery is connected in the circuit and maintains its terminal characteristics, the current (dc) through the circuit will not change in magnitude or direction.

Battery

E (volts)

$I_{\text{conventional}}$

I_{electron}

$I = \dfrac{V}{R} = \dfrac{E}{R}$

R V
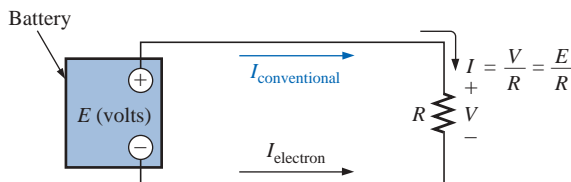
FIG. 5.1

Introducing the basic components of an electric circuit.

If we consider the wire to be an ideal conductor (that is, having no opposition to flow), the potential difference V across the resistor will equal the applied voltage of the battery: V (volts) $= E$ (volts).

For all one-voltage-
source dc circuits
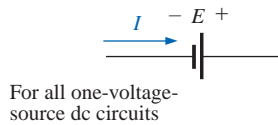
FIG. 5.2

*Defining the direction of conventional flow for
single-source dc circuits.*

For any combination of voltage
sources in the same dc circuit
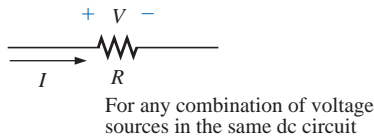
FIG. 5.3

*Defining the polarity resulting from a
conventional current I through a resistive
element.*

(a) Series circuit

(b) R_1 and R_2 are not in series

FIG. 5.4

*(a) Series circuit; (b) situation in which R_1
and R_2 are not in series.*

The current is limited only by the resistor R. The higher the resistance, the less the current, and conversely, as determined by Ohm's law.

By convention (as discussed in Chapter 2), the direction of **conventional current flow** $I_{conventional}$ as shown in Fig. 5.1 is opposite to that of **electron flow** ($I_{electron}$). Also, the uniform flow of charge dictates that the direct current I be the same everywhere in the circuit. By following the direction of conventional flow, we notice that there is a rise in potential across the battery ($-$ to $+$), and a drop in potential across the resistor ($+$ to $-$). For single-voltage-source dc circuits, conventional flow always passes from a low potential to a high potential when passing through a voltage source, as shown in Fig. 5.2. However, conventional flow always passes from a high to a low potential when passing through a resistor for any number of voltage sources in the same circuit, as shown in Fig. 5.3.

The circuit of Fig. 5.1 is the simplest possible configuration. This chapter and the chapters to follow will add elements to the system in a very specific manner to introduce a range of concepts that will form a major part of the foundation required to analyze the most complex system. Be aware that the laws, rules, and so on, introduced in Chapters 5 and 6 will be used throughout your studies of electrical, electronic, or computer systems. They will not be dropped for a more advanced set as you progress to more sophisticated material. It is therefore critical that the concepts be understood thoroughly and that the various procedures and methods be applied with confidence.

5.2 SERIES CIRCUITS

A **circuit** consists of any number of elements joined at terminal points, providing at least one closed path through which charge can flow. The circuit of Fig. 5.4(a) has three elements joined at three terminal points (*a, b,* and *c*) to provide a closed path for the current *I*.

Two elements are in series if

1. *They have only one terminal in common (i.e., one lead of one is connected to only one lead of the other).*
2. *The common point between the two elements is not connected to another current-carrying element.*

In Fig. 5.4(a), the resistors R_1 and R_2 are in series because they have *only* point *b* in common. The other ends of the resistors are connected elsewhere in the circuit. For the same reason, the battery *E* and resistor R_1 are in series (terminal *a* in common), and the resistor R_2 and the battery *E* are in series (terminal *c* in common). Since all the elements are in series, the network is called a **series circuit.** Two common examples of series connections include the tying of small pieces of rope together to form a longer rope and the connecting of pipes to get water from one point to another.

If the circuit of Fig. 5.4(a) is modified such that a current-carrying resistor R_3 is introduced, as shown in Fig. 5.4(b), the resistors R_1 and R_2 are no longer in series due to a violation of number 2 of the above definition of series elements.

The current is the same through series elements.

For the circuit of Fig. 5.4(a), therefore, the current I through each resistor is the same as that through the battery. The fact that the current is
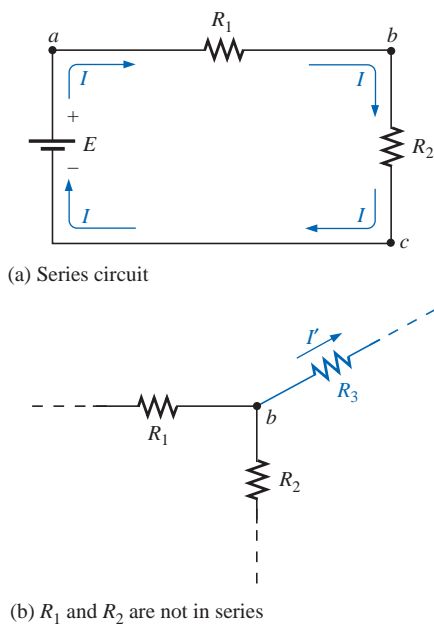
the same through series elements is often used as a path to determine whether two elements are in series or to confirm a conclusion.

A **branch** of a circuit is any portion of the circuit that has one or more elements in series. In Fig. 5.4(a), the resistor R_1 forms one branch of the circuit, the resistor R_2 another, and the battery E a third.

The total resistance of a series circuit is the sum of the resistance levels.

In Fig. 5.4(a), for example, the total resistance (R_T) is equal to $R_1 + R_2$. Note that the total resistance is actually the resistance "seen" by the battery as it "looks" into the series combination of elements as shown in Fig. 5.5.

In general, to find the total resistance of N resistors in series, the following equation is applied:

$$R_T = R_1 + R_2 + R_3 + \cdots + R_N \qquad \text{(ohms, } \Omega\text{)} \qquad \textbf{(5.1)}$$

Once the total resistance is known, the circuit of Fig. 5.4(a) can be redrawn as shown in Fig. 5.6, clearly revealing that the only resistance the source "sees" is the total resistance. It is totally unaware of how the elements are connected to establish R_T. Once R_T is known, the current drawn from the source can be determined using Ohm's law, as follows:

$$I_s = \frac{E}{R_T} \qquad \text{(amperes, A)} \qquad \textbf{(5.2)}$$

Since E is fixed, the magnitude of the source current will be totally dependent on the magnitude of R_T. A larger R_T will result in a relatively small value of I_s, while lesser values of R_T will result in increased current levels.

The fact that the current is the same through each element of Fig. 5.4(a) permits a direct calculation of the voltage across each resistor using Ohm's law; that is,

$$V_1 = IR_1, V_2 = IR_2, V_3 = IR_3, \ldots, V_N = IR_N \qquad \text{(volts, V)} \qquad \textbf{(5.3)}$$

The power delivered to each resistor can then be determined using any one of three equations as listed below for R_1:

$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1} \qquad \text{(watts, W)} \qquad \textbf{(5.4)}$$

The power delivered by the source is

$$P_{\text{del}} = EI \qquad \text{(watts, W)} \qquad \textbf{(5.5)}$$

The total power delivered to a resistive circuit is equal to the total power dissipated by the resistive elements.

That is,

$$P_{\text{del}} = P_1 + P_2 + P_3 + \cdots + P_N \qquad \textbf{(5.6)}$$
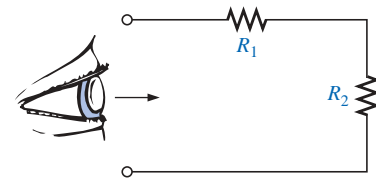
FIG. 5.5
Resistance "seen" by source.
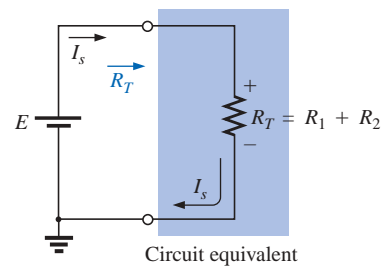
FIG. 5.6
Replacing the series resistors R_1 and R_2 of Fig. 5.5 with the total resistance.
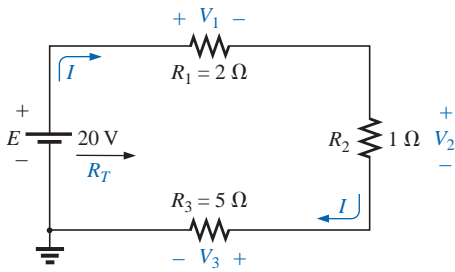
FIG. 5.7

Example 5.1.

EXAMPLE 5.1

a. Find the total resistance for the series circuit of Fig. 5.7.
b. Calculate the source current I_s.
c. Determine the voltages V_1, V_2, and V_3.
d. Calculate the power dissipated by R_1, R_2, and R_3.
e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d).

Solutions:

a. $R_T = R_1 + R_2 + R_3 = 2\ \Omega + 1\ \Omega + 5\ \Omega = \mathbf{8\ \Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{20\text{ V}}{8\ \Omega} = \mathbf{2.5\ A}$

c. $V_1 = IR_1 = (2.5\text{ A})(2\ \Omega) = \mathbf{5\ V}$
   $V_2 = IR_2 = (2.5\text{ A})(1\ \Omega) = \mathbf{2.5\ V}$
   $V_3 = IR_3 = (2.5\text{ A})(5\ \Omega) = \mathbf{12.5\ V}$

d. $P_1 = V_1 I_1 = (5\text{ V})(2.5\text{ A}) = \mathbf{12.5\ W}$
   $P_2 = I_2^2 R_2 = (2.5\text{ A})^2(1\ \Omega) = \mathbf{6.25\ W}$
   $P_3 = V_3^2/R_3 = (12.5\text{ V})^2/5\ \Omega = \mathbf{31.25\ W}$

e. $P_{\text{del}} = EI = (20\text{ V})(2.5\text{ A}) = \mathbf{50\ W}$
   $P_{\text{del}} = P_1 + P_2 + P_3$
   $50\text{ W} = 12.5\text{ W} + 6.25\text{ W} + 31.25\text{ W}$
   $\underline{50\text{ W} = 50\text{ W} \quad \text{(checks)}}$

To find the total resistance of *N* resistors of the same value in series, simply multiply the value of *one* of the resistors by the number in series; that is,

$$\boxed{R_T = NR} \qquad (5.7)$$

EXAMPLE 5.2 Determine R_T, I, and V_2 for the circuit of Fig. 5.8.

Solution: Note the current direction as established by the battery and the polarity of the voltage drops across R_2 as determined by the current direction. Since $R_1 = R_3 = R_4$,

$$R_T = NR_1 + R_2 = (3)(7\ \Omega) + 4\ \Omega = 21\ \Omega + 4\ \Omega = \mathbf{25\ \Omega}$$

$$I = \frac{E}{R_T} = \frac{50\text{ V}}{25\ \Omega} = \mathbf{2\ A}$$

$$V_2 = IR_2 = (2\text{ A})(4\ \Omega) = \mathbf{8\ V}$$
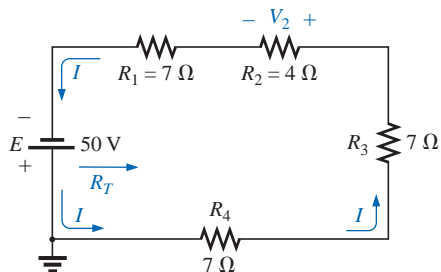
FIG. 5.8

Example 5.2.

Examples 5.1 and 5.2 are straightforward substitution-type problems that are relatively easy to solve with some practice. Example 5.3, however, is evidence of another type of problem that requires a firm grasp of the fundamental equations and an ability to identify which equation to use first. The best preparation for this type of exercise is simply to work through as many problems of this kind as possible.

EXAMPLE 5.3 Given R_T and I, calculate R_1 and E for the circuit of Fig. 5.9.

FIG. 5.9

Example 5.3.

Solution:

$$R_T = R_1 + R_2 + R_3$$
$$12\ \text{k}\Omega = R_1 + 4\ \text{k}\Omega + 6\ \text{k}\Omega$$
$$R_1 = 12\ \text{k}\Omega - 10\ \text{k}\Omega = \textbf{2 k}\boldsymbol{\Omega}$$
$$E = IR_T = (6 \times 10^{-3}\ \text{A})(12 \times 10^3\ \Omega) = \textbf{72 V}$$

(a)

(b)

FIG. 5.10
Reducing series dc voltage sources to a single source.

5.3 VOLTAGE SOURCES IN SERIES

Voltage sources can be connected in series, as shown in Fig. 5.10, to increase or decrease the total voltage applied to a system. The net voltage is determined simply by summing the sources with the same polarity and subtracting the total of the sources with the opposite "pressure." The net polarity is the polarity of the larger sum.

In Fig. 5.10(a), for example, the sources are all "pressuring" current to the right, so the net voltage is

$$E_T = E_1 + E_2 + E_3 = 10\ \text{V} + 6\ \text{V} + 2\ \text{V} = 18\ \text{V}$$

as shown in the figure. In Fig. 5.10(b), however, the greater "pressure" is to the left, with a net voltage of

$$E_T = E_2 + E_3 - E_1 = 9\ \text{V} + 3\ \text{V} - 4\ \text{V} = 8\ \text{V}$$

and the polarity shown in the figure.

5.4 KIRCHHOFF'S VOLTAGE LAW

Note Fig. 5.11.

Kirchhoff's voltage law *(KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.*

A **closed loop** is any continuous path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit. In Fig. 5.12, by following the current, we can trace a continuous path that leaves point *a* through R_1 and returns through E without leaving the circuit. Therefore, *abcda* is a closed loop. For us to be able to apply Kirchhoff's voltage law, the summation of potential rises and drops must be made in one direction around the closed loop.

For uniformity, the clockwise (CW) direction will be used throughout the text for all applications of Kirchhoff's voltage law. Be aware, however, that the same result will be obtained if the counterclockwise (CCW) direction is chosen and the law applied correctly.

A plus sign is assigned to a potential rise ($-$ to $+$), and a minus sign to a potential drop ($+$ to $-$). If we follow the current in Fig. 5.12 from point *a,* we first encounter a potential drop V_1 ($+$ to $-$) across R_1 and then another potential drop V_2 across R_2. Continuing through the voltage source, we have a potential rise E ($-$ to $+$) before returning to point *a*. In symbolic form, where Σ represents summation, ↻ the closed loop, and V the potential drops and rises, we have

$$\boxed{\Sigma_\circlearrowright V = 0}$$
(Kirchhoff's voltage law in symbolic form)   **(5.8)**
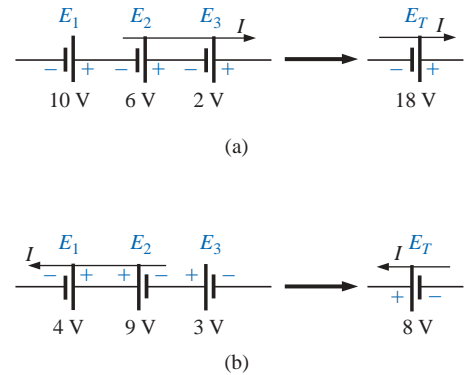
German (Königsberg, Berlin)
(1824–87)
Physicist
Professor of Physics, University of Heidelberg

Courtesy of the
Smithsonian Institution
Photo No. 58,283

Although a contributor to a number of areas in the physics domain, he is best known for his work in the electrical area with his definition of the relationships between the currents and voltages of a network in 1847. Did extensive research with German chemist Robert Bunsen (developed the *Bunsen burner*), resulting in the discovery of the important elements of *cesium* and *rubidium*.

FIG. 5.11
Gustav Robert Kirchhoff.
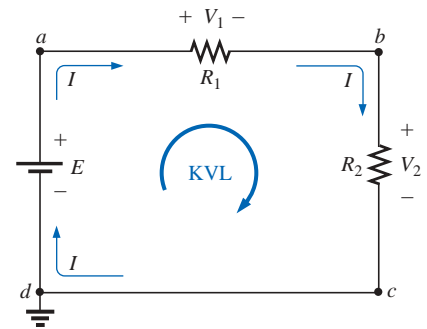
FIG. 5.12
Applying Kirchhoff's voltage law to a series dc circuit.

which for the circuit of Fig. 5.12 yields (clockwise direction, following the current I and starting at point d):

$$+E - V_1 - V_2 = 0$$

or

$$E = V_1 + V_2$$

revealing that

the applied voltage of a series circuit equals the sum of the voltage drops across the series elements.

Kirchhoff's voltage law can also be stated in the following form:

$$\boxed{\Sigma_{\circlearrowright} V_{\text{rises}} = \Sigma_{\circlearrowright} V_{\text{drops}}} \qquad \text{(5.9)}$$

which in words states that the sum of the rises around a closed loop must equal the sum of the drops in potential. The text will emphasize the use of Eq. (5.8), however.

If the loop were taken in the counterclockwise direction starting at point a, the following would result:

$$\Sigma_{\circlearrowright} V = 0$$

$$-E + V_2 + V_1 = 0$$

or, as before,

$$E = V_1 + V_2$$

The application of Kirchhoff's voltage law need not follow a path that includes current-carrying elements.

For example, in Fig. 5.13 there is a difference in potential between points a and b, even though the two points are not connected by a current-carrying element. Application of Kirchhoff's voltage law around the closed loop will result in a difference in potential of 4 V between the two points. That is, using the clockwise direction:

$$+12 \text{ V} - V_x - 8 \text{ V} = 0$$

and

$$V_x = \mathbf{4 \text{ V}}$$
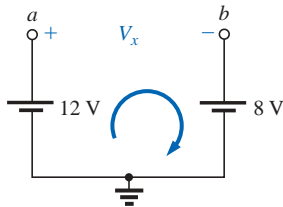
FIG. 5.13

Demonstration that a voltage can exist between two points not connected by a current-carrying conductor.

EXAMPLE 5.4 Determine the unknown voltages for the networks of Fig. 5.14.

(a)

(b)
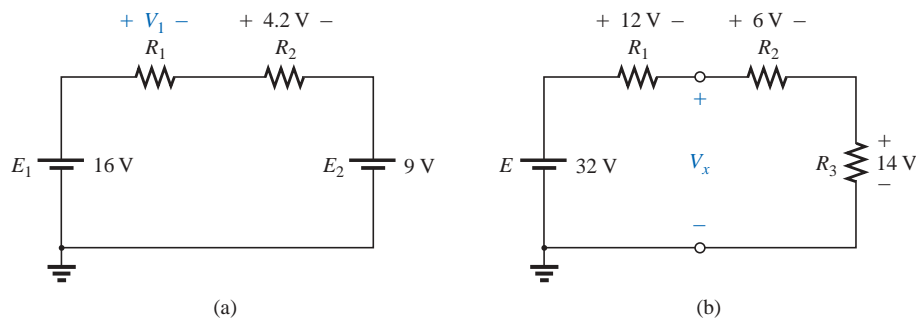
FIG. 5.14

Example 5.4.

Solution: When applying Kirchhoff's voltage law, be sure to concentrate on the polarities of the voltage rise or drop rather than on the

type of element. In other words, do not treat a voltage drop across a resistive element differently from a voltage drop across a source. If the polarity dictates that a drop has occurred, that is the important fact when applying the law. In Fig. 5.14(a), for instance, if we choose the clockwise direction, we will find that there is a drop across the resistors R_1 and R_2 and a drop across the source E_2. All will therefore have a minus sign when Kirchhoff's voltage law is applied.

Application of Kirchhoff's voltage law to the circuit of Fig. 5.14(a) in the clockwise direction will result in

$$+E_1 - V_1 - V_2 - E_2 = 0$$

and $\qquad V_1 = E_1 - V_2 - E_2 = 16\text{ V} - 4.2\text{ V} - 9\text{ V}$
$$= \textbf{2.8 V}$$

The result clearly indicates that there was no need to know the values of the resistors or the current to determine the unknown voltage. Sufficient information was carried by the other voltage levels to permit a determination of the unknown.

In Fig. 5.14(b) the unknown voltage is not across a current-carrying element. However, as indicated in the paragraphs above, Kirchhoff's voltage law is not limited to current-carrying elements. In this case there are two possible paths for finding the unknown. Using the clockwise path, including the voltage source E, will result in

$$+E - V_1 - V_x = 0$$

and $\qquad V_x = E - V_1 = 32\text{ V} - 12\text{ V}$
$$= \textbf{20 V}$$

Using the clockwise direction for the other loop involving R_2 and R_3 will result in

$$+V_x - V_2 - V_3 = 0$$

and $\qquad V_x = V_2 + V_3 = 6\text{ V} + 14\text{ V}$
$$= \textbf{20 V}$$

matching the result above.

EXAMPLE 5.5 Find V_1 and V_2 for the network of Fig. 5.15.

Solution: For path 1, starting at point a in a clockwise direction:

$$+25\text{ V} - V_1 + 15\text{ V} = 0$$

and $\qquad\qquad V_1 = \textbf{40 V}$

For path 2, starting at point a in a clockwise direction:

$$-V_2 - 20\text{ V} = 0$$

and $\qquad\qquad V_2 = \textbf{−20 V}$

The minus sign simply indicates that the actual polarities of the potential difference are opposite the assumed polarity indicated in Fig. 5.15.
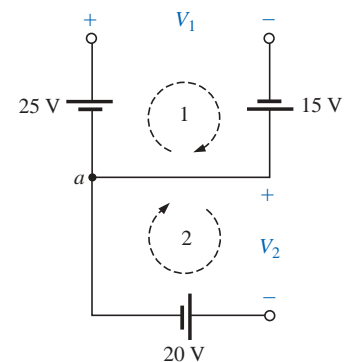
FIG. 5.15
Example 5.5.

The next example will emphasize the fact that when we are applying Kirchhoff's voltage law, it is the polarities of the voltage rise or drop that are the important parameters, and not the type of element involved.

EXAMPLE 5.6 Using Kirchhoff's voltage law, determine the unknown voltages for the network of Fig. 5.16.
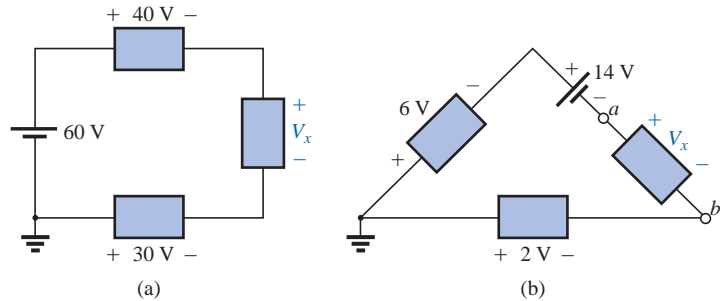
FIG. 5.16
Example 5.6.

Solution: Note in each circuit that there are various polarities across the unknown elements since they can contain any mixture of components. Applying Kirchhoff's voltage law to the network of Fig. 5.16(a) in the clockwise direction will result in

$$60 \text{ V} - 40 \text{ V} - V_x + 30 \text{ V} = 0$$

and $\quad V_x = 60 \text{ V} + 30 \text{ V} - 40 \text{ V} = 90 \text{ V} - 40 \text{ V}$
$$= \textbf{50 V}$$

In Fig. 5.16(b) the polarity of the unknown voltage is not provided. In such cases, make an assumption about the polarity, and apply Kirchhoff's voltage law as before. If the result has a plus sign, the assumed polarity was correct. If it has a minus sign, the magnitude is correct, but the assumed polarity has to be reversed. In this case if we assume *a* to be positive and *b* to be negative, an application of Kirchhoff's voltage law in the clockwise direction will result in

$$-6 \text{ V} - 14 \text{ V} - V_x + 2 \text{ V} = 0$$

and $\quad\quad\quad\quad\quad V_x = -20 \text{ V} + 2 \text{ V}$
$$= \textbf{-18 V}$$

Since the result is negative, we know that *a* should be negative and *b* should be positive, but the magnitude of 18 V is correct.
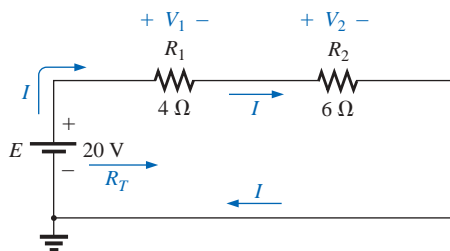
FIG. 5.17
Example 5.7.

EXAMPLE 5.7 For the circuit of Fig. 5.17:
a. Find R_T.
b. Find I.
c. Find V_1 and V_2.
d. Find the power to the 4-Ω and 6-Ω resistors.
e. Find the power delivered by the battery, and compare it to that dissipated by the 4-Ω and 6-Ω resistors combined.
f. Verify Kirchhoff's voltage law (clockwise direction).

Solutions:

a. $R_T = R_1 + R_2 = 4 \text{ Ω} + 6 \text{ Ω} = \textbf{10 Ω}$

b. $I = \dfrac{E}{R_T} = \dfrac{20 \text{ V}}{10 \text{ Ω}} = \textbf{2 A}$

c. $V_1 = IR_1 = (2 \text{ A})(4 \text{ }\Omega) = \textbf{8 V}$
   $V_2 = IR_2 = (2 \text{ A})(6 \text{ }\Omega) = \textbf{12 V}$

d. $P_{4\Omega} = \dfrac{V_1^2}{R_1} = \dfrac{(8 \text{ V})^2}{4} = \dfrac{64}{4} = \textbf{16 W}$

   $P_{6\Omega} = I^2R_2 = (2 \text{ A})^2(6 \text{ }\Omega) = (4)(6) = \textbf{24 W}$

e. $P_E = EI = (20 \text{ V})(2 \text{ A}) = \textbf{40 W}$
   $P_E = P_{4\Omega} + P_{6\Omega}$
   $40 \text{ W} = 16 \text{ W} + 24 \text{ W}$
   $40 \text{ W} = 40 \text{ W}$ (checks)

f. $\Sigma_C V = +E - V_1 - V_2 = 0$
   $E = V_1 + V_2$
   $20 \text{ V} = 8 \text{ V} + 12 \text{ V}$
   $20 \text{ V} = 20 \text{ V}$ (checks)
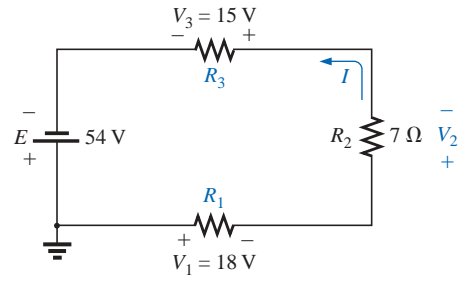
FIG. 5.18
Example 5.8.

EXAMPLE 5.8 For the circuit of Fig. 5.18:
a. Determine V_2 using Kirchhoff's voltage law.
b. Determine I.
c. Find R_1 and R_3.

Solutions:
a. Kirchhoff's voltage law (clockwise direction):

$$-E + V_3 + V_2 + V_1 = 0$$

or $\qquad\qquad E = V_1 + V_2 + V_3$

and $V_2 = E - V_1 - V_3 = 54 \text{ V} - 18 \text{ V} - 15 \text{ V} = \textbf{21 V}$

b. $I = \dfrac{V_2}{R_2} = \dfrac{21 \text{ V}}{7 \text{ }\Omega} = \textbf{3 A}$

c. $R_1 = \dfrac{V_1}{I} = \dfrac{18 \text{ V}}{3 \text{ A}} = \textbf{6 }\boldsymbol{\Omega}$

   $R_3 = \dfrac{V_3}{I} = \dfrac{15 \text{ V}}{3 \text{ A}} = \textbf{5 }\boldsymbol{\Omega}$

FIG. 5.19
Series dc circuit with elements to be inter-changed.

5.5 INTERCHANGING SERIES ELEMENTS

The elements of a series circuit can be interchanged without affecting the total resistance, current, or power to each element. For instance, the network of Fig. 5.19 can be redrawn as shown in Fig. 5.20 without affecting I or V_2. The total resistance R_T is 35 Ω in both cases, and $I = 70 \text{ V}/35 \text{ }\Omega = 2 \text{ A}$. The voltage $V_2 = IR_2 = (2 \text{ A})(5 \text{ }\Omega) = 10 \text{ V}$ for both configurations.

FIG. 5.20
Circuit of Fig. 5.19 with R_2 and R_3 interchanged.
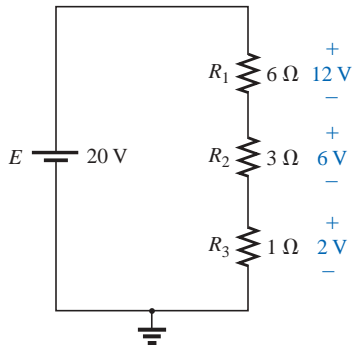
EXAMPLE 5.9 Determine I and the voltage across the 7-Ω resistor for the network of Fig. 5.21.

Solution: The network is redrawn in Fig. 5.22.

$$R_T = (2)(4 \text{ }\Omega) + 7 \text{ }\Omega = 15 \text{ }\Omega$$

$$I = \dfrac{E}{R_T} = \dfrac{37.5 \text{ V}}{15 \text{ }\Omega} = \textbf{2.5 A}$$

$$V_{7\Omega} = IR = (2.5 \text{ A})(7 \text{ }\Omega) = \textbf{17.5 V}$$
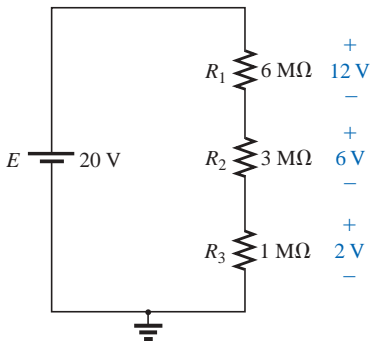
FIG. 5.21
Example 5.9.

FIG. 5.22
Redrawing the circuit of Fig. 5.21.

5.6 VOLTAGE DIVIDER RULE

In a series circuit,

the voltage across the resistive elements will divide as the magnitude of the resistance levels.

For example, the voltages across the resistive elements of Fig. 5.23 are provided. The largest resistor of 6 Ω captures the bulk of the applied voltage, while the smallest resistor R_3 has the least. Note in addition that, since the resistance level of R_1 is 6 times that of R_3, the voltage across R_1 is 6 times that of R_3. The fact that the resistance level of R_2 is 3 times that of R_1 results in three times the voltage across R_2. Finally, since R_1 is twice R_2, the voltage across R_1 is twice that of R_2. In general, therefore, the voltage across series resistors will have the same ratio as their resistance levels.

It is particularly interesting to note that, if the resistance levels of all the resistors of Fig. 5.23 are increased by the same amount, as shown in Fig. 5.24, the voltage levels will all remain the same. In other words, even though the resistance levels were increased by a factor of 1 million, the voltage ratios remain the same. Clearly, therefore, it is the ratio of resistor values that counts when it comes to voltage division and not the relative magnitude of all the resistors. The current level of the network will be severely affected by the change in resistance level from Fig. 5.23 to Fig. 5.24, but the voltage levels will remain the same.

Based on the above, a first glance at the series network of Fig. 5.25 should suggest that the major part of the applied voltage will appear across the 1-MΩ resistor and very little across the 100-Ω resistor. In fact, 1 MΩ = (1000)1 kΩ = (10,000)100 Ω, revealing that V_1 = $1000V_2$ = $10,000V_3$.

Solving for the current and then the three voltage levels will result in

$$I = \frac{E}{R_T} = \frac{100 \text{ V}}{1,001,100 \text{ }\Omega} \cong 99.89 \text{ }\mu\text{A}$$

and

$$V_1 = IR_1 = (99.89 \text{ }\mu\text{A})(1 \text{ M}\Omega) = \mathbf{99.89 \text{ V}}$$
$$V_2 = IR_2 = (99.89 \text{ }\mu\text{A})(1 \text{ k}\Omega) = \mathbf{99.89 \text{ mV}} = 0.09989 \text{ V}$$
$$V_3 = IR_3 = (99.89 \text{ }\mu\text{A})(100 \text{ }\Omega) = \mathbf{9.989 \text{ mV}} = 0.009989 \text{ V}$$

clearly substantiating the above conclusions. For the future, therefore, use this approach to estimate the share of the input voltage across series elements to act as a check against the actual calculations or to simply obtain an estimate with a minimum of effort.

FIG. 5.23
Revealing how the voltage will divide across series resistive elements.

FIG. 5.24
The ratio of the resistive values determines the voltage division of a series dc circuit.

FIG. 5.25
The largest of the series resistive elements will capture the major share of the applied voltage.

In the above discussion the current was determined before the voltages of the network were determined. There is, however, a method referred to as the **voltage divider rule** (VDR) that permits determining the voltage levels without first finding the current. The rule can be derived by analyzing the network of Fig. 5.26.

$$R_T = R_1 + R_2$$

and

$$I = \frac{E}{R_T}$$

Applying Ohm's law:

$$V_1 = IR_1 = \left(\frac{E}{R_T}\right)R_1 = \frac{R_1 E}{R_T}$$

with

$$V_2 = IR_2 = \left(\frac{E}{R_T}\right)R_2 = \frac{R_2 E}{R_T}$$

Note that the format for V_1 and V_2 is

$$V_x = \frac{R_x E}{R_T} \qquad \text{(voltage divider rule)} \qquad \textbf{(5.10)}$$

where V_x is the voltage across R_x, E is the impressed voltage across the series elements, and R_T is the total resistance of the series circuit.

In words, the **voltage divider rule** states that

the voltage across a resistor in a series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements.
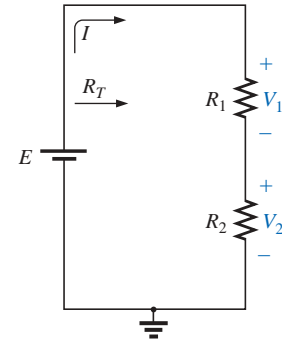
FIG. 5.26

Developing the voltage divider rule.

EXAMPLE 5.10 Determine the voltage V_1 for the network of Fig. 5.27.

Solution: Eq. (5.10):

$$V_1 = \frac{R_1 E}{R_T} = \frac{R_1 E}{R_1 + R_2} = \frac{(20\ \Omega)(64\ \text{V})}{20\ \Omega + 60\ \Omega} = \frac{1280\ \text{V}}{80} = \textbf{16 V}$$
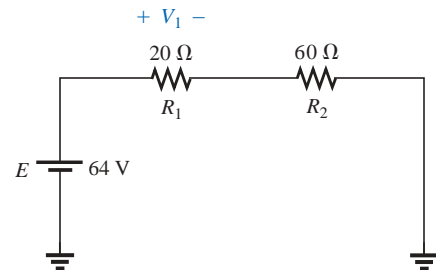
EXAMPLE 5.11 Using the voltage divider rule, determine the voltages V_1 and V_3 for the series circuit of Fig. 5.28.

Solution:

$$V_1 = \frac{R_1 E}{R_T} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{2\ \text{k}\Omega + 5\ \text{k}\Omega + 8\ \text{k}\Omega} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega}$$

$$= \frac{(2 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega} = \frac{90\ \text{V}}{15} = \textbf{6 V}$$

$$V_3 = \frac{R_3 E}{R_T} = \frac{(8\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega} = \frac{(8 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega}$$

$$= \frac{360\ \text{V}}{15} = \textbf{24 V}$$
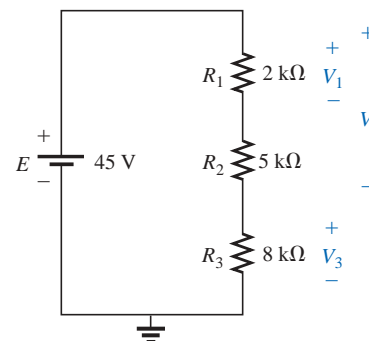
FIG. 5.27

Example 5.10.

FIG. 5.28

Example 5.11.

The rule can be extended to the voltage across two or more series elements if the resistance in the numerator of Eq. (5.10) is expanded to

include the total resistance of the series elements that the voltage is to be found across (R'); that is,

$$V' = \frac{R'E}{R_T} \qquad \text{(volts)} \qquad \textbf{(5.11)}$$

EXAMPLE 5.12 Determine the voltage V' in Fig. 5.28 across resistors R_1 and R_2.

Solution:

$$V' = \frac{R'E}{R_T} = \frac{(2 \text{ k}\Omega + 5 \text{ k}\Omega)(45 \text{ V})}{15 \text{ k}\Omega} = \frac{(7 \text{ k}\Omega)(45 \text{ V})}{15 \text{ k}\Omega} = \textbf{21 V}$$

There is also no need for the voltage E in the equation to be the source voltage of the network. For example, if V is the total voltage across a number of series elements such as those shown in Fig. 5.29, then

$$V_{2\Omega} = \frac{(2 \ \Omega)(27 \text{ V})}{4 \ \Omega + 2 \ \Omega + 3 \ \Omega} = \frac{54 \text{ V}}{9} = \textbf{6 V}$$
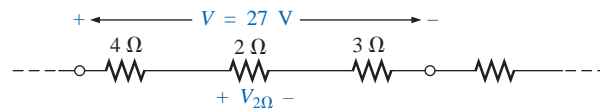
FIG. 5.29

The total voltage across series elements need not be an independent voltage source.
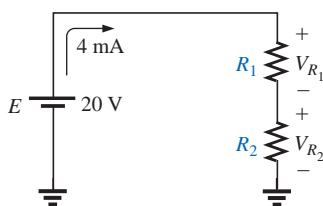
FIG. 5.30

Example 5.13.

EXAMPLE 5.13 Design the voltage divider of Fig. 5.30 such that $V_{R_1} = 4V_{R_2}$.

Solution: The total resistance is defined by

$$R_T = \frac{E}{I} = \frac{20 \text{ V}}{4 \text{ mA}} = 5 \text{ k}\Omega$$

Since $V_{R_1} = 4V_{R_2}$,

$$R_1 = 4R_2$$

Thus $\qquad R_T = R_1 + R_2 = 4R_2 + R_2 = 5R_2$

and $\qquad 5R_2 = 5 \text{ k}\Omega$
$$R_2 = \textbf{1 k}\boldsymbol{\Omega}$$

and $\qquad R_1 = 4R_2 = \textbf{4 k}\boldsymbol{\Omega}$

5.7 NOTATION

Notation will play an increasingly important role in the analysis to follow. It is important, therefore, that we begin to examine the notation used throughout the industry.

Voltage Sources and Ground

Except for a few special cases, electrical and electronic systems are grounded for reference and safety purposes. The symbol for the ground connection appears in Fig. 5.31 with its defined potential level—zero volts. None of the circuits discussed thus far have contained the ground connection. If Fig. 5.4(a) were redrawn with a grounded supply, it might appear as shown in Fig. 5.32(a), (b), or (c). In any case, it is understood that the negative terminal of the battery and the bottom of the resistor R_2 are at ground potential. Although Figure 5.32(c) shows

• 0 V

FIG. 5.31
Ground potential.

(a) (b) (c)
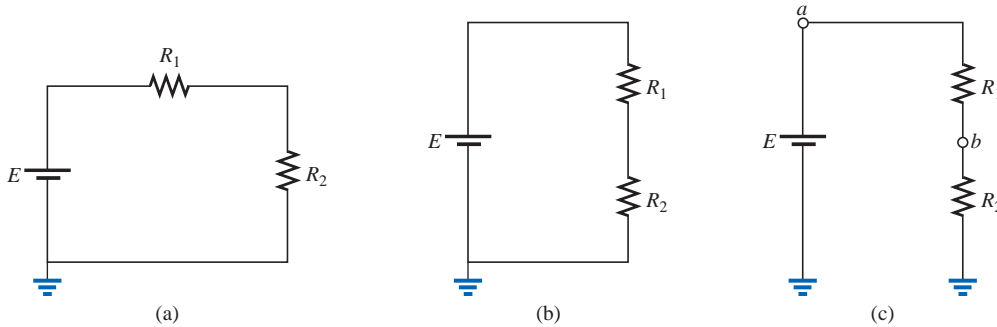
FIG. 5.32
Three ways to sketch the same series dc circuit.

no connection between the two grounds, it is recognized that such a connection exists for the continuous flow of charge. If $E = 12$ V, then point a is 12 V positive with respect to ground potential, and 12 V exist across the series combination of resistors R_1 and R_2. If a voltmeter placed from point b to ground reads 4 V, then the voltage across R_2 is 4 V, with the higher potential at point b.

On large schematics where space is at a premium and clarity is important, voltage sources may be indicated as shown in Figs. 5.33(a) and 5.34(a) rather than as illustrated in Figs. 5.33(b) and 5.34(b). In addition, potential levels may be indicated as in Fig. 5.35, to permit a rapid check of the potential levels at various points in a network with respect to ground to ensure that the system is operating properly.

(a) (b)
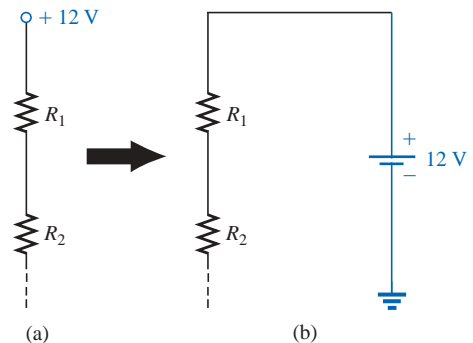
FIG. 5.33
Replacing the special notation for a dc voltage source with the standard symbol.

(a) (b)
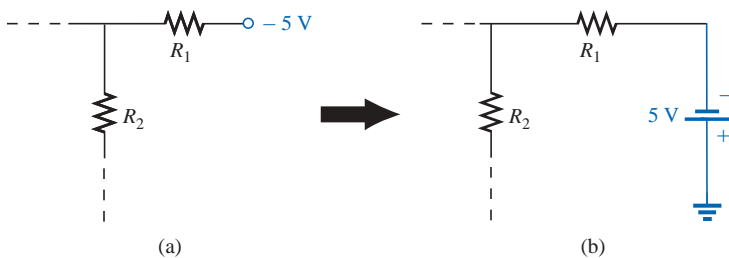
FIG. 5.34
Replacing the notation for a negative dc supply with the standard notation.
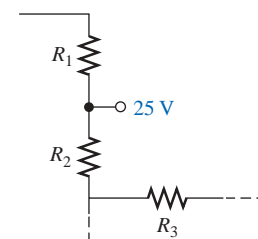
FIG. 5.35
The expected voltage level at a particular point in a network of the system is functioning properly.

Double-Subscript Notation

The fact that voltage is an *across* variable and exists between two points has resulted in a double-subscript notation that defines the first

subscript as the higher potential. In Fig. 5.36(a), the two points that define the voltage across the resistor R are denoted by a and b. Since a is the first subscript for V_{ab}, point a must have a higher potential than point b if V_{ab} is to have a positive value. If, in fact, point b is at a higher potential than point a, V_{ab} will have a negative value, as indicated in Fig. 5.36(b).
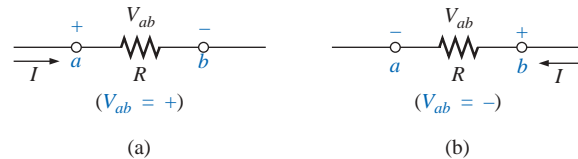
FIG. 5.36
Defining the sign for double-subscript notation.

In summary:

The double-subscript notation V_{ab} specifies point a as the higher potential. If this is not the case, a negative sign must be associated with the magnitude of V_{ab}.

In other words,

the voltage V_{ab} is the voltage at point a with respect to (w.r.t.) point b.

Single-Subscript Notation

If point b of the notation V_{ab} is specified as ground potential (zero volts), then a single-subscript notation can be employed that provides the voltage at a point with respect to ground.

In Fig. 5.37, V_a is the voltage from point a to ground. In this case it is obviously 10 V since it is right across the source voltage E. The voltage V_b is the voltage from point b to ground. Because it is directly across the 4-Ω resistor, $V_b = 4$ V.

In summary:

The single-subscript notation V_a specifies the voltage at point a with respect to ground (zero volts). If the voltage is less than zero volts, a negative sign must be associated with the magnitude of V_a.
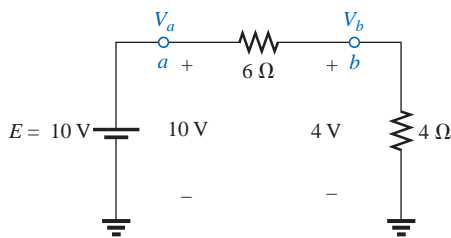
FIG. 5.37
Defining the use of single-subscript notation for voltage levels.

General Comments

A particularly useful relationship can now be established that will have extensive applications in the analysis of electronic circuits. For the above notational standards, the following relationship exists:

$$V_{ab} = V_a - V_b \qquad \textbf{(5.12)}$$

In other words, if the voltage at points a and b is known with respect to ground, then the voltage V_{ab} can be determined using Eq. (5.12). In Fig. 5.37, for example,

$$V_{ab} = V_a - V_b = 10 \text{ V} - 4 \text{ V}$$
$$= 6 \text{ V}$$

EXAMPLE 5.14 Find the voltage V_{ab} for the conditions of Fig. 5.38.

Solution: Applying Eq. (5.12):

$$V_{ab} = V_a - V_b = 16 \text{ V} - 20 \text{ V}$$
$$= -4 \text{ V}$$

$V_a = +16 \text{ V}$ $V_b = +20 \text{ V}$

FIG. 5.38
Example 5.14.

Note the negative sign to reflect the fact that point b is at a higher potential than point a.

EXAMPLE 5.15 Find the voltage V_a for the configuration of Fig. 5.39.

Solution: Applying Eq. (5.12):

$$V_{ab} = V_a - V_b$$

and
$$V_a = V_{ab} + V_b = 5 \text{ V} + 4 \text{ V}$$
$$= 9 \text{ V}$$

V_a $V_{ab} = +5 \text{ V}$ $V_b = 4 \text{ V}$

FIG. 5.39
Example 5.15.

EXAMPLE 5.16 Find the voltage V_{ab} for the configuration of Fig. 5.40.

Solution: Applying Eq. (5.12):

$$V_{ab} = V_a - V_b = 20 \text{ V} - (-15 \text{ V}) = 20 \text{ V} + 15 \text{ V}$$
$$= 35 \text{ V}$$

$V_a = +20 \text{ V}$

R $10 \text{ k}\Omega$ V_{ab}

$V_b = -15 \text{ V}$

FIG. 5.40
Example 5.16.

Note in Example 5.16 the care that must be taken with the signs when applying the equation. The voltage is dropping from a high level of $+20$ V to a negative voltage of -15 V. As shown in Fig. 5.41, this represents a drop in voltage of 35 V. In some ways it's like going from a positive checking balance of $20 to owing $15; the total expenditure is $35.

EXAMPLE 5.17 Find the voltages V_b, V_c, and V_{ac} for the network of Fig. 5.42.
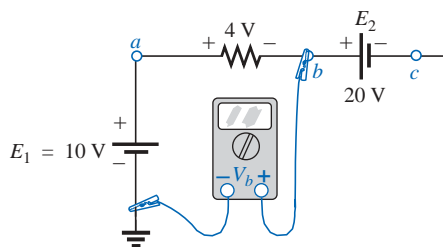
FIG. 5.42
Example 5.17.

Solution: Starting at ground potential (zero volts), we proceed through a rise of 10 V to reach point a and then pass through a drop in potential of 4 V to point b. The result is that the meter will read

$$V_b = +10 \text{ V} - 4 \text{ V} = 6 \text{ V}$$

as clearly demonstrated by Fig. 5.43.

$V_a = 20 \text{ V}$

$V_{ab} = 35 \text{ V}$

Gnd (0 V)

$V_b = -15 \text{ V}$
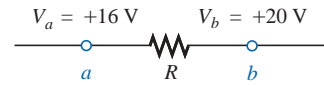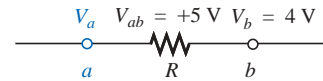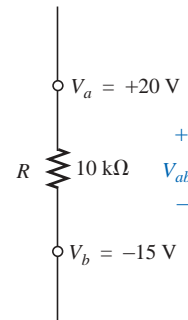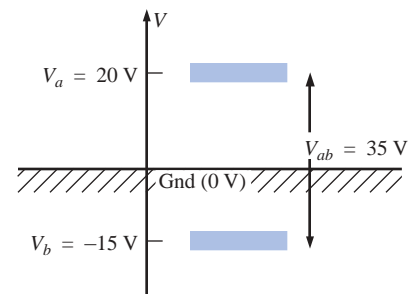
FIG. 5.41
The impact of positive and negative voltages on the total voltage drop.

10 V

4 V

6 V

Gnd (0 V)
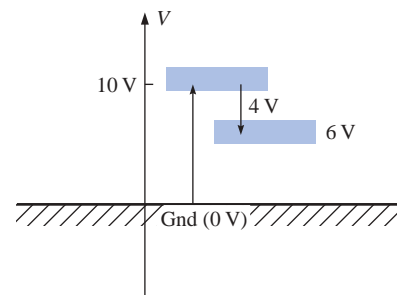
FIG. 5.43
Determining V_b using the defined voltage levels.

If we then proceed to point c, there is an additional drop of 20 V, resulting in

$$V_c = V_b - 20\,\text{V} = 6\,\text{V} - 20\,\text{V} = \mathbf{-14\,V}$$

as shown in Fig. 5.44.
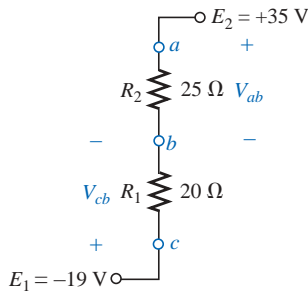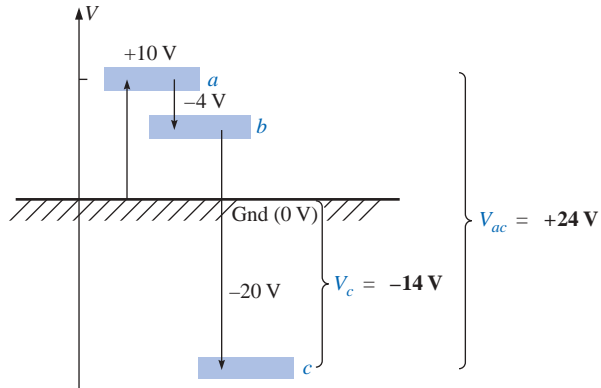
FIG. 5.44

Review of the potential levels for the circuit of Fig. 5.42.

FIG. 5.45

Example 5.18.

The voltage V_{ac} can be obtained using Eq. (5.12) or by simply referring to Fig. 5.44:

$$V_{ac} = V_a - V_c = 10\,\text{V} - (-14\,\text{V})$$
$$= \mathbf{24\,V}$$

EXAMPLE 5.18 Determine V_{ab}, V_{cb}, and V_c for the network of Fig. 5.45.

Solution: There are two ways to approach this problem. The first is to sketch the diagram of Fig. 5.46 and note that there is a 54-V drop across the series resistors R_1 and R_2. The current can then be determined using Ohm's law and the voltage levels as follows:

$$I = \frac{54\,\text{V}}{45\,\Omega} = 1.2\,\text{A}$$

$$V_{ab} = IR_2 = (1.2\,\text{A})(25\,\Omega) = \mathbf{30\,V}$$
$$V_{cb} = -IR_1 = -(1.2\,\text{A})(20\,\Omega) = \mathbf{-24\,V}$$
$$V_c = E_1 = \mathbf{-19\,V}$$

FIG. 5.46

Determining the total voltage drop across the resistive elements of Fig. 5.45.

The other approach is to redraw the network as shown in Fig. 5.47 to clearly establish the aiding effect of E_1 and E_2 and then solve the resulting series circuit.

$$I = \frac{E_1 + E_2}{R_T} = \frac{19\,\text{V} + 35\,\text{V}}{45\,\Omega} = \frac{54\,\text{V}}{45\,\Omega} = 1.2\,\text{A}$$

and $\qquad V_{ab} = \mathbf{30\,V} \qquad V_{cb} = \mathbf{-24\,V} \qquad V_c = \mathbf{-19\,V}$
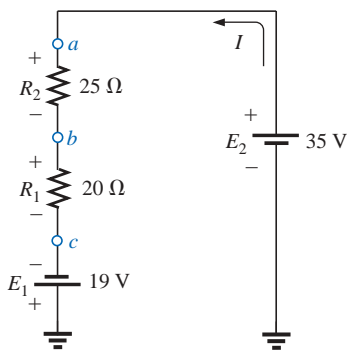
FIG. 5.47

Redrawing the circuit of Fig. 5.45 using standard dc voltage supply symbols.

EXAMPLE 5.19 Using the voltage divider rule, determine the voltages V_1 and V_2 of Fig. 5.48.

Solution: Redrawing the network with the standard battery symbol will result in the network of Fig. 5.49. Applying the voltage divider rule,

$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(4\ \Omega)(24\ \text{V})}{4\ \Omega + 2\ \Omega} = \textbf{16 V}$$

$$V_2 = \frac{R_2 E}{R_1 + R_2} = \frac{(2\ \Omega)(24\ \text{V})}{4\ \Omega + 2\ \Omega} = \textbf{8 V}$$
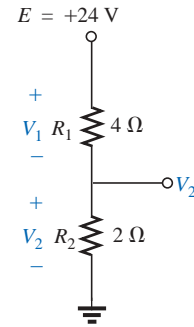
EXAMPLE 5.20 For the network of Fig. 5.50:
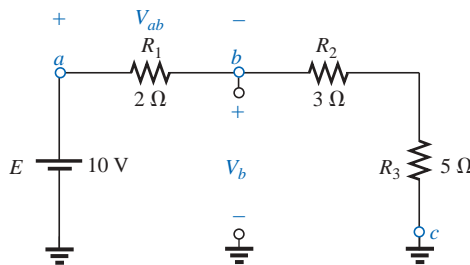
FIG. 5.50
Example 5.20.

a. Calculate V_{ab}.
b. Determine V_b.
c. Calculate V_c.

Solutions:
a. Voltage divider rule:

$$V_{ab} = \frac{R_1 E}{R_T} = \frac{(2\ \Omega)(10\ \text{V})}{2\ \Omega + 3\ \Omega + 5\ \Omega} = \textbf{+2 V}$$

b. Voltage divider rule:

$$V_b = V_{R_2} + V_{R_3} = \frac{(R_2 + R_3)E}{R_T} = \frac{(3\ \Omega + 5\ \Omega)(10\ \text{V})}{10\ \Omega} = \textbf{8 V}$$

$$\text{or} \quad V_b = V_a - V_{ab} = E - V_{ab} = 10\ \text{V} - 2\ \text{V} = \textbf{8 V}$$

c. $V_c = $ ground potential $= \textbf{0 V}$

5.8 INTERNAL RESISTANCE OF VOLTAGE SOURCES

Every source of voltage, whether a generator, battery, or laboratory supply as shown in Fig. 5.51(a), will have some **internal resistance.** The equivalent circuit of any source of voltage will therefore appear as shown in Fig. 5.51(b). In this section, we will examine the effect of the internal resistance on the output voltage so that any unexpected changes in terminal characteristics can be explained.

In all the circuit analyses to this point, the ideal voltage source (no internal resistance) was used [see Fig. 5.52(a)]. The ideal voltage source has no internal resistance and an output voltage of E volts with no load or full load. In the practical case [Fig. 5.52(b)], where we con-

FIG. 5.48
Example 5.19.

FIG. 5.49
Circuit of Fig. 5.48 redrawn.
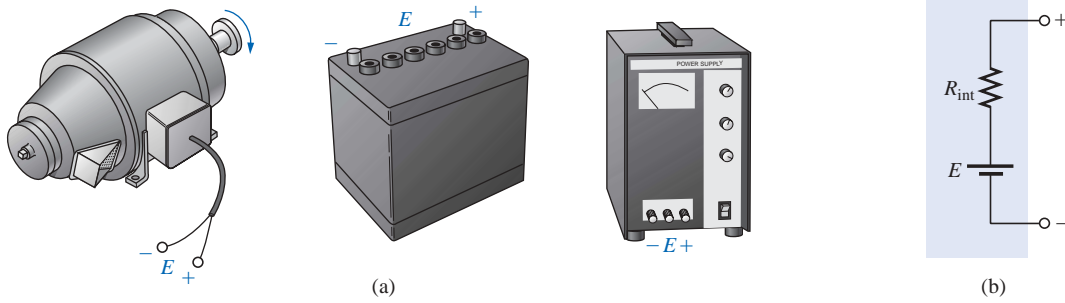
FIG. 5.51
(a) Sources of dc voltage; (b) equivalent circuit.
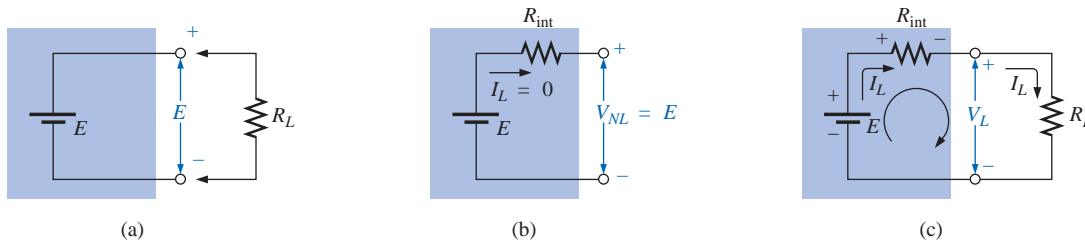
FIG. 5.52
Voltage source: (a) ideal, $R_{int} = 0\ \Omega$; (b) determining V_{NL}; (c) determining R_{int}.

sider the effects of the internal resistance, the output voltage will be E volts only when no-load ($I_L = 0$) conditions exist. When a load is connected [Fig. 5.52(c)], the output voltage of the voltage source will decrease due to the voltage drop across the internal resistance.

By applying Kirchhoff's voltage law around the indicated loop of Fig. 5.52(c), we obtain

$$E - I_L R_{int} - V_L = 0$$

or, since

$$E = V_{NL}$$

we have

$$V_{NL} - I_L R_{int} - V_L = 0$$

and

$$\boxed{V_L = V_{NL} - I_L R_{int}} \tag{5.13}$$

If the value of R_{int} is not available, it can be found by first solving for R_{int} in the equation just derived for V_L; that is,

$$R_{int} = \frac{V_{NL} - V_L}{I_L} = \frac{V_{NL}}{I_L} - \frac{I_L R_L}{I_L}$$

and

$$\boxed{R_{int} = \frac{V_{NL}}{I_L} - R_L} \tag{5.14}$$

A plot of the output voltage versus current appears in Fig. 5.53 for the dc generator having the circuit representation of Fig. 5.51(b). Note that any increase in load demand, starting at any level, causes an additional drop in terminal voltage due to the increasing loss in potential across the internal resistance. At maximum current, denoted by I_{FL}, the
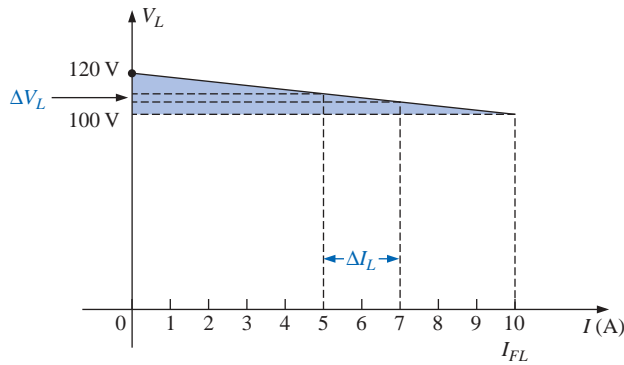
FIG. 5.53

V_L versus I_L for a dc generator with $R_{int} = 2\ \Omega$.

voltage across the internal resistance is $V_{int} = I_{FL}R_{int} = (10\text{ A})(2\ \Omega) = 20$ V, and the terminal voltage has dropped to 100 V—a significant difference when you can ideally expect a 120-V generator to provide the full 120 V if you stay below the listed full-load current. Eventually, if the load current were permitted to increase without limit, the voltage across the internal resistance would equal the supply voltage, and the terminal voltage would be zero. The larger the internal resistance, the steeper is the slope of the characteristics of Fig. 5.53. In fact, for any chosen interval of voltage or current, the magnitude of the internal resistance is given by

$$R_{int} = \frac{\Delta V_L}{\Delta I_L} \tag{5.15}$$

For the chosen interval of 5–7 A ($\Delta I_L = 2$ A) on Fig. 5.53, ΔV_L is 4 V, and $R_{int} = \Delta V_L/\Delta I_L = 4$ V/2 A $= 2\ \Omega$.

A direct consequence of the loss in output voltage is a loss in power delivered to the load. Multiplying both sides of Eq. (5.13) by the current I_L in the circuit, we obtain

$$
\underbrace{I_L V_L}_{\substack{\text{Power} \\ \text{to load}}} = \underbrace{I_L V_{NL}}_{\substack{\text{Power output} \\ \text{by battery}}} - \underbrace{I_L^2 R_{int}}_{\substack{\text{Power loss in} \\ \text{the form of heat}}} \tag{5.16}
$$

EXAMPLE 5.21 Before a load is applied, the terminal voltage of the power supply of Fig. 5.54(a) is set to 40 V. When a load of 500 Ω is attached, as shown in Fig. 5.54(b), the terminal voltage drops to 38.5 V. What happened to the remainder of the no-load voltage, and what is the internal resistance of the source?

Solution: The difference of 40 V − 38.5 V = 1.5 V now appears across the internal resistance of the source. The load current is 38.5 V/0.5 kΩ = 77 mA. Applying Eq. (5.14),

$$R_{int} = \frac{V_{NL}}{I_L} - R_L = \frac{40\text{ V}}{77\text{ mA}} - 0.5\text{ k}\Omega$$

$$= 519.48\ \Omega - 500\ \Omega = \mathbf{19.48\ \Omega}$$
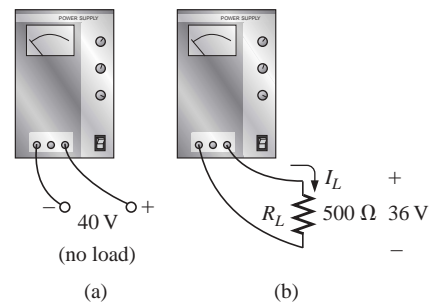
FIG. 5.54

Example 5.21.
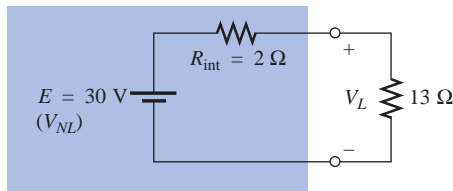
FIG. 5.55
Example 5.22.

EXAMPLE 5.22 The battery of Fig. 5.55 has an internal resistance of 2 Ω. Find the voltage V_L and the power lost to the internal resistance if the applied load is a 13-Ω resistor.

Solution:

$$I_L = \frac{30 \text{ V}}{2 \text{ Ω} + 13 \text{ Ω}} = \frac{30 \text{ V}}{15 \text{ Ω}} = 2 \text{ A}$$

$$V_L = V_{NL} - I_L R_{int} = 30 \text{ V} - (2 \text{ A})(2 \text{ Ω}) = \textbf{26 V}$$

$$P_{lost} = I_L^2 R_{int} = (2 \text{ A})^2(2 \text{ Ω}) = (4)(2) = \textbf{8 W}$$

Procedures for measuring R_{int} will be described in Section 5.10.

5.9 VOLTAGE REGULATION

For any supply, ideal conditions dictate that for the range of load demand (I_L), the terminal voltage remain fixed in magnitude. In other words, if a supply is set for 12 V, it is desirable that it maintain this terminal voltage, even though the current demand on the supply may vary. A measure of how close a supply will come to ideal conditions is given by the voltage regulation characteristic. By definition, the **voltage regulation** (*VR*) of a supply between the limits of full-load and no-load conditions (Fig. 5.56) is given by the following:
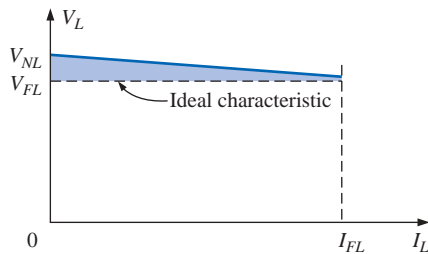
FIG. 5.56
Defining voltage regulation.

$$\text{Voltage regulation } (VR)\% = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% \quad \textbf{(5.17)}$$

For ideal conditions, $V_{FL} = V_{NL}$ and $VR\% = 0$. Therefore, *the smaller the voltage regulation, the less the variation in terminal voltage with change in load.*

It can be shown with a short derivation that the voltage regulation is also given by

$$VR\% = \frac{R_{int}}{R_L} \times 100\% \quad \textbf{(5.18)}$$

In other words, the smaller the internal resistance for the same load, the smaller the regulation and the more ideal the output.

EXAMPLE 5.23 Calculate the voltage regulation of a supply having the characteristics of Fig. 5.53.

Solution:

$$VR\% = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% = \frac{120 \text{ V} - 100 \text{ V}}{100 \text{ V}} \times 100\%$$

$$= \frac{20}{100} \times 100\% = \textbf{20\%}$$

EXAMPLE 5.24 Determine the voltage regulation of the supply of Fig. 5.54.

Solution:

$$VR\% = \frac{R_{\text{int}}}{R_L} \times 100\% = \frac{19.48\ \Omega}{500\ \Omega} \times 100\% \cong \mathbf{3.9\%}$$

5.10 MEASUREMENT TECHNIQUES

In Chapter 2, it was noted that ammeters are inserted in the branch in which the current is to be measured. We now realize that such a condition specifies that

ammeters are placed in series with the branch in which the current is to be measured

as shown in Fig. 5.57.

    If the ammeter is to have minimal impact on the behavior of the network, its resistance should be very small (ideally zero ohms) compared to the other series elements of the branch such as the resistor R of Fig. 5.57. If the meter resistance approaches or exceeds 10% of R, it will naturally have a significant impact on the current level it is measuring. It is also noteworthy that the resistances of the separate current scales of the same meter are usually not the same. In fact, the meter resistance normally increases with decreasing current levels. However, for the majority of situations one can simply assume that the internal ammeter resistance is small enough compared to the other circuit elements that it can be ignored.

    For an up-scale (analog meter) or positive (digital meter) reading, an ammeter must be connected with current entering the positive terminal of the meter and leaving the negative terminal, as shown in Fig. 5.58. Since most meters employ a red lead for the positive terminal and a black lead for the negative, simply ensure that current enters the red lead and leaves the black one.

Voltmeters are always hooked up across the element for which the voltage is to be determined.

An up-scale or positive reading on a voltmeter is obtained by being sure that the positive terminal (red lead) is connected to the point of higher potential and the negative terminal (black lead) is connected to the lower potential, as shown in Fig. 5.59.
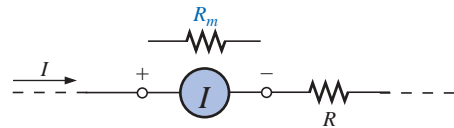
FIG. 5.57
Series connection of an ammeter.
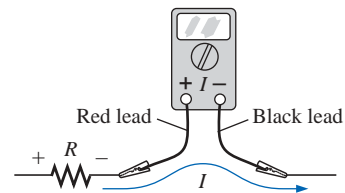
FIG. 5.58
Connecting an ammeter for an up-scale (positive) reading.
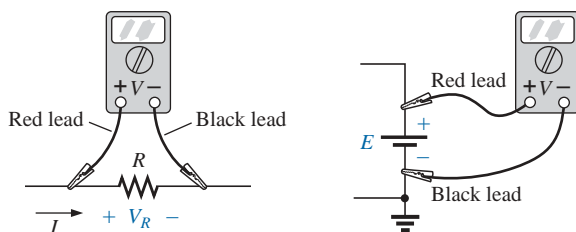
FIG. 5.59
Hooking up a voltmeter to obtain an up-scale (positive) reading.

    For the double-subscript notation, always hook up the red lead to the first subscript and the black lead to the second; that is, to measure the voltage V_{ab} in Fig. 5.60, connect the red lead to point a and the black

lead to point *b*. For single-subscript notation, hook up the red lead to the point of interest and the black lead to ground, as shown in Fig. 5.60 for V_a and V_b.
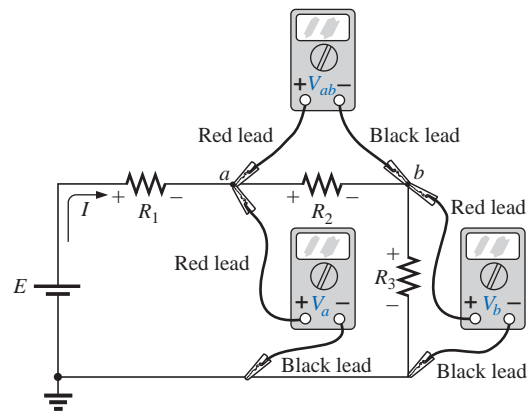
FIG. 5.60

Measuring voltages with double- and single-subscript notation.

The internal resistance of a supply cannot be measured with an ohm-meter due to the voltage present. However, the no-load voltage can be measured by simply hooking up the voltmeter as shown in Fig. 5.61(a). Do not be concerned about the apparent path for current that the meter seems to provide by completing the circuit. The internal resistance of the meter is usually sufficiently high to ensure that the resulting current is so small that it can be ignored. (Voltmeter loading effects will be discussed in detail in Section 6.9.) An ammeter could then be placed directly across the supply, as shown in Fig. 5.61(b), to measure the short-circuit current I_{sc} and R_{int} as determined by Ohm's law: $R_{int} = E_{NL}/I_{sc}$. However, since the internal resistance of the supply may be very low, performing the measurement could result in high current levels that could damage the meter and supply and possibly cause dangerous side effects. The setup of Fig. 5.61(b) is therefore *not* suggested. A better approach would be to apply a resistive load that will result in a supply current of about half the maximum rated value and measure the terminal voltage. Then use Eq. (5.14).
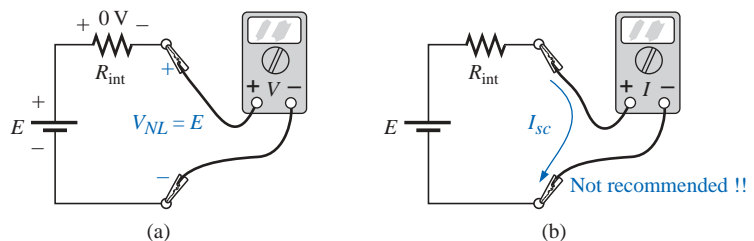
FIG. 5.61

(a) Measuring the no-load voltage E; (b) measuring the short-circuit current.

PROBLEMS

SECTION 5.2   Series Circuits

1. Find the total resistance and current I for each circuit of Fig. 5.71.
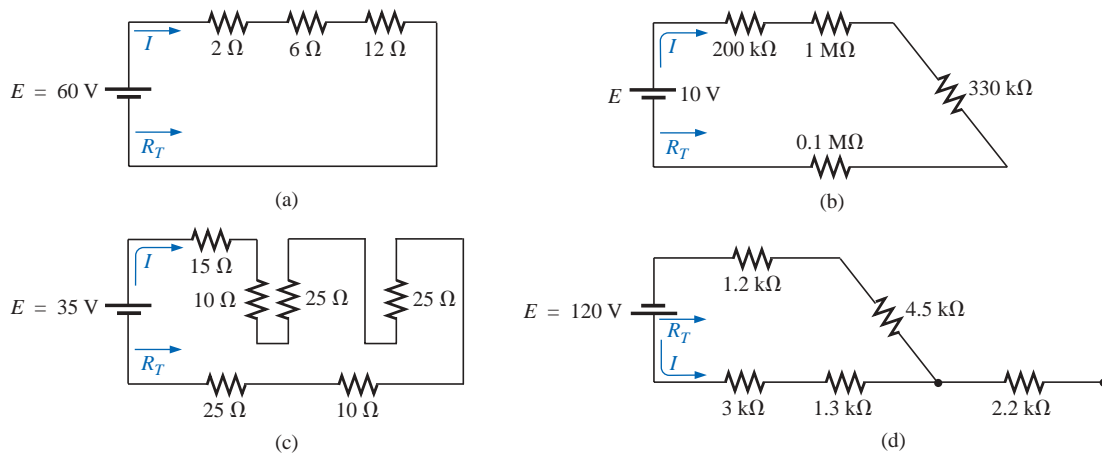
FIG. 5.71
Problems 1 and 36.

2. For the circuits of Fig. 5.72, the total resistance is specified. Find the unknown resistances and the current I for each circuit.
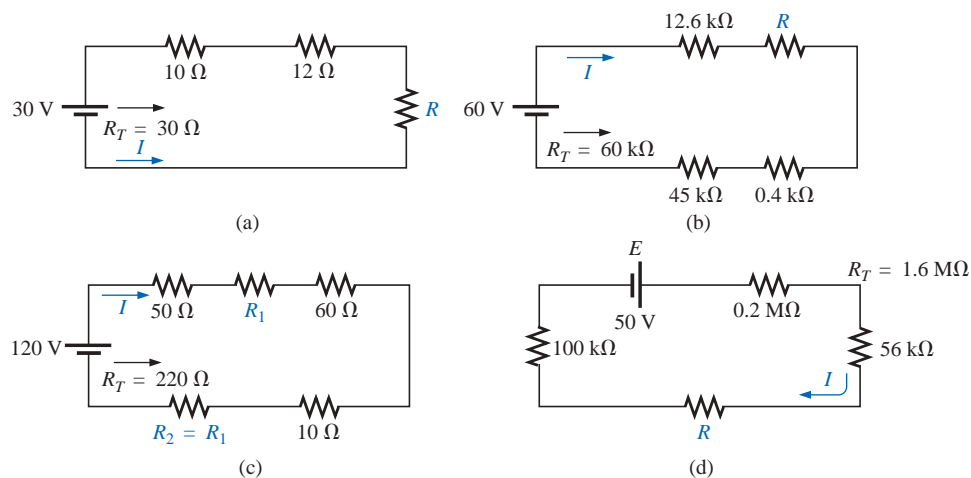
FIG. 5.72
Problem 2.

3. Find the applied voltage E necessary to develop the current specified in each network of Fig. 5.73.

(a)

(b)
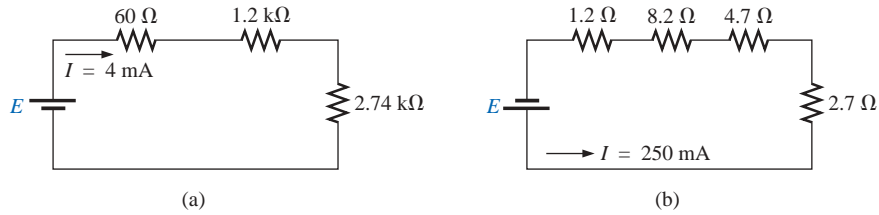
FIG. 5.73
Problem 3.

*4. For each network of Fig. 5.74, determine the current I, the source voltage E, the unknown resistance, and the voltage across each element.

(a)

(b)
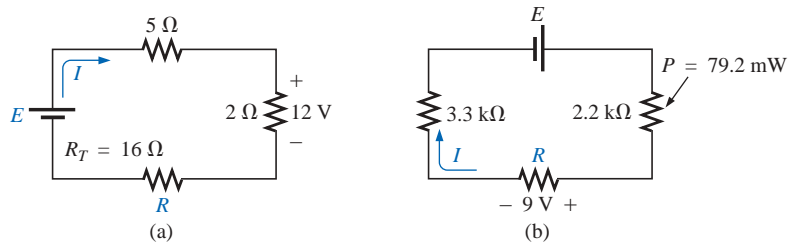
FIG. 5.74
Problem 4.

SECTION 5.3   Voltage Sources in Series

5. Determine the current I and its direction for each network of Fig. 5.75. Before solving for I, redraw each network with a single voltage source.

(a)

(b)
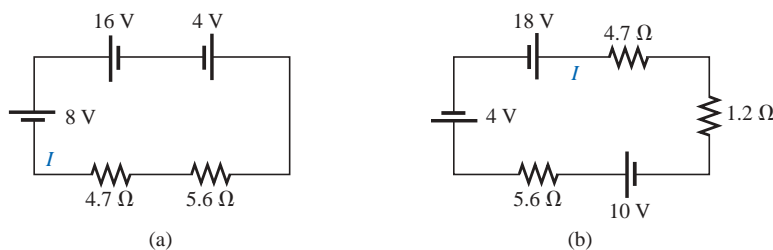
FIG. 5.75
Problem 5.

*6. Find the unknown voltage source and resistor for the networks of Fig. 5.76. Also indicate the direction of the resulting current.
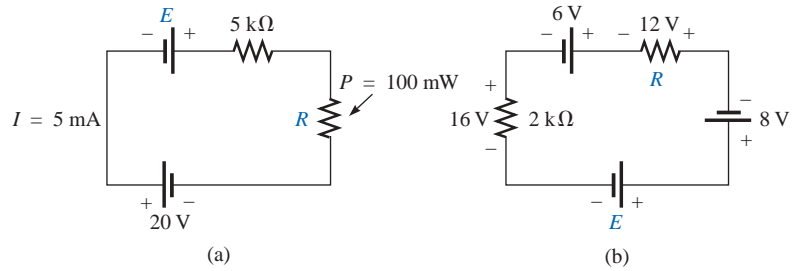
FIG. 5.76
Problem 6.

SECTION 5.4   Kirchhoff's Voltage Law

7. Find V_{ab} with polarity for the circuits of Fig. 5.77. Each box can contain a load or a power supply, or a combination of both.
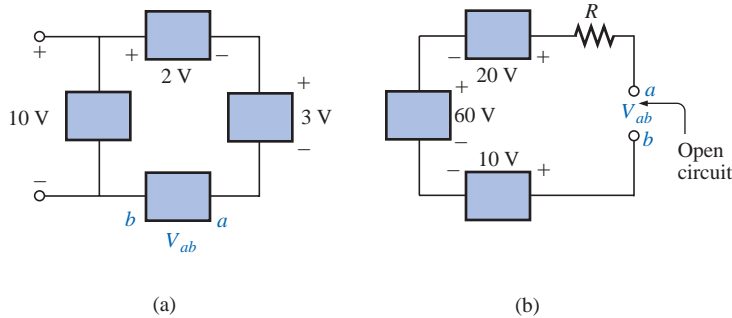
FIG. 5.77
Problem 7.

8. Although the networks of Fig. 5.78 are not simply series circuits, determine the unknown voltages using Kirchhoff's voltage law.
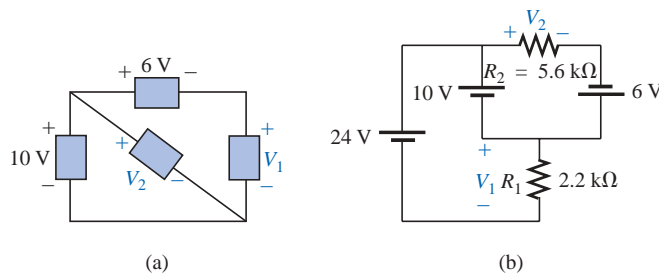
FIG. 5.78
Problem 8.

9. Determine the current I and the voltage V_1 for the network of Fig. 5.79.
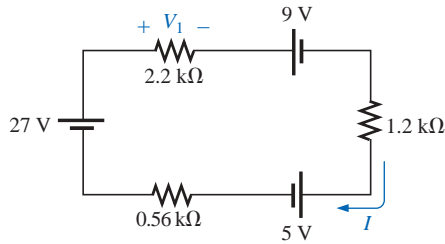
FIG. 5.79
Problem 9.
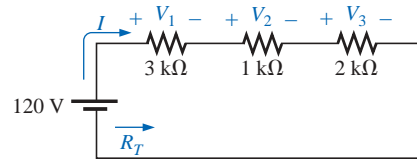
FIG. 5.80
Problem 10.

10. For the circuit of Fig. 5.80:
   a. Find the total resistance, current, and unknown voltage drops.
   b. Verify Kirchhoff's voltage law around the closed loop.
   c. Find the power dissipated by each resistor, and note whether the power delivered is equal to the power dissipated.
   d. If the resistors are available with wattage ratings of 1/2, 1, and 2 W, what minimum wattage rating can be used for each resistor in this circuit?
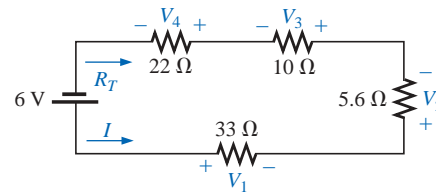
11. Repeat Problem 10 for the circuit of Fig. 5.81.

*12. Find the unknown quantities in the circuits of Fig. 5.82 using the information provided.
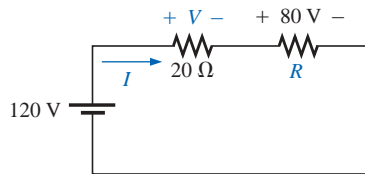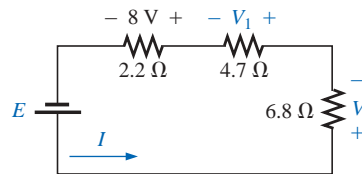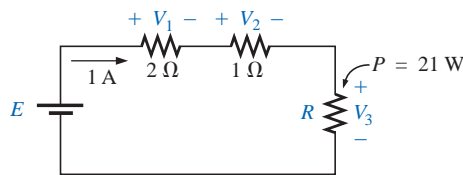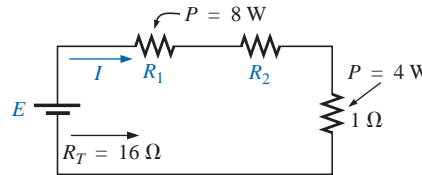
FIG. 5.81
Problem 11.

FIG. 5.82
Problem 12.

13. Eight holiday lights are connected in series as shown in Fig. 5.83.
   a. If the set is connected to a 120-V source, what is the current through the bulbs if each bulb has an internal resistance of $28\frac{1}{8}\,\Omega$?
   b. Determine the power delivered to each bulb.
   c. Calculate the voltage drop across each bulb.
   d. If one bulb burns out (that is, the filament opens), what is the effect on the remaining bulbs?
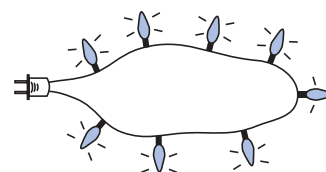
FIG. 5.83
Problem 13.
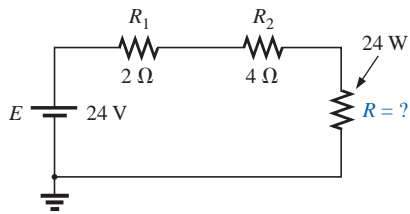
FIG. 5.84

Problem 14.

*14. For the conditions specified in Fig. 5.84, determine the unknown resistance.

SECTION 5.6   Voltage Divider Rule

15. Using the voltage divider rule, find V_{ab} (with polarity) for the circuits of Fig. 5.85.

(a)

(b)

(c)

(d)
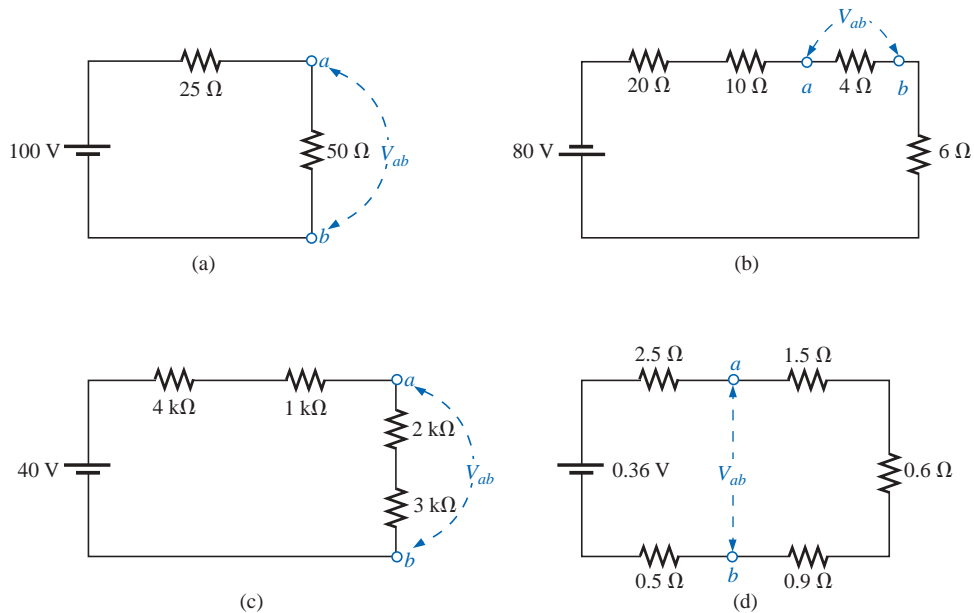
FIG. 5.85

Problems 15 and 37.

16. Find the unknown resistance using the voltage divider rule and the information provided for the circuits of Fig. 5.86.

(a)

(b)

FIG. 5.86

Problem 16.

40 V

+

R_3 V_3

−

+

R_2 30 Ω V_2

−

+

R_1 10 Ω 4 V

−

FIG. 5.87
Problem 17.

+12 V

R_1

+4 V

R_2 8 Ω

−4 V

R_3

−8 V
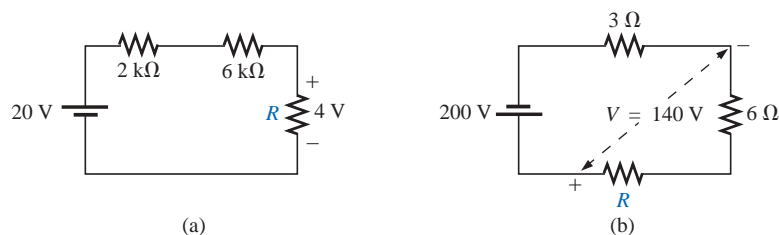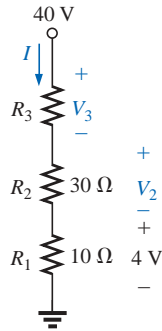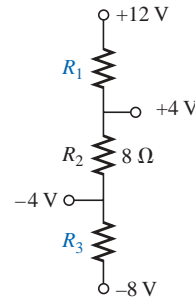
FIG. 5.88
Problem 18.

17. Referring to Fig. 5.87:
    a. Determine V_2 by simply noting that $R_2 = 3R_1$.
    b. Calculate V_3.
    c. Noting the magnitude of V_3 compared to V_2 or V_1, determine R_3 by inspection.
    d. Calculate the source current I.
    e. Calculate the resistance R_3 using Ohm's law, and compare it to the result of part (c).

18. Given the information appearing in Fig. 5.88, find the level of resistance for R_1 and R_3.

19. **a.** Design a voltage divider circuit that will permit the use of an 8-V, 50-mA bulb in an automobile with a 12-V electrical system.
    b. What is the minimum wattage rating of the chosen resistor if ¼-W, ½-W and 1-W resistors are available?

20. Determine the values of R_1, R_2, R_3, and R_4 for the voltage divider of Fig. 5.89 if the source current is 16 mA.

21. Design the voltage divider of Fig. 5.90 such that $V_{R_1} = (1/5)V_{R_2}$ if $I = 4$ mA.

22. Find the voltage across each resistor of Fig. 5.91 if $R_1 = 2R_3$ and $R_2 = 7R_3$.

23. **a.** Design the circuit of Fig. 5.92 such that $V_{R_2} = 3V_{R_1}$ and $V_{R_3} = 4V_{R_2}$.
    b. If the current I is reduced to 10 μA, what are the new values of R_1, R_2, and R_3? How do they compare to the results of part (a)?

+48 V

R_1

R_2

+12 V

16 mA

E 100 V R_3

R_4

−20 V

FIG. 5.89
Problem 20.

+ ←———— 72 V ————→ −

I R_1 R_2

FIG. 5.90
Problem 21.

+
R_1 V_1
−

+
60 V R_2 V_2
−

+
R_3 V_3
−

FIG. 5.91
Problem 22.

$I = 10$ mA R_1 R_2

+
E 64 V R_3
−
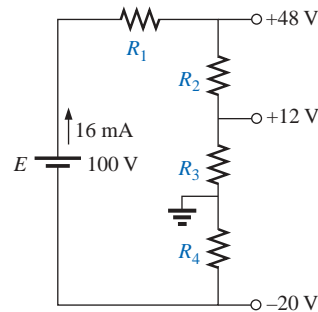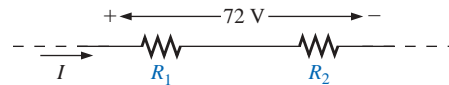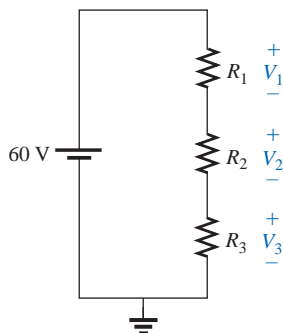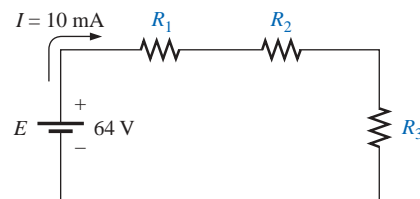
FIG. 5.92
Problem 23.

SECTION 5.7 Notation

24. Determine the voltages V_a, V_b, and V_{ab} for the networks of Fig. 5.93.

(a)   (b)   (c)

FIG. 5.93
Problem 24.

(a)   (b)
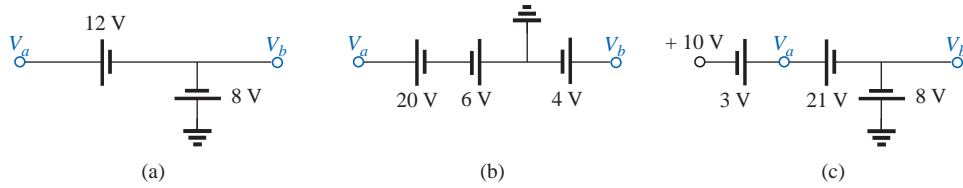
FIG. 5.94
Problem 25.

25. Determine the current I (with direction) and the voltage V (with polarity) for the networks of Fig. 5.94.
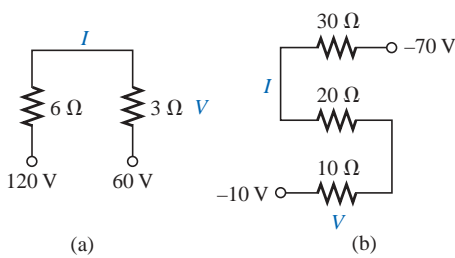
26. Determine the voltages V_a and V_1 for the networks of Fig. 5.95.

(a)   (b)
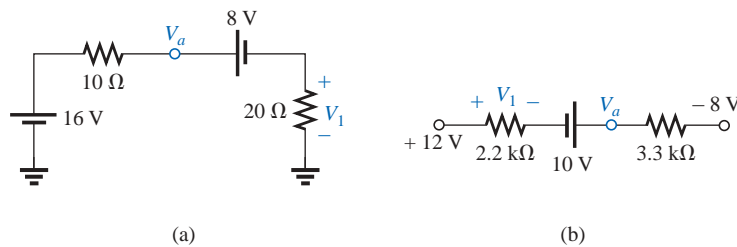
FIG. 5.95
Problem 26.

FIG. 5.96
Problem 27.

*27. For the network of Fig. 5.96, determine the voltages:
  a. V_a, V_b, V_c, V_d, V_e
  b. V_{ab}, V_{dc}, V_{cb}
  c. V_{ac}, V_{db}
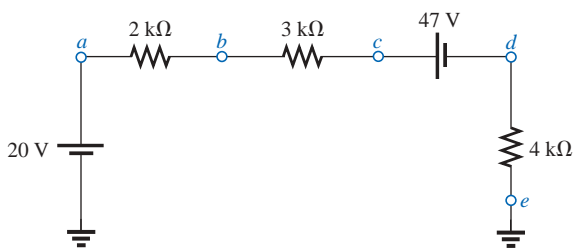
*28. For the network of Fig. 5.97, determine the voltages:
   a. V_a, V_b, V_c, V_d
   b. V_{ab}, V_{cb}, V_{cd}
   c. V_{ad}, V_{ca}
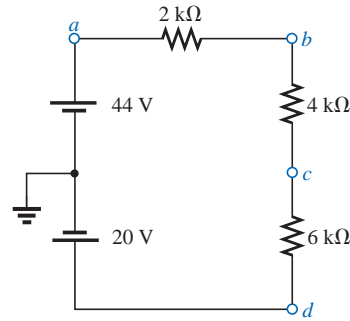
FIG. 5.97
Problem 28.

*29. For the integrated circuit of Fig. 5.98, determine V_0, V_4, V_7, V_{10}, V_{23}, V_{30}, V_{67}, V_{56}, and I (magnitude and direction).
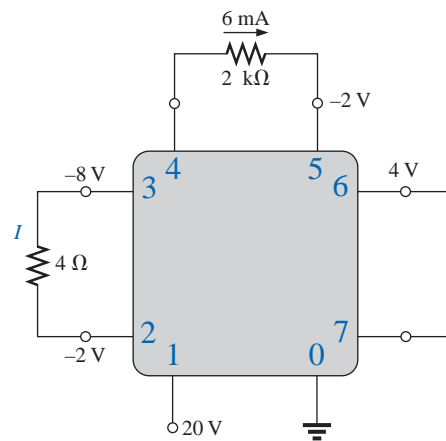
FIG. 5.98
Problem 29.

*30. For the integrated circuit of Fig. 5.99, determine V_0, V_{03}, V_2, V_{23}, V_{12}, and I_i.
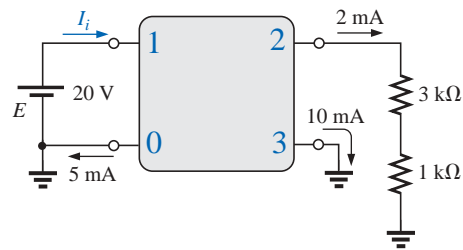
SECTION 5.8   Internal Resistance of Voltage Sources

31. Find the internal resistance of a battery that has a no-load output voltage of 60 V and that supplies a current of 2 A to a load of 28 Ω.

32. Find the voltage V_L and the power loss in the internal resistance for the configuration of Fig. 5.100.

33. Find the internal resistance of a battery that has a no-load output voltage of 6 V and supplies a current of 10 mA to a load of 1/2 kΩ.

SECTION 5.9   Voltage Regulation

34. Determine the voltage regulation for the battery of Problem 31.

35. Calculate the voltage regulation for the supply of Fig. 5.100.

FIG. 5.99
Problem 30.
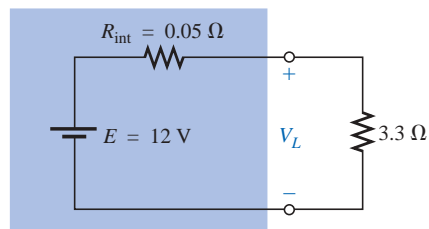
FIG. 5.100
Problems 32 and 35.

FIG. 5.101
Problem 40.

PSpice or Electronics Workbench

36. Using schematics, determine the current *I* and the voltage across each resistor for the network of Fig. 5.71(a).

37. Using schematics, determine the voltage V_{ab} for the network of Fig. 5.85(d).

Programming Language   (C++, QBASIC, Pascal, etc.)

38. Write a program to determine the total resistance of any number of resistors in series.

39. Write a program that will apply the voltage divider rule to either resistor of a series circuit with a single source and two series resistors.

40. Write a program to tabulate the current and power to the resistor R_L of the network of Fig. 5.101 for a range of values for R_L from 1 Ω to 20 Ω. Print out the value of R_L that results in maximum power to R_L.

GLOSSARY

Branch   The portion of a circuit consisting of one or more elements in series.

Circuit   A combination of a number of elements joined at terminal points providing at least one closed path through which charge can flow.

Closed loop   Any continuous connection of branches that allows tracing of a path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit.

Conventional current flow   A defined direction for the flow of charge in an electrical system that is opposite to that of the motion of electrons.

Electron flow   The flow of charge in an electrical system having the same direction as the motion of electrons.

Internal resistance   The inherent resistance found internal to any source of energy.

Kirchhoff's voltage law (KVL)   The algebraic sum of the potential rises and drops around a closed loop (or path) is zero.

Series circuit   A circuit configuration in which the elements have only one point in common and each terminal is not connected to a third, current-carrying element.

Voltage divider rule (VDR)   A method by which a voltage in a series circuit can be determined without first calculating the current in the circuit.

Voltage regulation (*VR*)   A value, given as a percent, that provides an indication of the change in terminal voltage of a supply with a change in load demand.

6
Parallel Circuits

6.1 INTRODUCTION

Two network configurations, series and parallel, form the framework for some of the most complex network structures. A clear understanding of each will pay enormous dividends as more complex methods and networks are examined. The series connection was discussed in detail in the last chapter. We will now examine the **parallel circuit** and all the methods and laws associated with this important configuration.

6.2 PARALLEL ELEMENTS

Two elements, branches, or networks are in **parallel** *if they have two points in common.*

In Fig. 6.1, for example, elements 1 and 2 have terminals *a* and *b* in common; they are therefore in parallel.
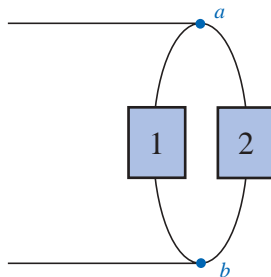
FIG. 6.1
Parallel elements.

In Fig. 6.2, all the elements are in parallel because they satisfy the above criterion. Three configurations are provided to demonstrate how the parallel networks can be drawn. Do not let the squaring of the con-
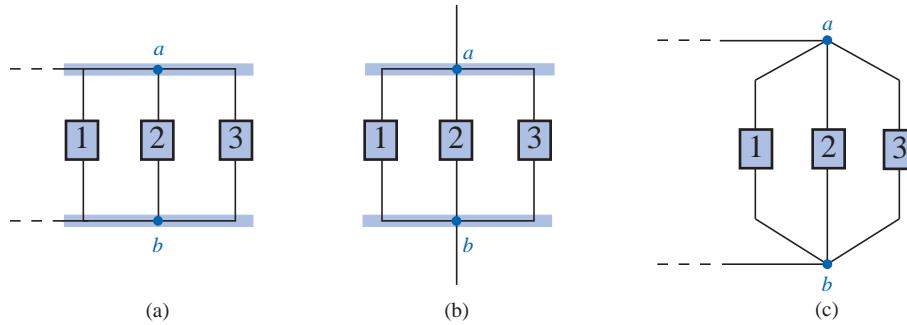
FIG. 6.2

Different ways in which three parallel elements may appear.
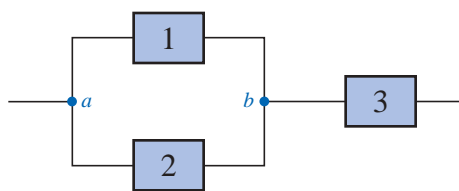
FIG. 6.3

Network in which 1 and 2 are in parallel and 3 is in series with the parallel combination of 1 and 2.
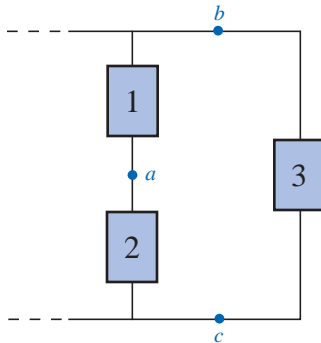
FIG. 6.4

Network in which 1 and 2 are in series and 3 is in parallel with the series combination of 1 and 2.

nection at the top and bottom of Fig. 6.2(a) and (b) cloud the fact that all the elements are connected to one terminal point at the top and bottom, as shown in Fig. 6.2(c).

In Fig. 6.3, elements 1 and 2 are in parallel because they have terminals a and b in common. The parallel combination of 1 and 2 is then in series with element 3 due to the common terminal point b.

In Fig. 6.4, elements 1 and 2 are in series due to the common point a, but the series combination of 1 and 2 is in parallel with element 3 as defined by the common terminal connections at b and c.

In Figs. 6.1 through 6.4, the numbered boxes were used as a general symbol representing single resistive elements, or batteries, or complex network configurations.

Common examples of parallel elements include the rungs of a ladder, the tying of more than one rope between two points to increase the strength of the connection, and the use of pipes between two points to split the water between the two points at a ratio determined by the area of the pipes.

6.3 TOTAL CONDUCTANCE AND RESISTANCE

Recall that for series resistors, the total resistance is the sum of the resistor values.

For parallel elements, the total conductance is the sum of the individual conductances.

That is, for the parallel network of Fig. 6.5, we write

$$G_T = G_1 + G_2 + G_3 + \cdots + G_N \qquad \textbf{(6.1)}$$

Since increasing levels of conductance will establish higher current levels, the more terms appearing in Eq. (6.1), the higher the input cur-
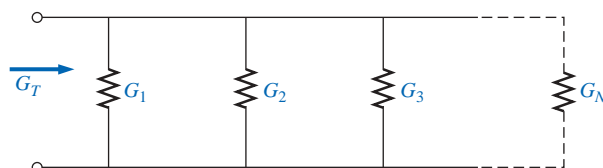
FIG. 6.5

Determining the total conductance of parallel conductances.

rent level. In other words, as the number of resistors in parallel increases, the input current level will increase for the same applied voltage—the opposite effect of increasing the number of resistors in series.

Substituting resistor values for the network of Fig. 6.5 will result in the network of Fig. 6.6. Since $G = 1/R$, the total resistance for the network can be determined by direct substitution into Eq. (6.1):
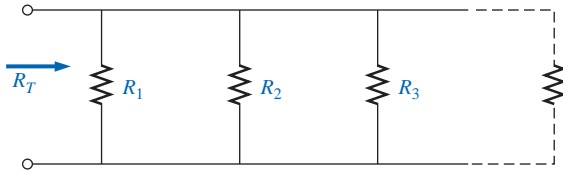
FIG. 6.6

Determining the total resistance of parallel resistors.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N} \qquad (6.2)$$

Note that the equation is for 1 divided by the total resistance rather than the total resistance. Once the sum of the terms to the right of the equals sign has been determined, it will then be necessary to divide the result into 1 to determine the total resistance. The following examples will demonstrate the additional calculations introduced by the inverse relationship.

EXAMPLE 6.1 Determine the total conductance and resistance for the parallel network of Fig. 6.7.

Solution:

$$G_T = G_1 + G_2 = \frac{1}{3\ \Omega} + \frac{1}{6\ \Omega} = 0.333\ \text{S} + 0.167\ \text{S} = \mathbf{0.5\ S}$$

and

$$R_T = \frac{1}{G_T} = \frac{1}{0.5\ \text{S}} = \mathbf{2\ \Omega}$$
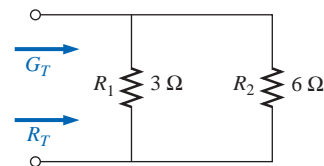
FIG. 6.7

Example 6.1.

EXAMPLE 6.2 Determine the effect on the total conductance and resistance of the network of Fig. 6.7 if another resistor of 10 Ω were added in parallel with the other elements.

Solution:

$$G_T = 0.5\ \text{S} + \frac{1}{10\ \Omega} = 0.5\ \text{S} + 0.1\ \text{S} = \mathbf{0.6\ S}$$

$$R_T = \frac{1}{G_T} = \frac{1}{0.6\ \text{S}} \cong \mathbf{1.667\ \Omega}$$

Note, as mentioned above, that adding additional terms increases the conductance level and decreases the resistance level.

EXAMPLE 6.3 Determine the total resistance for the network of Fig. 6.8.
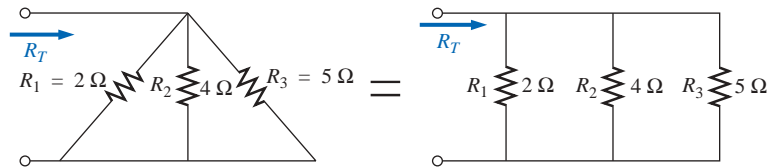
FIG. 6.8
Example 6.3.

Solution:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{2\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{5\ \Omega} = 0.5\text{ S} + 0.25\text{ S} + 0.2\text{ S}$$

$$= 0.95\text{ S}$$

and
$$R_T = \frac{1}{0.95\text{ S}} = \mathbf{1.053\ \Omega}$$

The above examples demonstrate an interesting and useful (for checking purposes) characteristic of parallel resistors:

The total resistance of parallel resistors is always less than the value of the smallest resistor.

In addition, the wider the spread in numerical value between two parallel resistors, the closer the total resistance will be to the smaller resistor. For instance, the total resistance of 3 Ω in parallel with 6 Ω is 2 Ω, as demonstrated in Example 6.1. However, the total resistance of 3 Ω in parallel with 60 Ω is 2.85 Ω, which is much closer to the value of the smaller resistor.

For *equal* resistors in parallel, the equation becomes significantly easier to apply. For N equal resistors in parallel, Equation (6.2) becomes

$$\frac{1}{R_T} = \underbrace{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \cdots + \frac{1}{R}}_{N}$$

$$= N\left(\frac{1}{R}\right)$$

and
$$\boxed{R_T = \frac{R}{N}} \tag{6.3}$$

In other words, the total resistance of N parallel resistors of equal value is the resistance of *one* resistor divided by the number (N) of parallel elements.

For conductance levels, we have

$$\boxed{G_T = NG} \tag{6.4}$$

EXAMPLE 6.4

a. Find the total resistance of the network of Fig. 6.9.
b. Calculate the total resistance for the network of Fig. 6.10.

Solutions:

a. Figure 6.9 is redrawn in Fig. 6.11:
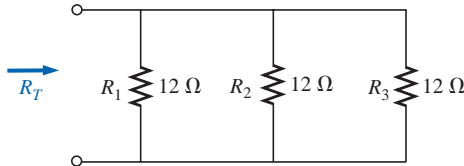
FIG. 6.11
Redrawing the network of Fig. 6.9.

$$R_T = \frac{R}{N} = \frac{12\ \Omega}{3} = \mathbf{4\ \Omega}$$

b. Figure 6.10 is redrawn in Fig. 6.12:

$$R_T = \frac{R}{N} = \frac{2\ \Omega}{4} = \mathbf{0.5\ \Omega}$$

FIG. 6.9
*Example 6.4: three parallel resistors
of equal value.*
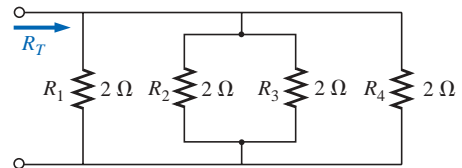
FIG. 6.10
*Example 6.4: four parallel resistors
of equal value.*
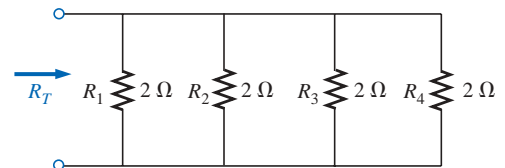
FIG. 6.12
Redrawing the network of Fig. 6.10.

In the vast majority of situations, only two or three parallel resistive elements need to be combined. With this in mind, the following equations were developed to reduce the effects of the inverse relationship when determining R_T.

For two parallel resistors, we write

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Multiplying the top and bottom of each term of the right side of the equation by the other resistor will result in

$$\frac{1}{R_T} = \left(\frac{R_2}{R_2}\right)\frac{1}{R_1} + \left(\frac{R_1}{R_1}\right)\frac{1}{R_2} = \frac{R_2}{R_1 R_2} + \frac{R_1}{R_1 R_2}$$

$$= \frac{R_2 + R_1}{R_1 R_2}$$

and

$$R_T = \frac{R_1 R_2}{R_1 + R_2} \qquad\qquad \textbf{(6.5)}$$

In words,

the total resistance of two parallel resistors is the product of the two divided by their sum.

For three parallel resistors, the equation for R_T becomes

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} \qquad\qquad \textbf{(6.6a)}$$

requiring that we be careful with all the divisions into 1.

Equation (6.6a) can also be expanded into the form of Eq. (6.5), resulting in Eq. (6.6b):

$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$
(6.6b)

with the denominator showing all the possible product combinations of the resistors taken two at a time. An alternative to either form of Eq. (6.6) is to simply apply Eq. (6.5) twice, as will be demonstrated in Example 6.6.

EXAMPLE 6.5 Repeat Example 6.1 using Eq. (6.5).

Solution:

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{9} = \mathbf{2\ \Omega}$$

EXAMPLE 6.6 Repeat Example 6.3 using Eq. (6.6a).

Solution:

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

$$= \frac{1}{\dfrac{1}{2\ \Omega} + \dfrac{1}{4\ \Omega} + \dfrac{1}{5\ \Omega}} = \frac{1}{0.5 + 0.25 + 0.2}$$

$$= \frac{1}{0.95} = \mathbf{1.053\ \Omega}$$

Applying Eq. (6.5) twice yields

$$R'_T = 2\ \Omega\ ||\ 4\ \Omega = \frac{(2\ \Omega)(4\ \Omega)}{2\ \Omega + 4\ \Omega} = \frac{4}{3}\ \Omega$$

$$R_T = R'_T\ ||\ 5\ \Omega = \frac{\left(\dfrac{4}{3}\ \Omega\right)\left(5\ \Omega\right)}{\dfrac{4}{3}\ \Omega + 5\ \Omega} = \mathbf{1.053\ \Omega}$$

Recall that series elements can be interchanged without affecting the magnitude of the total resistance or current. In parallel networks,

parallel elements can be interchanged without changing the total resistance or input current.

Note in the next example how redrawing the network can often clarify which operations and equations should be applied.

EXAMPLE 6.7 Calculate the total resistance of the parallel network of Fig. 6.13.
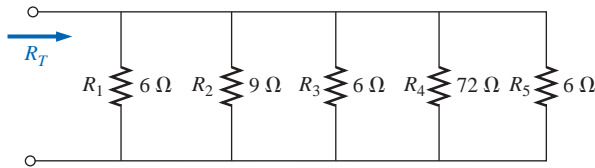
FIG. 6.13
Example 6.7.

Solution: The network is redrawn in Fig. 6.14:
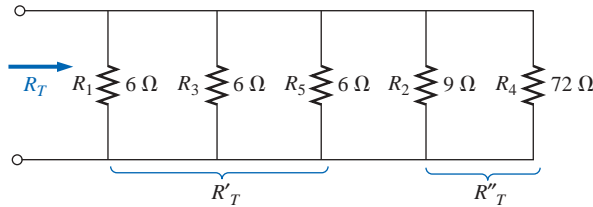
FIG. 6.14
Network of Fig. 6.13 redrawn.

$$R'_T = \frac{R}{N} = \frac{6\ \Omega}{3} = 2\ \Omega$$

$$R''_T = \frac{R_2 R_4}{R_2 + R_4} = \frac{(9\ \Omega)(72\ \Omega)}{9\ \Omega + 72\ \Omega} = \frac{648\ \Omega}{81} = 8\ \Omega$$

and
$$R_T = R'_T \ ||\ R''_T$$
$$\uparrow\!\!\!\!\text{—— In parallel with}$$

$$= \frac{R'_T R''_T}{R'_T + R''_T} = \frac{(2\ \Omega)(8\ \Omega)}{2\ \Omega + 8\ \Omega} = \frac{16\ \Omega}{10} = \mathbf{1.6\ \Omega}$$

The preceding examples show direct substitution, in which once the proper equation is defined, it is only a matter of plugging in the numbers and performing the required algebraic maneuvers. The next two examples have a design orientation, where specific network parameters are defined and the circuit elements must be determined.

EXAMPLE 6.8 Determine the value of R_2 in Fig. 6.15 to establish a total resistance of 9 kΩ.

Solution:

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_T(R_1 + R_2) = R_1 R_2$$

$$R_T R_1 + R_T R_2 = R_1 R_2$$

$$R_T R_1 = R_1 R_2 - R_T R_2$$

$$R_T R_1 = (R_1 - R_T) R_2$$

FIG. 6.15
Example 6.8.

and
$$\boxed{R_2 = \frac{R_T R_1}{R_1 - R_T}}$$ **(6.7)**

Substituting values:

$$R_2 = \frac{(9 \text{ k}\Omega)(12 \text{ k}\Omega)}{12 \text{ k}\Omega - 9 \text{ k}\Omega}$$

$$= \frac{108 \text{ k}\Omega}{3} = \textbf{36 k}\boldsymbol{\Omega}$$

FIG. 6.16
Example 6.9.

EXAMPLE 6.9 Determine the values of R_1, R_2, and R_3 in Fig. 6.16 if $R_2 = 2R_1$ and $R_3 = 2R_2$ and the total resistance is 16 kΩ.

Solution:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{16 \text{ k}\Omega} = \frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{4R_1}$$

since

$$R_3 = 2R_2 = 2(2R_1) = 4R_1$$

and

$$\frac{1}{16 \text{ k}\Omega} = \frac{1}{R_1} + \frac{1}{2}\left(\frac{1}{R_1}\right) + \frac{1}{4}\left(\frac{1}{R_1}\right)$$

$$\frac{1}{16 \text{ k}\Omega} = 1.75\left(\frac{1}{R_1}\right)$$

with

$$R_1 = 1.75(16 \text{ k}\Omega) = \textbf{28 k}\boldsymbol{\Omega}$$
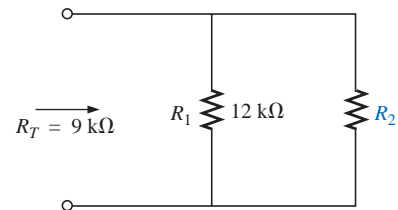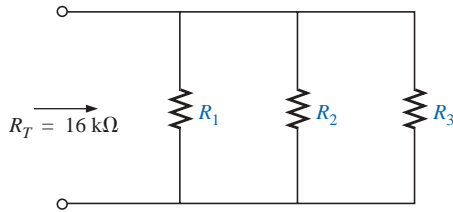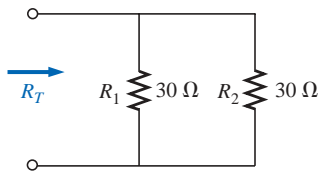
FIG. 6.17
Example 6.10: two equal, parallel resistors.
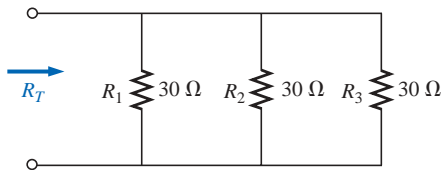
FIG. 6.18
Adding a third parallel resistor of equal value to the network of Fig. 6.17.
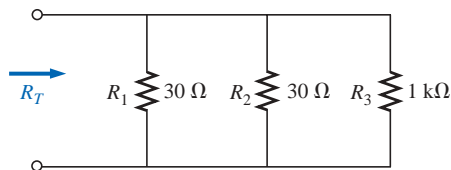
FIG. 6.19
Adding a much larger parallel resistor to the network of Fig. 6.17.

Recall for series circuits that the total resistance will always increase as additional elements are added in series.

For parallel resistors, the total resistance will always decrease as additional elements are added in parallel.

The next example demonstrates this unique characteristic of parallel resistors.

EXAMPLE 6.10
a. Determine the total resistance of the network of Fig. 6.17.
b. What is the effect on the total resistance of the network of Fig. 6.17 if an additional resistor of the same value is added, as shown in Fig. 6.18?
c. What is the effect on the total resistance of the network of Fig. 6.17 if a very large resistance is added in parallel, as shown in Fig. 6.19?
d. What is the effect on the total resistance of the network of Fig. 6.17 if a very small resistance is added in parallel, as shown in Fig. 6.20?

Solutions:

a. $R_T = 30 \text{ }\Omega \parallel 30 \text{ }\Omega = \dfrac{30 \text{ }\Omega}{2} = \textbf{15 }\boldsymbol{\Omega}$

b. $R_T = 30 \text{ }\Omega \parallel 30 \text{ }\Omega \parallel 30 \text{ }\Omega = \dfrac{30 \text{ }\Omega}{3} = \textbf{10 }\boldsymbol{\Omega} < 15 \text{ }\Omega$

R_T decreased

c. $R_T = 30 \text{ }\Omega \parallel 30 \text{ }\Omega \parallel 1 \text{ k}\Omega = 15 \text{ }\Omega \parallel 1 \text{ k}\Omega$

$= \dfrac{(15 \text{ }\Omega)(1000 \text{ }\Omega)}{15 \text{ }\Omega + 1000 \text{ }\Omega} = \textbf{14.778 }\boldsymbol{\Omega} < 15 \text{ }\Omega$

Small decrease in R_T

FIG. 6.20
Adding a much smaller parallel resistor to the network of Fig. 6.17.

d. $R_T = 30 \ \Omega \parallel 30 \ \Omega \parallel 0.1 \ \Omega = 15 \ \Omega \parallel 0.1 \ \Omega$

$$= \frac{(15 \ \Omega)(0.1 \ \Omega)}{15 \ \Omega + 0.1 \ \Omega} = \mathbf{0.099 \ \Omega} < 15 \ \Omega$$

Significant decrease in R_T

In each case the total resistance of the network decreased with the increase of an additional parallel resistive element, no matter how large or small. Note also that the total resistance is also smaller than that of the smallest parallel element.

6.4 PARALLEL CIRCUITS

The network of Fig. 6.21 is the simplest of parallel circuits. All the elements have terminals a and b in common. The total resistance is determined by $R_T = R_1 R_2/(R_1 + R_2)$, and the source current by $I_s = E/R_T$. Throughout the text, the subscript s will be used to denote a property of the source. Since the terminals of the battery are connected directly across the resistors R_1 and R_2, the following should be obvious:

The voltage across parallel elements is the same.

Using this fact will result in

$$V_1 = V_2 = E$$

and

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1}$$

with

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}$$

If we take the equation for the total resistance and multiply both sides by the applied voltage, we obtain

$$E\left(\frac{1}{R_T}\right) = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

and

$$\frac{E}{R_T} = \frac{E}{R_1} + \frac{E}{R_2}$$

Substituting the Ohm's law relationships appearing above, we find that the source current

$$I_s = I_1 + I_2$$

permitting the following conclusion:

For single-source parallel networks, the source current (I_s) is equal to the sum of the individual branch currents.

The power dissipated by the resistors and delivered by the source can be determined from

$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}$$

$$P_2 = V_2 I_2 = I_2^2 R_2 = \frac{V_2^2}{R_2}$$

$$P_s = E I_s = I_s^2 R_T = \frac{E^2}{R_T}$$
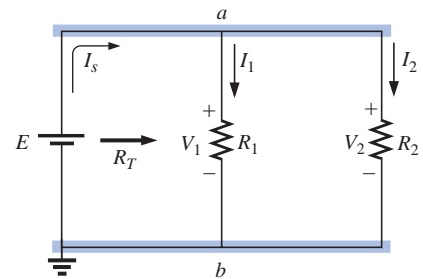
FIG. 6.21
Parallel network.
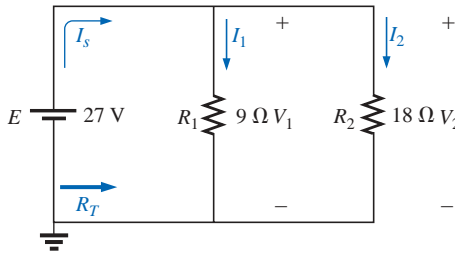
FIG. 6.22
Example 6.11.

EXAMPLE 6.11 For the parallel network of Fig. 6.22:

a. Calculate R_T.
b. Determine I_s.
c. Calculate I_1 and I_2, and demonstrate that $I_s = I_1 + I_2$.
d. Determine the power to each resistive load.
e. Determine the power delivered by the source, and compare it to the total power dissipated by the resistive elements.

Solutions:

a. $R_T = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{(9\ \Omega)(18\ \Omega)}{9\ \Omega + 18\ \Omega} = \dfrac{162\ \Omega}{27} = \textbf{6}\ \boldsymbol{\Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{27\ \text{V}}{6\ \Omega} = \textbf{4.5 A}$

c. $\quad I_1 = \dfrac{V_1}{R_1} = \dfrac{E}{R_1} = \dfrac{27\ \text{V}}{9\ \Omega} = \textbf{3 A}$

$\quad I_2 = \dfrac{V_2}{R_2} = \dfrac{E}{R_2} = \dfrac{27\ \text{V}}{18\ \Omega} = \textbf{1.5 A}$

$\quad I_s = I_1 + I_2$

$\quad 4.5\ \text{A} = 3\ \text{A} + 1.5\ \text{A}$

$\quad \textbf{4.5 A} = \textbf{4.5 A} \quad \text{(checks)}$

d. $P_1 = V_1 I_1 = EI_1 = (27\ \text{V})(3\ \text{A}) = \textbf{81 W}$
$\quad P_2 = V_2 I_2 = EI_2 = (27\ \text{V})(1.5\ \text{A}) = \textbf{40.5 W}$
e. $P_s = EI_s = (27\ \text{V})(4.5\ \text{A}) = \textbf{121.5 W}$
$\quad = P_1 + P_2 = 81\ \text{W} + 40.5\ \text{W} = \textbf{121.5 W}$

EXAMPLE 6.12 Given the information provided in Fig. 6.23:
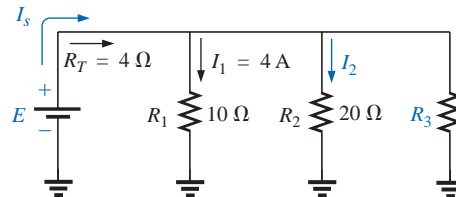
FIG. 6.23
Example 6.12.

a. Determine R_3.
b. Calculate E.
c. Find I_s.
d. Find I_2.
e. Determine P_2.

Solutions:

a. $\quad \dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$

$\quad \dfrac{1}{4\ \Omega} = \dfrac{1}{10\ \Omega} + \dfrac{1}{20\ \Omega} + \dfrac{1}{R_3}$

$0.25\ \text{S} = 0.1\ \text{S} + 0.05\ \text{S} + \dfrac{1}{R_3}$

$0.25\ \text{S} = 0.15\ \text{S} + \dfrac{1}{R_3}$

$\dfrac{1}{R_3} = 0.1\ \text{S}$

$R_3 = \dfrac{1}{0.1\ \text{S}} = \mathbf{10\ \Omega}$

b. $E = V_1 = I_1 R_1 = (4\ \text{A})(10\ \Omega) = \mathbf{40\ V}$

c. $I_s = \dfrac{E}{R_T} = \dfrac{40\ \text{V}}{4\ \Omega} = \mathbf{10\ A}$

d. $I_2 = \dfrac{V_2}{R_2} = \dfrac{E}{R_2} = \dfrac{40\ \text{V}}{20\ \Omega} = \mathbf{2\ A}$

e. $P_2 = I_2^2 R_2 = (2\ \text{A})^2 (20\ \Omega) = \mathbf{80\ W}$

Mathcad Solution:　This example provides an excellent opportunity to practice our skills using Mathcad. As shown in Fig. 6.24, the known parameters and quantities of the network are entered first, followed by an equation for the unknown resistor R_3. Note that after the first division operator was selected, a left bracket was established (to be followed eventually by a right enclosure bracket) to tell the computer that the mathematical operations in the denominator must be carried out first before the division into 1. In addition, each individual division into 1 is separated by brackets to ensure that the division operation is performed before each quantity is added to the neighboring factor. Finally, keep in mind that the Mathcad bracket must encompass each individual expression of the denominator before you place the right bracket in place.
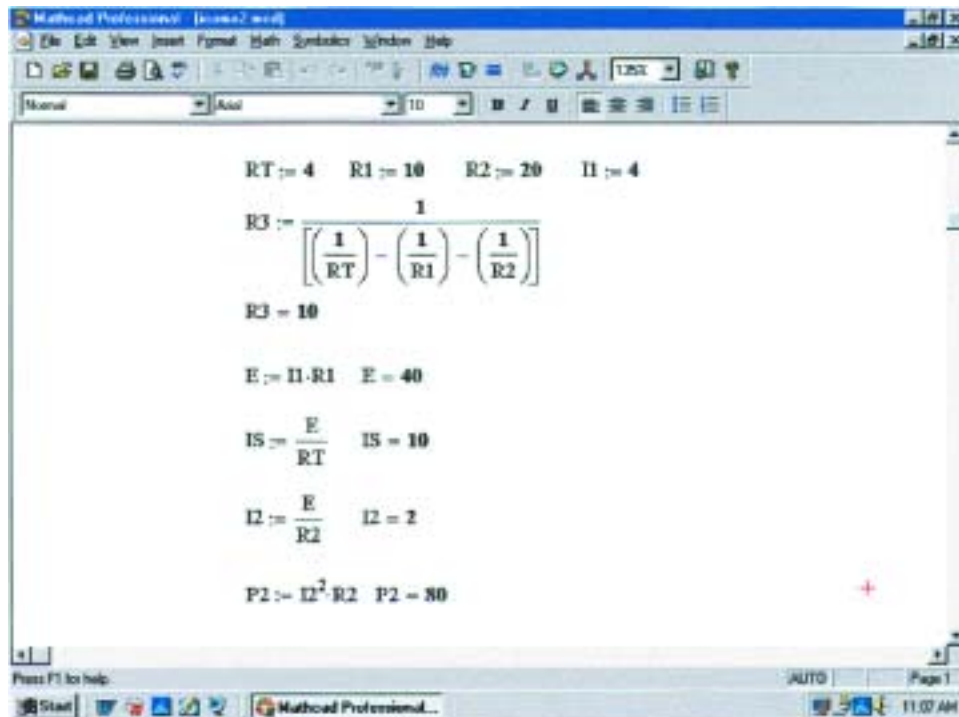
FIG. 6.24
Using Mathcad to confirm the results of Example 6.12.

In each case, the quantity of interest was entered below the defining equation to obtain the numerical result by selecting an equal sign. As expected, all the results match the longhand solution.

6.5 KIRCHHOFF'S CURRENT LAW

Kirchhoff's voltage law provides an important relationship among voltage levels around any closed loop of a network. We now consider **Kirchhoff's current law (KCL),** which provides an equally important relationship among current levels at any junction.

Kirchhoff's current law (KCL) *states that the algebraic sum of the currents entering and leaving an area, system, or junction is zero.*

In other words,

the sum of the currents entering an area, system, or junction must equal the sum of the currents leaving the area, system, or junction.

In equation form:

$$\Sigma \, I_{entering} = \Sigma \, I_{leaving} \tag{6.8}$$

In Fig. 6.25, for instance, the shaded area can enclose an entire system, a complex network, or simply a junction of two or more paths. In each case the current entering must equal that leaving, as witnessed by the fact that

$$I_1 + I_4 = I_2 + I_3$$
$$4 \, A + 8 \, A = 2 \, A + 10 \, A$$
$$12 \, A = 12 \, A$$

FIG. 6.25
Introducing Kirchhoff's current law.

The most common application of the law will be at the junction of two or more paths of current flow, as shown in Fig. 6.26. For some students it is difficult initially to determine whether a current is entering or leaving a junction. One approach that may help is to picture yourself as standing on the junction and treating the path currents as arrows. If the arrow appears to be heading toward you, as is the case for I_1 in Fig. 6.26, then it is entering the junction. If you see the tail of the arrow (from the junction) as it travels down its path away from you, it is leaving the junction, as is the case for I_2 and I_3 in Fig. 6.26.

Applying Kirchhoff's current law to the junction of Fig. 6.26:

$$\Sigma \, I_{entering} = \Sigma \, I_{leaving}$$
$$6 \, A = 2 \, A + 4 \, A$$
$$\underline{6 \, A = 6 \, A \quad \text{(checks)}}$$

FIG. 6.26
Demonstrating Kirchhoff's current law.

In the next two examples, unknown currents can be determined by applying Kirchhoff's current law. Simply remember to place all current levels entering a junction to the left of the equals sign and the sum of all currents leaving a junction to the right of the equals sign. The water-in-the-pipe analogy is an excellent one for supporting and clarifying the preceding law. Quite obviously, the sum total of the water entering a junction must equal the total of the water leaving the exit pipes.

In technology the term **node** is commonly used to refer to a junction of two or more branches. Therefore, this term will be used frequently in the analyses that follow.

EXAMPLE 6.13 Determine the currents I_3 and I_4 of Fig. 6.27 using Kirchhoff's current law.

Solution: We must first work with junction a since the only unknown is I_3. At junction b there are two unknowns, and both cannot be determined from one application of the law.
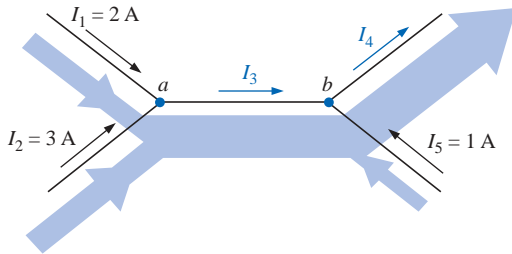
FIG. 6.27
Example 6.13.

At a:

$$\Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}$$
$$I_1 + I_2 = I_3$$
$$2\,\text{A} + 3\,\text{A} = I_3$$
$$I_3 = \mathbf{5\,A}$$

At b:

$$\Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}$$
$$I_3 + I_5 = I_4$$
$$5\,\text{A} + 1\,\text{A} = I_4$$
$$I_4 = \mathbf{6\,A}$$

EXAMPLE 6.14 Determine I_1, I_3, I_4, and I_5 for the network of Fig. 6.28.
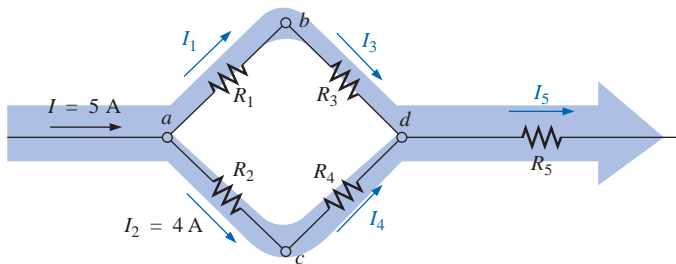
FIG. 6.28
Example 6.14.

Solution: At a:

$$\Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}$$
$$I = I_1 + I_2$$
$$5\,\text{A} = I_1 + 4\,\text{A}$$

Subtracting 4 A from both sides gives

$$5\,\text{A} - 4\,\text{A} = I_1 + 4\,\cancel{\text{A}} - 4\,\cancel{\text{A}}$$
$$I_1 = 5\,\text{A} - 4\,\text{A} = \mathbf{1\,A}$$

At *b*:

$$\Sigma I_{entering} = \Sigma I_{leaving}$$
$$I_1 = I_3 = \mathbf{1\,A}$$

as it should, since R_1 and R_3 are in series and the current is the same in series elements.

At *c*:

$$I_2 = I_4 = \mathbf{4\,A}$$

for the same reasons given for junction *b*.

At *d*:

$$\Sigma I_{entering} = \Sigma I_{leaving}$$
$$I_3 + I_4 = I_5$$
$$1\,A + 4\,A = I_5$$
$$I_5 = \mathbf{5\,A}$$

If we enclose the entire network, we find that the current entering is $I = 5$ A; the net current leaving from the far right is $I_5 = 5$ A. The two must be equal since the net current entering any system must equal that leaving.
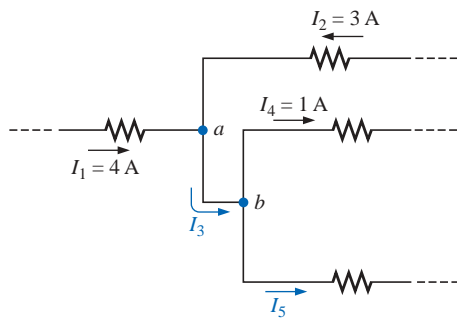
FIG. 6.29
Example 6.15.

EXAMPLE 6.15 Determine the currents I_3 and I_5 of Fig. 6.29 through applications of Kirchhoff's current law.

Solution: Note that since node *b* has two unknown quantities and node *a* has only one, we must first apply Kirchhoff's current law to node *a*. The result can then be applied to node *b*.

For node *a*,

$$I_1 + I_2 = I_3$$
$$4\,A + 3\,A = I_3$$

and

$$I_3 = \mathbf{7\,A}$$

For node *b*,

$$I_3 = I_4 + I_5$$
$$7\,A = 1\,A + I_5$$

and

$$I_5 = 7\,A - 1\,A = \mathbf{6\,A}$$
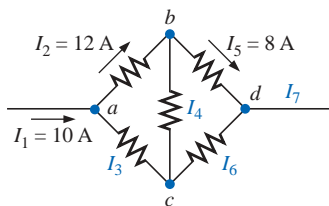
FIG. 6.30
Example 6.16.

EXAMPLE 6.16 Find the magnitude and direction of the currents I_3, I_4, I_6, and I_7 for the network of Fig. 6.30. Even though the elements are not in series or parallel, Kirchhoff's current law can be applied to determine all the unknown currents.

Solution: Considering the overall system, we know that the current entering must equal that leaving. Therefore,

$$I_7 = I_1 = \mathbf{10\,A}$$

Since 10 A are entering node *a* and 12 A are leaving, I_3 must be supplying current to the node.

Applying Kirchhoff's current law at node *a*,

$$I_1 + I_3 = I_2$$
$$10\,A + I_3 = 12\,A$$

and

$$I_3 = 12\,A - 10\,A = \mathbf{2\,A}$$

At node *b*, since 12 A are entering and 8 A are leaving, I_4 must be leaving. Therefore,

$$I_2 = I_4 + I_5$$
$$12 \text{ A} = I_4 + 8 \text{ A}$$

and
$$I_4 = 12 \text{ A} - 8 \text{ A} = \textbf{4 A}$$

At node c, I_3 is leaving at 2 A and I_4 is entering at 4 A, requiring that I_6 be leaving. Applying Kirchhoff's current law at node c,

$$I_4 = I_3 + I_6$$
$$4 \text{ A} = 2 \text{ A} + I_6$$

and
$$I_6 = 4 \text{ A} - 2 \text{ A} = \textbf{2 A}$$

As a check at node d,

$$I_5 + I_6 = I_7$$
$$8 \text{ A} + 2 \text{ A} = 10 \text{ A}$$
$$\underline{\textbf{10 A} = \textbf{10 A}} \quad \text{(checks)}$$

Looking back at Example 6.11, we find that the current entering the top node is 4.5 A and the current leaving the node is $I_1 + I_2 = 3 \text{ A} + 1.5 \text{ A} = 4.5 \text{ A}$. For Example 6.12, we have

$$I_s = I_1 + I_2 + I_3$$
$$10 \text{ A} = 4 \text{ A} + 2 \text{ A} + I_3$$

and
$$I_3 = 10 \text{ A} - 6 \text{ A} = 4 \text{ A}$$

The application of Kirchhoff's current law is not limited to networks where all the internal connections are known or visible. For instance, all the currents of the integrated circuit of Fig. 6.31 are known except I_1. By treating the system as a single node, we can apply Kirchhoff's current law using the following values to ensure an accurate listing of all known quantities:

I_i	I_o
10 mA	5 mA
4 mA	4 mA
8 mA	2 mA
22 mA	6 mA
	17 mA

FIG. 6.31
Integrated circuit.

Noting the total input current versus that leaving clearly reveals that I_1 is a current of 22 mA − 17 mA = 5 mA leaving the system.

6.6 CURRENT DIVIDER RULE

As the name suggests, the **current divider rule (CDR)** will determine how the current entering a set of parallel branches will split between the elements.

For two parallel elements of equal value, the current will divide equally.

For parallel elements with different values, the smaller the resistance, the greater the share of input current.

For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistor values.

For example, if one of two parallel resistors is twice the other, then the current through the larger resistor will be half the other.

In Fig. 6.32, since I_1 is 1 mA and R_1 is six times R_3, the current through R_3 must be 6 mA (without making any other calculations including the total current or the actual resistance levels). For R_2 the current must be 2 mA since R_1 is twice R_2. The total current must then be the sum of I_1, I_2, and I_3, or 9 mA. In total, therefore, knowing only the current through R_1, we were able to find all the other currents of the configuration without knowing anything more about the network.
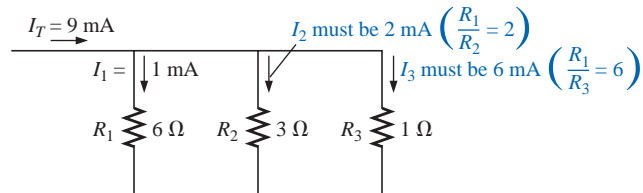
FIG. 6.32

Demonstrating how current will divide between unequal resistors.

For networks in which only the resistor values are given along with the input current, the current divider rule should be applied to determine the various branch currents. It can be derived using the network of Fig. 6.33.
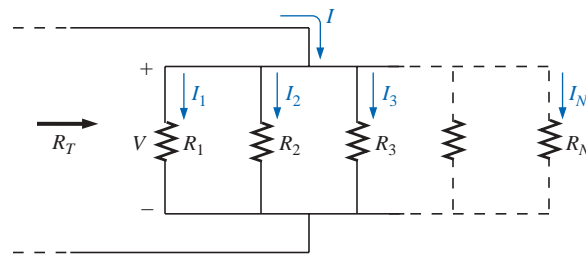
FIG. 6.33

Deriving the current divider rule.

The input current I equals V/R_T, where R_T is the total resistance of the parallel branches. Substituting $V = I_x R_x$ into the above equation, where I_x refers to the current through a parallel branch of resistance R_x, we have

$$I = \frac{V}{R_T} = \frac{I_x R_x}{R_T}$$

and

$$\boxed{I_x = \frac{R_T}{R_x} I} \tag{6.9}$$

which is the general form for the current divider rule. In words, the current through any parallel branch is equal to the product of the *total* resistance of the parallel branches and the input current divided by the resistance of the branch through which the current is to be determined.

For the current I_1,

$$I_1 = \frac{R_T}{R_1} I$$

and for I_2,

$$I_2 = \frac{R_T}{R_2} I$$

and so on.

For the particular case of *two parallel resistors,* as shown in Fig. 6.34,

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

and

$$I_1 = \frac{R_T}{R_1} I = \frac{\dfrac{R_1 R_2}{R_1 + R_2}}{R_1} I$$

and

Note difference in subscripts.

$$I_1 = \frac{R_2 I}{R_1 + R_2}$$ **(6.10)**

Similarly for I_2,

$$I_2 = \frac{R_1 I}{R_1 + R_2}$$ **(6.11)**

In words, for two parallel branches, the current through either branch is equal to the product of the *other* parallel resistor and the input current divided by the *sum* (not the total parallel resistance) of the two parallel resistances.

FIG. 6.34
Developing an equation for current division between two parallel resistors.

EXAMPLE 6.17 Determine the current I_2 for the network of Fig. 6.35 using the current divider rule.

Solution:

$$I_2 = \frac{R_1 I_s}{R_1 + R_2} = \frac{(4 \text{ k}\Omega)(6 \text{ A})}{4 \text{ k}\Omega + 8 \text{ k}\Omega} = \frac{4}{12}(6 \text{ A}) = \frac{1}{3}(6 \text{ A})$$
$$= \textbf{2 A}$$
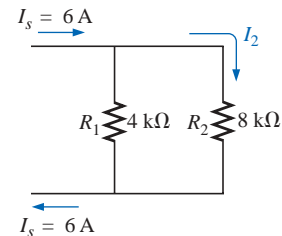
FIG. 6.35
Example 6.17.

EXAMPLE 6.18 Find the current I_1 for the network of Fig. 6.36.
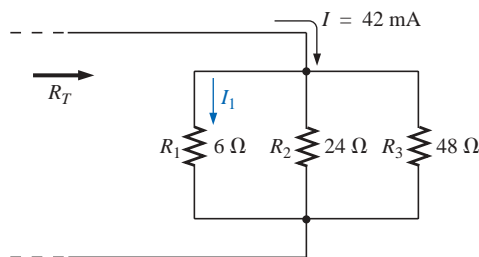
FIG. 6.36
Example 6.18

Solution: There are two options for solving this problem. The first is to use Eq. (6.9) as follows:

$$\frac{1}{R_T} = \frac{1}{6 \text{ } \Omega} + \frac{1}{24 \text{ } \Omega} + \frac{1}{48 \text{ } \Omega} = 0.1667 \text{ S} + 0.0417 \text{ S} + 0.0208 \text{ S}$$
$$= 0.2292 \text{ S}$$

and $$R_T = \frac{1}{0.2292 \text{ S}} = 4.363 \ \Omega$$

with $$I_1 = \frac{R_T}{R_1} I = \frac{4.363 \ \Omega}{6 \ \Omega} (42 \text{ mA}) = \textbf{30.54 mA}$$

The second option is to apply Eq. (6.10) once after combining R_2 and R_3 as follows:

$$24 \ \Omega \parallel 48 \ \Omega = \frac{(24 \ \Omega)(48 \ \Omega)}{24 \ \Omega + 48 \ \Omega} = 16 \ \Omega$$

and $$I_1 = \frac{16 \ \Omega(42 \text{ mA})}{16 \ \Omega + 6 \ \Omega} = \textbf{30.54 mA}$$

Both options generated the same answer, leaving you with a choice for future calculations involving more than two parallel resistors.

EXAMPLE 6.19 Determine the magnitude of the currents I_1, I_2, and I_3 for the network of Fig. 6.37.
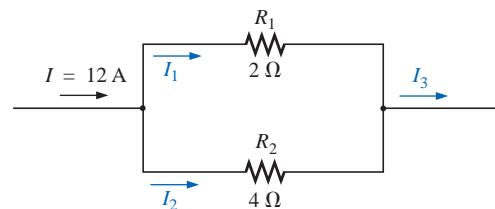
FIG. 6.37
Example 6.19.

Solution: By Eq. (6.10), the current divider rule,

$$I_1 = \frac{R_2 I}{R_1 + R_2} = \frac{(4 \ \Omega)(12 \text{ A})}{2 \ \Omega + 4 \ \Omega} = \textbf{8 A}$$

Applying Kirchhoff's current law,

$$I = I_1 + I_2$$

and $$I_2 = I - I_1 = 12 \text{ A} - 8 \text{ A} = \textbf{4 A}$$

or, using the current divider rule again,

$$I_2 = \frac{R_1 I}{R_1 + R_2} = \frac{(2 \ \Omega)(12 \text{ A})}{2 \ \Omega + 4 \ \Omega} = \textbf{4 A}$$

The total current entering the parallel branches must equal that leaving. Therefore,

$$I_3 = I = \textbf{12 A}$$

or $$I_3 = I_1 + I_2 = 8 \text{ A} + 4 \text{ A} = \textbf{12 A}$$
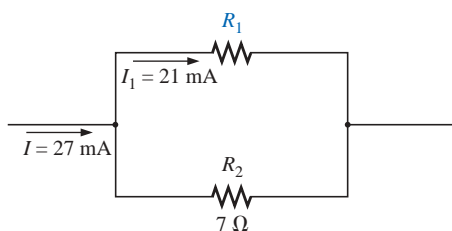
EXAMPLE 6.20 Determine the resistance R_1 to effect the division of current in Fig. 6.38.

Solution: Applying the current divider rule,

$$I_1 = \frac{R_2 I}{R_1 + R_2}$$

FIG. 6.38
Example 6.20.

and

$$(R_1 + R_2)I_1 = R_2 I$$
$$R_1 I_1 + R_2 I_1 = R_2 I$$
$$R_1 I_1 = R_2 I - R_2 I_1$$
$$R_1 = \frac{R_2(I - I_1)}{I_1}$$

Substituting values:

$$R_1 = \frac{7 \ \Omega(27 \ \text{mA} - 21 \ \text{mA})}{21 \ \text{mA}}$$

$$= 7 \ \Omega\left(\frac{6}{21}\right) = \frac{42 \ \Omega}{21} = \mathbf{2 \ \Omega}$$

An alternative approach is

$$I_2 = I - I_1 \quad \text{(Kirchhoff's current law)}$$
$$= 27 \ \text{mA} - 21 \ \text{mA} = 6 \ \text{mA}$$
$$V_2 = I_2 R_2 = (6 \ \text{mA})(7 \ \Omega) = 42 \ \text{mV}$$
$$V_1 = I_1 R_1 = V_2 = 42 \ \text{mV}$$

and

$$R_1 = \frac{V_1}{I_1} = \frac{42 \ \text{mV}}{21 \ \text{mA}} = \mathbf{2 \ \Omega}$$

From the examples just described, note the following:

Current seeks the path of least resistance.

That is,

1. More current passes through the smaller of two parallel resistors.
2. The current entering any number of parallel resistors divides into these resistors as the inverse ratio of their ohmic values. This relationship is depicted in Fig. 6.39.
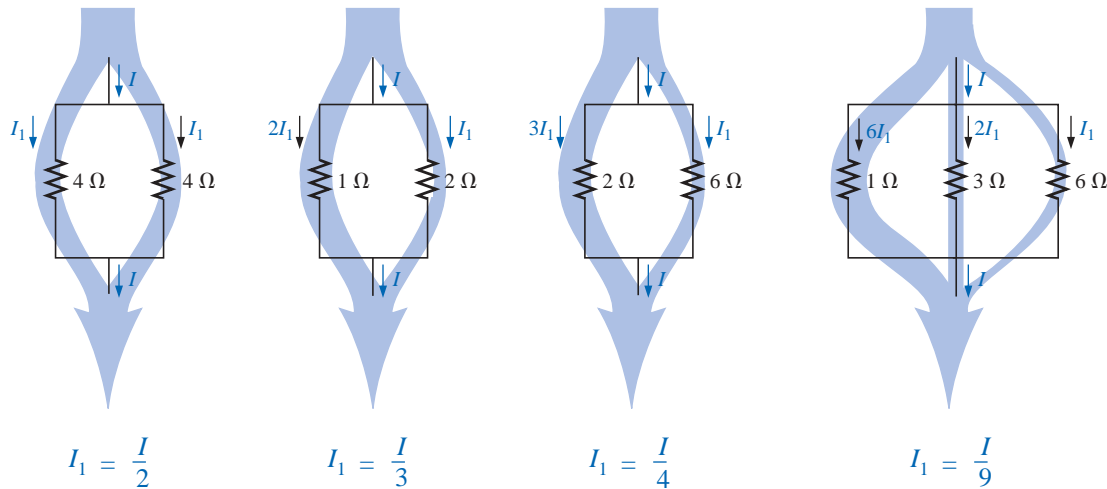
FIG. 6.39
Current division through parallel branches.

6.7 VOLTAGE SOURCES IN PARALLEL

Voltage sources are placed in parallel as shown in Fig. 6.40 only if they have the same voltage rating. The primary reason for placing two or more batteries in parallel of the same terminal voltage would be to
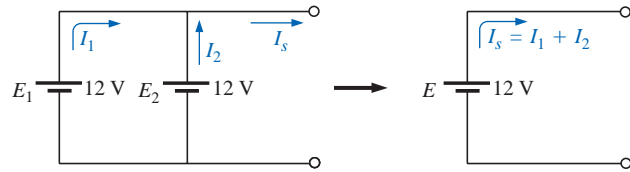
FIG. 6.40
Parallel voltage sources.
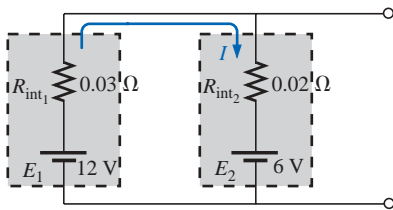
FIG. 6.41
Parallel batteries of different terminal voltages.

increase the current rating (and, therefore, the power rating) of the source. As shown in Fig. 6.40, the current rating of the combination is determined by $I_s = I_1 + I_2$ at the same terminal voltage. The resulting power rating is twice that available with one supply.

If two batteries of different terminal voltages were placed in parallel, both would be left ineffective or damaged because the terminal voltage of the larger battery would try to drop rapidly to that of the lower supply. Consider two lead-acid car batteries of different terminal voltage placed in parallel, as shown in Fig. 6.41.

The relatively small internal resistances of the batteries are the only current-limiting elements of the resulting series circuit. The current is

$$I = \frac{E_1 - E_2}{R_{int_1} + R_{int_2}} = \frac{12 \text{ V} - 6 \text{ V}}{0.03 \text{ } \Omega + 0.02 \text{ } \Omega} = \frac{6 \text{ V}}{0.05 \text{ } \Omega} = \textbf{120 A}$$

which far exceeds the continuous drain rating of the larger supply, resulting in a rapid discharge of E_1 and a destructive impact on the smaller supply.

6.8 OPEN AND SHORT CIRCUITS

Open circuits and short circuits can often cause more confusion and difficulty in the analysis of a system than standard series or parallel configurations. This will become more obvious in the chapters to follow when we apply some of the methods and theorems.

An **open circuit** is simply two isolated terminals not connected by an element of any kind, as shown in Fig. 6.42(a). Since a path for conduction does not exist, the current associated with an open circuit must always be zero. The voltage across the open circuit, however, can be any value, as determined by the system it is connected to. In summary, therefore,

an open circuit can have a potential difference (voltage) across its terminals, but the current is always zero amperes.
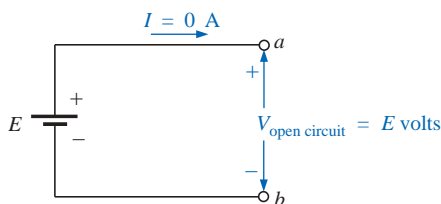
FIG. 6.43
Demonstrating the characteristics of an open circuit.
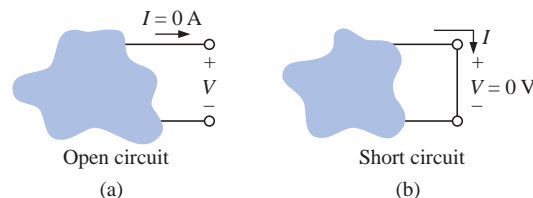
FIG. 6.42
Two special network configurations.

In Fig. 6.43, an open circuit exists between terminals *a* and *b*. As shown in the figure, the voltage across the open-circuit terminals is the supply voltage, but the current is zero due to the absence of a complete circuit.

A **short circuit** is a very low resistance, direct connection between two terminals of a network, as shown in Fig. 6.42(b). The current through the short circuit can be any value, as determined by the system it is connected to, but the voltage across the short circuit will always be zero volts because the resistance of the short circuit is assumed to be essentially zero ohms and $V = IR = I(0\ \Omega) = 0$ V.

In summary, therefore,

a short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.

In Fig. 6.44(a), the current through the 2-Ω resistor is 5 A. If a short circuit should develop across the 2-Ω resistor, the total resistance of the parallel combination of the 2-Ω resistor and the short (of essentially zero ohms) will be $2\ \Omega \parallel 0\ \Omega = \dfrac{(2\ \Omega)(0\ \Omega)}{2\ \Omega + 0\ \Omega} = 0\ \Omega$, and the current will rise to very high levels, as determined by Ohm's law:

$$I = \frac{E}{R} = \frac{10\ \text{V}}{0\ \Omega} \longrightarrow \infty\ \text{A}$$

(a)   (b)
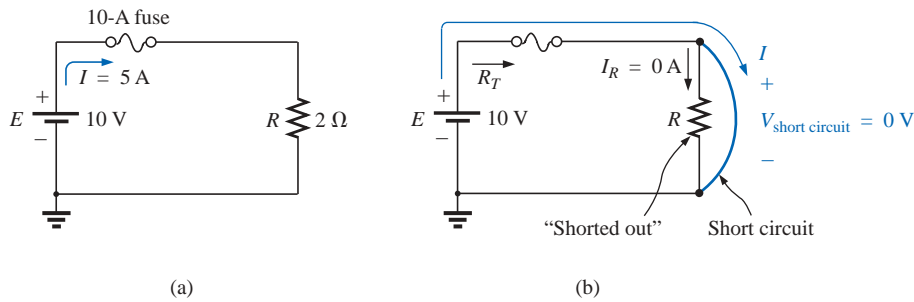
FIG. 6.44
Demonstrating the effect of a short circuit on current levels.

The effect of the 2-Ω resistor has effectively been "shorted out" by the low-resistance connection. The maximum current is now limited only by the circuit breaker or fuse in series with the source.

For the layperson, the terminology *short circuit* or *open circuit* is usually associated with dire situations such as power loss, smoke, or fire. However, in network analysis both can play an integral role in determining specific parameters about a system. Most often, however, if a short-circuit condition is to be established, it is accomplished with a *jumper*—a lead of negligible resistance to be connected between the points of interest. Establishing an open circuit simply requires making sure that the terminals of interest are isolated from each other.

EXAMPLE 6.21 Determine the voltage V_{ab} for the network of Fig. 6.45.

Solution: The open circuit requires that I be zero amperes. The voltage drop across both resistors is therefore zero volts since $V = IR = (0)R = 0$ V. Applying Kirchhoff's voltage law around the closed loop,

$$V_{ab} = E = \textbf{20 V}$$
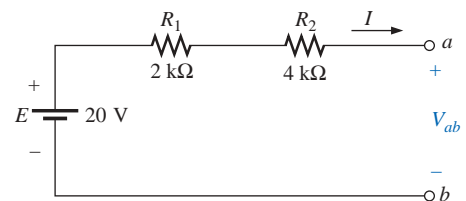
FIG. 6.45
Example 6.21.
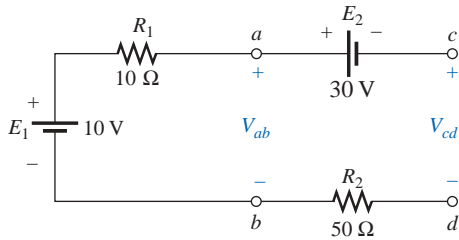
FIG. 6.46
Example 6.22.
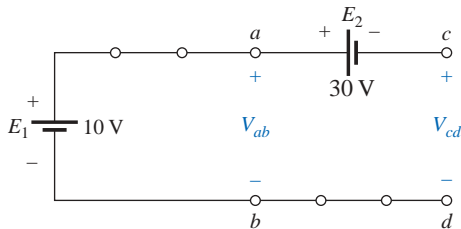
FIG. 6.47
Circuit of Fig. 6.46 redrawn.

EXAMPLE 6.22 Determine the voltages V_{ab} and V_{cd} for the network of Fig. 6.46.

Solution: The current through the system is zero amperes due to the open circuit, resulting in a 0-V drop across each resistor. Both resistors can therefore be replaced by short circuits, as shown in Fig. 6.47. The voltage V_{ab} is then directly across the 10-V battery, and

$$V_{ab} = E_1 = \mathbf{10\ V}$$

The voltage V_{cd} requires an application of Kirchhoff's voltage law:

$$+E_1 - E_2 - V_{cd} = 0$$

or $$V_{cd} = E_1 - E_2 = 10\ \text{V} - 30\ \text{V} = \mathbf{-20\ V}$$

The negative sign in the solution simply indicates that the actual voltage V_{cd} has the opposite polarity of that appearing in Fig. 6.46.

EXAMPLE 6.23 Determine the unknown voltage and current for each network of Fig. 6.48.

(a)

(b)
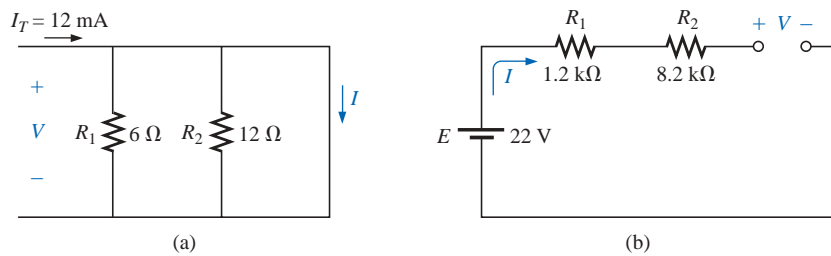
FIG. 6.48
Example 6.23.

Solution: For the network of Fig. 6.48(a), the current I_T will take the path of least resistance, and, since the short-circuit condition at the end of the network is the least-resistance path, all the current will pass through the short circuit. This conclusion can be verified using Eq. (6.9). The voltage across the network is the same as that across the short circuit and will be zero volts, as shown in Fig. 6.49(a).

(a)

(b)
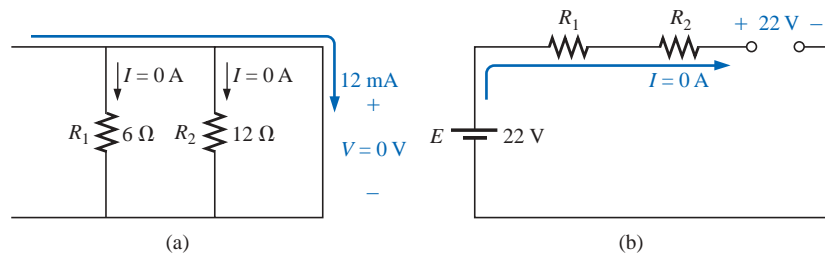
FIG. 6.49
Solutions to Example 6.23.

For the network of Fig. 6.48(b), the open-circuit condition requires that the current be zero amperes. The voltage drops across the resistors

must therefore be zero volts, as determined by Ohm's law [$V_R = IR = (0)R = 0$ V], with the resistors simply acting as a connection from the supply to the open circuit. The result is that the open-circuit voltage will be $E = 22$ V, as shown in Fig. 6.49(b).

FIG. 6.50
Example 6.24.

EXAMPLE 6.24 Calculate the current I and the voltage V for the network of Fig. 6.50.

Solution: The 10-kΩ resistor has been effectively shorted out by the jumper, resulting in the equivalent network of Fig. 6.51. Using Ohm's law,

$$I = \frac{E}{R_1} = \frac{18 \text{ V}}{5 \text{ k}\Omega} = \textbf{3.6 mA}$$

and
$$V = E = \textbf{18 V}$$

FIG. 6.51
Network of Fig. 6.50 redrawn.

EXAMPLE 6.25 Determine V and I for the network of Fig. 6.52 if the resistor R_2 is shorted out.

Solution: The redrawn network appears in Fig. 6.53. The current through the 3-Ω resistor is zero due to the open circuit, causing all the current I to pass through the jumper. Since $V_{3\Omega} = IR = (0)R = 0$ V, the voltage V is directly across the short, and

$$V = \textbf{0 V}$$

with
$$I = \frac{E}{R_1} = \frac{6 \text{ V}}{2 \text{ }\Omega} = \textbf{3 A}$$

FIG. 6.52
Example 6.25.

6.9 VOLTMETERS: LOADING EFFECT

In Chapters 2 and 5, it was noted that voltmeters are always placed across an element to measure the potential difference. We now realize that this connection is synonymous with placing the voltmeter in parallel with the element. The insertion of a meter in parallel with a resistor results in a combination of parallel resistors as shown in Fig. 6.54. Since the resistance of two parallel branches is always less than the smaller parallel resistance, the resistance of the voltmeter should be as large as possible (ideally infinite). In Fig. 6.54, a DMM with an internal resistance of 11 MΩ is measuring the voltage across a 10-kΩ resistor. The total resistance of the combination is

$$R_T = 10 \text{ k}\Omega \parallel 11 \text{ M}\Omega = \frac{(10^4 \text{ }\Omega)(11 \times 10^6 \text{ }\Omega)}{10^4 \text{ }\Omega + (11 \times 10^6 \text{ }\Omega)} = 9.99 \text{ k}\Omega$$

and we find that the network is essentially undisturbed. However, if we use a VOM with an internal resistance of 50 kΩ on the 2.5-V scale, the parallel resistance is

$$R_T = 10 \text{ k}\Omega \parallel 50 \text{ k}\Omega = \frac{(10^4 \text{ }\Omega)(50 \times 10^3 \text{ }\Omega)}{10^4 \text{ }\Omega + (50 \times 10^3 \text{ }\Omega)} = 8.33 \text{ k}\Omega$$

and the behavior of the network will be altered somewhat since the 10-kΩ resistor will now appear to be 8.33 kΩ to the rest of the network.

The loading of a network by the insertion of meters is not to be taken lightly, especially in research efforts where accuracy is a primary consideration. It is good practice always to check the meter resistance level

FIG. 6.53
Network of Fig. 6.52 with R_2 replaced by a jumper.
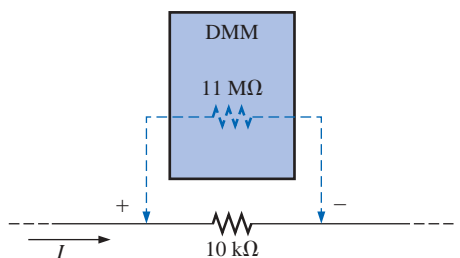
FIG. 6.54
Voltmeter loading.

against the resistive elements of the network before making measurements. A factor of 10 between resistance levels will usually provide fairly accurate meter readings for a wide range of applications.

Most DMMs have internal resistance levels in excess of 10 MΩ on all voltage scales, while the internal resistance of VOMs is sensitive to the chosen scale. To determine the resistance of each scale setting of a VOM in the voltmeter mode, simply multiply the maximum voltage of the scale setting by the **ohm/volt (Ω/V) rating** of the meter, normally found at the bottom of the face of the meter.

For a typical ohm/volt rating of 20,000, the 2.5-V scale would have an internal resistance of

$$(2.5 \text{ V})(20,000 \text{ Ω/V}) = 50 \text{ kΩ}$$

whereas for the 100-V scale, it would be

$$(100 \text{ V})(20,000 \text{ Ω/V}) = 2 \text{ MΩ}$$

and for the 250-V scale,

$$(250 \text{ V})(20,000 \text{ Ω/V}) = 5 \text{ MΩ}$$
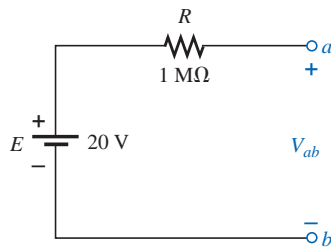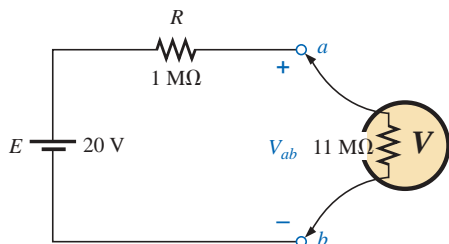
FIG. 6.55
Example 6.26.

FIG. 6.56
Applying a DMM to the circuit of Fig. 6.55.

EXAMPLE 6.26 For the relatively simple network of Fig. 6.55:
a. What is the open-circuit voltage V_{ab}?
b. What will a DMM indicate if it has an internal resistance of 11 MΩ? Compare your answer to the results of part (a).
c. Repeat part (b) for a VOM with an Ω/V rating of 20,000 on the 100-V scale.

Solutions:
a. $V_{ab} = $ **20 V**
b. The meter will complete the circuit as shown in Fig. 6.56. Using the voltage divider rule,

$$V_{ab} = \frac{11 \text{ MΩ}(20 \text{ V})}{11 \text{ MΩ} + 1 \text{ MΩ}} = \textbf{18.33 V}$$

c. For the VOM, the internal resistance of the meter is

$$R_m = 100 \text{ V}(20,000 \text{ Ω/V}) = 2 \text{ MΩ}$$

and

$$V_{ab} = \frac{2 \text{ MΩ}(20 \text{ V})}{2 \text{ MΩ} + 1 \text{ MΩ}} = \textbf{13.33 V}$$

revealing the need to consider carefully the internal resistance of the meter in some instances.

Measurement Techniques

For components in series, the placement of ammeters and voltmeters was quite straightforward if a few simple rules were followed. For parallel circuits, however, some of the measurements can require a little extra care. For any configuration keep in mind that all voltage measurements can be made without disturbing the network at all. For ammeters, however, the branch in which the current is to be measured must be opened and the meter inserted.

Since the voltage is the same across parallel elements, only one voltmeter will be required as shown in Fig. 6.57. It is a two-point measurement, with the negative or black lead connected to the point of lower potential and the positive or red lead to the point of higher potential to

This time, rather than using meters to make the measurements, we will use indicators. The **Indicators** key pad is the tenth down on the left toolbar. It has the appearance of an LCD display with the number 8. Once it has been selected, eight possible indicators will appear. For this example, the **A** indicator, representing an ammeter, will be used since we are interested only in the curent levels. When **A** has been selected, a **Component Browser** will appear with four choices under the **Component Name List;** each option refers to a position for the ammeter. The **H** means "horizontal" as shown in the picture window when the dialog box is first opened. The **HR** means "horizontal," but with the polarity reversed. The **V** is for a vertical configuration with the positive sign at the top, and the **VR** is the vertical position with the positive sign at the bottom. Simply select the one you want followed by an **OK,** and your choice will appear in that position on the screen. Click it into position, and you can return for the next indicator. Once all the elements are in place and their values set, simulation can be initiated with the sequence **Simulate-Run.** The results shown in Fig. 6.66 will appear.

Note that all the results appear with the indicator boxes. All are positive results because the ammeters were all entered with a configuration that would result in conventional current entering the positive current. Also note that as was true for inserting the meters, the indicators are placed in series with the branch in which the current is to be measured.

PROBLEMS

SECTION 6.2   Parallel Elements

1. For each configuration of Fig. 6.67, determine which elements are in series and which are in parallel.

(a)   (b)   (c)
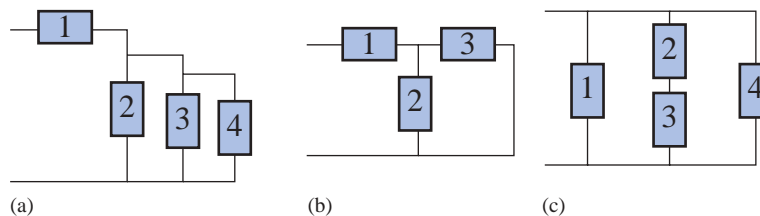
FIG. 6.67
Problem 1.

2. For the network of Fig. 6.68:
    a. Which elements are in parallel?
    b. Which elements are in series?
    c. Which branches are in parallel?
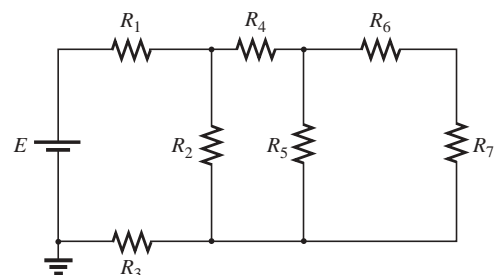
FIG. 6.68
Problem 2.

SECTION 6.3 Total Conductance and Resistance

3. Find the total conductance and resistance for the networks of Fig. 6.69.
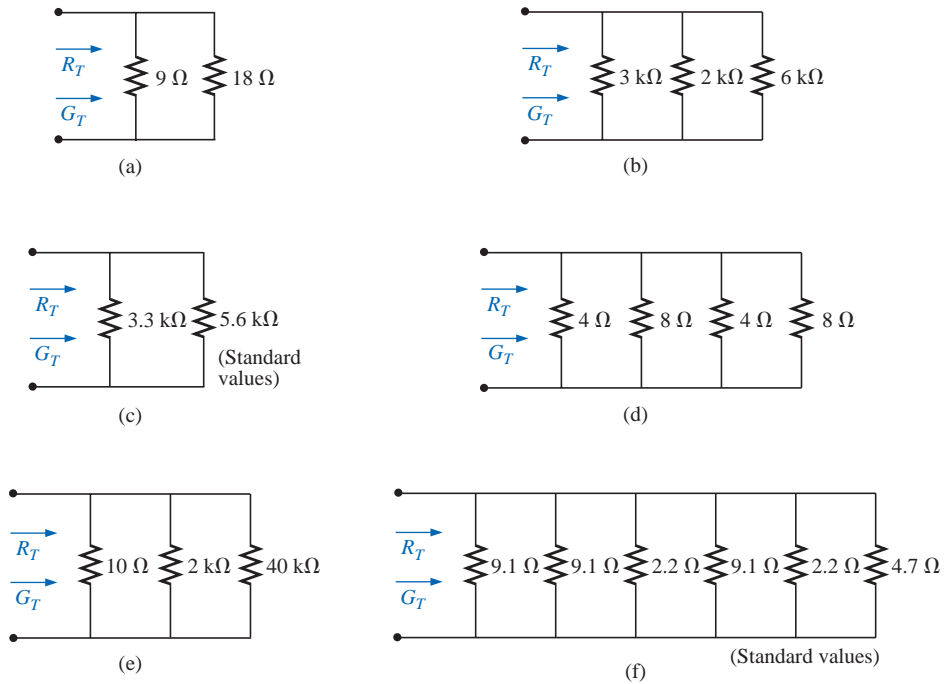
FIG. 6.69
Problem 3.

4. The total conductance of each network of Fig. 6.70 is specified. Find the value in ohms of the unknown resistances.
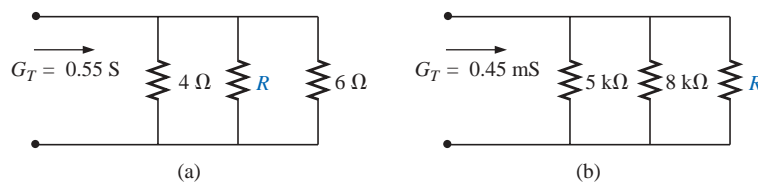
FIG. 6.70
Problem 4.

5. The total resistance of each circuit of Fig. 6.71 is specified. Find the value in ohms of the unknown resistances.
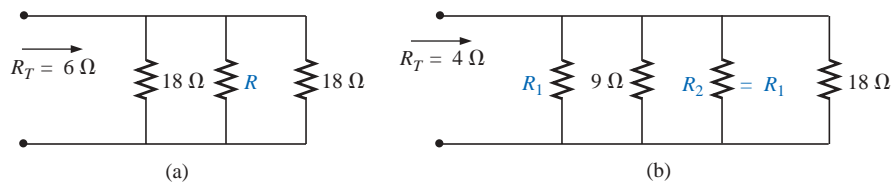
FIG. 6.71
Problem 5.

*6. Determine the unknown resistors of Fig. 6.72 given the
fact that $R_2 = 5R_1$ and $R_3 = (1/2)R_1$.

*7. Determine R_1 for the network of Fig. 6.73.
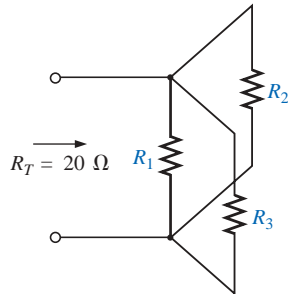
FIG. 6.72
Problem 6.
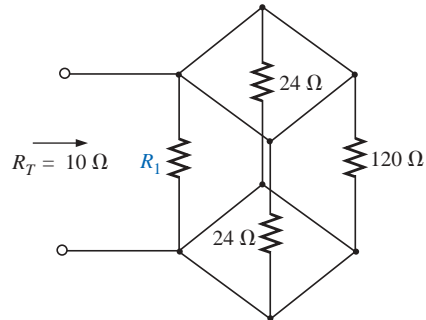
FIG. 6.73
Problem 7.

SECTION 6.4 Parallel Circuits

8. For the network of Fig. 6.74:
   a. Find the total conductance and resistance.
   b. Determine I_s and the current through each parallel
   branch.
   c. Verify that the source current equals the sum of the
   parallel branch currents.
   d. Find the power dissipated by each resistor, and note
   whether the power delivered is equal to the power dis-
   sipated.
   e. If the resistors are available with wattage ratings of
   1/2, 1, 2, and 50 W, what is the minimum wattage rat-
   ing for each resistor?

9. Repeat Problem 8 for the network of Fig. 6.75.
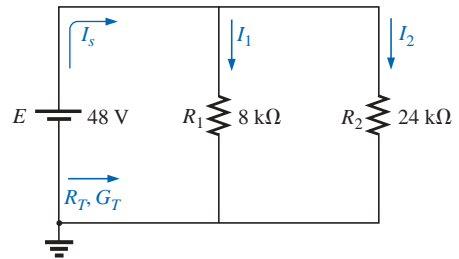
FIG. 6.74
Problem 8.
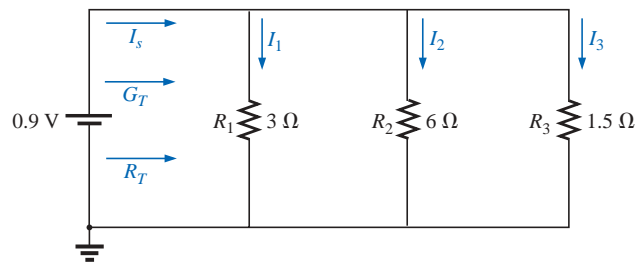
FIG. 6.75
Problem 9.

10. Repeat Problem 8 for the network of Fig. 6.76 con-
structed of standard resistor values.
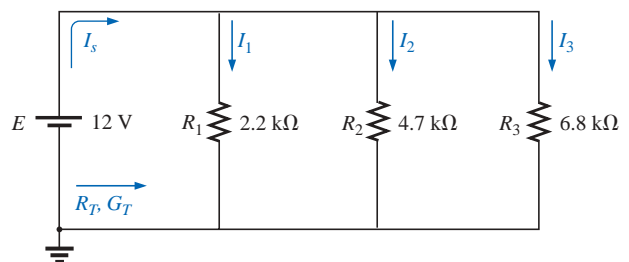
FIG. 6.76
Problem 10.
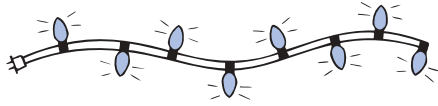
FIG. 6.77
Problem 11.

11. Eight holiday lights are connected in parallel as shown in Fig. 6.77.
   a. If the set is connected to a 120-V source, what is the current through each bulb if each bulb has an internal resistance of 1.8 kΩ?
   b. Determine the total resistance of the network.
   c. Find the power delivered to each bulb.
   d.. If one bulb burns out (that is, the filament opens), what is the effect on the remaining bulbs?
   e. Compare the parallel arrangement of Fig. 6.77 to the series arrangement of Fig. 5.87. What are the relative advantages and disadvantages of the parallel system compared to the series arrangement?

12. A portion of a residential service to a home is depicted in Fig. 6.78.
   a. Determine the current through each parallel branch of the network.
   b. Calculate the current drawn from the 120-V source. Will the 20-A circuit breaker trip?
   c. What is the total resistance of the network?
   d. Determine the power supplied by the 120-V source. How does it compare to the total power of the load?
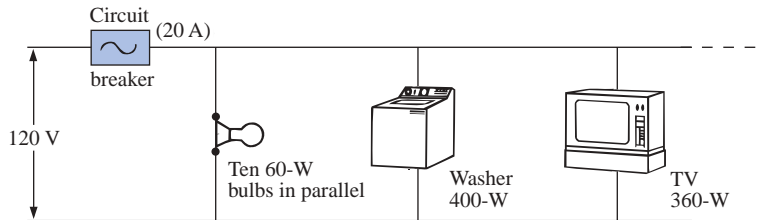
FIG. 6.78
Problems 12 and 37.

13. Determine the currents I_1 and I_s for the networks of Fig. 6.79.
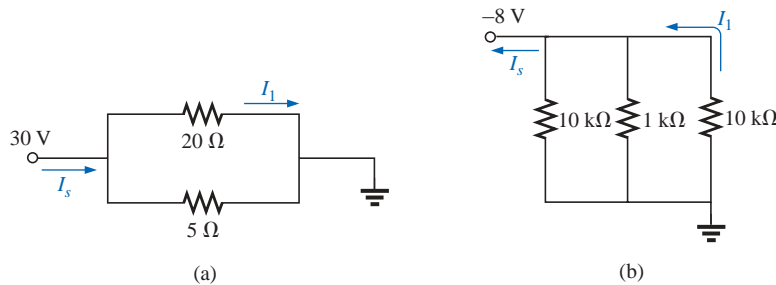
FIG. 6.79
Problem 13.

14. Using the information provided, determine the resistance R_1 for the network of Fig. 6.80.

*15. Determine the power delivered by the dc battery in Fig. 6.81.
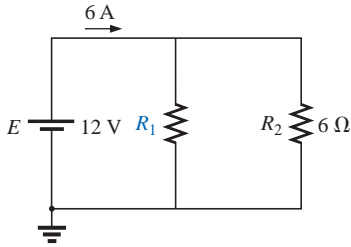
FIG. 6.80
Problem 14.
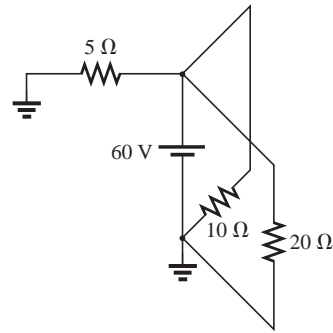
FIG. 6.81
Problem 15.

*16. For the network of Fig. 6.82:
  a. Find the current I_1.
  b. Calculate the power dissipated by the 4-Ω resistor.
  c. Find the current I_2.
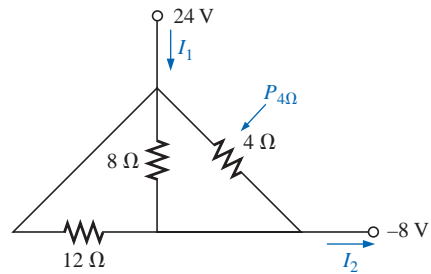
FIG. 6.82
Problem 16.

*17. For the network of Fig. 6.83:
  a. Find the current I.
  b. Determine the voltage V.
  c. Calculate the source current I_s.
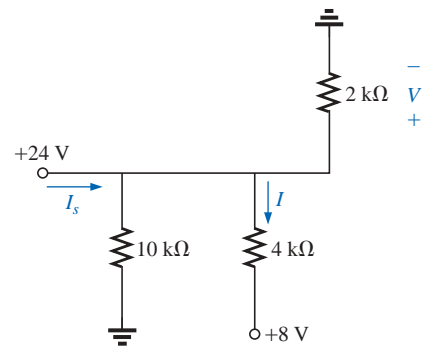
FIG. 6.83
Problem 17.

SECTION 6.5   Kirchhoff's Current Law

18. Find all unknown currents and their directions in the circuits of Fig. 6.84.
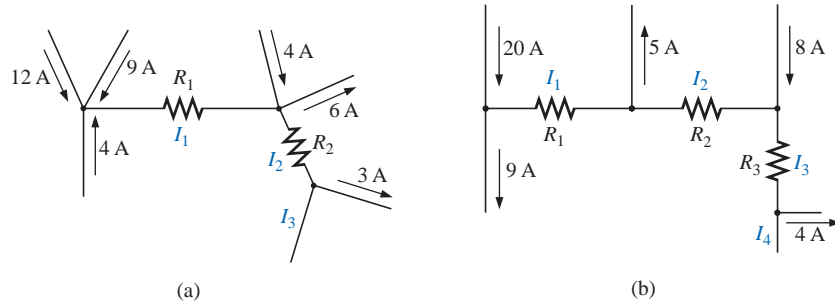
FIG. 6.84
Problems 18 and 38.

*19. Using Kirchhoff's current law, determine the unknown currents for the networks of Fig. 6.85.
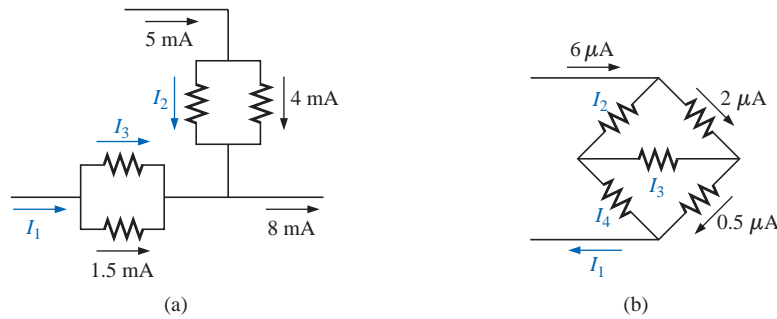
FIG. 6.85
Problem 19.

20. Using the information provided in Fig. 6.86, find the branch resistors R_1 and R_3, the total resistance R_T, and the voltage source E.
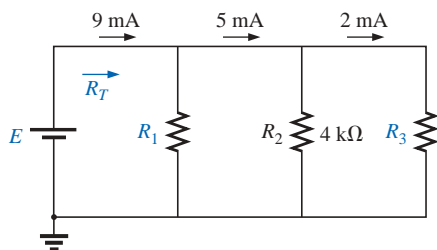
FIG. 6.86
Problem 20.

*21. Find the unknown quantities for the circuits of Fig. 6.87 using the information provided.

(a) (b) (c) (d)
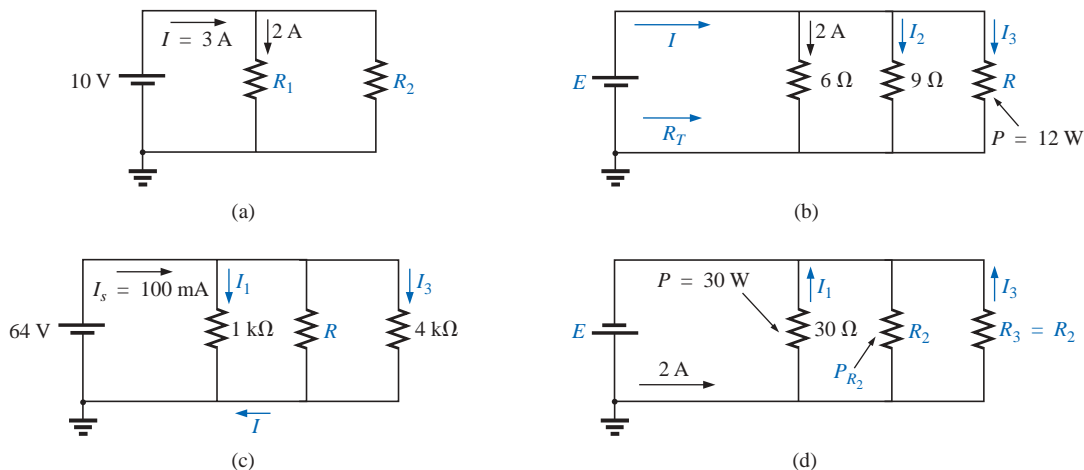
FIG. 6.87
Problem 21.

SECTION 6.6   Current Divider Rule

22. Using the information provided in Fig. 6.88, determine the current through each branch using simply the ratio of parallel resistor values. Then determine the total current I_T.

23. Using the current divider rule, find the unknown currents for the networks of Fig. 6.89.

FIG. 6.88
Problem 22.

(a) (b) (c) (d)
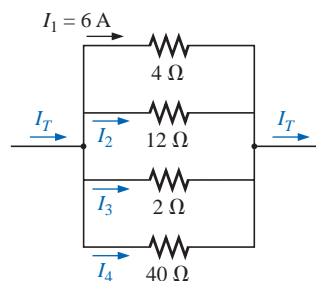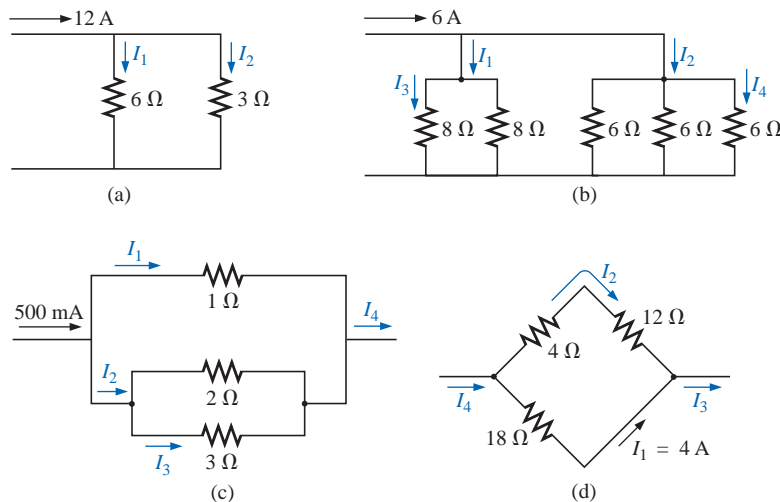
FIG. 6.89
Problem 23.
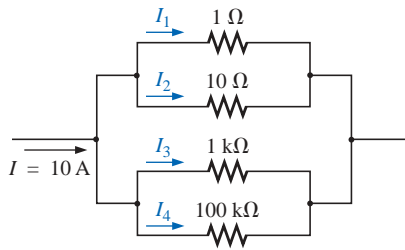
FIG. 6.90
Problem 24.

*24. Parts (a), (b), and (c) of this problem should be done by inspection—that is, mentally. The intent is to obtain a solution without a lengthy series of calculations. For the network of Fig. 6.90:

   a. What is the approximate value of I_1 considering the magnitude of the parallel elements?
   b. What are the ratios I_1/I_2 and I_3/I_4?
   c. What are the ratios I_2/I_3 and I_1/I_4?
   d. Calculate the current I_1, and compare it to the result of part (a).
   e. Determine the current I_4 and calculate the ratio I_1/I_4. How does the ratio compare to the result of part (c)?

25. Find the unknown quantities using the information provided for the networks of Fig. 6.91.

(a)

(b)
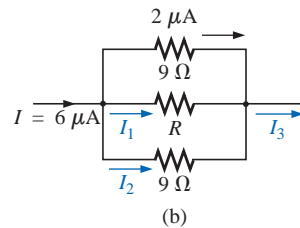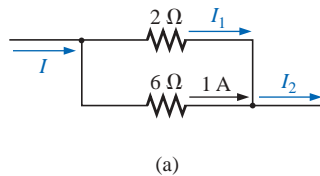
FIG. 6.91
Problem 25.

*26. For the network of Fig. 6.92, calculate the resistor R that will ensure the current $I_1 = 3I_2$.

*27. Design the network of Fig. 6.93 such that $I_2 = 4I_1$ and $I_3 = 3I_2$.
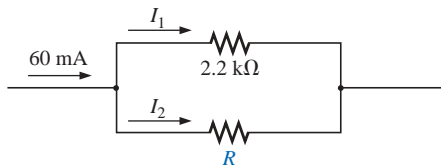
FIG. 6.92
Problem 26.
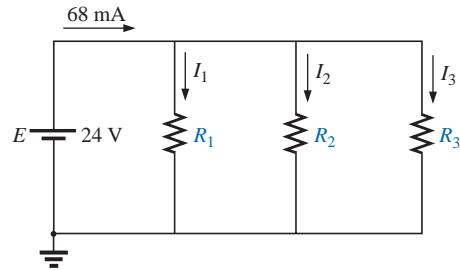
FIG. 6.93
Problem 27.

SECTION 6.7   Voltage Sources in Parallel

28. Assuming identical supplies, determine the currents I_1 and I_2 for the network of Fig. 6.94.

29. Assuming identical supplies, determine the current I and resistance R for the parallel network of Fig. 6.95.
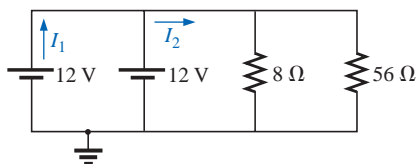
FIG. 6.94
Problem 28.
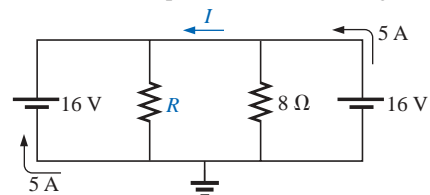
FIG. 6.95
Problem 29.

SECTION 6.8   Open and Short Circuits

30. For the network of Fig. 6.96:
   a. Determine I_s and V_L.
   b. Determine I_s if R_L is shorted out.
   c. Determine V_L if R_L is replaced by an open circuit.
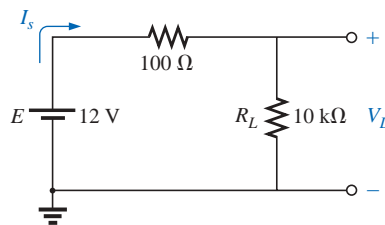
FIG. 6.96
Problem 30.

31. For the network of Fig. 6.97:
   a. Determine the open-circuit voltage V_L.
   b. If the 2.2-kΩ resistor is short circuited, what is the new value of V_L?
   c. Determine V_L if the 4.7-kΩ resistor is replaced by an open circuit.
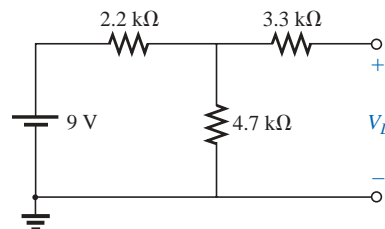
FIG. 6.97
Problem 31.

*32. For the network of Fig. 6.98, determine
   a. the short-circuit currents I_1 and I_2.
   b. the voltages V_1 and V_2.
   c. the source current I_s.
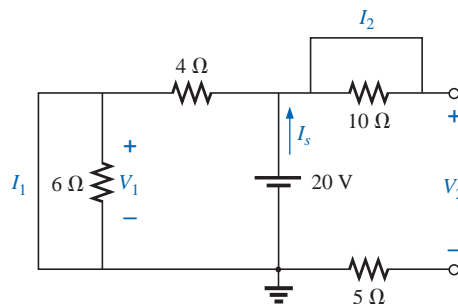
FIG. 6.98
Problem 32.

SECTION 6.9   Voltmeters: Loading Effect

33. For the network of Fig. 6.99:
   a. Determine the voltage V_2.
   b. Determine the reading of a DMM having an internal resistance of 11 MΩ when used to measure V_2.
   c. Repeat part (b) with a VOM having an ohm/volt rating of 20,000 using the 10-V scale. Compare the results of parts (b) and (c). Explain any difference.
   d. Repeat part (c) with $R_1 = 100$ kΩ and $R_2 = 200$ kΩ.
   e. Based on the above, can you make any general conclusions about the use of a voltmeter?
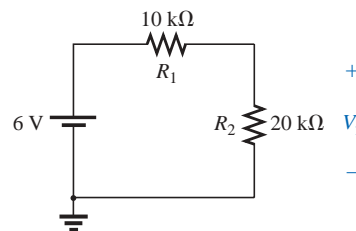
FIG. 6.99
Problems 33 and 40.

SECTION 6.10   Troubleshooting Techniques

34. Based on the measurements of Fig. 6.100, determine whether the network is operating correctly. If not, try to determine why.
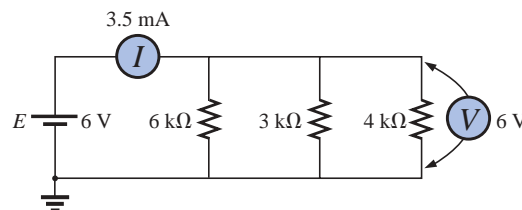
FIG. 6.100
Problem 34.

35. Referring to the network of Fig. 6.101, is $V_a = 8.8$ V the correct reading for the given configuration? If not, which element has been connected incorrectly in the network?
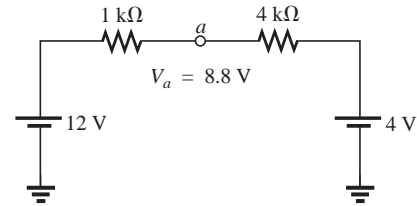
FIG. 6.101
Problem 35.
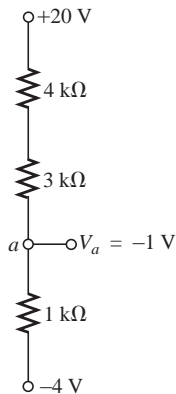
36. a. The voltage V_a for the network of Fig. 6.102 is **−1 V**. If it suddenly jumped to 20 V, what could have happened to the circuit structure? Localize the problem area.
  b. If the voltage V_a is 6 V rather than −1 V, try to explain what is wrong about the network construction.

SECTION 6.12 Computer Analysis

PSpice or Electronics Workbench

37. Using schematics, determine all the currents for the network of Fig. 6.78.

38. Using schematics, determine the unknown quantities for the network of Fig. 6.84.

Programming Language (C++, QBASIC, Pascal, etc.)

39. Write a program to determine the total resistance and conductance of any number of elements in parallel.

40. Write a program that will tabulate the voltage V_2 of Fig. 6.99 measured by a VOM with an internal resistance of 200 kΩ as R_2 varies from 10 kΩ to 200 kΩ in increments of 10 kΩ.

FIG. 6.102
Problem 36.

GLOSSARY

Current divider rule (CDR)   A method by which the current through parallel elements can be determined without first finding the voltage across those parallel elements.

Kirchhoff's current law (KCL)   The algebraic sum of the currents entering and leaving a node is zero.

Node   A junction of two or more branches.

Ohm/volt (Ω/V) rating   A rating used to determine both the current sensitivity of the movement and the internal resistance of the meter.

Open circuit   The absence of a direct connection between two points in a network.

Parallel circuit   A circuit configuration in which the elements have two points in common.

Short circuit   A direct connection of low resistive value that can significantly alter the behavior of an element or system.

7
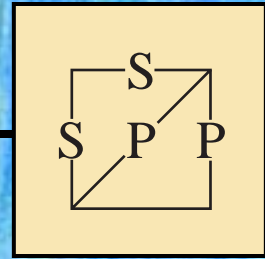
Series-Parallel Networks

7.1 SERIES-PARALLEL NETWORKS

A firm understanding of the basic principles associated with series and parallel circuits is a sufficient background to begin an investigation of any single-source dc network having a combination of series and parallel elements or branches. Multisource networks are considered in detail in Chapters 8 and 9. In general,

series-parallel networks *are networks that contain both series and parallel circuit configurations.*

One can become proficient in the analysis of series-parallel networks only through exposure, practice, and experience. In time the path to the desired unknown becomes more obvious as one recalls similar configurations and the frustration resulting from choosing the wrong approach. There are a few steps that can be helpful in getting started on the first few exercises, although the value of each will become apparent only with experience.

General Approach

1. Take a moment to study the problem "in total" and make a brief mental sketch of the overall approach you plan to use. The result may be time- and energy-saving shortcuts.
2. Next examine each region of the network independently before tying them together in series-parallel combinations. This will usually simplify the network and possibly reveal a direct approach toward obtaining one or more desired unknowns. It also eliminates many of the errors that might result due to the lack of a systematic approach.
3. Redraw the network as often as possible with the reduced branches and undisturbed unknown quantities to maintain clarity and provide the reduced networks for the trip back to unknown quantities from the source.
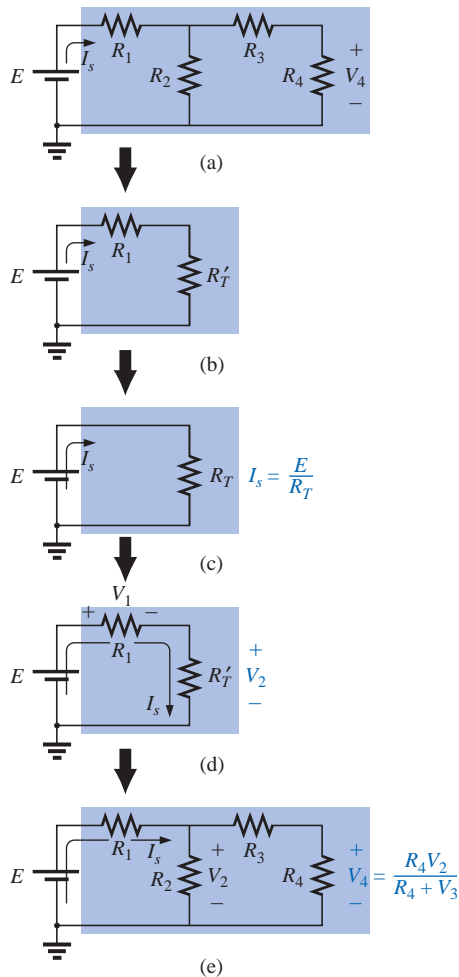
FIG. 7.1

Introducing the reduce and return approach.

4. When you have a solution, check that it is reasonable by considering the magnitudes of the energy source and the elements in the network. If it does not seem reasonable, either solve the circuit using another approach or check over your work very carefully.

Reduce and Return Approach

For many single-source, series-parallel networks, the analysis is one that works back to the source, determines the source current, and then finds its way to the desired unknown. In Fig. 7.1(a), for instance, the voltage V_4 is desired. The absence of a single series or parallel path to V_4 from the source immediately reveals that the methods introduced in the last two chapters cannot be applied here. First, series and parallel elements must be combined to establish the reduced circuit of Fig. 7.1(b). Then series elements are combined to form the simplest of configurations in Fig. 7.1(c). The source current can now be determined using Ohm's law, and we can proceed back through the network as shown in Fig. 7.1(d). The voltage V_2 can be determined and then the original network can be redrawn, as shown in Fig. 7.1(e). Since V_2 is now known, the voltage divider rule can be used to find the desired voltage V_4. Because of the similarities between the networks of Figs. 7.1(a) and 7.1(e), and between 7.1(b) and 7.1(d), the networks drawn during the reduction phase are often used for the return path.

Although all the details of the analysis were not described above, the general procedure for a number of series-parallel network problems employs the procedure described above: Work back for I_s and then follow the return path for the specific unknown. Not every problem will follow this path; some will have simpler, more direct solutions. However, the reduce and return approach will handle one type of problem that does surface over and over again.

Block Diagram Approach

The block diagram approach will be employed throughout to emphasize the fact that combinations of elements, not simply single resistive elements, can be in series or parallel. The approach will also reveal the number of seemingly different networks that have the same basic structure and therefore can involve similar analysis techniques.

Initially, there will be some concern about identifying series and parallel elements and branches and choosing the best procedure to follow toward a solution. However, as you progress through the examples and try a few problems, a common path toward most solutions will surface that can actually make the analysis of such systems an interesting, enjoyable experience.

In Fig. 7.2, blocks B and C are in parallel (points b and c in common), and the voltage source E is in series with block A (point a in common). The parallel combination of B and C is also in series with A and the voltage source E due to the common points b and $c,$ respectively.

To ensure that the analysis to follow is as clear and uncluttered as possible, the following notation will be used for series and parallel combinations of elements. For series resistors R_1 and R_2, a comma will be inserted between their subscript notations, as shown here:

$$R_{1,2} = R_1 + R_2$$
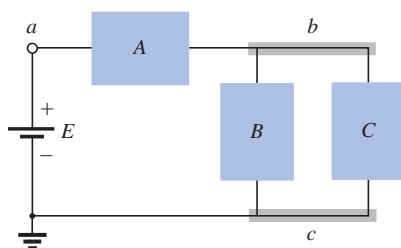
FIG. 7.2

Introducing the block diagram approach.

For parallel resistors R_1 and R_2, the parallel symbol will be inserted between their subscript notations, as follows:

$$R_{1\|2} = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

EXAMPLE 7.1 If each block of Fig. 7.2 were a single resistive element, the network of Fig. 7.3 might result.

The parallel combination of R_B and R_C results in

$$R_{B\|C} = R_B \| R_C = \frac{(12 \text{ k}\Omega)(6 \text{ k}\Omega)}{12 \text{ k}\Omega + 6 \text{ k}\Omega} = 4 \text{ k}\Omega$$

The equivalent resistance $R_{B\|C}$ is then in series with R_A, and the total resistance "seen" by the source is

$$R_T = R_A + R_{B\|C}$$
$$= 2 \text{ k}\Omega + 4 \text{ k}\Omega = \mathbf{6 \text{ k}\Omega}$$

The result is an equivalent network, as shown in Fig. 7.4, permitting the determination of the source current I_s.

$$I_s = \frac{E}{R_T} = \frac{54 \text{ V}}{6 \text{ k}\Omega} = \mathbf{9 \text{ mA}}$$

and, since the source and R_A are in series,

$$I_A = I_s = 9 \text{ mA}$$

We can then use the equivalent network of Fig. 7.5 to determine I_B and I_C using the current divider rule:

$$I_B = \frac{6 \text{ k}\Omega(I_s)}{6 \text{ k}\Omega + 12 \text{ k}\Omega} = \frac{6}{18}I_s = \frac{1}{3}(9 \text{ mA}) = \mathbf{3 \text{ mA}}$$

$$I_C = \frac{12 \text{ k}\Omega(I_s)}{12 \text{ k}\Omega + 6 \text{ k}\Omega} = \frac{12}{18}I_s = \frac{2}{3}(9 \text{ mA}) = \mathbf{6 \text{ mA}}$$

or, applying Kirchhoff's current law,

$$I_C = I_s - I_B = 9 \text{ mA} - 3 \text{ mA} = \mathbf{6 \text{ mA}}$$

Note that in this solution, we worked back to the source to obtain the source current or total current supplied by the source. The remaining unknowns were then determined by working back through the network to find the other unknowns.

FIG. 7.3
Example 7.1.

FIG. 7.4
Reduced equivalent of Fig. 7.3.
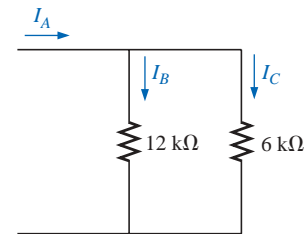
FIG. 7.5
Determining I_B and I_C for the network of Fig. 7.3.

EXAMPLE 7.2 It is also possible that the blocks *A*, *B*, and *C* of Fig. 7.2 contain the elements and configurations of Fig. 7.6. Working with each region:

$$A: \quad R_A = 4 \text{ } \Omega$$

$$B: \quad R_B = R_2 \| R_3 = R_{2\|3} = \frac{R}{N} = \frac{4 \text{ } \Omega}{2} = 2 \text{ } \Omega$$

$$C: \quad R_C = R_4 + R_5 = R_{4,5} = 0.5 \text{ } \Omega + 1.5 \text{ } \Omega = 2 \text{ } \Omega$$

Blocks *B* and *C* are still in parallel, and

$$R_{B\|C} = \frac{R}{N} = \frac{2 \text{ } \Omega}{2} = 1 \text{ } \Omega$$
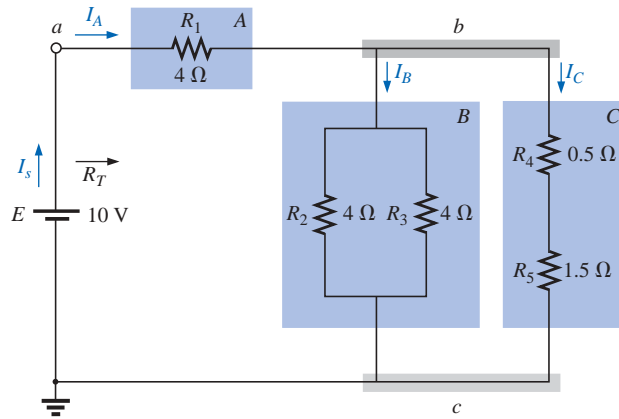
FIG. 7.6

Example 7.2.

with

$$R_T = R_A + R_{B\|C}$$
$$= 4\ \Omega + 1\ \Omega = \mathbf{5\ \Omega}$$

(Note the similarity between this equation and that obtained for Example 7.1.)

and

$$I_s = \frac{E}{R_T} = \frac{10\ \text{V}}{5\ \Omega} = \mathbf{2\ A}$$

We can find the currents I_A, I_B, and I_C using the reduction of the network of Fig. 7.6 (recall Step 3) as found in Fig. 7.7. Note that I_A, I_B, and I_C are the same in Figs. 7.6 and 7.7 and therefore also appear in Fig. 7.7. In other words, the currents I_A, I_B, and I_C of Fig. 7.7 will have the same magnitude as the same currents of Fig. 7.6.

$$I_A = I_s = \mathbf{2\ A}$$

and
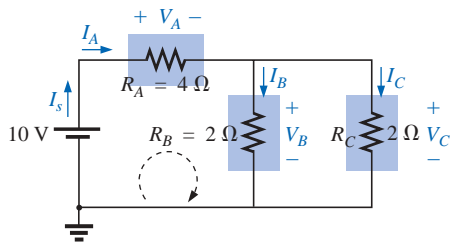
$$I_B = I_C = \frac{I_A}{2} = \frac{I_s}{2} = \frac{2\ \text{A}}{2} = \mathbf{1\ A}$$

FIG. 7.7

Reduced equivalent of Fig. 7.6.

Returning to the network of Fig. 7.6, we have

$$I_{R_2} = I_{R_3} = \frac{I_B}{2} = \mathbf{0.5\ A}$$

The voltages V_A, V_B, and V_C from either figure are

$$V_A = I_A R_A = (2\ \text{A})(4\ \Omega) = \mathbf{8\ V}$$
$$V_B = I_B R_B = (1\ \text{A})(2\ \Omega) = \mathbf{2\ V}$$
$$V_C = V_B = \mathbf{2\ V}$$

Applying Kirchhoff's voltage law for the loop indicated in Fig. 7.7, we obtain

$$\Sigma_{\circlearrowleft} V = E - V_A - V_B = 0$$
$$E = V_A + V_B = 8\ \text{V} + 2\ \text{V}$$

or

$$\underline{10\ \text{V} = 10\ \text{V}\quad \text{(checks)}}$$

EXAMPLE 7.3 Another possible variation of Fig. 7.2 appears in Fig. 7.8.

$$R_A = R_{1\|2} = \frac{(9\ \Omega)(6\ \Omega)}{9\ \Omega + 6\ \Omega} = \frac{54\ \Omega}{15} = 3.6\ \Omega$$
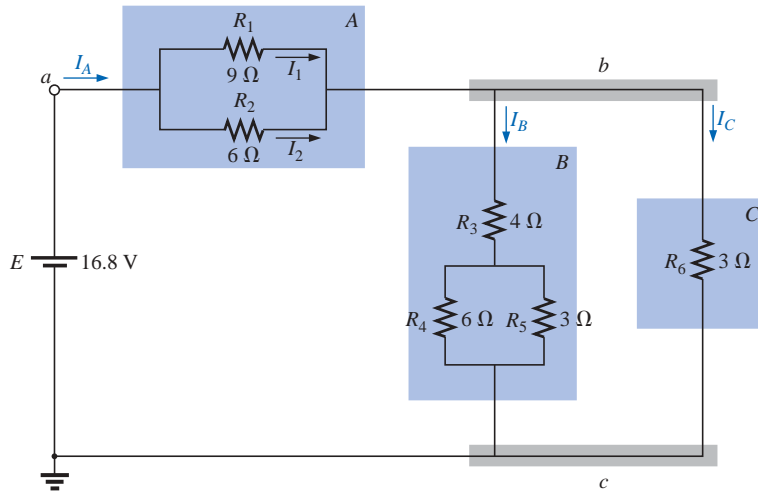
FIG. 7.8
Example 7.3.

$$R_B = R_3 + R_{4\|5} = 4\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 4\ \Omega + 2\ \Omega = 6\ \Omega$$

$$R_C = 3\ \Omega$$

The network of Fig. 7.8 can then be redrawn in reduced form, as shown in Fig. 7.9. Note the similarities between this circuit and the circuits of Figs. 7.3 and 7.7.

$$R_T = R_A + R_{B\|C} = 3.6\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega}$$

$$= 3.6\ \Omega + 2\ \Omega = \textbf{5.6}\ \boldsymbol{\Omega}$$

$$I_s = \frac{E}{R_T} = \frac{16.8\ \text{V}}{5.6\ \Omega} = \textbf{3 A}$$

$$I_A = I_s = \textbf{3 A}$$

FIG. 7.9
Reduced equivalent of Fig. 7.8.

Applying the current divider rule yields

$$I_B = \frac{R_C I_A}{R_C + R_B} = \frac{(3\ \Omega)(3\ \text{A})}{3\ \Omega + 6\ \Omega} = \frac{9\ \text{A}}{9} = \textbf{1 A}$$

By Kirchhoff's current law,

$$I_C = I_A - I_B = 3\ \text{A} - 1\ \text{A} = \textbf{2 A}$$

By Ohm's law,

$$V_A = I_A R_A = (3\ \text{A})(3.6\ \Omega) = \textbf{10.8 V}$$

$$V_B = I_B R_B = V_C = I_C R_C = (2\ \text{A})(3\ \Omega) = \textbf{6 V}$$

Returning to the original network (Fig. 7.8) and applying the current divider rule,

$$I_1 = \frac{R_2 I_A}{R_2 + R_1} = \frac{(6\ \Omega)(3\ \text{A})}{6\ \Omega + 9\ \Omega} = \frac{18\ \text{A}}{15} = \textbf{1.2 A}$$

By Kirchhoff's current law,

$$I_2 = I_A - I_1 = 3\ \text{A} - 1.2\ \text{A} = \textbf{1.8 A}$$

Figures 7.3, 7.6, and 7.8 are only a few of the infinite variety of configurations that the network can assume starting with the basic arrangement of Fig. 7.2. They were included in our discussion to emphasize the
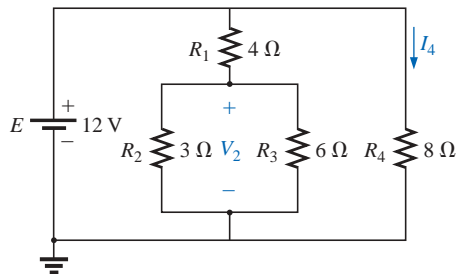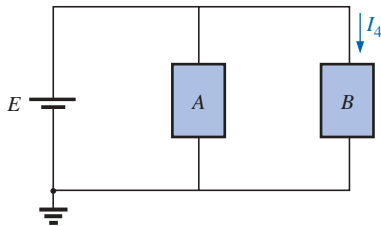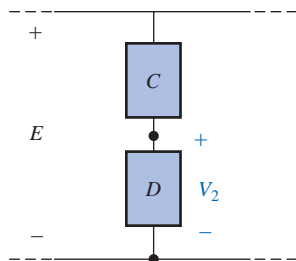
importance of considering each region of the network independently before finding the solution for the network as a whole.

The blocks of Fig. 7.2 can be arranged in a variety of ways. In fact, there is no limit on the number of series-parallel configurations that can appear within a given network. In reverse, the block diagram approach can be used effectively to reduce the apparent complexity of a system by identifying the major series and parallel components of the network. This approach will be demonstrated in the next few examples.

7.2 DESCRIPTIVE EXAMPLES

EXAMPLE 7.4 Find the current I_4 and the voltage V_2 for the network of Fig. 7.10.

Solution: In this case, particular unknowns are requested instead of a complete solution. It would, therefore, be a waste of time to find all the currents and voltages of the network. The method employed should concentrate on obtaining only the unknowns requested. With the block diagram approach, the network has the basic structure of Fig. 7.11, clearly indicating that the three branches are in parallel and the voltage across A and B is the supply voltage. The current I_4 is now immediately obvious as simply the supply voltage divided by the resultant resistance for B. If desired, block A could be broken down further, as shown in Fig. 7.12, to identify C and D as series elements, with the voltage V_2 capable of being determined using the voltage divider rule once the resistance of C and D is reduced to a single value. This is an example of how a mental sketch of the approach might be made before applying laws, rules, and so on, to avoid dead ends and growing frustration.

Applying Ohm's law,

$$I_4 = \frac{E}{R_B} = \frac{E}{R_4} = \frac{12\text{ V}}{8\text{ }\Omega} = \mathbf{1.5\text{ A}}$$

Combining the resistors R_2 and R_3 of Fig. 7.10 will result in

$$R_D = R_2 \parallel R_3 = 3\text{ }\Omega \parallel 6\text{ }\Omega = \frac{(3\text{ }\Omega)(6\text{ }\Omega)}{3\text{ }\Omega + 6\text{ }\Omega} = \frac{18\text{ }\Omega}{9} = 2\text{ }\Omega$$

and, applying the voltage divider rule,

$$V_2 = \frac{R_D E}{R_D + R_C} = \frac{(2\text{ }\Omega)(12\text{ V})}{2\text{ }\Omega + 4\text{ }\Omega} = \frac{24\text{ V}}{6} = \mathbf{4\text{ V}}$$

EXAMPLE 7.5 Find the indicated currents and voltages for the network of Fig. 7.13.

Solution: Again, only specific unknowns are requested. When the network is redrawn, it will be particularly important to note which unknowns are preserved and which will have to be determined using the original configuration. The block diagram of the network may appear as shown in Fig. 7.14, clearly revealing that A and B are in series. Note in this form the number of unknowns that have been preserved. The voltage V_1 will be the same across the three parallel branches of Fig. 7.13, and V_5 will be the same across R_4 and R_5. The unknown currents I_2 and I_4 are lost since they represent the currents through only one of the parallel branches. However, once V_1 and V_5 are known, the required currents can be found using Ohm's law.

FIG. 7.10

Example 7.4.

FIG. 7.11

Block diagram of Fig. 7.10.

FIG. 7.12

Alternative block diagram for the first parallel branch of Fig. 7.10.
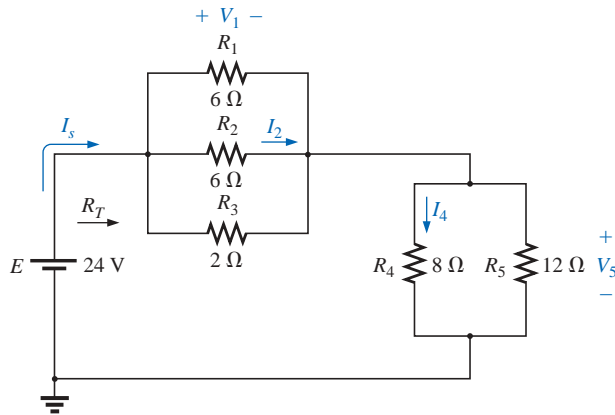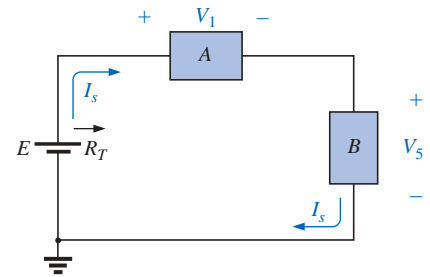
FIG. 7.13
Example 7.5.

FIG. 7.14
Block diagram for Fig. 7.13.

$$R_{1\|2} = \frac{R}{N} = \frac{6\ \Omega}{2} = 3\ \Omega$$

$$R_A = R_{1\|2\|3} = \frac{(3\ \Omega)(2\ \Omega)}{3\ \Omega + 2\ \Omega} = \frac{6\ \Omega}{5} = 1.2\ \Omega$$

$$R_B = R_{4\|5} = \frac{(8\ \Omega)(12\ \Omega)}{8\ \Omega + 12\ \Omega} = \frac{96\ \Omega}{20} = 4.8\ \Omega$$

The reduced form of Fig. 7.13 will then appear as shown in Fig. 7.15, and

$$R_T = R_{1\|2\|3} + R_{4\|5} = 1.2\ \Omega + 4.8\ \Omega = \mathbf{6\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{24\ \text{V}}{6\ \Omega} = \mathbf{4\ A}$$

with

$$V_1 = I_s R_{1\|2\|3} = (4\ \text{A})(1.2\ \Omega) = \mathbf{4.8\ V}$$
$$V_5 = I_s R_{4\|5} = (4\ \text{A})(4.8\ \Omega) = \mathbf{19.2\ V}$$

Applying Ohm's law,

$$I_4 = \frac{V_5}{R_4} = \frac{19.2\ \text{V}}{8\ \Omega} = \mathbf{2.4\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_1}{R_2} = \frac{4.8\ \text{V}}{6\ \Omega} = \mathbf{0.8\ A}$$
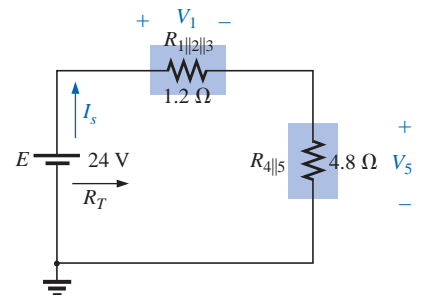
FIG. 7.15
Reduced form of Fig. 7.13.

The next example demonstrates that unknown voltages do not have to be across elements but can exist between any two points in a network. In addition, the importance of redrawing the network in a more familiar form is clearly revealed by the analysis to follow.

EXAMPLE 7.6
a. Find the voltages V_1, V_3, and V_{ab} for the network of Fig. 7.16.
b. Calculate the source current I_s.

Solutions: This is one of those situations where it might be best to redraw the network before beginning the analysis. Since combining both sources will not affect the unknowns, the network is redrawn as shown in Fig. 7.17, establishing a parallel network with the total source voltage across each parallel branch. The net source voltage is the difference between the two with the polarity of the larger.
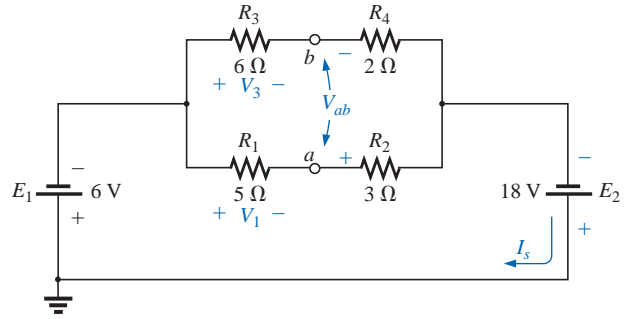
FIG. 7.16
Example 7.6.
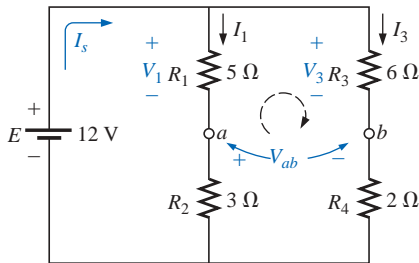
FIG. 7.17
Network of Fig. 7.16 redrawn.

a. Note the similarities with Fig. 7.12, permitting the use of the voltage divider rule to determine V_1 and V_3:

$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(5\ \Omega)(12\ \text{V})}{5\ \Omega + 3\ \Omega} = \frac{60\ \text{V}}{8} = \textbf{7.5 V}$$

$$V_3 = \frac{R_3 E}{R_3 + R_4} = \frac{(6\ \Omega)(12\ \text{V})}{6\ \Omega + 2\ \Omega} = \frac{72\ \text{V}}{8} = \textbf{9 V}$$

The open-circuit voltage V_{ab} is determined by applying Kirchhoff's voltage law around the indicated loop of Fig. 7.17 in the clockwise direction starting at terminal a.

$$+V_1 - V_3 + V_{ab} = 0$$

and $\qquad V_{ab} = V_3 - V_1 = 9\ \text{V} - 7.5\ \text{V} = \textbf{1.5 V}$

b. By Ohm's law,

$$I_1 = \frac{V_1}{R_1} = \frac{7.5\ \text{V}}{5\ \Omega} = 1.5\ \text{A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{9\ \text{V}}{6\ \Omega} = 1.5\ \text{A}$$

Applying Kirchhoff's current law,

$$I_s = I_1 + I_3 = 1.5\ \text{A} + 1.5\ \text{A} = \textbf{3 A}$$
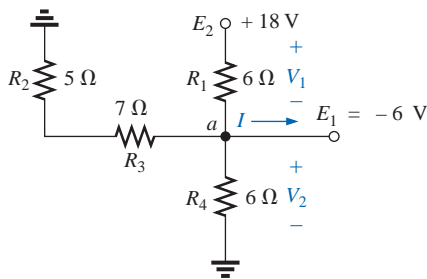
FIG. 7.18
Example 7.7.

EXAMPLE 7.7 For the network of Fig. 7.18, determine the voltages V_1 and V_2 and the current I.

Solution: It would indeed be difficult to analyze the network in the form of Fig. 7.18 with the symbolic notation for the sources and the reference or ground connection in the upper left-hand corner of the diagram. However, when the network is redrawn as shown in Fig. 7.19, the unknowns and the relationship between branches become significantly clearer. Note the common connection of the grounds and the replacing of the terminal notation by actual supplies.

It is now obvious that

$$V_2 = -E_1 = \textbf{-6 V}$$

The minus sign simply indicates that the chosen polarity for V_2 in Fig. 7.18 is opposite to that of the actual voltage. Applying Kirchhoff's voltage law to the loop indicated, we obtain

$$-E_1 + V_1 - E_2 = 0$$
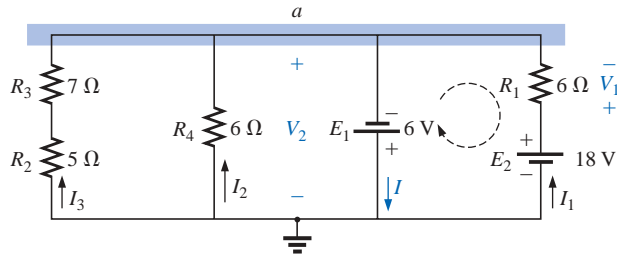
FIG. 7.19
Network of Fig. 7.18 redrawn.

and
$$V_1 = E_2 + E_1 = 18\ \text{V} + 6\ \text{V} = \textbf{24 V}$$

Applying Kirchhoff's current law to node a yields

$$I = I_1 + I_2 + I_3$$

$$= \frac{V_1}{R_1} + \frac{E_1}{R_4} + \frac{E_1}{R_2 + R_3}$$

$$= \frac{24\ \text{V}}{6\ \Omega} + \frac{6\ \text{V}}{6\ \Omega} + \frac{6\ \text{V}}{12\ \Omega}$$

$$= 4\ \text{A} + 1\ \text{A} + 0.5\ \text{A}$$

$$I = \textbf{5.5 A}$$

The next example is clear evidence of the fact that techniques learned in the current chapters will have far-reaching applications and will not be dropped for improved methods. Even though the **transistor** has not been introduced in this text, the dc levels of a transistor network can be examined using the basic rules and laws introduced in the early chapters of this text.

EXAMPLE 7.8 For the transistor configuration of Fig. 7.20, in which V_B and V_{BE} have been provided:
a. Determine the voltage V_E and the current I_E.
b. Calculate V_1.
c. Determine V_{BC} using the fact that the approximation $I_C = I_E$ is often applied to transistor networks.
d. Calculate V_{CE} using the information obtained in parts (a) through (c).

Solutions:
a. From Fig. 7.20, we find

$$V_2 = V_B = 2\ \text{V}$$

Writing Kirchhoff's voltage law around the lower loop yields

$$V_2 - V_{BE} - V_E = 0$$

or
$$V_E = V_2 - V_{BE} = 2\ \text{V} - 0.7\ \text{V} = \textbf{1.3 V}$$

and
$$I_E = \frac{V_E}{R_E} = \frac{1.3\ \text{V}}{1000\ \Omega} = \textbf{1.3 mA}$$

b. Applying Kirchhoff's voltage law to the input side (left-hand region of the network) will result in

$$V_2 + V_1 - V_{CC} = 0$$

FIG. 7.20
Example 7.8.

and $V_1 = V_{CC} - V_2$

but $V_2 = V_B$

and $V_1 = V_{CC} - V_2 = 22\text{ V} - 2\text{ V} = \textbf{20 V}$

c. Redrawing the section of the network of immediate interest will result in Fig. 7.21, where Kirchhoff's voltage law yields

$$V_C + V_{R_C} - V_{CC} = 0$$

and $V_C = V_{CC} - V_{R_C} = V_{CC} - I_C R_C$

but $I_C = I_E$

and $V_C = V_{CC} - I_E R_C = 22\text{ V} - (1.3\text{ mA})(10\text{ k}\Omega)$
        $= 9\text{ V}$

Then $V_{BC} = V_B - V_C$
        $= 2\text{ V} - 9\text{ V}$
        $= \textbf{−7 V}$

d. $V_{CE} = V_C - V_E$
        $= 9\text{ V} - 1.3\text{ V}$
        $= \textbf{7.7 V}$
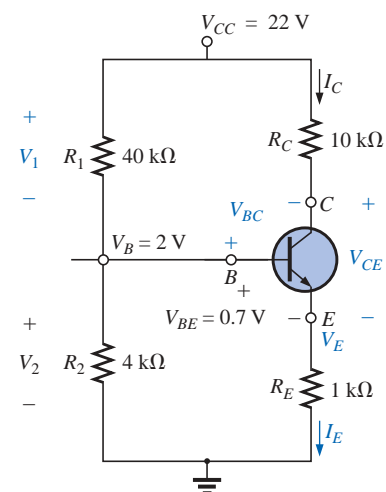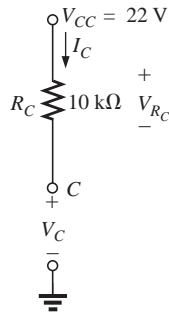
FIG. 7.21
Determining V_C for the network of Fig. 7.20.

EXAMPLE 7.9 Calculate the indicated currents and voltage of Fig. 7.22.
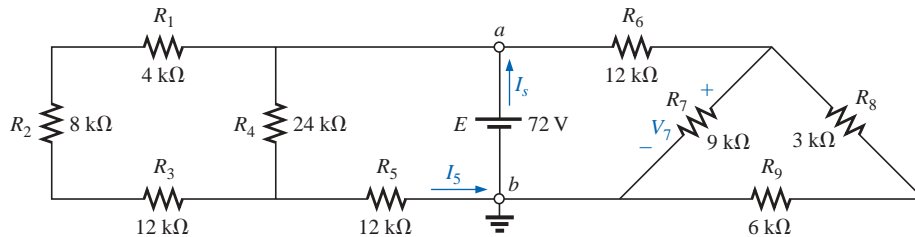
FIG. 7.22
Example 7.9.

Solution: Redrawing the network after combining series elements yields Fig. 7.23, and
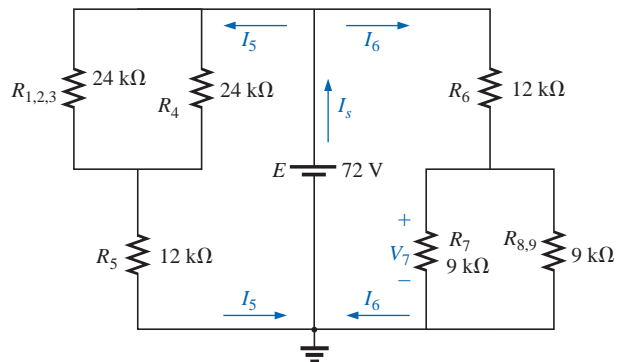
FIG. 7.23
Network of Fig. 7.22 redrawn.

$$I_5 = \frac{E}{R_{(1,2,3)\|4} + R_5} = \frac{72 \text{ V}}{12 \text{ k}\Omega + 12 \text{ k}\Omega} = \frac{72 \text{ V}}{24 \text{ k}\Omega} = \textbf{3 mA}$$

with

$$V_7 = \frac{R_{7\|(8,9)}E}{R_{7\|(8,9)} + R_6} = \frac{(4.5 \text{ k}\Omega)(72 \text{ V})}{4.5 \text{ k}\Omega + 12 \text{ k}\Omega} = \frac{324 \text{ V}}{16.5} = \textbf{19.6 V}$$

$$I_6 = \frac{V_7}{R_{7\|(8,9)}} = \frac{19.6 \text{ V}}{4.5 \text{ k}\Omega} = \textbf{4.35 mA}$$

and $\qquad I_s = I_5 + I_6 = 3 \text{ mA} + 4.35 \text{ mA} = \textbf{7.35 mA}$

Since the potential difference between points a and b of Fig. 7.22 is fixed at E volts, the circuit to the right or left is unaffected if the network is reconstructed as shown in Fig. 7.24.
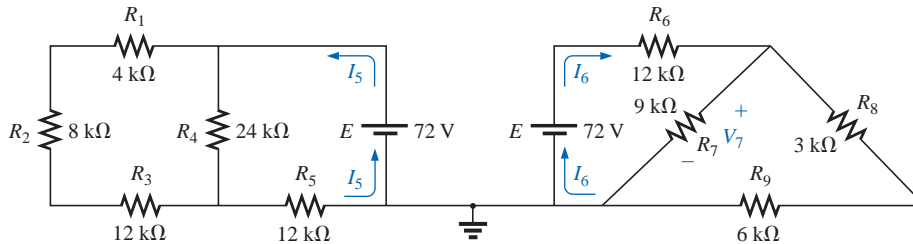
FIG. 7.24
An alternative approach to Example 7.9.

We can find each quantity required, except I_s, by analyzing each circuit independently. To find I_s, we must find the source current for each circuit and add it as in the above solution; that is, $I_s = I_5 + I_6$.

EXAMPLE 7.10 This example demonstrates the power of Kirchhoff's voltage law by determining the voltages V_1, V_2, and V_3 for the network of Fig. 7.25. For path 1 of Fig. 7.26,

$$E_1 - V_1 - E_3 = 0$$
and $\qquad V_1 = E_1 - E_3 = 20 \text{ V} - 8 \text{ V} = \textbf{12 V}$

For path 2,

$$E_2 - V_1 - V_2 = 0$$
and $\qquad V_2 = E_2 - V_1 = 5 \text{ V} - 12 \text{ V} = \textbf{−7 V}$

indicating that V_2 has a magnitude of 7 V but a polarity opposite to that appearing in Fig. 7.25. For path 3,

$$V_3 + V_2 - E_3 = 0$$
and $\qquad V_3 = E_3 - V_2 = 8 \text{ V} - (-7 \text{ V}) = 8 \text{ V} + 7 \text{ V} = \textbf{15 V}$

Note that the polarity of V_2 was maintained as originally assumed, requiring that -7 V be substituted for V_2.

7.3 LADDER NETWORKS

A three-section **ladder network** appears in Fig. 7.27. The reason for the terminology is quite obvious for the repetitive structure. Basically two approaches are used to solve networks of this type.
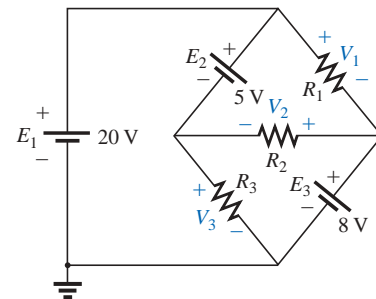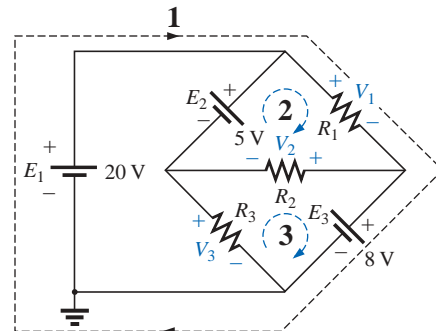
FIG. 7.25
Example 7.10.

FIG. 7.26
Defining the paths for Kirchhoff's voltage law.
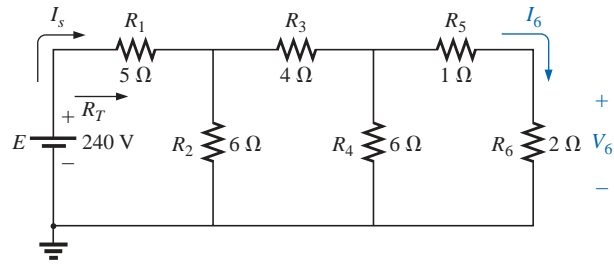
FIG. 7.27
Ladder network.

Method 1

Calculate the total resistance and resulting source current, and then work back through the ladder until the desired current or voltage is obtained. This method is now employed to determine V_6 in Fig. 7.27.
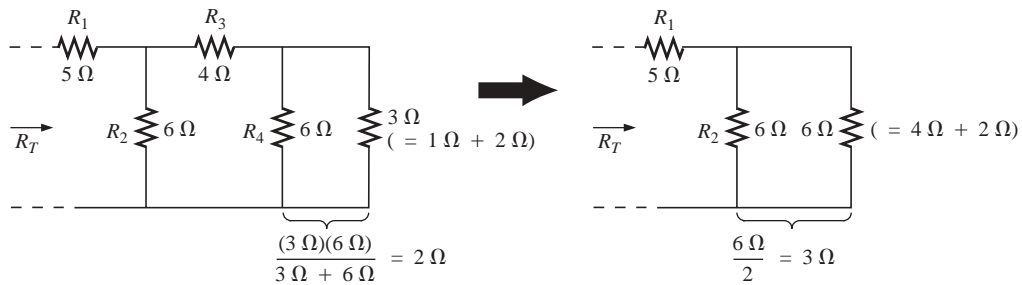
FIG. 7.28
Working back to the source to determine R_T for the network of Fig. 7.27.
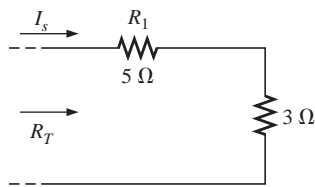
FIG. 7.29
Calculating R_T and I_s.

FIG. 7.30
Working back toward I_6.

Combining parallel and series elements as shown in Fig. 7.28 will result in the reduced network of Fig. 7.29, and

$$R_T = 5 \ \Omega + 3 \ \Omega = 8 \ \Omega$$

$$I_s = \frac{E}{R_T} = \frac{240 \ \text{V}}{8 \ \Omega} = 30 \ \text{A}$$

Working our way back to I_6 (Fig. 7.30), we find that

$$I_1 = I_s$$

and

$$I_3 = \frac{I_s}{2} = \frac{30 \ \text{A}}{2} = 15 \ \text{A}$$

and, finally (Fig. 7.31),
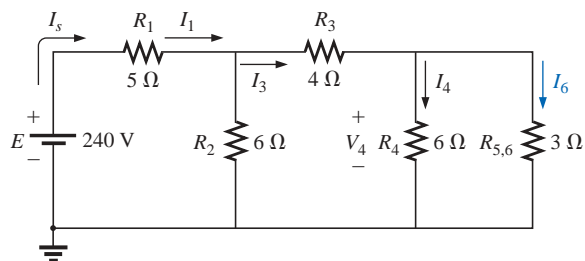
FIG. 7.31
Calculating I_6.

$$I_6 = \frac{(6\ \Omega)I_3}{6\ \Omega + 3\ \Omega} = \frac{6}{9}(15\ \text{A}) = 10\ \text{A}$$

and $\quad V_6 = I_6 R_6 = (10\ \text{A})(2\ \Omega) = \textbf{20 V}$

Method 2

Assign a letter symbol to the last branch current and work back through the network to the source, maintaining this assigned current or other current of interest. The desired current can then be found directly. This method can best be described through the analysis of the same network considered above in Fig. 7.27, redrawn in Fig. 7.32.
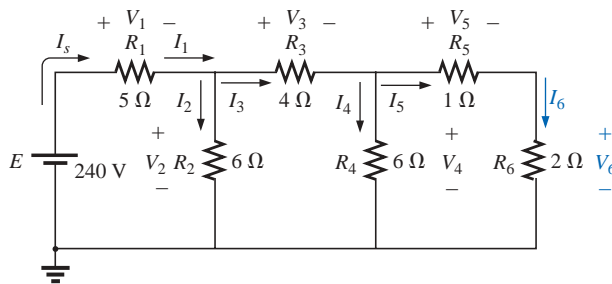
FIG. 7.32
An alternative approach for ladder networks.

The assigned notation for the current through the final branch is I_6:

$$I_6 = \frac{V_4}{R_5 + R_6} = \frac{V_4}{1\ \Omega + 2\ \Omega} = \frac{V_4}{3\ \Omega}$$

or $\quad V_4 = (3\ \Omega)I_6$

so that $\quad I_4 = \dfrac{V_4}{R_4} = \dfrac{(3\ \Omega)I_6}{6\ \Omega} = 0.5I_6$

and $\quad I_3 = I_4 + I_6 = 0.5I_6 + I_6 = 1.5I_6$

$\quad V_3 = I_3 R_3 = (1.5I_6)(4\ \Omega) = (6\ \Omega)I_6$

Also, $\quad V_2 = V_3 + V_4 = (6\ \Omega)I_6 + (3\ \Omega)I_6 = (9\ \Omega)I_6$

so that $\quad I_2 = \dfrac{V_2}{R_2} = \dfrac{(9\ \Omega)I_6}{6\ \Omega} = 1.5I_6$

and $\quad I_s = I_2 + I_3 = 1.5I_6 + 1.5I_6 = 3I_6$

with $\quad V_1 = I_1 R_1 = I_s R_1 = (5\ \Omega)I_s$

so that $\quad E = V_1 + V_2 = (5\ \Omega)I_s + (9\ \Omega)I_6$

$\quad = (5\ \Omega)(3I_6) + (9\ \Omega)I_6 = (24\ \Omega)I_6$

and $\quad I_6 = \dfrac{E}{24\ \Omega} = \dfrac{240\ \text{V}}{24\ \Omega} = 10\ \text{A}$

with $\quad V_6 = I_6 R_6 = (10\ \text{A})(2\ \Omega) = \textbf{20 V}$

as was obtained using method 1.

Mathcad

Mathcad will now be used to analyze the ladder network of Fig. 7.27 using method 1. It will provide an excellent opportunity to practice the basic maneuvers introduced in earlier chapters.

First, as shown in Fig. 7.33, all the parameters of the network must be defined. Then the same sequence is followed as included in the text

material. For Mathcad, however, we must be sure that the defining sequence for each new variable flows from left to right, as shown in Fig. 7.33, until R_{10} is defined. We are then ready to write the equation for the total resistance and display the result. All the remaining parameters are then defined and displayed as shown. The results are an exact match with the longhand solution.

The wonderful thing about Mathcad is that this sequence can be put in memory and called for as the need arises for different networks. Simply redefine the parameters of the network, and all the new values for the important parameters of the network will be displayed immediately.
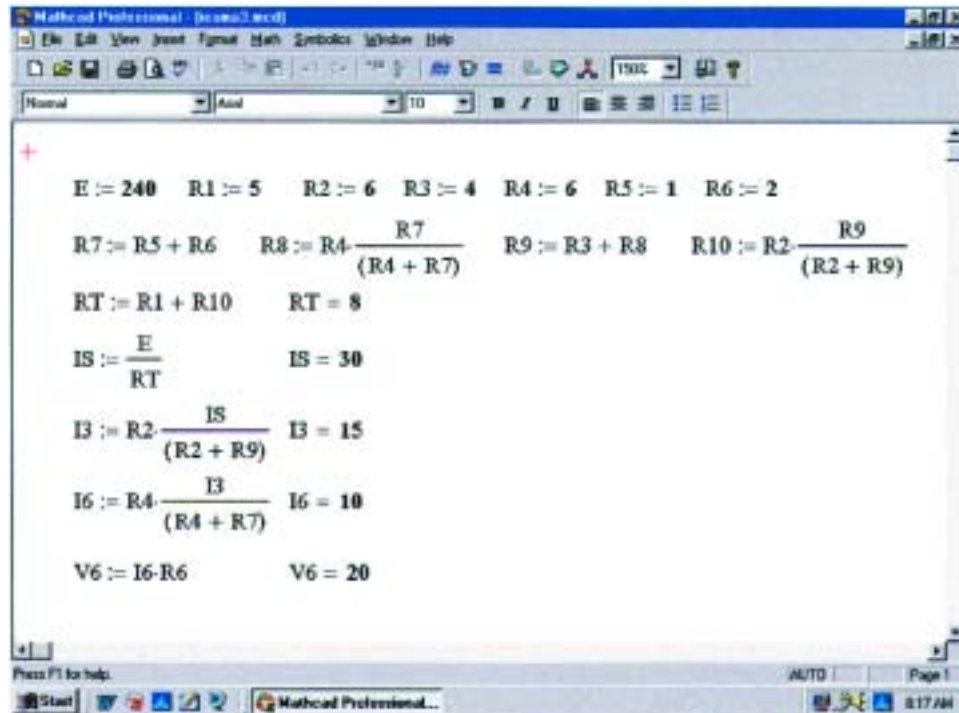
FIG. 7.33
Using Mathcad to analyze the ladder network of Fig. 7.27.
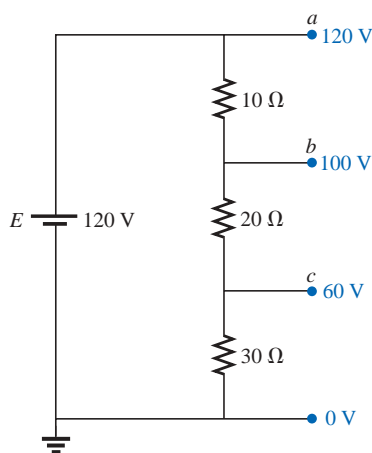
FIG. 7.34
Voltage divider supply.

7.4 VOLTAGE DIVIDER SUPPLY (UNLOADED AND LOADED)

The term *loaded* appearing in the title of this section refers to the application of an element, network, or system to a supply that will draw current from the supply. As pointed out in Section 5.8, the application of a load can affect the terminal voltage of the supply.

Through a voltage divider network such as the one in Fig. 7.34, a number of terminal voltages can be made available from a single supply. The voltage levels shown (with respect to ground) are determined by a direct application of the voltage divider rule. Figure 7.34 reflects a no-load situation due to the absence of any current-drawing elements connected between terminals *a, b,* or *c* and ground.

The larger the resistance level of the applied loads compared to the resistance level of the voltage divider network, the closer the resulting terminal voltage to the no-load levels. In other words, the lower the current demand from a supply, the closer the terminal characteristics are to the no-load levels.

To demonstrate the validity of the above statement, let us consider the network of Fig. 7.34 with resistive loads that are the average value of the resistive elements of the voltage divider network, as shown in Fig. 7.35.
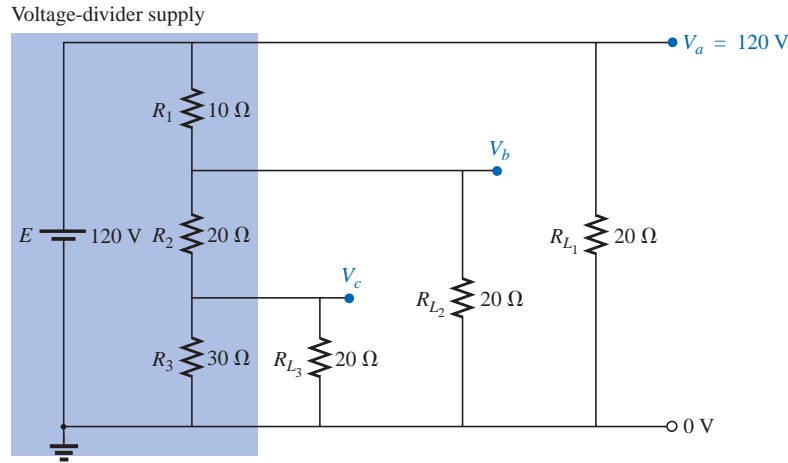
FIG. 7.35

Voltage divider supply with loads equal to the average value of the resistive elements that make up the supply.

The voltage V_a is unaffected by the load R_{L_1} since the load is in parallel with the supply voltage E. The result is $V_a = 120$ V, which is the same as the no-load level. To determine V_b, we must first note that R_3 and R_{L_3} are in parallel and $R'_3 = R_3 \parallel R_{L_3} = 30\ \Omega \parallel 20\ \Omega = 12\ \Omega$. The parallel combination $R'_2 = (R_2 + R'_3) \parallel R_{L_2} = (20\ \Omega + 12\ \Omega) \parallel 20\ \Omega = 32\ \Omega \parallel 20\ \Omega = 12.31\ \Omega$. Applying the voltage divider rule gives

$$V_b = \frac{(12.31\ \Omega)(120\ \text{V})}{12.31\ \Omega + 10\ \Omega} = 66.21\ \text{V}$$

versus 100 V under no-load conditions.

The voltage V_c is

$$V_c = \frac{(12\ \Omega)(66.21\ \text{V})}{12\ \Omega + 20\ \Omega} = 24.83\ \text{V}$$

versus 60 V under no-load conditions.

The effect of load resistors close in value to the resistor employed in the voltage divider network is, therefore, to decrease significantly some of the terminal voltages.

If the load resistors are changed to the 1-kΩ level, the terminal voltages will all be relatively close to the no-load values. The analysis is similar to the above, with the following results:

$$V_a = 120\ \text{V} \qquad V_b = 98.88\ \text{V} \qquad V_c = 58.63\ \text{V}$$

If we compare current drains established by the applied loads, we find for the network of Fig. 7.35 that

$$I_{L_2} = \frac{V_{L_2}}{R_{L_2}} = \frac{66.21\ \text{V}}{20\ \Omega} = 3.31\ \text{A}$$

and for the 1-kΩ level,

$$I_{L_2} = \frac{98.88\ \text{V}}{1\ \text{k}\Omega} = 98.88\ \text{mA} < 0.1\ \text{A}$$

As noted above in the highlighted statement, the more the current drain, the greater the change in terminal voltage with the application of the load. This is certainly verified by the fact that I_{L_2} is about 33.5 times larger with the 20-Ω loads.

The next example is a design exercise. The voltage and current ratings of each load are provided, along with the terminal ratings of the supply. The required voltage divider resistors must be found.

EXAMPLE 7.11 Determine R_1, R_2, and R_3 for the voltage divider supply of Fig. 7.36. Can 2-W resistors be used in the design?
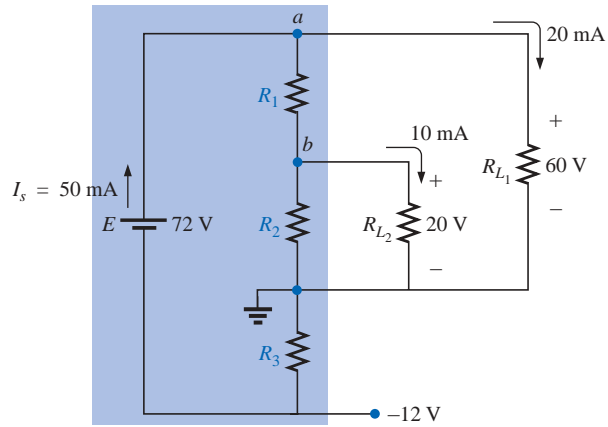
FIG. 7.36
Example 7.11.

Solution: R_3:

$$R_3 = \frac{V_{R_3}}{I_{R_3}} = \frac{V_{R_3}}{I_s} = \frac{12\text{ V}}{50\text{ mA}} = \textbf{240 }\boldsymbol{\Omega}$$

$$P_{R_3} = (I_{R_3})^2 R_3 = (50\text{ mA})^2\ 240\ \Omega = 0.6\text{ W} < 2\text{ W}$$

R_1: Applying Kirchhoff's current law to node a:

$$I_s - I_{R_1} - I_{L_1} = 0$$

and

$$I_{R_1} = I_s - I_{L_1} = 50\text{ mA} - 20\text{ mA}$$
$$= 30\text{ mA}$$

$$R_1 = \frac{V_{R_1}}{I_{R_1}} = \frac{V_{L_1} - V_{L_2}}{I_{R_1}} = \frac{60\text{ V} - 20\text{ V}}{30\text{ mA}} = \frac{40\text{ V}}{30\text{ mA}}$$
$$= \textbf{1.33 k}\boldsymbol{\Omega}$$

$$P_{R_1} = (I_{R_1})^2 R_1 = (30\text{ mA})^2\ 1.33\text{ k}\Omega = 1.197\text{ W} < 2\text{ W}$$

R_2: Applying Kirchhoff's current law at node b:

$$I_{R_1} - I_{R_2} - I_{L_2} = 0$$

and

$$I_{R_2} = I_{R_1} - I_{L_2} = 30\text{ mA} - 10\text{ mA}$$
$$= 20\text{ mA}$$

$$R_2 = \frac{V_{R_2}}{I_{R_2}} = \frac{20\text{ V}}{20\text{ mA}} = \textbf{1 k}\boldsymbol{\Omega}$$

$$P_{R_2} = (I_{R_2})^2 R_2 = (20\text{ mA})^2\ 1\text{ k}\Omega = 0.4\text{ W} < 2\text{ W}$$

Since P_{R_1}, P_{R_2}, and P_{R_3} are less than 2 W, 2-W resistors can be used for the design.

7.5 POTENTIOMETER LOADING

For the unloaded potentiometer of Fig. 7.37, the output voltage is determined by the voltage divider rule, with R_T in the figure representing the total resistance of the potentiometer. Too often it is assumed that the voltage across a load connected to the wiper arm is determined solely by the potentiometer, and the effect of the load can be ignored. This is definitely not the case, as is demonstrated in the next few paragraphs.

When a load is applied as shown in Fig. 7.38, the output voltage V_L is now a function of the magnitude of the load applied since R_1 is not as shown in Fig. 7.37 but is instead the parallel combination of R_1 and R_L.

The output voltage is now

$$V_L = \frac{R'E}{R' + R_2} \qquad \text{with } R' = R_1 \, \| \, R_L \qquad \textbf{(7.1)}$$

If it is desired to have good control of the output voltage V_L through the controlling dial, knob, screw, or whatever, it is advisable to choose a load or potentiometer that satisfies the following relationship:

$$\boxed{R_L \geq R_T} \qquad \textbf{(7.2)}$$

For example, if we disregard Eq. (7.2) and choose a 1-MΩ potentiometer with a 100-Ω load and set the wiper arm to 1/10 the total resistance, as shown in Fig. 7.39, then

$$R' = 100 \text{ k}\Omega \, \| \, 100 \, \Omega = 99.9 \, \Omega$$

and

$$V_L = \frac{99.9 \, \Omega (10 \text{ V})}{99.9 \, \Omega + 900 \text{ k}\Omega} \cong 0.001 \text{ V} = 1 \text{ mV}$$

which is extremely small compared to the expected level of 1 V.

In fact, if we move the wiper arm to the midpoint,

$$R' = 500 \text{ k}\Omega \, \| \, 100 \, \Omega = 99.98 \, \Omega$$

and

$$V_L = \frac{(99.98 \, \Omega)(10 \text{ V})}{99.98 \, \Omega + 500 \text{ k}\Omega} \cong 0.002 \text{ V} = 2 \text{ mV}$$

which is negligible compared to the expected level of 5 V. Even at $R_1 = 900$ kΩ, V_L is simply 0.01 V, or 1/1000 of the available voltage.

Using the reverse situation of $R_T = 100 \, \Omega$ and $R_L = 1 \, M\Omega$ and the wiper arm at the 1/10 position, as in Fig. 7.40, we find

$$R' = 10 \, \Omega \, \| \, 1 \, M\Omega \cong 10 \, \Omega$$

and

$$V_L = \frac{10 \, \Omega (10 \text{ V})}{10 \, \Omega + 90 \, \Omega} = 1 \text{ V}$$

as desired.

For the lower limit (worst-case design) of $R_L = R_T = 100 \, \Omega$, as defined by Eq. (7.2) and the halfway position of Fig. 7.38,

$$R' = 50 \, \Omega \, \| \, 100 \, \Omega = 33.33 \, \Omega$$

and

$$V_L = \frac{33.33 \, \Omega (10 \text{ V})}{33.33 \, \Omega + 50 \, \Omega} \cong 4 \text{ V}$$

It may not be the ideal level of 5 V, but at least 40% of the voltage E has been achieved at the halfway position rather than the 0.02% obtained with $R_L = 100 \, \Omega$ and $R_T = 1 \, M\Omega$.

In general, therefore, try to establish a situation for potentiometer control in which Equation (7.2) is satisfied to the highest degree possible.
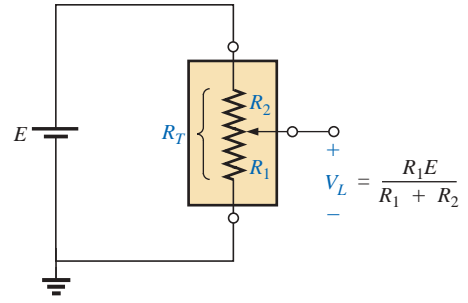
FIG. 7.37
Unloaded potentiometer.

$$V_L = \frac{R_1 E}{R_1 + R_2}$$
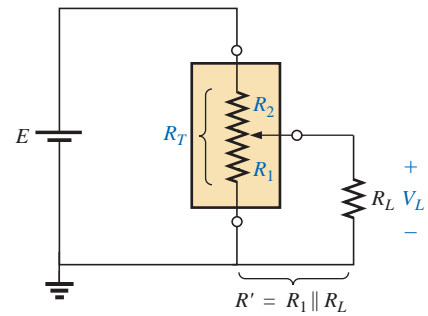
FIG. 7.38
Loaded potentiometer.

$$R' = R_1 \, \| \, R_L$$
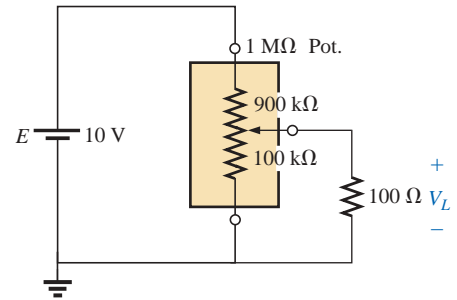
FIG. 7.39
$R_T > R_L$.
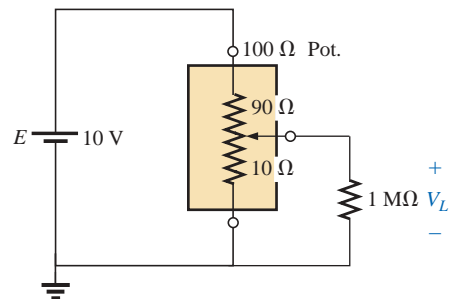
FIG. 7.40
$R_L > R_T$.

Someone might suggest that we make R_T as small as possible to bring the percent result as close to the ideal as possible. Keep in mind, however, that the potentiometer has a power rating, and for networks such as Fig. 7.40, $P_{max} \cong E^2/R_T = (10\text{ V})^2/100\ \Omega = 1$ W. If R_T is reduced to 10 Ω, $P_{max} = (10\text{ V})^2/10\ \Omega = 10$ W, which would require a *much larger* unit.
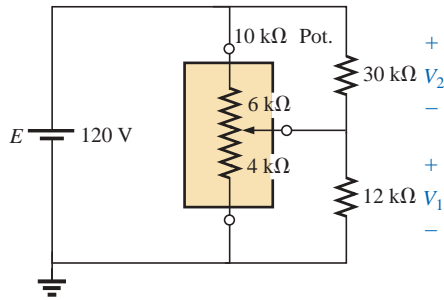
EXAMPLE 7.12 Find the voltages V_1 and V_2 for the loaded potentiometer of Fig. 7.41.

Solution:

$$\text{Ideal (no load):}\quad V_1 = \frac{4\text{ k}\Omega(120\text{ V})}{10\text{ k}\Omega} = 48\text{ V}$$

$$V_2 = \frac{6\text{ k}\Omega(120\text{ V})}{10\text{ k}\Omega} = 72\text{ V}$$

$$\text{Loaded:}\quad R' = 4\text{ k}\Omega \parallel 12\text{ k}\Omega = 3\text{ k}\Omega$$

$$R'' = 6\text{ k}\Omega \parallel 30\text{ k}\Omega = 5\text{ k}\Omega$$

$$V_1 = \frac{3\text{ k}\Omega(120\text{ V})}{8\text{ k}\Omega} = \mathbf{45\ V}$$

$$V_2 = \frac{5\text{ k}\Omega(120\text{ V})}{8\text{ k}\Omega} = \mathbf{75\ V}$$

The ideal and loaded voltage levels are so close that the design can be considered a good one for the applied loads. A slight variation in the position of the wiper arm will establish the ideal voltage levels across the two loads.

FIG. 7.41
Example 7.12.

7.6 AMMETER, VOLTMETER, AND OHMMETER DESIGN

Now that the fundamentals of series, parallel, and series-parallel networks have been introduced, we are prepared to investigate the fundamental design of an ammeter, voltmeter, and ohmmeter. Our design of each will employ the **d'Arsonval analog movement** of Fig. 7.42. The movement consists basically of an iron-core coil mounted on bearings between a permanent magnet. The helical springs limit the turning motion of the coil and provide a path for the current to reach the coil. When a current is passed through the movable coil, the fluxes of the coil and permanent magnet will interact to develop a torque on the coil that will cause it to rotate on its bearings. The movement is adjusted to indicate zero deflection on a meter scale when the current through the coil is zero. The direction of current through the coil will then determine whether the pointer will display an up-scale or below-zero indication. For this reason, ammeters and voltmeters have an assigned polarity on their terminals to ensure an up-scale reading.

D'Arsonval movements are usually rated by current and resistance. The specifications of a typical movement might be 1 mA, 50 Ω. The 1 mA is the *current sensitivity* (*CS*) of the movement, which is the current required for a full-scale deflection. It will be denoted by the symbol I_{CS}. The 50 Ω represents the internal resistance (R_m) of the movement. A common notation for the movement and its specifications is provided in Fig. 7.43.

FIG. 7.42
d'Arsonval analog movement. (Courtesy of Weston Instruments, Inc.)

1 mA, 50 Ω
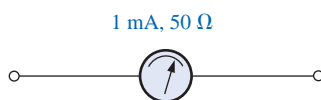
FIG. 7.43
Movement notation.

The Ammeter

The maximum current that the d'Arsonval movement can read independently is equal to the current sensitivity of the movement. However, higher currents can be measured if additional circuitry is introduced. This additional circuitry, as shown in Fig. 7.44, results in the basic construction of an ammeter.
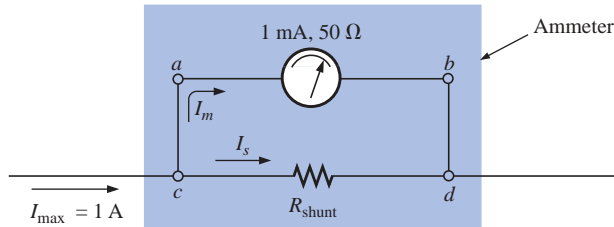
FIG. 7.44
Basic ammeter.

The resistance R_{shunt} is chosen for the ammeter of Fig. 7.44 to allow 1 mA to flow through the movement when a maximum current of 1 A enters the ammeter. If less than 1 A should flow through the ammeter, the movement will have less than 1 mA flowing through it and will indicate less than full-scale deflection.

Since the voltage across parallel elements must be the same, the potential drop across a-b in Fig. 7.44 must equal that across c-d; that is,

$$(1 \text{ mA})(50 \text{ Ω}) = R_{shunt}I_s$$

Also, I_s must equal $1 \text{ A} - 1 \text{ mA} = 999 \text{ mA}$ if the current is to be limited to 1 mA through the movement (Kirchhoff's current law). Therefore,

$$(1 \text{ mA})(50 \text{ Ω}) = R_{shunt}(999 \text{ mA})$$

$$R_{shunt} = \frac{(1 \text{ mA})(50 \text{ Ω})}{999 \text{ mA}}$$

$$\cong 0.05 \text{ Ω}$$

In general,

$$R_{shunt} = \frac{R_m I_{CS}}{I_{max} - I_{CS}} \tag{7.3}$$

One method of constructing a multirange ammeter is shown in Fig. 7.45, where the rotary switch determines the R_{shunt} to be used for the
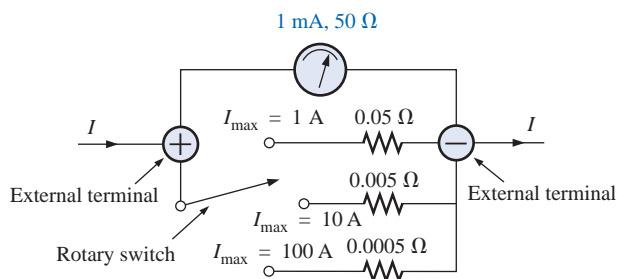
FIG. 7.45
Multirange ammeter.

maximum current indicated on the face of the meter. Most meters employ the same scale for various values of maximum current. If you read 375 on the 0–5 mA scale with the switch on the 5 setting, the current is 3.75 mA; on the 50 setting, the current is 37.5 mA; and so on.

The Voltmeter

A variation in the additional circuitry will permit the use of the d'Arsonval movement in the design of a voltmeter. The 1-mA, 50-Ω movement can also be rated as a 50-mV (1 mA $\times$ 50 Ω), 50-Ω movement, indicating that the maximum voltage that the movement can measure independently is 50 mV. The millivolt rating is sometimes referred to as the *voltage sensitivity* (*VS*). The basic construction of the voltmeter is shown in Fig. 7.46.
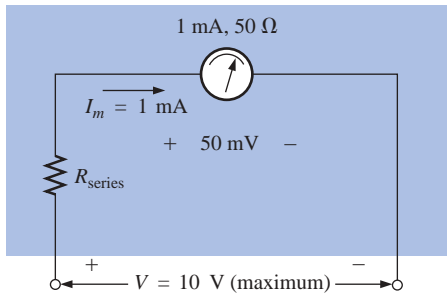
The R_{series} is adjusted to limit the current through the movement to 1 mA when the maximum voltage is applied across the voltmeter. A lesser voltage would simply reduce the current in the circuit and thereby the deflection of the movement.

Applying Kirchhoff's voltage law around the closed loop of Fig. 7.46, we obtain

$$[10 \text{ V} - (1 \text{ mA})(R_{\text{series}})] - 50 \text{ mV} = 0$$

or

$$R_{\text{series}} = \frac{10 \text{ V} - (50 \text{ mV})}{1 \text{ mA}} = 9950 \ \Omega$$

In general,

$$\boxed{R_{\text{series}} = \frac{V_{\text{max}} - V_{VS}}{I_{CS}}} \tag{7.4}$$

One method of constructing a multirange voltmeter is shown in Fig. 7.47. If the rotary switch is at 10 V, $R_{\text{series}} = 9.950$ kΩ; at 50 V, $R_{\text{series}} = 40$ kΩ + 9.950 kΩ = 49.950 kΩ; and at 100 V, $R_{\text{series}} = 50$ kΩ + 40 kΩ + 9.950 kΩ = 99.950 kΩ.

The Ohmmeter

In general, ohmmeters are designed to measure resistance in the low, mid-, or high range. The most common is the **series ohmmeter,** designed to read resistance levels in the midrange. It employs the series configuration of Fig. 7.48. The design is quite different from that of the

FIG. 7.46
Basic voltmeter.
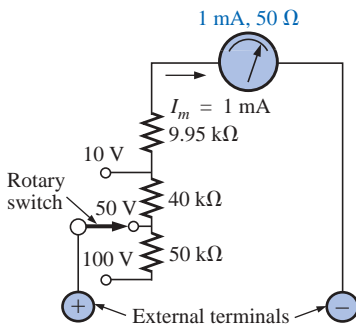
FIG. 7.47
Multirange voltmeter.
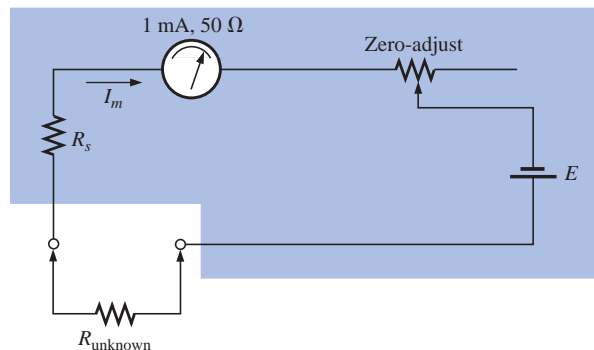
FIG. 7.48
Series ohmmeter.

ammeter or voltmeter because it will show a full-scale deflection for zero ohms and no deflection for infinite resistance.

To determine the series resistance R_s, the external terminals are shorted (a direct connection of zero ohms between the two) to simulate zero ohms, and the zero-adjust is set to half its maximum value. The resistance R_s is then adjusted to allow a current equal to the current sensitivity of the movement (1 mA) to flow in the circuit. The zero-adjust is set to half its value so that any variation in the components of the meter that may produce a current more or less than the current sensitivity can be compensated for. The current I_m is

$$I_m \text{ (full scale)} = I_{CS} = \frac{E}{R_s + R_m + \dfrac{\text{zero-adjust}}{2}} \qquad (7.5)$$

and
$$R_s = \frac{E}{I_{CS}} - R_m - \frac{\text{zero-adjust}}{2} \qquad (7.6)$$

If an unknown resistance is then placed between the external terminals, the current will be reduced, causing a deflection less than full scale. If the terminals are left open, simulating infinite resistance, the pointer will not deflect since the current through the circuit is zero.

An instrument designed to read very low values of resistance appears in Fig. 7.49. Because of its low-range capability, the network design must be a great deal more sophisticated than described above. It employs electronic components that eliminate the inaccuracies introduced by lead and contact resistances. It is similar to the above system in the sense that it is completely portable and does require a dc battery to establish measurement conditions. Special leads are employed to limit any introduced resistance levels. The maximum scale setting can be set as low as 0.00352 (3.52 mΩ).

FIG. 7.49
Milliohmmeter. (Courtesy of Keithley Instruments, Inc.)

The **megohmmeter** (often called a *megger*) is an instrument for measuring very high resistance values. Its primary function is to test the insulation found in power transmission systems, electrical machinery, transformers, and so on. To measure the high-resistance values, a high dc voltage is established by a hand-driven generator. If the shaft is rotated above some set value, the output of the generator will be fixed at one selectable voltage, typically 250 V, 500 V, or 1000 V. A photograph of a commercially available tester is shown in Fig. 7.50. For this instrument, the range is zero to 5000 MΩ.

FIG. 7.50
Megohmmeter. (Courtesy of AEMC Corp.)

```
Enter R1: 10
Enter R2: 220
Enter R3: 12
Enter R4: 100
Enter R5: 1e30
Enter R6: 1e30
Enter E: -60

The total resistance is 84.216866 ohms.
The source current is -0.712446 Amperes.
I3 equals -0.472103 Amperes.
I6 equals -2.360515e-29 Amperes.
```

FIG. 7.63

C++ response to an analysis of the ladder network of Fig. 7.60 without the elements R_5 and R_6.

PROBLEMS

SECTION 7.2   Descriptive Examples

1. Which elements of the networks in Fig. 7.64 are in series or parallel? In other words, which elements of the given networks have the same current (series) or voltage (parallel)? Restrict your decision to single elements, and do not include combined elements.
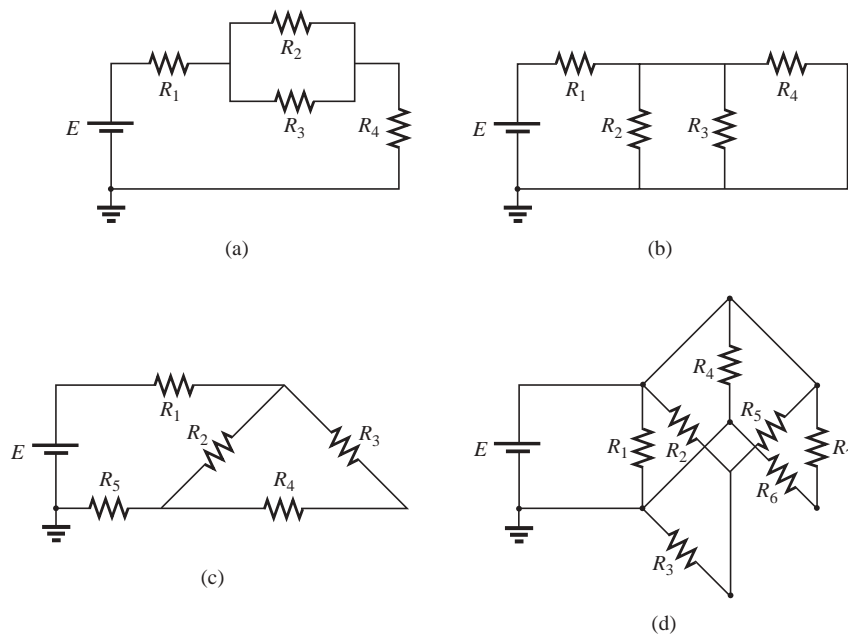
FIG. 7.64
Problem 1.

2. Determine R_T for the networks of Fig. 7.65.

(a)

(b)

(c)

(d)
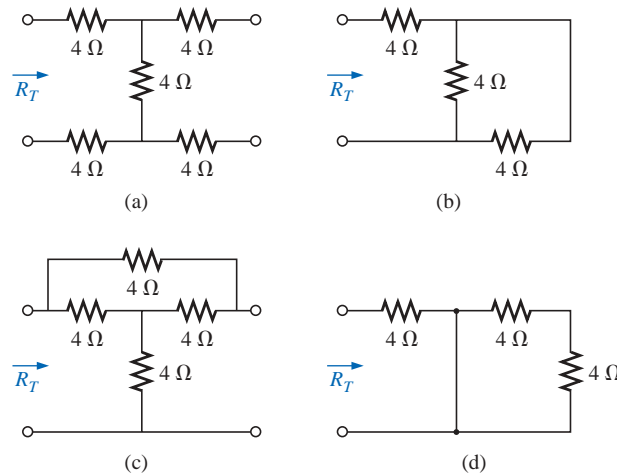
FIG. 7.65
Problem 2.

3. For the network of Fig. 7.66:
  a. Does $I = I_3 = I_6$? Explain.
  b. If $I = 5$ A and $I_1 = 2$ A, find I_2.
  c. Does $I_1 + I_2 = I_4 + I_5$? Explain.
  d. If $V_1 = 6$ V and $E = 10$ V, find V_2.
  e. If $R_1 = 3$ Ω, $R_2 = 2$ Ω, $R_3 = 4$ Ω, and $R_4 = 1$ Ω, what is R_T?
  f. If the resistors have the values given in part (e) and $E = 10$ V, what is the value of I in amperes?
  g. Using values given in parts (e) and (f), find the power delivered by the battery E and dissipated by the resistors R_1 and R_2.

4. For the network of Fig. 7.67:
  a. Calculate R_T.
  b. Determine I and I_1.
  c. Find V_3.

5. For the network of Fig. 7.68:
  a. Determine R_T.
  b. Find I_s, I_1, and I_2.
  c. Calculate V_a.
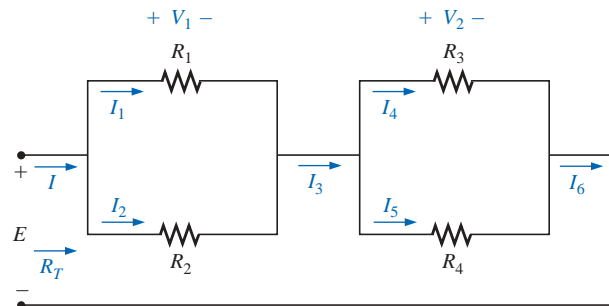
FIG. 7.66
Problem 3.
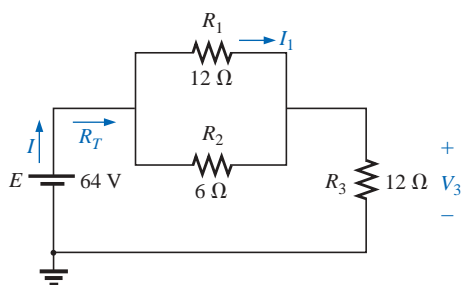
FIG. 7.67
Problem 4.
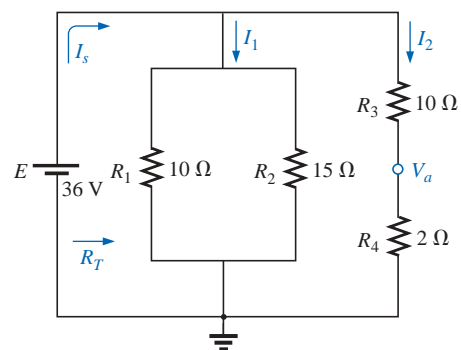
FIG. 7.68
Problem 5.

6. Determine the currents I_1 and I_2 for the network of Fig. 7.69.
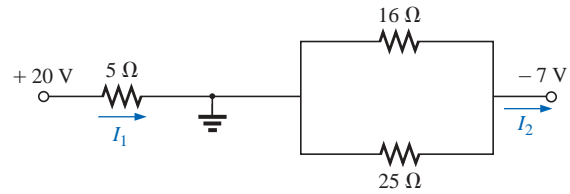
FIG. 7.69
Problem 6.

7. a. Find the magnitude and direction of the currents I, I_1, I_2, and I_3 for the network of Fig. 7.70.
   b. Indicate their direction on Fig. 7.70.

*8. For the network of Fig. 7.71:
   a. Determine the currents I_s, I_1, I_3, and I_4.
   b. Calculate V_a and V_{bc}.
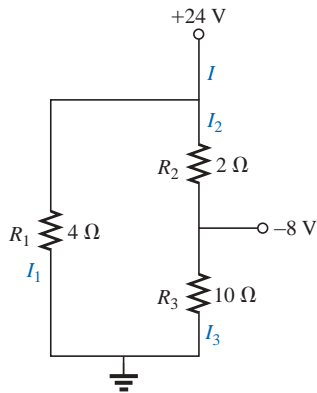
FIG. 7.70
Problem 7.
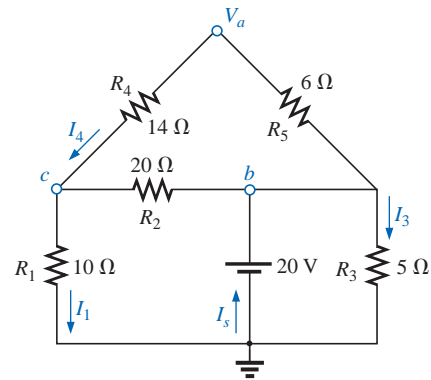
FIG. 7.71
Problem 8.
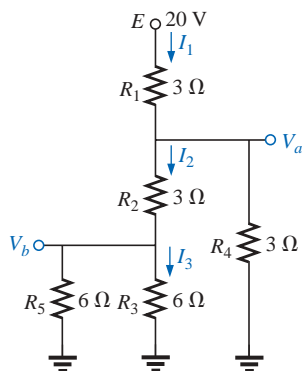
9. For the network of Fig. 7.72:
   a. Determine the current I_1.
   b. Calculate the currents I_2 and I_3.
   c. Determine the voltage levels V_a and V_b.

FIG. 7.72
Problem 9.

10. For the network of Fig. 7.73:
   a. Find the currents I and I_6.
   b. Find the voltages V_1 and V_5.
   c. Find the power delivered to the 6-kΩ resistor.
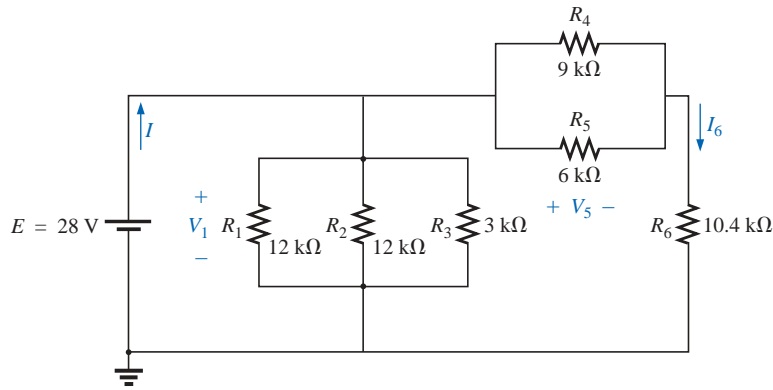
FIG. 7.73
Problem 10.

*11. For the series-parallel network of Fig. 7.74:
   a. Find the current I.
   b. Find the currents I_3 and I_9.
   c. Find the current I_8.
   d. Find the voltage V_{ab}.
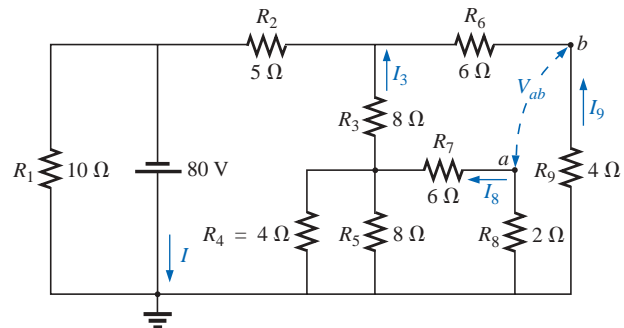
FIG. 7.74
Problem 11.

*12. Determine the dc levels for the transistor network of Fig. 7.75 using the fact that $V_{BE} = 0.7$ V, $V_E = 2$ V, and $I_C = I_E$. That is:
   a. Determine I_E and I_C.
   b. Calculate I_B.
   c. Determine V_B and V_C.
   d. Find V_{CE} and V_{BC}.

FIG. 7.75
Problem 12.

$V_{DD} = 16$ V

R_1 2 MΩ

R_D 2.5 kΩ

$V_{GS} = -1.75$ V
$I_G = 0$ A
$I_D = I_S$

R_2 270 kΩ

R_S 1.5 kΩ
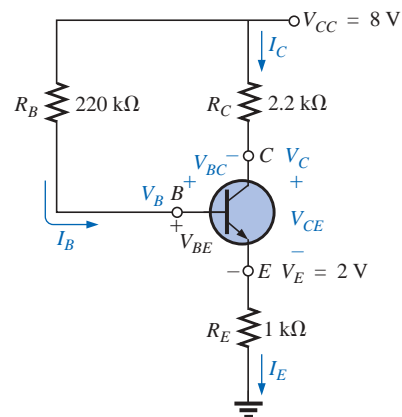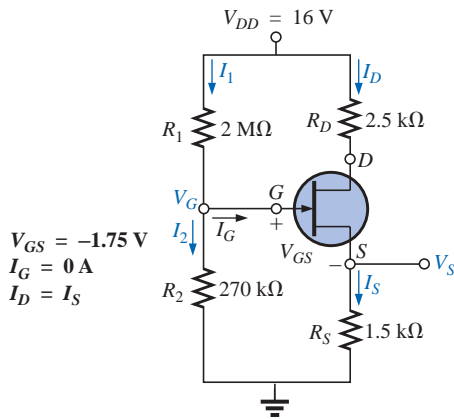
FIG. 7.76
Problem 13.

*13. The network of Fig. 7.76 is the basic biasing arrangement for the *field-effect transistor* (FET), a device of increasing importance in electronic design. (*Biasing* simply means the application of dc levels to establish a particular set of operating conditions.) Even though you may be unfamiliar with the FET, you can perform the following analysis using only the basic laws introduced in this chapter and the information provided on the diagram.

    a. Determine the voltages V_G and V_S.

    b. Find the currents I_1, I_2, I_D, and I_S.

    c. Determine V_{DS}.

    d. Calculate V_{DG}.
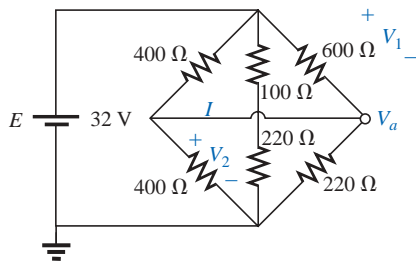
*14. For the network of Fig. 7.77:

    a. Determine R_T.

    b. Calculate V_a.

    c. Find V_1.

    d. Calculate V_2.

    e. Determine I (with direction).

15. For the network of Fig. 7.78:

    a. Determine the current I.
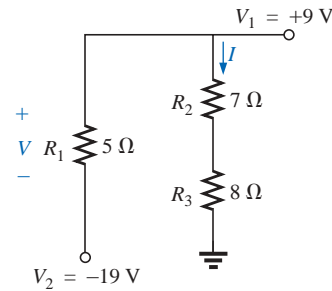
    b. Find V.
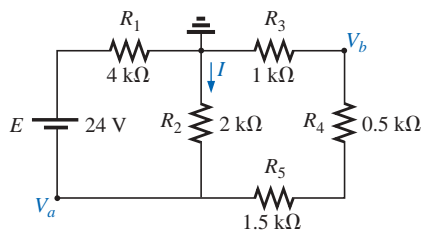
FIG. 7.77
Problem 14.

FIG. 7.78
Problem 15.

*16. Determine the current I and the voltages V_a, V_b, and V_{ab} for the network of Fig. 7.79.

FIG. 7.79
Problem 16.

17. For the configuration of Fig. 7.80:
  a. Find the currents I_2, I_6, and I_8.
  b. Find the voltages V_4 and V_8.
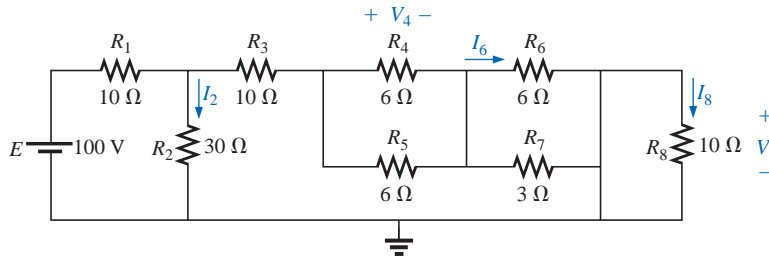
FIG. 7.80
Problem 17.

18. Determine the voltage V and the current I for the network of Fig. 7.81.
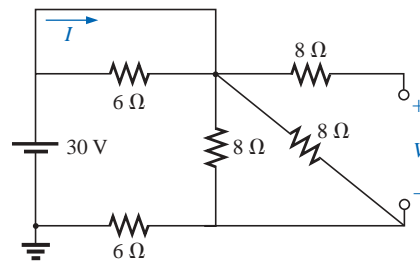
FIG. 7.81
Problem 18.

*19. For the network of Fig. 7.82:
  a. Determine R_T by combining resistive elements.
  b. Find V_1 and V_4.
  c. Calculate I_3 (with direction).
  d. Determine I_s by finding the current through each element and then applying Kirchhoff's current law. Then calculate R_T from $R_T = E/I_s$, and compare the answer with the solution of part (a).
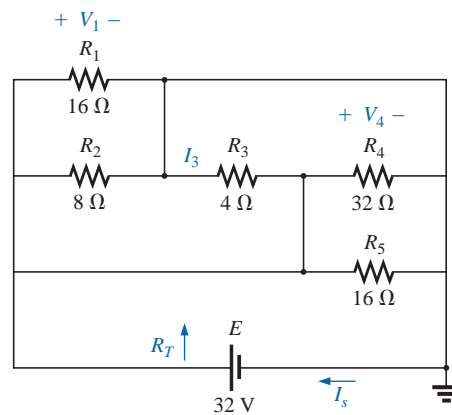
FIG. 7.82
Problem 19.

20. For the network of Fig. 7.83:
  a. Determine the voltage V_{ab}. (*Hint:* Just use Kirchhoff's voltage law.)
  b. Calculate the current I.

FIG. 7.83
Problem 20.

*21. For the network of Fig. 7.84:
  a. Determine the current *I*.
  b. Calculate the open-circuit voltage *V*.

*22. For the network of Fig. 7.85, find the resistance R_3 if the current through it is 2 A.
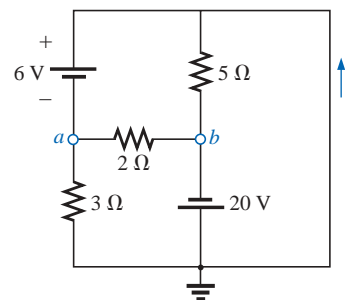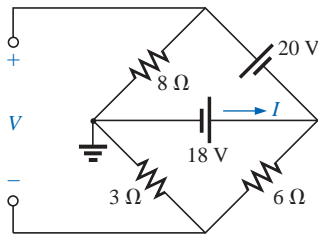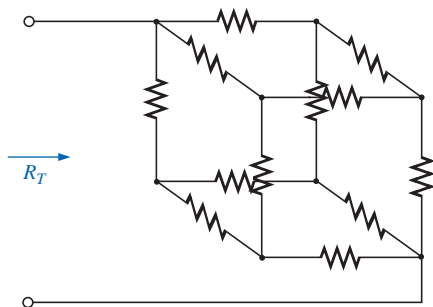
FIG. 7.84
Problem 21.

FIG. 7.85
Problem 22.

*23. If all the resistors of the cube in Fig. 7.86 are 10 Ω, what is the total resistance? (*Hint:* Make some basic assumptions about current division through the cube.)

*24. Given the voltmeter reading $V = 27$ V in Fig. 7.87:
  a. Is the network operating properly?
  b. If not, what could be the cause of the incorrect reading?
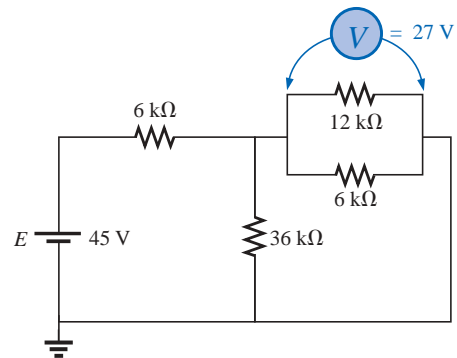
FIG. 7.86
Problem 23.
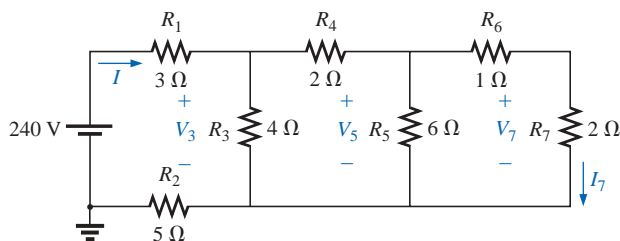
FIG. 7.87
Problem 24.

FIG. 7.88
Problem 25.

SECTION 7.3 Ladder Networks

25. For the ladder network of Fig. 7.88:
  a. Find the current *I*.
  b. Find the current I_7.
  c. Determine the voltages V_3, V_5, and V_7.
  d. Calculate the power delivered to R_7, and compare it to the power delivered by the 240-V supply.

26. For the ladder network of Fig. 7.89:
   a. Determine R_T.
   b. Calculate I.
   c. Find I_8.
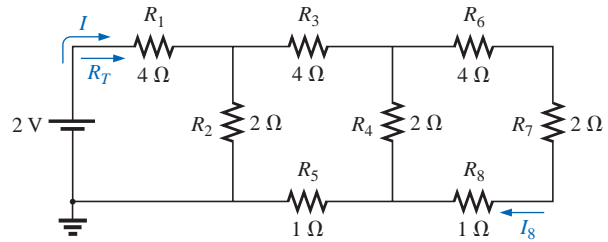
FIG. 7.89
Problem 26.

*27. Determine the power delivered to the 10-Ω load of Fig. 7.90.
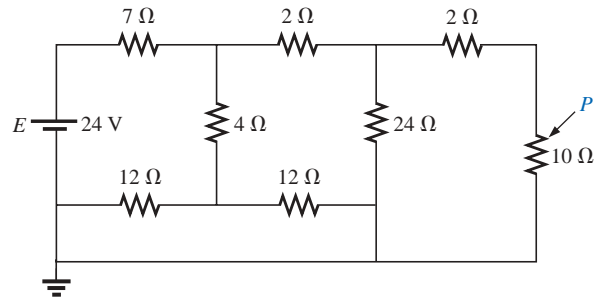
FIG. 7.90
Problem 27.

*28. For the multiple ladder configuration of Fig. 7.91:
   a. Determine I.
   b. Calculate I_4.
   c. Find I_6.
   d. Find I_{10}.
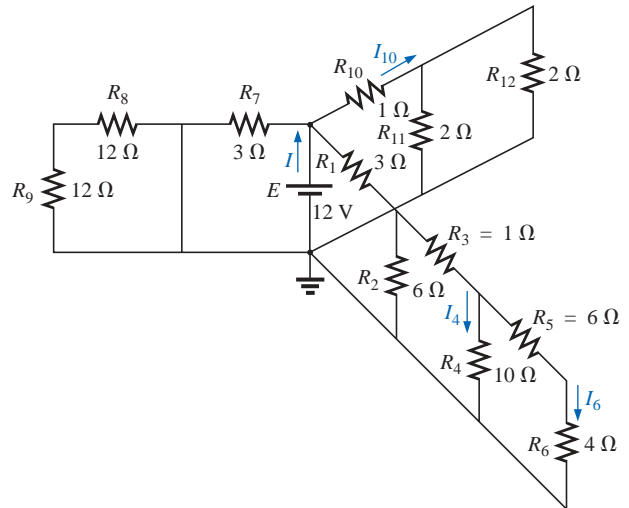
SECTION 7.4   Voltage Divider Supply
(Unloaded and Loaded)

29. Given the voltage divider supply of Fig. 7.92:
   a. Determine the supply voltage E.
   b. Find the load resistors R_{L_2} and R_{L_3}.
   c. Determine the voltage divider resistors R_1, R_2, and R_3.
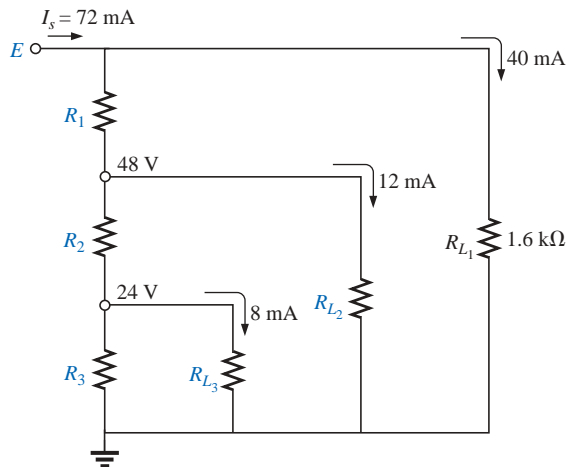
FIG. 7.91
Problem 28.

FIG. 7.92
Problem 29.

FIG. 7.93
Problem 30.
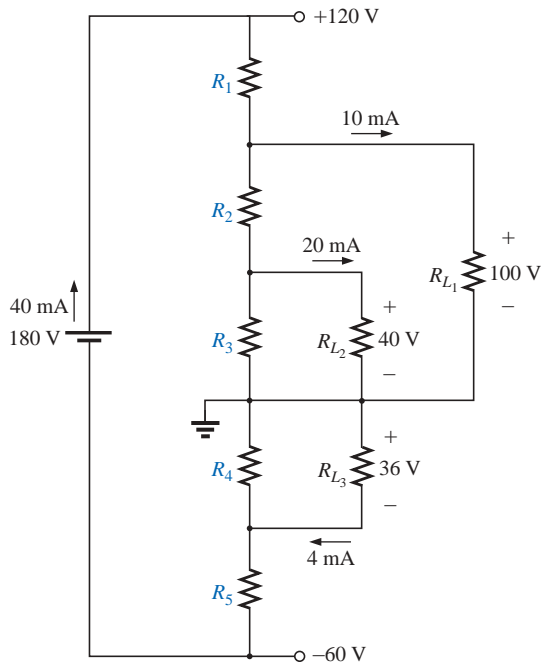
FIG. 7.94
Problem 31.

FIG. 7.95
Problem 32.

*30. Determine the voltage divider supply resistors for the configuration of Fig. 7.93. Also determine the required wattage rating for each resistor, and compare their levels.
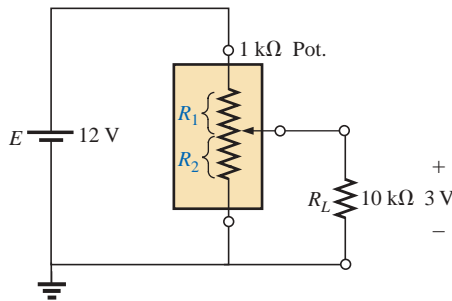
SECTION 7.5 Potentiometer Loading

*31. For the system of Fig. 7.94:
   a. At first exposure, does the design appear to be a good one?
   b. In the absence of the 10-kΩ load, what are the values of R_1 and R_2 to establish 3 V across R_2?
   c. Determine the values of R_1 and R_2 when the load is applied, and compare them to the results of part (b).
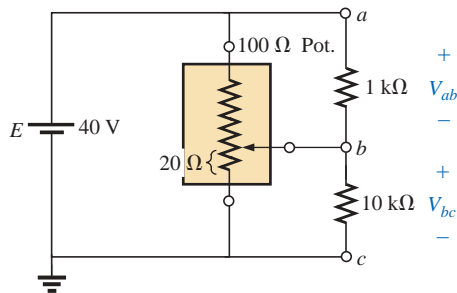
*32. For the potentiometer of Fig. 7.95:
   a. What are the voltages V_{ab} and V_{bc} with no load applied?
   b. What are the voltages V_{ab} and V_{bc} with the indicated loads applied?
   c. What is the power dissipated by the potentiometer under the loaded conditions of Fig. 7.95?
   d. What is the power dissipated by the potentiometer with no loads applied? Compare it to the results of part (c).

SECTION 7.6 Ammeter, Voltmeter, and Ohmmeter Design

33. A d'Arsonval movement is rated 1 mA, 100 Ω.
   a. What is the current sensitivity?
   b. Design a 20-A ammeter using the above movement. Show the circuit and component values.

34. Using a 50-μA, 1000-Ω d'Arsonval movement, design a multirange milliammeter having scales of 25 mA, 50 mA, and 100 mA. Show the circuit and component values.

35. A d'Arsonval movement is rated 50 μA, 1000 Ω.
   a. Design a 15-V dc voltmeter. Show the circuit and component values.
   b. What is the ohm/volt rating of the voltmeter?

36. Using a 1-mA, 100-Ω d'Arsonval movement, design a multirange voltmeter having scales of 5 V, 50 V, and 500 V. Show the circuit and component values.

37. A digital meter has an internal resistance of 10 MΩ on its 0.5-V range. If you had to build a voltmeter with a d'Arsonval movement, what current sensitivity would you need if the meter were to have the same internal resistance on the same voltage scale?

*38. a. Design a series ohmmeter using a 100-μA, 1000-Ω movement; a zero-adjust with a maximum value of 2 kΩ; a battery of 3 V; and a series resistor whose value is to be determined.
   b. Find the resistance required for full-scale, 3/4-scale, 1/2-scale, and 1/4-scale deflection.
   c. Using the results of part (b), draw the scale to be used with the ohmmeter.

39. Describe the basic construction and operation of the megohmmeter.

*40. Determine the reading of the ohmmeter for the configuration of Fig. 7.96.
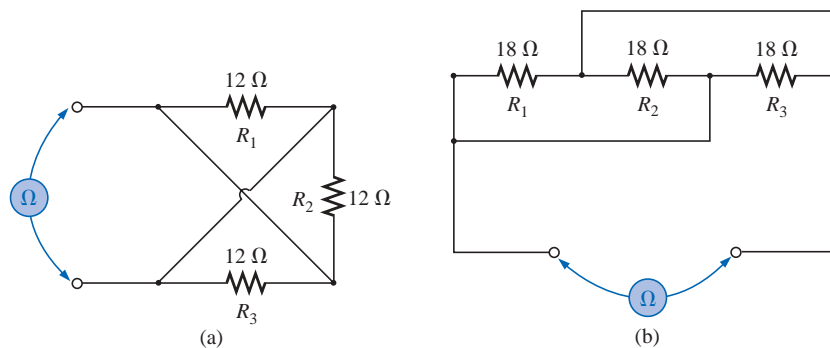
FIG. 7.96
Problem 40.

SECTION 7.9   Computer Analysis

PSpice or Electronics Workbench

41. Using schematics, determine V_1, V_3, V_{ab}, and I_s for the network of Fig. 7.16.

42. Using schematics, determine I_s, I_5, and V_7 for the network of Fig. 7.22.

Programming Language (C++, QBASIC, Pascal, etc.)

43. Write a program that will find the complete solution for the network of Fig. 7.6. That is, given all the parameters of the network, calculate the current, voltage, and power to each element.

44. Write a program to find all the quantities of Example 7.8 given the network parameters.

GLOSSARY

d'Arsonval movement An iron-core coil mounted on bearings between a permanent magnet. A pointer connected to the movable core indicates the strength of the current passing through the coil.

Ladder network A network that consists of a cascaded set of series-parallel combinations and has the appearance of a ladder.

Megohmmeter An instrument for measuring very high resistance levels, such as in the megohm range.

Series ohmmeter A resistance-measuring instrument in which the movement is placed in series with the unknown resistance.

Series-parallel network A network consisting of a combination of both series and parallel branches.

Transistor A three-terminal semiconductor electronic device that can be used for amplification and switching purposes.

8
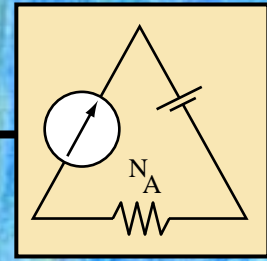
Methods of Analysis and Selected Topics (dc)

8.1 INTRODUCTION

The circuits described in the previous chapters had only one source or two or more sources in series or parallel present. The step-by-step procedure outlined in those chapters cannot be applied if the sources are not in series or parallel. There will be an interaction of sources that will not permit the reduction technique used in Chapter 7 to find quantities such as the total resistance and source current.

Methods of analysis have been developed that allow us to approach, in a systematic manner, a network with any number of sources in any arrangement. Fortunately, these methods can also be applied to networks with only one source. The methods to be discussed in detail in this chapter include **branch-current analysis, mesh analysis,** and **nodal analysis.** Each can be applied to the same network. The "best" method cannot be defined by a set of rules but can be determined only by acquiring a firm understanding of the relative advantages of each. All the methods can be applied to *linear bilateral* networks. The term *linear* indicates that the characteristics of the network elements (such as the resistors) are independent of the voltage across or current through them. The second term, *bilateral,* refers to the fact that there is no change in the behavior or characteristics of an element if the current through or voltage across the element is reversed. Of the three methods listed above, the branch-current method is the only one not restricted to bilateral devices. Before discussing the methods in detail, we shall consider the current source and conversions between voltage and current sources. At the end of the chapter we shall consider bridge networks and Δ-Y and Y-Δ conversions. Chapter 9 will present the important theorems of network analysis that can also be employed to solve networks with more than one source.

8.2 CURRENT SOURCES

The concept of the **current source** was introduced in Section 2.4 with the photograph of a commercially available unit. We must now investi-

gate its characteristics in greater detail so that we can properly determine its effect on the networks to be examined in this chapter.

The current source is often referred to as the *dual* of the voltage source. A battery supplies a *fixed* voltage, and the source current can vary; but the current source supplies a *fixed* current to the branch in which it is located, while its terminal voltage may vary as determined by the network to which it is applied. Note from the above that *duality* simply implies an interchange of current and voltage to distinguish the characteristics of one source from the other.

The interest in the current source is due primarily to semiconductor devices such as the transistor. In the basic electronics courses, you will find that the transistor is a current-controlled device. In the physical model (equivalent circuit) of a transistor used in the analysis of transistor networks, there appears a current source as indicated in Fig. 8.1. The symbol for a current source appears in Fig. 8.1(a). The direction of the arrow within the circle indicates the direction in which current is being supplied.

(a) Transistor symbol   (b) Transistor equivalent circuit
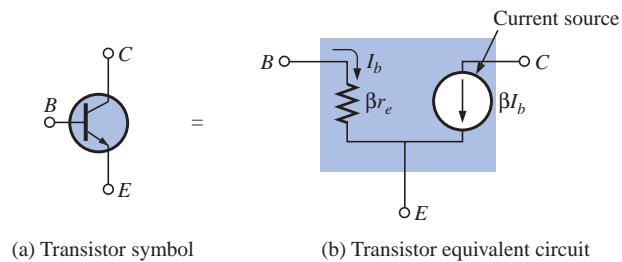
FIG. 8.1

Current source within the transistor equivalent circuit.

For further comparison, the terminal characteristics of an *ideal dc* voltage and current source are presented in Fig. 8.2, *ideal* implying perfect sources, or no internal losses sensitive to the demand from the applied load. Note that for the voltage source, the terminal voltage is fixed at E volts independent of the direction of the current I. The direction and magnitude of I will be determined by the network to which the supply is connected.
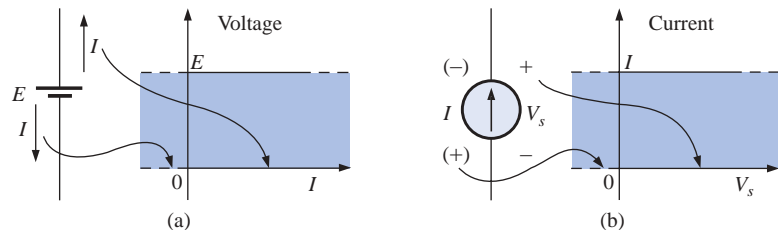
FIG. 8.2

Comparing the characteristics of an ideal voltage and current source.

The characteristics of the ideal current source, shown in Fig. 8.2(b), reveal that the magnitude of the supply current is independent of the polarity of the voltage across the source. The polarity and magnitude of the source voltage V_s will be determined by the network to which the source is connected.

For all one-voltage-source networks the current will have the direction indicated to the right of the battery in Fig. 8.2(a). For all single-

current-source networks, it will have the polarity indicated to the right of the current source in Fig. 8.2(b).

In review:

A current source determines the current in the branch in which it is located

and

the magnitude and polarity of the voltage across a current source are a function of the network to which it is applied.

EXAMPLE 8.1 Find the source voltage V_s and the current I_1 for the circuit of Fig. 8.3.

Solution:

$$I_1 = I = \textbf{10 mA}$$
$$V_s = V_1 = I_1R_1 = (10 \text{ mA})(20 \text{ k}\Omega) = \textbf{200 V}$$

EXAMPLE 8.2 Find the voltage V_s and the currents I_1 and I_2 for the network of Fig. 8.4.

Solution:

$$V_s = E = \textbf{12 V}$$
$$I_2 = \frac{V_R}{R} = \frac{E}{R} = \frac{12 \text{ V}}{4 \text{ }\Omega} = \textbf{3 A}$$

Applying Kirchhoff's current law:

$$I = I_1 + I_2$$

and

$$I_1 = I - I_2 = 7 \text{ A} - 3 \text{ A} = \textbf{4 A}$$

FIG. 8.3
Example 8.1.

FIG. 8.4
Example 8.2.

EXAMPLE 8.3 Determine the current I_1 and the voltage V_s for the network of Fig. 8.5.

Solution: Using the current divider rule:

$$I_1 = \frac{R_2I}{R_2 + R_1} = \frac{(1 \text{ }\Omega)(6 \text{ A})}{1 \text{ }\Omega + 2 \text{ }\Omega} = \textbf{2 A}$$

The voltage V_1 is

$$V_1 = I_1R_1 = (2 \text{ A})(2 \text{ }\Omega) = 4 \text{ V}$$

and, applying Kirchhoff's voltage law,

$$+V_s - V_1 - 20 \text{ V} = 0$$

and

$$V_s = V_1 + 20 \text{ V} = 4 \text{ V} + 20 \text{ V}$$
$$= \textbf{24 V}$$

Note the polarity of V_s as determined by the multisource network.
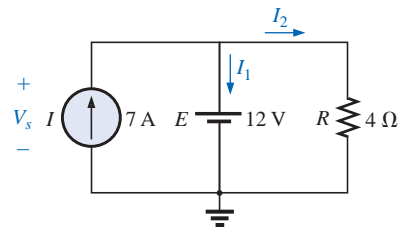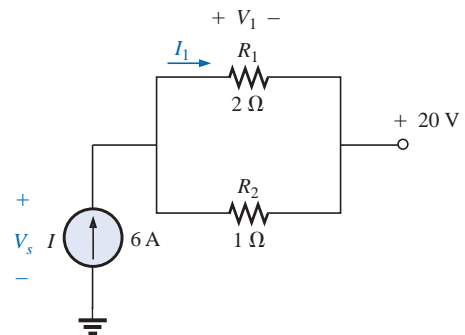
FIG. 8.5
Example 8.3.

8.3 SOURCE CONVERSIONS

The current source described in the previous section is called an *ideal source* due to the absence of any internal resistance. In reality, all
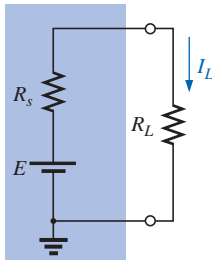
FIG. 8.6

Practical voltage source.
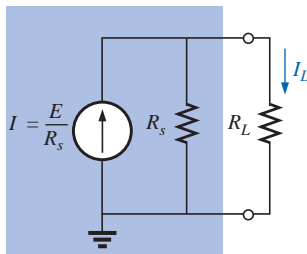
FIG. 8.7

Practical current source.

sources—whether they are voltage or current—have some internal resistance in the relative positions shown in Figs. 8.6 and 8.7. For the voltage source, if $R_s = 0 \ \Omega$ or is so small compared to any series resistor that it can be ignored, then we have an "ideal" voltage source. For the current source, if $R_s = \infty \ \Omega$ or is large enough compared to other parallel elements that it can be ignored, then we have an "ideal" current source.

If the internal resistance is included with either source, then that source can be converted to the other type using the procedure to be described in this section. Since it is often advantageous to make such a maneuver, this entire section is devoted to being sure that the steps are understood. It is important to realize, however, as we proceed through this section, that

source conversions are equivalent only at their external terminals.

The internal characteristics of each are quite different.

We want the equivalence to ensure that the applied load of Figs. 8.6 and 8.7 will receive the same current, voltage, and power from each source and in effect not know, or care, which source is present.

In Fig. 8.6 if we solve for the load current I_L, we obtain

$$I_L = \frac{E}{R_s + R_L} \tag{8.1}$$

If we multiply this by a factor of 1, which we can choose to be R_s/R_s, we obtain

$$I_L = \frac{(1)E}{R_s + R_L} = \frac{(R_s/R_s)E}{R_s + R_L} = \frac{R_s(E/R_s)}{R_s + R_L} = \frac{R_s I}{R_s + R_L} \tag{8.2}$$

If we define $I = E/R_s$, Equation (8.2) is the same as that obtained by applying the current divider rule to the network of Fig. 8.7. The result is an equivalence between the networks of Figs. 8.6 and 8.7 that simply requires that $I = E/R_s$ and the series resistor R_s of Fig. 8.6 be placed in parallel, as in Fig. 8.7. The validity of this is demonstrated in Example 8.4 of this section.

For clarity, the equivalent sources, *as far as terminals a and b are concerned,* are repeated in Fig. 8.8 with the equations for converting in either direction. Note, as just indicated, that the resistor R_s is the same in each source; only its position changes. The current of the current source or the voltage of the voltage source is determined using Ohm's law and the parameters of the other configuration. It was pointed out in some detail in Chapter 6 that every source of voltage has some internal series resistance. *For the current source, some internal parallel resistance will always exist in the practical world.* However, in many cases, it is an
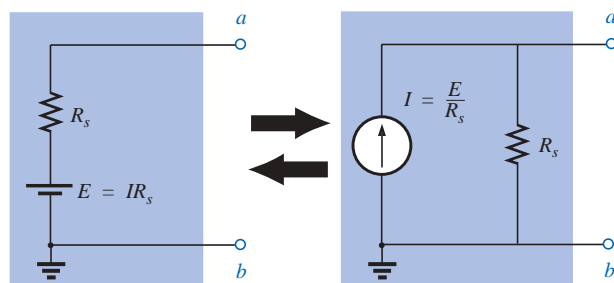
FIG. 8.8

Source conversion.

excellent approximation to drop the internal resistance of a source due to the magnitude of the elements of the network to which it is applied. For this reason, in the analyses to follow, voltage sources may appear without a series resistor, and current sources may appear without a parallel resistance. Realize, however, that for us to perform a conversion from one type of source to another, a voltage source must have a resistor in series with it, and a current source must have a resistor in parallel.

EXAMPLE 8.4

a. Convert the voltage source of Fig. 8.9(a) to a current source, and calculate the current through the 4-Ω load for each source.
b. Replace the 4-Ω load with a 1-kΩ load, and calculate the current I_L for the voltage source.
c. Repeat the calculation of part (b) assuming that the voltage source is ideal ($R_s = 0$ Ω) because R_L is so much larger than R_s. Is this one of those situations where assuming that the source is ideal is an appropriate approximation?

Solutions:

a. See Fig. 8.9.

$$\text{Fig. 8.9(a):} \quad I_L = \frac{E}{R_s + R_L} = \frac{6\text{ V}}{2\text{ }\Omega + 4\text{ }\Omega} = \mathbf{1\text{ A}}$$

$$\text{Fig. 8.9(b):} \quad I_L = \frac{R_s I}{R_s + R_L} = \frac{(2\text{ }\Omega)(3\text{ A})}{2\text{ }\Omega + 4\text{ }\Omega} = \mathbf{1\text{ A}}$$

b. $I_L = \dfrac{E}{R_s + R_L} = \dfrac{6\text{ V}}{2\text{ }\Omega + 1\text{ k}\Omega} \cong \mathbf{5.99\text{ mA}}$

c. $I_L = \dfrac{E}{R_L} = \dfrac{6\text{ V}}{1\text{ k}\Omega} = \mathbf{6\text{ mA}} \cong 5.99\text{ mA}$

Yes, $R_L \gg R_s$ (voltage source).

(a)

(b)
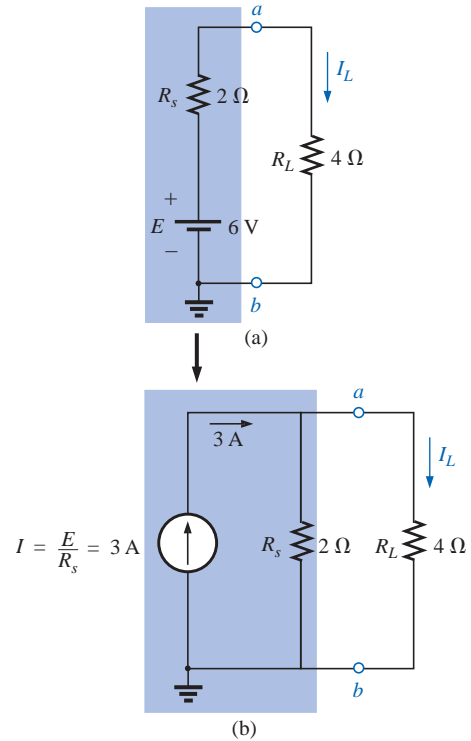
FIG. 8.9
Example 8.4.

EXAMPLE 8.5

a. Convert the current source of Fig. 8.10(a) to a voltage source, and find the load current for each source.
b. Replace the 6-kΩ load with a 10-Ω load, and calculate the current I_L for the current source.
c. Repeat the calculation of part (b) assuming that the current source is ideal ($R_s = \infty$ Ω) because R_L is so much smaller than R_s. Is this one of those situations where assuming that the source is ideal is an appropriate approximation?

(a)   (b)
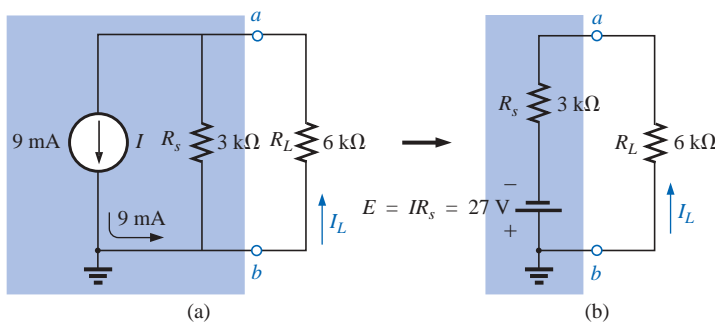
FIG. 8.10
Example 8.5.

Solutions:

a. See Fig. 8.10.

Fig. 8.10(a): $I_L = \dfrac{R_s I}{R_s + R_L} = \dfrac{(3 \text{ k}\Omega)(9 \text{ mA})}{3 \text{ k}\Omega + 6 \text{ k}\Omega} = \textbf{3 mA}$

Fig. 8.10(b): $I_L = \dfrac{E}{R_s + R_L} = \dfrac{27 \text{ V}}{3 \text{ k}\Omega + 6 \text{ k}\Omega} = \dfrac{27 \text{ V}}{9 \text{ k}\Omega} = \textbf{3 mA}$

b. $I_L = \dfrac{R_s I}{R_s + R_L} = \dfrac{(3 \text{ k}\Omega)(9 \text{ mA})}{3 \text{ k}\Omega + 10 \text{ }\Omega} = \textbf{8.97 mA}$

c. $I_L = I = \textbf{9 mA} \cong 8.97 \text{ mA}$

Yes, $R_s \gg R_L$ (current source).

8.4   CURRENT SOURCES IN PARALLEL

If two or more current sources are in parallel, they may all be replaced by one current source having the magnitude and direction of the resultant, which can be found by summing the currents in one direction and subtracting the sum of the currents in the opposite direction. The new parallel resistance is determined by methods described in the discussion of parallel resistors in Chapter 5. Consider the following examples.

EXAMPLE 8.6 Reduce the parallel current sources of Figs. 8.11 and 8.12 to a single current source.

$I_s = 10 \text{ A} - 6 \text{ A} = \textbf{4 A}$
$R_s = 3 \text{ }\Omega \parallel 6 \text{ }\Omega = \textbf{2 }\Omega$

FIG. 8.11
Example 8.6.

$I_s = 7 \text{ A} + 4 \text{ A} - 3 \text{ A} = \textbf{8 A}$
$R_s = R_1 = \textbf{4 }\Omega$
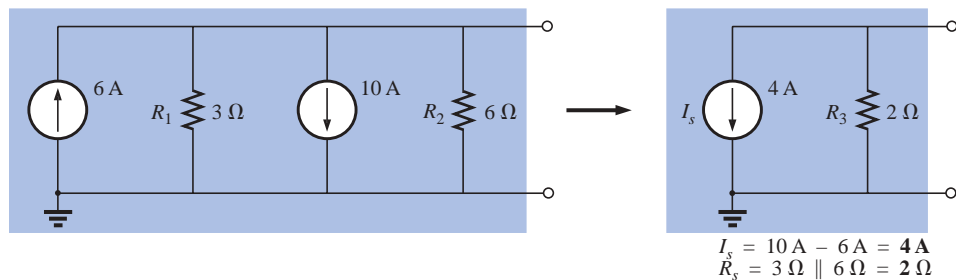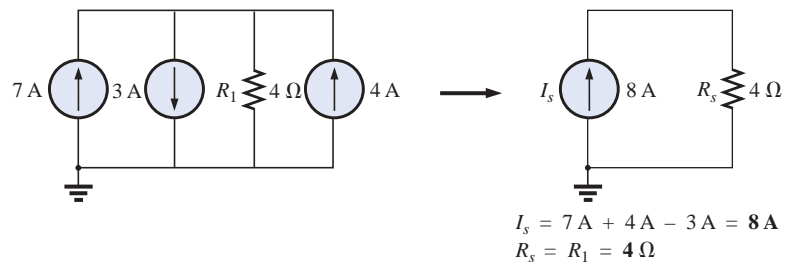
FIG. 8.12
Example 8.6.

Solution: Note the solution in each figure.

EXAMPLE 8.7 Reduce the network of Fig. 8.13 to a single current source, and calculate the current through R_L.

Solution: In this example, the voltage source will first be converted to a current source as shown in Fig. 8.14. Combining current sources,

$$I_1 = \frac{E_1}{R_1} = \frac{32 \text{ V}}{8 \text{ }\Omega} = 4 \text{ A}$$
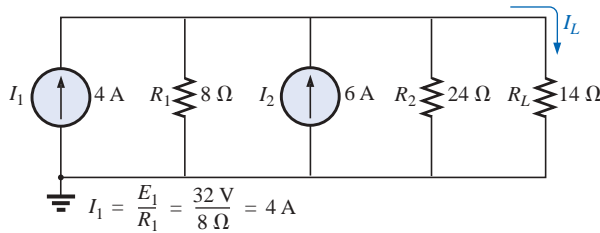
FIG. 8.14
Network of Fig. 8.13 following the conversion of the voltage source to a current source.

$$I_s = I_1 + I_2 = 4 \text{ A} + 6 \text{ A} = \textbf{10 A}$$

and $$R_s = R_1 \| R_2 = 8 \text{ }\Omega \| 24 \text{ }\Omega = \textbf{6 }\boldsymbol{\Omega}$$

Applying the current divider rule to the resulting network of Fig. 8.15,

$$I_L = \frac{R_s I_s}{R_s + R_L} = \frac{(6 \text{ }\Omega)(10 \text{ A})}{6 \text{ }\Omega + 14 \text{ }\Omega} = \frac{60 \text{ A}}{20} = \textbf{3 A}$$

EXAMPLE 8.8 Determine the current I_2 in the network of Fig. 8.16.

Solution: Although it might appear that the network cannot be solved using methods introduced thus far, one source conversion as shown in Fig. 8.17 will result in a simple series circuit:

$$E_s = I_1 R_1 = (4 \text{ A})(3 \text{ }\Omega) = 12 \text{ V}$$

and $$R_s = R_1 = 3 \text{ }\Omega$$

and $$I_2 = \frac{E_s + E_2}{R_s + R_2} = \frac{12 \text{ V} + 5 \text{ V}}{3 \text{ }\Omega + 2 \text{ }\Omega} = \frac{17 \text{ V}}{5 \text{ }\Omega} = \textbf{3.4 A}$$

8.5 CURRENT SOURCES IN SERIES

The current through any branch of a network can be only single-valued. For the situation indicated at point a in Fig. 8.18, we find by application of Kirchhoff's current law that the current leaving that point is greater than that entering—an impossible situation. Therefore,

current sources of different current ratings are not connected in series,

just as voltage sources of different voltage ratings are not connected in parallel.

8.6 BRANCH-CURRENT ANALYSIS

We will now consider the first in a series of methods for solving networks with two or more sources. Once the **branch-current method** is
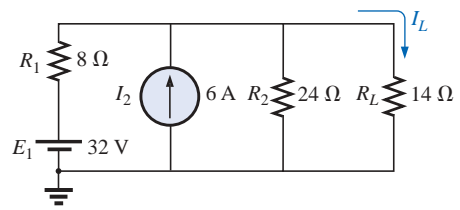
FIG. 8.13
Example 8.7.
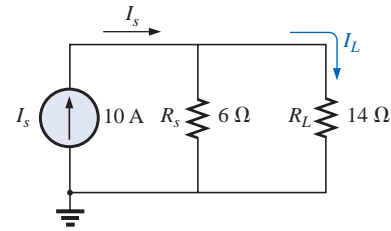
FIG. 8.15
Network of Fig. 8.14 reduced to its simplest form.
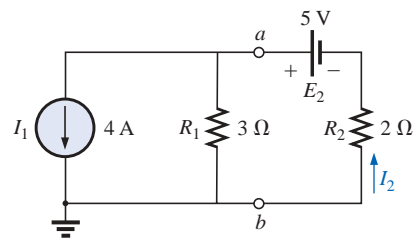
FIG. 8.16
Example 8.8.
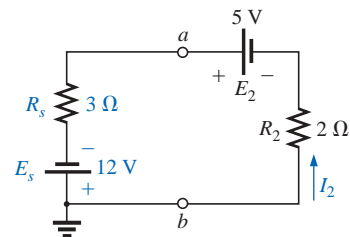
FIG. 8.17
Network of Fig. 8.16 following the conversion of the current source to a voltage source.

No!
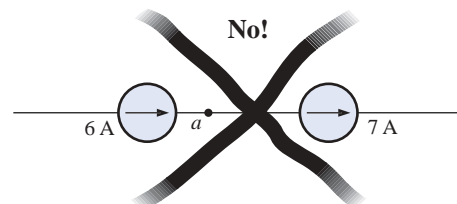
FIG. 8.18
Invalid situation.

mastered, there is no linear dc network for which a solution cannot be found. Keep in mind that networks with two isolated voltage sources cannot be solved using the approach of Chapter 7. For additional evidence of this fact, try solving for the unknown elements of Example 8.9 using the methods introduced in Chapter 7. The network of Fig. 8.21 can be solved using the source conversions described in the last section, but the method to be described in this section has applications far beyond the configuration of this network. The most direct introduction to a method of this type is to list the series of steps required for its application. There are four steps, as indicated below. Before continuing, understand that this method will produce the current through each branch of the network, the *branch current*. Once this is known, all other quantities, such as voltage or power, can be determined.

1. *Assign a distinct current of arbitrary direction to each branch of the network.*
2. *Indicate the polarities for each resistor as determined by the assumed current direction.*
3. *Apply Kirchhoff's voltage law around each closed, independent loop of the network.*

The best way to determine how many times Kirchhoff's voltage law will have to be applied is to determine the number of "windows" in the network. The network of Example 8.9 has a definite similarity to the two-window configuration of Fig. 8.19(a). The result is a need to apply Kirchhoff's voltage law twice. For networks with three windows, as shown in Fig. 8.19(b), three applications of Kirchhoff's voltage law are required, and so on.

(a)   (b)
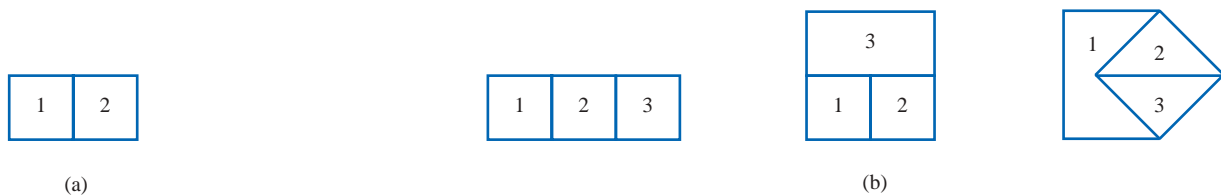
FIG. 8.19
Determining the number of independent closed loops.

4. *Apply Kirchhoff's current law at the minimum number of nodes that will include all the branch currents of the network.*

The minimum number is one less than the number of independent nodes of the network. For the purposes of this analysis, a **node** is a junction of two or more branches, where a branch is any combination
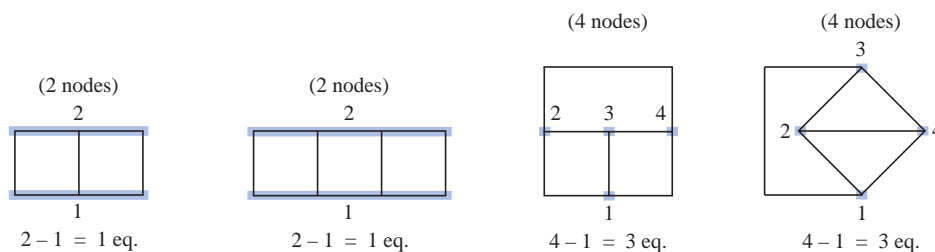
FIG. 8.20
Determining the number of applications of Kirchhoff's current law required.

of series elements. Figure 8.20 defines the number of applications of Kirchhoff's current law for each configuration of Fig. 8.19.

5. *Solve the resulting simultaneous linear equations for assumed branch currents.*

It is assumed that the use of the **determinants method** to solve for the currents I_1, I_2, and I_3 is understood and is a part of the student's mathematical background. If not, a detailed explanation of the procedure is provided in Appendix C. Calculators and computer software packages such as Mathcad can find the solutions quickly and accurately.

EXAMPLE 8.9 Apply the branch-current method to the network of Fig. 8.21.

Solution 1:

Step 1: Since there are three distinct branches (*cda, cba, ca*), three currents of arbitrary directions (I_1, I_2, I_3) are chosen, as indicated in Fig. 8.21. The current directions for I_1 and I_2 were chosen to match the "pressure" applied by sources E_1 and E_2, respectively. Since both I_1 and I_2 enter node *a*, I_3 is leaving.

Step 2: Polarities for each resistor are drawn to agree with assumed current directions, as indicated in Fig. 8.22.
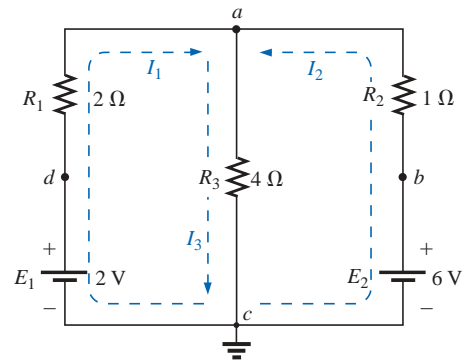
FIG. 8.21
Example 8.9.
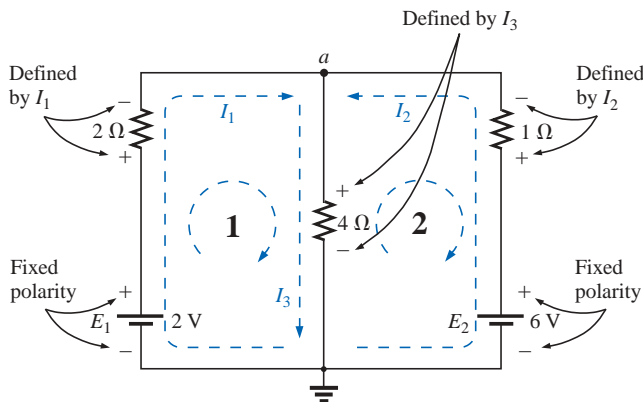
FIG. 8.22
Inserting the polarities across the resistive elements as defined by the chosen branch currents.

Step 3: Kirchhoff's voltage law is applied around each closed loop (1 and 2) in the clockwise direction:

loop 1: $\Sigma_{\circlearrowright} V = +E_1 - V_{R_1} - V_{R_3} = 0$

  (Rise in potential / Drop in potential)

loop 2: $\Sigma_{\circlearrowright} V = +V_{R_3} + V_{R_2} - E_2 = 0$

  (Rise in potential / Drop in potential)

and

loop 1: $\Sigma_{\circlearrowright} V = +2\text{ V} - (2\ \Omega)I_1 - (4\ \Omega)I_3 = 0$

  (Battery potential / Voltage drop across 2-Ω resistor / Voltage drop across 4-Ω resistor)

loop 2: $\Sigma_{\circlearrowright} V = (4\ \Omega)I_3 + (1\ \Omega)I_2 - 6\text{ V} = 0$

Step 4: Applying Kirchhoff's current law at node *a* (in a two-node network, the law is applied at only one node),

$$I_1 + I_2 = I_3$$

Step 5: There are three equations and three unknowns (units removed for clarity):

$$
\begin{aligned}
2 - 2I_1 - 4I_3 &= 0 \\
4I_3 + 1I_2 - 6 &= 0 \\
I_1 + I_2 &= I_3
\end{aligned}
\qquad \text{Rewritten:} \qquad
\begin{aligned}
2I_1 + 0 + 4I_3 &= 2 \\
0 + I_2 + 4I_3 &= 6 \\
I_1 + I_2 - I_3 &= 0
\end{aligned}
$$

Using third-order determinants (Appendix C), we have

$$
I_1 = \frac{\begin{vmatrix} 2 & 0 & 4 \\ 6 & 1 & 4 \\ 0 & 1 & -1 \end{vmatrix}}{D} = -1\,\text{A}
$$

A negative sign in front of a branch current indicates only that the actual current is in the direction opposite to that assumed.

$$
D = \begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}
$$

$$
I_2 = \frac{\begin{vmatrix} 2 & 2 & 4 \\ 0 & 6 & 4 \\ 1 & 0 & -1 \end{vmatrix}}{D} = 2\,\text{A}
$$

$$
I_3 = \frac{\begin{vmatrix} 2 & 0 & 2 \\ 0 & 1 & 6 \\ 1 & 1 & 0 \end{vmatrix}}{D} = 1\,\text{A}
$$

Mathcad Solution: Once you understand the procedure for entering the parameters, you can use Mathcad to solve determinants such as
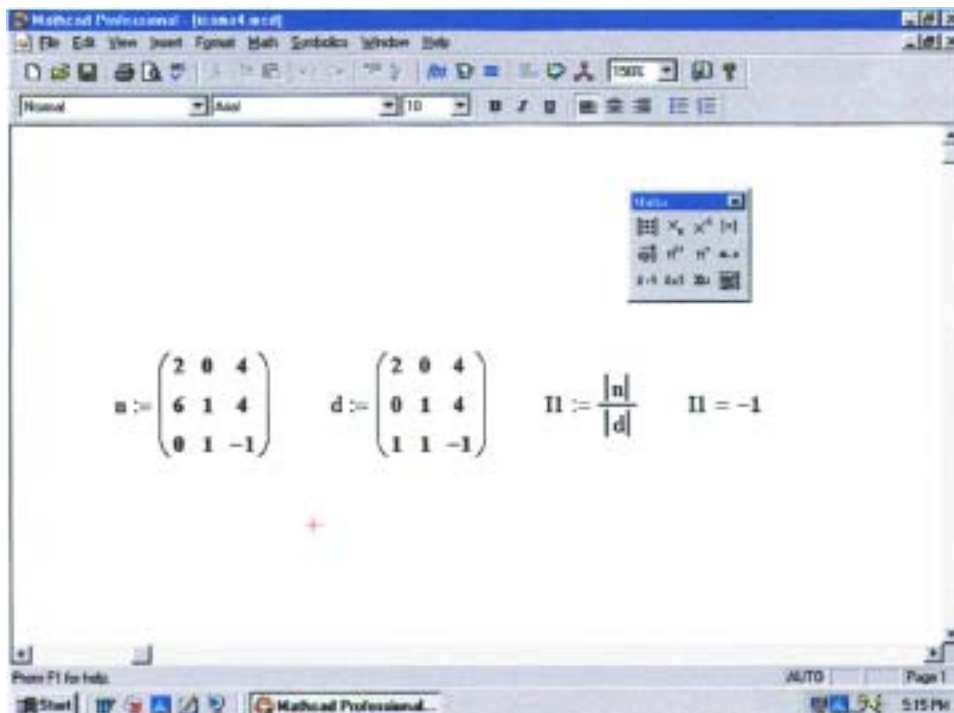
FIG. 8.23
Using Mathcad to verify the numerical calculations of Example 8.9.

appearing in Solution 1 in a very short time frame. The numerator is defined by n in the same manner described for earlier exercises. Then the sequence **View-Toolbars-Matrix** is applied to obtain the **Matrix** toolbar appearing in Fig. 8.23. Selecting the top left option called **Matrix** will result in the **Insert Matrix** dialog box in which **3 × 3** is selected. The 3 × 3 matrix will appear with a bracket to signal which parameter should be entered. Enter that parameter, and then use the left click of the mouse to select the next parameter you want to enter. When you have finished, move on to define the denominator d in the same manner. Then define the current of interest, select **Determinant** from the **Matrix** toolbar, and insert the numerator variable n. Follow with a division sign, and enter the **Determinant** of the denominator as shown in Fig. 8.23. Retype **I1** and select the equal sign; the correct result of **−1** will appear.

Once you have mastered the rather simple and direct process just described, the availability of Mathcad as a checking tool or solving mechanism will be deeply appreciated.

Solution 2: Instead of using third-order determinants as in Solution 1, we could reduce the three equations to two by substituting the third equation in the first and second equations:

$$2 - 2I_1 - 4\overbrace{(I_1 + I_2)}^{I_3} = 0 \qquad 2 - 2I_1 - 4I_1 - 4I_2 = 0$$

$$4\overbrace{(I_1 + I_2)}^{I_3} + I_2 - 6 = 0 \qquad 4I_1 + 4I_2 + I_2 - 6 = 0$$

or

$$-6I_1 - 4I_2 = -2$$
$$+4I_1 + 5I_2 = +6$$

Multiplying through by -1 in the top equation yields

$$6I_1 + 4I_2 = +2$$
$$4I_1 + 5I_2 = +6$$

and using determinants,

$$I_1 = \frac{\begin{vmatrix} 2 & 4 \\ 6 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & 4 \\ 4 & 5 \end{vmatrix}} = \frac{10 - 24}{30 - 16} = \frac{-14}{14} = \mathbf{-1A}$$

Using the TI-86 calculator:

det[[2,4][6,5]]/det[[6,4][4,5]] (ENTER) −1

CALC. 8.1

Note the det (determinant) obtained from a Math listing under a MATRX menu and the fact that each determinant must be determined individually. The first set of brackets within the overall determinant brackets of the first determinant defines the first row of the determinant, while the second set of brackets within the same determinant defines the second row. A comma separates the entries for each row. Obviously, the time to learn how to enter the parameters is minimal when you consider the savings in time and the accuracy obtained.

$$I_2 = \frac{\begin{vmatrix} 6 & 2 \\ 4 & 6 \end{vmatrix}}{14} = \frac{36 - 8}{14} = \frac{28}{14} = \textbf{2 A}$$

$$I_3 = I_1 + I_2 = -1 + 2 = \textbf{1 A}$$

It is now important that the impact of the results obtained be understood. The currents I_1, I_2, and I_3 are the actual currents in the branches in which they were defined. A negative sign in the solution simply reveals that the actual current has the opposite direction than initially defined—the magnitude is correct. Once the actual current directions and their magnitudes are inserted in the original network, the various voltages and power levels can be determined. For this example, the actual current directions and their magnitudes have been entered on the original network in Fig. 8.24. Note that the current through the series elements R_1 and E_1 is 1 A; the current through R_3, 1 A; and the current through the series elements R_2 and E_2, 2 A. Due to the minus sign in the solution, the direction of I_1 is opposite to that shown in Fig. 8.21. The voltage across any resistor can now be found using Ohm's law, and the power delivered by either source or to any one of the three resistors can be found using the appropriate power equation.
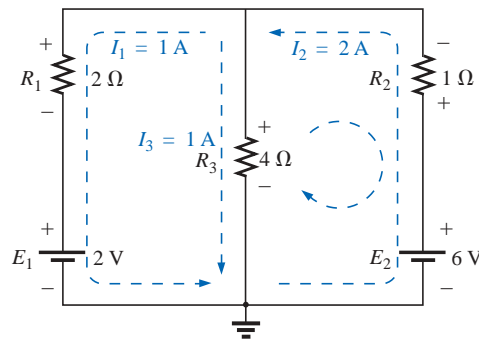
FIG. 8.24

Reviewing the results of the analysis of the network of Fig. 8.21.

Applying Kirchhoff's voltage law around the loop indicated in Fig. 8.24,

$$\Sigma_{\circlearrowright} V = +(4\ \Omega)I_3 + (1\ \Omega)I_2 - 6\ V = 0$$

or

$$(4\ \Omega)I_3 + (1\ \Omega)I_2 = 6\ V$$

and

$$(4\ \Omega)(1\ A) + (1\ \Omega)(2\ A) = 6\ V$$
$$4\ V + 2\ V = 6\ V$$
$$\underline{6\ V = 6\ V \quad \text{(checks)}}$$

EXAMPLE 8.10 Apply branch-current analysis to the network of Fig. 8.25.

Solution: Again, the current directions were chosen to match the "pressure" of each battery. The polarities are then added and Kirchhoff's voltage law is applied around each closed loop in the clockwise direction. The result is as follows:

$$\text{loop 1:} \quad +15\ V - (4\ \Omega)I_1 + (10\ \Omega)I_3 - 20\ V = 0$$
$$\text{loop 2:} \quad +20\ V - (10\ \Omega)I_3 - (5\ \Omega)I_2 + 40\ V = 0$$
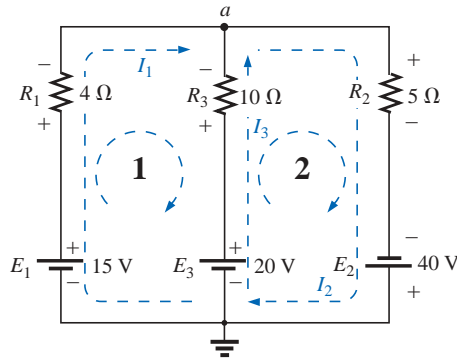
FIG. 8.25
Example 8.10.

Applying Kirchhoff's current law at node *a,*

$$I_1 + I_3 = I_2$$

Substituting the third equation into the other two yields (with units removed for clarity)

$$\left. \begin{array}{l} 15 - 4I_1 + 10I_3 - 20 = 0 \\ 20 - 10I_3 - 5(I_1 + I_3) + 40 = 0 \end{array} \right\}$$ Substituting for I_2 (since it occurs only once in the two equations)

or

$$-4I_1 + 10I_3 = 5$$
$$-5I_1 - 15I_3 = -60$$

Multiplying the lower equation by -1, we have

$$-4I_1 + 10I_3 = 5$$
$$5I_1 + 15I_3 = 60$$

$$I_1 = \frac{\begin{vmatrix} 5 & 10 \\ 60 & 15 \end{vmatrix}}{\begin{vmatrix} -4 & 10 \\ 5 & 15 \end{vmatrix}} = \frac{75 - 600}{-60 - 50} = \frac{-525}{-110} = \mathbf{4.773\ A}$$

$$I_3 = \frac{\begin{vmatrix} -4 & 5 \\ 5 & 60 \end{vmatrix}}{-110} = \frac{-240 - 25}{-110} = \frac{-265}{-110} = \mathbf{2.409\ A}$$

$$I_2 = I_1 + I_3 = 4.773 + 2.409 = \mathbf{7.182\ A}$$

revealing that the assumed directions were the actual directions, with I_2 equal to the sum of I_1 and I_3.

8.7 MESH ANALYSIS (GENERAL APPROACH)

The second method of analysis to be described is called **mesh analysis.** The term *mesh* is derived from the similarities in appearance between the closed loops of a network and a wire mesh fence. Although this approach is on a more sophisticated plane than the branch-current method, it incorporates many of the ideas just developed. Of the two methods, mesh analysis is the one more frequently applied today. Branch-current analysis is introduced as a stepping stone to mesh analysis because branch currents are initially more "real" to the student than the **mesh (loop) currents** employed in mesh analysis. Essentially,

the mesh-analysis approach simply eliminates the need to substitute the results of Kirchhoff's current law into the equations derived from Kirchhoff's voltage law. It is now accomplished in the initial writing of the equations. The systematic approach outlined below should be followed when applying this method.

1. *Assign a distinct current in the clockwise direction to each independent, closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current. In fact, any direction can be chosen for each loop current with no loss in accuracy, as long as the remaining steps are followed properly. However, by choosing the clockwise direction as a standard, we can develop a shorthand method (Section 8.8) for writing the required equations that will save time and possibly prevent some common errors.*

This first step is accomplished most effectively by placing a loop current *within* each "window" of the network, as demonstrated in the previous section, to ensure that they are all independent. A variety of other loop currents can be assigned. In each case, however, be sure that the information carried by any one loop equation is not included in a combination of the other network equations. This is the crux of the terminology: *independent*. No matter how you choose your loop currents, the number of loop currents required is always equal to the number of windows of a planar (no-crossovers) network. On occasion a network may appear to be nonplanar. However, a redrawing of the network may reveal that it is, in fact, planar. Such may be the case in one or two problems at the end of the chapter.

Before continuing to the next step, let us ensure that the concept of a loop current is clear. For the network of Fig. 8.26, the loop current I_1 is the branch current of the branch containing the 2-Ω resistor and 2-V battery. The current through the 4-Ω resistor is not I_1, however, since there is also a loop current I_2 through it. Since they have opposite directions, $I_{4\Omega}$ equals the difference between the two, $I_1 - I_2$ or $I_2 - I_1$, depending on which you choose to be the defining direction. In other words, *a loop current is a branch current only when it is the only loop current assigned to that branch.*

2. *Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop. Note the requirement that the polarities be placed within each loop. This requires, as shown in Fig. 8.26, that the 4-Ω resistor have two sets of polarities across it.*

3. *Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity and prepare us for the method to be introduced in the next section.*

   a. *If a resistor has two or more assumed currents through it, the total current through the resistor is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the* **same** *direction, minus the assumed currents through in the* **opposite** *direction.*

   b. *The polarity of a voltage source is unaffected by the direction of the assigned loop currents.*

4. *Solve the resulting simultaneous linear equations for the assumed loop currents.*

FIG. 8.26

Defining the mesh currents for a "two-window" network.

EXAMPLE 8.11 Consider the same basic network as in Example 8.9 of the preceding section, now appearing in Fig. 8.26.

Solution:

Step 1: Two loop currents (I_1 and I_2) are assigned in the clockwise direction in the windows of the network. A third loop (I_3) could have been included around the entire network, but the information carried by this loop is already included in the other two.

Step 2: Polarities are drawn within each window to agree with assumed current directions. Note that for this case, the polarities across the 4-Ω resistor are the opposite for each loop current.

Step 3: Kirchhoff's voltage law is applied around each loop in the clockwise direction. Keep in mind as this step is performed that the law is concerned only with the magnitude and polarity of the voltages around the closed loop and not with whether a voltage rise or drop is due to a battery or a resistive element. The voltage across each resistor is determined by $V = IR$, and for a resistor with more than one current through it, the current is the loop current of the loop being examined plus or minus the other loop currents as determined by their directions. If clockwise applications of Kirchhoff's voltage law are always chosen, the other loop currents will always be subtracted from the loop current of the loop being analyzed.

loop 1: $\quad +E_1 - V_1 - V_3 = 0 \qquad$ (clockwise starting at point *a*)

$$+2\text{ V} - (2\text{ }\Omega)I_1 - \overbrace{(4\text{ }\Omega)(I_1 - I_2)}^{\substack{\text{Voltage drop across}\\\text{4-}\Omega\text{ resistor}}} = 0$$

Total current through 4-Ω resistor

Subtracted since I_2 is opposite in direction to I_1.

loop 2: $\quad -V_3 - V_2 - E_2 = 0 \quad$ (clockwise starting at point *b*)

$$-(4\text{ }\Omega)(I_2 - I_1) - (1\text{ }\Omega)I_2 - 6\text{ V} = 0$$

Step 4: The equations are then rewritten as follows (without units for clarity):

$$\begin{aligned}\text{loop 1:} \quad &+2 - 2I_1 - 4I_1 + 4I_2 = 0\\\text{loop 2:} \quad &-4I_2 + 4I_1 - 1I_2 - 6 = 0\end{aligned}$$

and

$$\begin{aligned}\text{loop 1:} \quad &+2 - 6I_1 + 4I_2 = 0\\\text{loop 2:} \quad &-5I_2 + 4I_1 - 6 = 0\end{aligned}$$

or

$$\begin{aligned}\text{loop 1:} \quad &-6I_1 + 4I_2 = -2\\\text{loop 2:} \quad &+4I_1 - 5I_2 = +6\end{aligned}$$

Applying determinants will result in

$$I_1 = -\mathbf{1\,A} \quad \text{and} \quad I_2 = -\mathbf{2\,A}$$

The minus signs indicate that the currents have a direction opposite to that indicated by the assumed loop current.

The actual current through the 2-V source and 2-Ω resistor is therefore 1 A in the other direction, and the current through the 6-V source and 1-Ω resistor is 2 A in the opposite direction indicated on the circuit. The current through the 4-Ω resistor is determined by the following equation from the original network:

loop 1: $I_{4\Omega} = I_1 - I_2 = -1\,\text{A} - (-2\,\text{A}) = -1\,\text{A} + 2\,\text{A}$
$= \mathbf{1\,A}$ (in the direction of I_1)

The outer loop (I_3) and *one* inner loop (either I_1 or I_2) would also have produced the correct results. This approach, however, will often lead to errors since the loop equations may be more difficult to write. The best method of picking these loop currents is to use the window approach.

EXAMPLE 8.12 Find the current through each branch of the network of Fig. 8.27.

Solution:

Steps 1 and 2 are as indicated in the circuit. Note that the polarities of the 6-Ω resistor are different for each loop current.

Step 3: Kirchhoff's voltage law is applied around each closed loop in the clockwise direction:

loop 1: $+E_1 - V_1 - V_2 - E_2 = 0$ (clockwise starting at point a)
$+5\,\text{V} - (1\,\Omega)I_1 - (6\,\Omega)(I_1 \underset{\uparrow}{-} I_2) - 10\,\text{V} = 0$

I_2 flows through the 6-Ω resistor
in the direction opposite to I_1.

loop 2: $E_2 - V_2 - V_3 = 0$ (clockwise starting at point b)
$+10\,\text{V} - (6\,\Omega)(I_2 - I_1) - (2\,\Omega)I_2 = 0$

The equations are rewritten as

$$\left.\begin{array}{l} 5 - I_1 - 6I_1 + 6I_2 - 10 = 0 \\ 10 - 6I_2 + 6I_1 - 2I_2 = 0 \end{array}\right\} \begin{array}{l} -7I_1 + 6I_2 = 5 \\ +6I_1 - 8I_2 = -10 \end{array}$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} 5 & 6 \\ -10 & -8 \end{vmatrix}}{\begin{vmatrix} -7 & 6 \\ 6 & -8 \end{vmatrix}} = \frac{-40 + 60}{56 - 36} = \frac{20}{20} = \mathbf{1\,A}$$

$$I_2 = \frac{\begin{vmatrix} -7 & 5 \\ 6 & -10 \end{vmatrix}}{20} = \frac{70 - 30}{20} = \frac{40}{20} = \mathbf{2\,A}$$

Since I_1 and I_2 are positive and flow in opposite directions through the 6-Ω resistor and 10-V source, the total current in this branch is equal to the difference of the two currents in the direction of the larger:

$$I_2 > I_1 \quad (2\,\text{A} > 1\,\text{A})$$

Therefore,

$$I_{R_2} = I_2 - I_1 = 2\,\text{A} - 1\,\text{A} = \mathbf{1\,A} \quad \text{in the direction of } I_2$$

It is sometimes impractical to draw all the branches of a circuit at right angles to one another. The next example demonstrates how a portion of a network may appear due to various constraints. The method of analysis does not change with this change in configuration.

FIG. 8.27
Example 8.12.

EXAMPLE 8.13 Find the branch currents of the network of Fig. 8.28.

Solution:

Steps 1 and 2 are as indicated in the circuit.

Step 3: Kirchhoff's voltage law is applied around each closed loop:

loop 1: $-E_1 - I_1R_1 - E_2 - V_2 = 0$ (clockwise from point a)

$-6\,\text{V} - (2\,\Omega)I_1 - 4\,\text{V} - (4\,\Omega)(I_1 - I_2) = 0$

loop 2: $-V_2 + E_2 - V_3 - E_3 = 0$ (clockwise from point b)

$-(4\,\Omega)(I_2 - I_1) + 4\,\text{V} - (6\,\Omega)(I_2) - 3\,\text{V} = 0$

which are rewritten as

$$\left. \begin{array}{l} -10 - 4I_1 - 2I_1 + 4I_2 = 0 \\ +\,1 + 4I_1 - 4I_2 - 6I_2 = 0 \end{array} \right\} \begin{array}{l} -6I_1 + 4I_2 = +10 \\ +4I_1 - 10I_2 = -1 \end{array}$$

or, by multiplying the top equation by -1, we obtain

$$6I_1 - 4I_2 = -10$$
$$4I_1 - 10I_2 = -1$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} -10 & -4 \\ -1 & -10 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ 4 & -10 \end{vmatrix}} = \frac{100 - 4}{-60 + 16} = \frac{96}{-44} = \mathbf{-2.182\ A}$$

Using the TI-86 calculator:

det[[−10,−4][−1,−10]]/det[[6,−4][4,−10]] (ENTER) −2.182

CALC. 8.2

$$I_2 = \frac{\begin{vmatrix} 6 & -10 \\ 4 & -1 \end{vmatrix}}{-44} = \frac{-6 + 40}{-44} = \frac{34}{-44} = \mathbf{-0.773\ A}$$

The current in the 4-Ω resistor and 4-V source for loop 1 is

$$\begin{aligned} I_1 - I_2 &= -2.182\,\text{A} - (-0.773\,\text{A}) \\ &= -2.182\,\text{A} + 0.773\,\text{A} \\ &= \mathbf{-1.409\ A} \end{aligned}$$

revealing that it is 1.409 A in a direction opposite (due to the minus sign) to I_1 in loop 1.
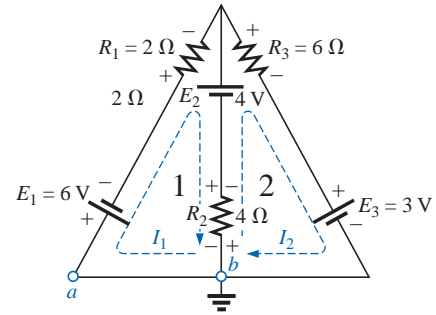
FIG. 8.28
Example 8.13.

Supermesh Currents

On occasion there will be current sources in the network to which mesh analysis is to be applied. In such cases one can convert the current source to a voltage source (if a parallel resistor is present) and proceed as before or utilize a *supermesh* current and proceed as follows.

Start as before and assign a mesh current to each independent loop, including the current sources, as if they were resistors or voltage sources. Then mentally (redraw the network if necessary) remove the current sources (replace with open-circuit equivalents), and apply

Kirchhoff's voltage law to all the remaining independent paths of the network using the mesh currents just defined. Any resulting path, including two or more mesh currents, is said to be the path of a *super-mesh* current. Then relate the chosen mesh currents of the network to the independent current sources of the network, and solve for the mesh currents. The next example will clarify the definition of a *supermesh* current and the procedure.

EXAMPLE 8.14 Using mesh analysis, determine the currents of the network of Fig. 8.29.
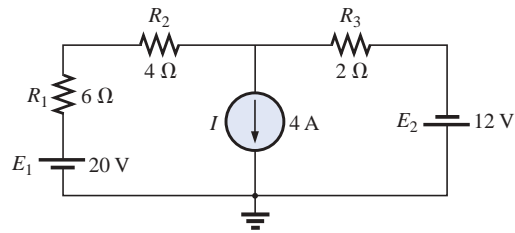
FIG. 8.29
Example 8.14.

Solution: First, the mesh currents for the network are defined, as shown in Fig. 8.30. Then the current source is mentally removed, as shown in Fig. 8.31, and Kirchhoff's voltage law is applied to the resulting network. The single path now including the effects of two mesh currents is referred to as the path of a *supermesh* current.
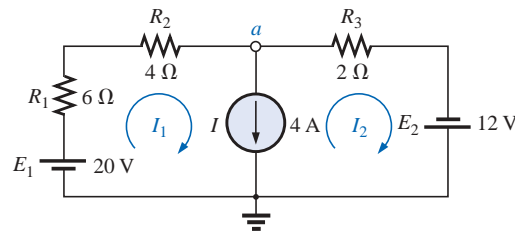
FIG. 8.30
Defining the mesh currents for the network of Fig. 8.29.
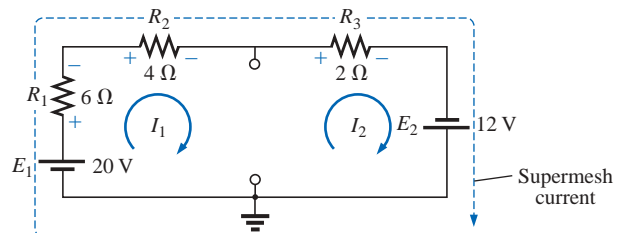
FIG. 8.31
Defining the supermesh current.

Applying Kirchhoff's law:

$$20 \text{ V} - I_1(6 \text{ } \Omega) - I_1(4 \text{ } \Omega) - I_2(2 \text{ } \Omega) + 12 \text{ V} = 0$$

or

$$10I_1 + 2I_2 = 32$$

Node a is then used to relate the mesh currents and the current source using Kirchhoff's current law:

$$I_1 = I + I_2$$

The result is two equations and two unknowns:

$$10I_1 + 2I_2 = 32$$
$$I_1 - I_2 = 4$$

Applying determinants:

$$I_1 = \frac{\begin{vmatrix} 32 & 2 \\ 4 & -1 \end{vmatrix}}{\begin{vmatrix} 10 & 2 \\ 1 & -1 \end{vmatrix}} = \frac{(32)(-1) - (2)(4)}{(10)(-1) - (2)(1)} = \frac{40}{12} = \textbf{3.33 A}$$

and
$$I_2 = I_1 - I = 3.33\text{ A} - 4\text{ A} = \textbf{-0.67 A}$$

In the above analysis, it might appear that when the current source was removed, $I_1 = I_2$. However, the supermesh approach requires that we stick with the original definition of each mesh current and not alter those definitions when current sources are removed.

EXAMPLE 8.15 Using mesh analysis, determine the currents for the network of Fig. 8.32.
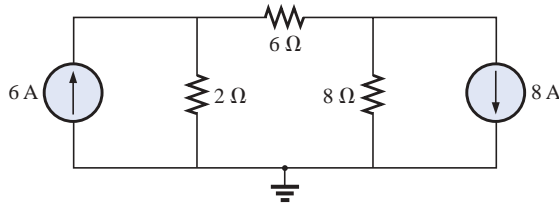
FIG. 8.32
Example 8.15.

Solution: The mesh currents are defined in Fig. 8.33. The current sources are removed, and the single supermesh path is defined in Fig. 8.34.
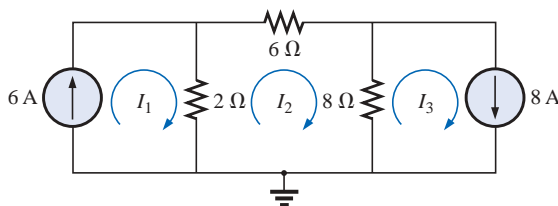
FIG. 8.33
Defining the mesh currents for the network of Fig. 8.32.
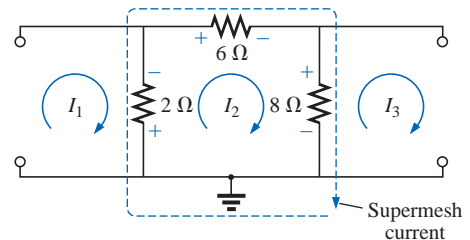
FIG. 8.34
Defining the supermesh current for the network of Fig. 8.32.

Applying Kirchhoff's voltage law around the supermesh path:

$$-V_{2\Omega} - V_{6\Omega} - V_{8\Omega} = 0$$
$$-(I_2 - I_1)2\ \Omega - I_2(6\ \Omega) - (I_2 - I_3)8\ \Omega = 0$$
$$-2I_2 + 2I_1 - 6I_2 - 8I_2 + 8I_3 = 0$$
$$2I_1 - 16I_2 + 8I_3 = 0$$

Introducing the relationship between the mesh currents and the current sources:

$$I_1 = 6\,\text{A}$$
$$I_3 = 8\,\text{A}$$

results in the following solutions:

$$2I_1 - 16I_2 + 8I_3 = 0$$
$$2(6\,\text{A}) - 16I_2 + 8(8\,\text{A}) = 0$$

and
$$I_2 = \frac{76\,\text{A}}{16} = \mathbf{4.75\,A}$$

Then
$$I_{2\Omega}\downarrow = I_1 - I_2 = 6\,\text{A} - 4.75\,\text{A} = \mathbf{1.25\,A}$$

and
$$I_{8\Omega}\uparrow = I_3 - I_2 = 8\,\text{A} - 4.75\,\text{A} = \mathbf{3.25\,A}$$

Again, note that you must stick with your original definitions of the various mesh currents when applying Kirchhoff's voltage law around the resulting supermesh paths.

8.8 MESH ANALYSIS (FORMAT APPROACH)

Now that the basis for the mesh-analysis approach has been established, we will now examine a technique for writing the mesh equations more rapidly and usually with fewer errors. As an aid in introducing the procedure, the network of Example 8.12 (Fig. 8.27) has been redrawn in Fig. 8.35 with the assigned loop currents. (Note that each loop current has a clockwise direction.)

The equations obtained are

$$-7I_1 + 6I_2 = 5$$
$$6I_1 - 8I_2 = -10$$

which can also be written as

$$7I_1 - 6I_2 = -5$$
$$8I_2 - 6I_1 = 10$$

and expanded as

$$\begin{array}{ccc} \text{Col. 1} & \text{Col. 2} & \text{Col. 3} \\ (1+6)I_1 - & 6I_2 & = (5-10) \\ (2+6)I_2 - & 6I_1 & = 10 \end{array}$$

Note in the above equations that column 1 is composed of a loop current times the sum of the resistors through which that loop current passes. Column 2 is the product of the resistors common to another loop current times that other loop current. Note that in each equation, this column is subtracted from column 1. Column 3 is the *algebraic* sum of the voltage sources through which the loop current of interest passes. A source is assigned a positive sign if the loop current passes from the negative to the positive terminal, and a negative value is assigned if the polarities are reversed. The comments above are correct only for a standard direction of loop current in each window, the one chosen being the clockwise direction.

The above statements can be extended to develop the following *format approach* to mesh analysis:

FIG. 8.35
Network of Fig. 8.27 redrawn with assigned loop currents.

1. *Assign a loop current to each independent, closed loop (as in the previous section) in a clockwise direction.*
2. *The number of required equations is equal to the number of chosen independent, closed loops. Column 1 of each equation is formed by summing the resistance values of those resistors through which the loop current of interest passes and multiplying the result by that loop current.*
3. *We must now consider the mutual terms, which, as noted in the examples above, are always subtracted from the first column. A* **mutual term** *is simply any resistive element having an additional loop current passing through it. It is possible to have more than one mutual term if the loop current of interest has an element in common with more than one other loop current. This will be demonstrated in an example to follow. Each term is the product of the mutual resistor and the other loop current passing through the same element.*
4. *The column to the right of the equality sign is the algebraic sum of the voltage sources through which the loop current of interest passes. Positive signs are assigned to those sources of voltage having a polarity such that the loop current passes from the negative to the positive terminal. A negative sign is assigned to those potentials for which the reverse is true.*
5. *Solve the resulting simultaneous equations for the desired loop currents.*

Before considering a few examples, be aware that since the column to the right of the equals sign is the algebraic sum of the voltage sources in that loop, *the format approach can be applied only to networks in which all current sources have been converted to their equivalent voltage source.*

EXAMPLE 8.16 Write the mesh equations for the network of Fig. 8.36, and find the current through the 7-Ω resistor.

Solution:

Step 1: As indicated in Fig. 8.36, each assigned loop current has a clockwise direction.

Steps 2 to 4:

$$I_1: \quad (8\ \Omega + 6\ \Omega + 2\ \Omega)I_1 - (2\ \Omega)I_2 = 4\ \text{V}$$
$$I_2: \quad (7\ \Omega + 2\ \Omega)I_2 - (2\ \Omega)I_1 = -9\ \text{V}$$

and

$$16I_1 - 2I_2 = 4$$
$$9I_2 - 2I_1 = -9$$

which, for determinants, are

$$16I_1 - 2I_2 = 4$$
$$-2I_1 + 9I_2 = -9$$

and

$$I_2 = I_{7\Omega} = \frac{\begin{vmatrix} 16 & 4 \\ -2 & -9 \end{vmatrix}}{\begin{vmatrix} 16 & -2 \\ -2 & 9 \end{vmatrix}} = \frac{-144 + 8}{144 - 4} = \frac{-136}{140}$$

$$= -0.971\ \text{A}$$
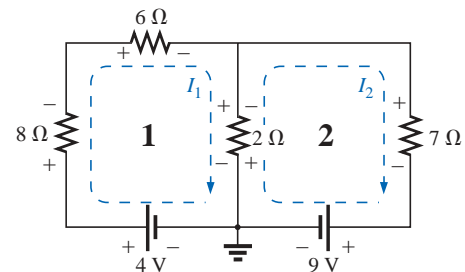
FIG. 8.36
Example 8.16.

EXAMPLE 8.17 Write the mesh equations for the network of Fig. 8.37.
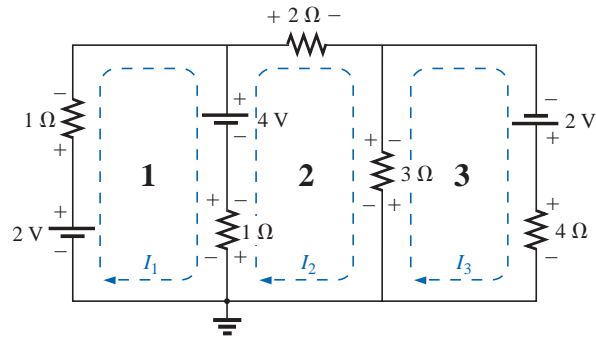
FIG. 8.37
Example 8.17.

Solution:

Step 1: Each window is assigned a loop current in the clockwise direction:

I_1 does not pass through an element mutual with I_3.

$$I_1: \qquad\qquad (1\ \Omega + 1\ \Omega)I_1 - (1\ \Omega)I_2 + 0 = 2\ V - 4\ V$$
$$I_2: \quad (1\ \Omega + 2\ \Omega + 3\ \Omega)I_2 - (1\ \Omega)I_1 - (3\ \Omega)I_3 = 4\ V$$
$$I_3: \qquad\qquad (3\ \Omega + 4\ \Omega)I_3 - (3\ \Omega)I_2 + 0 = 2\ V$$

I_3 does not pass through an element mutual with I_1.

Summing terms yields

$$2I_1 - I_2 + 0 = -2$$
$$6I_2 - I_1 - 3I_3 = 4$$
$$7I_3 - 3I_2 + 0 = 2$$

which are rewritten for determinants as

$$
\begin{array}{ccccccc}
c & & b & & a & & \\
2I_1 & - & I_2 & + & 0 & & = -2 \\
& & & & & & \\
-I_1 & + & 6I_2 & - & 3I_3 & & = 4 \\
& & & & & & \\
0 & - & 3I_2 & + & 7I_3 & & = 2 \\
\end{array}
$$

Note that the coefficients of the *a* and *b* diagonals are equal. This *symmetry* about the *c*-axis will always be true for equations written using the format approach. It is a check on whether the equations were obtained correctly.

We will now consider a network with only one source of voltage to point out that mesh analysis can be used to advantage in other than multi-source networks.

EXAMPLE 8.18 Find the current through the 10-Ω resistor of the network of Fig. 8.38.
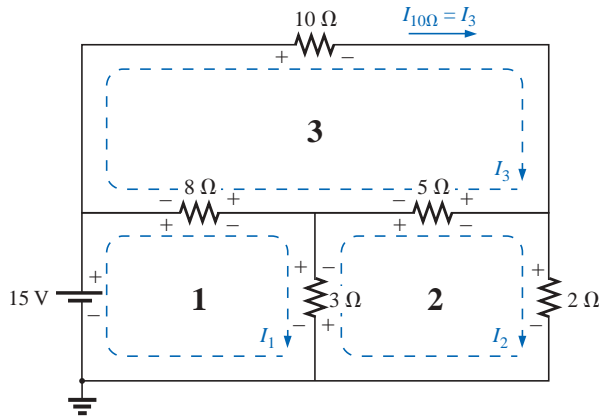
FIG. 8.38
Example 8.18.

Solution 1:

I_1: $\qquad (8\ \Omega + 3\ \Omega)I_1 - (8\ \Omega)I_3 - (3\ \Omega)I_2 = 15\ \text{V}$
I_2: $\quad (3\ \Omega + 5\ \Omega + 2\ \Omega)I_2 - (3\ \Omega)I_1 - (5\ \Omega)I_3 = 0$
I_3: $\ (8\ \Omega + 10\ \Omega + 5\ \Omega)I_3 - (8\ \Omega)I_1 - (5\ \Omega)I_2 = 0$

$$11I_1 - 8I_3 - 3I_2 = 15$$
$$10I_2 - 3I_1 - 5I_3 = 0$$
$$23I_3 - 8I_1 - 5I_2 = 0$$

or

$$11I_1 - \ 3I_2 - \ 8I_3 = 15$$
$$-3I_1 + 10I_2 - \ 5I_3 = 0$$
$$-8I_1 - \ 5I_2 + 23I_3 = 0$$

and

$$I_3 = I_{10\Omega} = \frac{\begin{vmatrix} 11 & -3 & 15 \\ -3 & 10 & 0 \\ -8 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 11 & -3 & -8 \\ -3 & 10 & -5 \\ -8 & -5 & 23 \end{vmatrix}} = \textbf{1.220 A}$$

Mathcad Solution: For this example, rather than take the time to develop the determinant form for each variable, we will apply Mathcad directly to the resulting equations. As shown in Fig. 8.39, a **Guess** value for each variable must first be defined. Such guessing helps the computer begin its iteration process as it searches for the solution. By providing a rough estimate of 1, the computer recognizes that the result will probably be a number with a magnitude less than 100 rather than have to worry about solutions that extend into the thousands or tens of thousands—the search has been narrowed considerably.

Next, as shown, the word **Given** must be entered to tell the computer that the defining equations will follow. Finally, each equation must be carefully entered and set equal to the constant on the right using the **Ctrl=** operation.

The results are then obtained with the **Find(I1,I2,I3)** expression and an equal sign. As shown, the results are available with an acceptable degree of accuracy even though entering the equations and performing the analysis took only a minute or two (with practice).
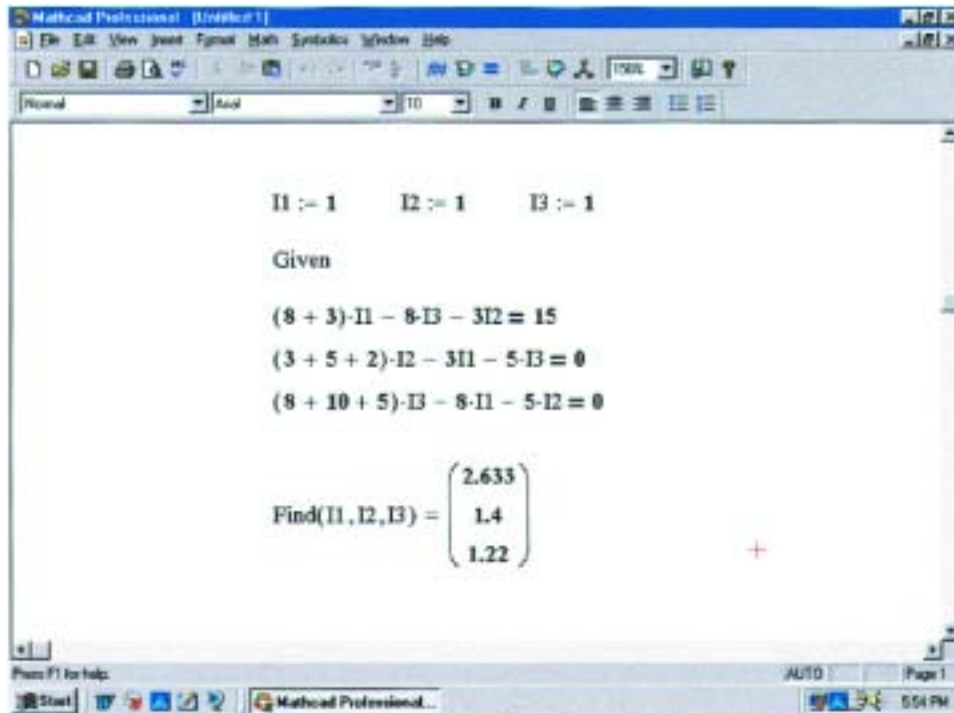
FIG. 8.39
Using Mathcad to verify the numerical calculations of Example 8.18.

Solution 2: Using the TI-86 calculator:

det[[11,−3,15][−3,10,0][−8,−5,0]]/det[[11,−3,−8][−3,10,−5][−8,−5,23]] (ENTER)   1.220

CALC. 8.3

This display certainly requires some care in entering the correct sequence of brackets in the required format, but it is still a rather neat, compact format.

8.9 NODAL ANALYSIS (GENERAL APPROACH)

Recall from the development of loop analysis that the general network equations were obtained by applying Kirchhoff's voltage law around each closed loop. We will now employ Kirchhoff's current law to develop a method referred to as **nodal analysis.**

A **node** is defined as a junction of two or more branches. If we now define one node of any network as a reference (that is, a point of zero potential or ground), the remaining nodes of the network will all have a fixed potential relative to this reference. For a network of N nodes, therefore, there will exist $(N-1)$ nodes with a fixed potential relative to the assigned reference node. Equations relating these nodal voltages can be written by applying Kirchhoff's current law at each of the $(N-1)$ nodes. To obtain the complete solution of a network, these nodal voltages are then evaluated in the same manner in which loop currents were found in loop analysis.

The nodal analysis method is applied as follows:

1. *Determine the number of nodes within the network.*
2. *Pick a reference node, and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.*
3. *Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law. In other words, for each node, don't be influenced by the direction that an unknown current for another node may have had. Each node is to be treated as a separate entity, independent of the application of Kirchhoff's current law to the other nodes.*
4. *Solve the resulting equations for the nodal voltages.*

A few examples will clarify the procedure defined by step 3. It will initially take some practice writing the equations for Kirchhoff's current law correctly, but in time the advantage of assuming that all the currents leave a node rather than identifying a specific direction for each branch will become obvious. (The same type of advantage is associated with assuming that all the mesh currents are clockwise when applying mesh analysis.)

EXAMPLE 8.19 Apply nodal analysis to the network of Fig. 8.40.

Solution:

Steps 1 and 2: The network has two nodes, as shown in Fig. 8.41. The lower node is defined as the reference node at ground potential (zero volts), and the other node as V_1, the voltage from node 1 to ground.

Step 3: I_1 and I_2 are defined as leaving the node in Fig. 8.42, and Kirchhoff's current law is applied as follows:

$$I = I_1 + I_2$$

The current I_2 is related to the nodal voltage V_1 by Ohm's law:

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1}{R_2}$$

The current I_1 is also determined by Ohm's law as follows:

$$I_1 = \frac{V_{R_1}}{R_1}$$

with

$$V_{R_1} = V_1 - E$$

Substituting into the Kirchhoff's current law equation:

$$I = \frac{V_1 - E}{R_1} + \frac{V_1}{R_2}$$

and rearranging, we have

$$I = \frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{E}{R_1}$$

or

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_1} + I$$
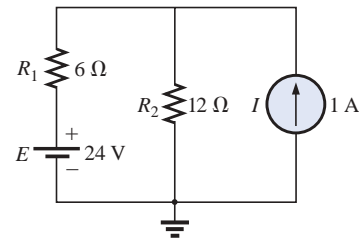
FIG. 8.40
Example 8.19.
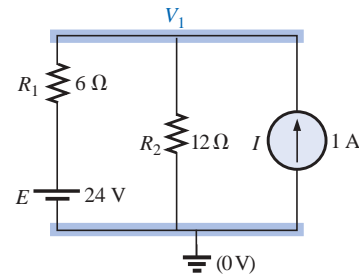
FIG. 8.41
Network of Fig. 8.40 with assigned nodes.
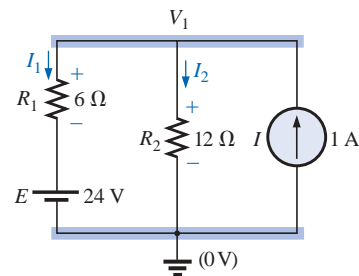
FIG. 8.42
Applying Kirchhoff's current law to the node V_1.
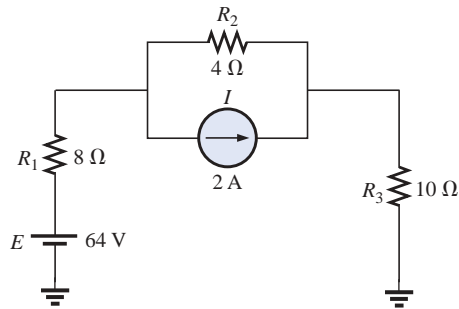
FIG. 8.43
Example 8.20.
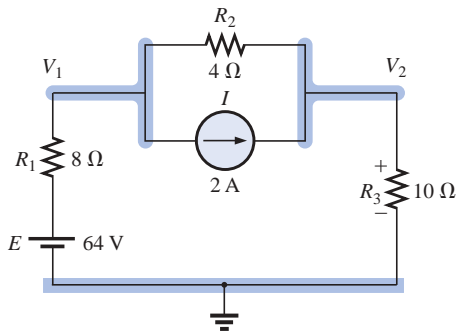
FIG. 8.44
Defining the nodes for the network of Fig. 8.43.
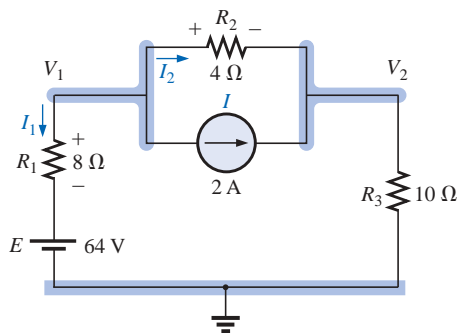
FIG. 8.45
Applying Kirchhoff's current law to node V_1.
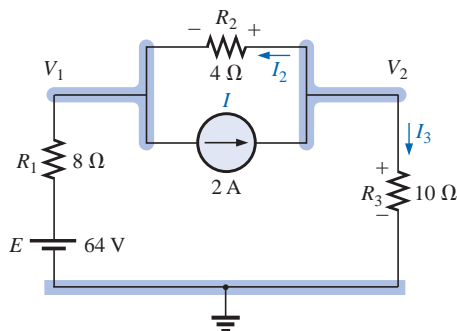
FIG. 8.46
Applying Kirchhoff's current law to node V_2.

Substituting numerical values, we obtain

$$V_1\left(\frac{1}{6\ \Omega} + \frac{1}{12\ \Omega}\right) = \frac{24\ \text{V}}{6\ \Omega} + 1\ \text{A} = 4\ \text{A} + 1\ \text{A}$$

$$V_1\left(\frac{1}{4\ \Omega}\right) = 5\ \text{A}$$

$$V_1 = \mathbf{20\ V}$$

The currents I_1 and I_2 can then be determined using the preceding equations:

$$I_1 = \frac{V_1 - E}{R_1} = \frac{20\ \text{V} - 24\ \text{V}}{6\ \Omega} = \frac{-4\ \text{V}}{6\ \Omega}$$

$$= \mathbf{-0.667\ A}$$

The minus sign indicates simply that the current I_1 has a direction opposite to that appearing in Fig. 8.42.

$$I_2 = \frac{V_1}{R_2} = \frac{20\ \text{V}}{12\ \Omega} = \mathbf{1.667\ A}$$

EXAMPLE 8.20 Apply nodal analysis to the network of Fig. 8.43.

Solution 1:

Steps 1 and 2: The network has three nodes, as defined in Fig. 8.44, with the bottom node again defined as the reference node (at ground potential, or zero volts), and the other nodes as V_1 and V_2.

Step 3: For node V_1 the currents are defined as shown in Fig. 8.45, and Kirchhoff's current law is applied:

$$0 = I_1 + I_2 + I$$

with

$$I_1 = \frac{V_1 - E}{R_1}$$

and

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1 - V_2}{R_2}$$

so that

$$\frac{V_1 - E}{R_1} + \frac{V_1 - V_2}{R_2} + I = 0$$

or

$$\frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2} + I = 0$$

and

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) = -I + \frac{E}{R_1}$$

Substituting values:

$$V_1\left(\frac{1}{8\ \Omega} + \frac{1}{4\ \Omega}\right) - V_2\left(\frac{1}{4\ \Omega}\right) = -2\ \text{A} + \frac{64\ \text{V}}{8\ \Omega} = 6\ \text{A}$$

For node V_2 the currents are defined as shown in Fig. 8.46, and Kirchhoff's current law is applied:

$$I = I_2 + I_3$$

with

$$I = \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3}$$

or

$$I = \frac{V_2}{R_2} - \frac{V_1}{R_2} + \frac{V_2}{R_3}$$

and

$$V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{R_2}\right) = I$$

Substituting values:

$$V_2\left(\frac{1}{4\ \Omega} + \frac{1}{10\ \Omega}\right) - V_1\left(\frac{1}{4\ \Omega}\right) = 2\ A$$

Step 4: The result is two equations and two unknowns:

$$V_1\left(\frac{1}{8\ \Omega} + \frac{1}{4\ \Omega}\right) - V_2\left(\frac{1}{4\ \Omega}\right) = 6\ A$$

$$-V_1\left(\frac{1}{4\ \Omega}\right) + V_2\left(\frac{1}{4\ \Omega} + \frac{1}{10\ \Omega}\right) = 2\ A$$

which become

$$0.375V_1 - 0.25V_2 = 6$$
$$-0.25V_1 + 0.35V_2 = 2$$

Using determinants,

$$V_1 = \textbf{37.818 V}$$

$$V_2 = \textbf{32.727 V}$$

Since E is greater than V_1, the current I_1 flows from ground to V_1 and is equal to

$$I_{R_1} = \frac{E - V_1}{R_1} = \frac{64\ V - 37.818\ V}{8\ \Omega} = \textbf{3.273 A}$$

The positive value for V_2 results in a current I_{R_3} from node V_2 to ground equal to

$$I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{V_2}{R_3} = \frac{32.727\ V}{10\ \Omega} = \textbf{3.273 A}$$

Since V_1 is greater than V_2, the current I_{R_2} flows from V_1 to V_2 and is equal to

$$I_{R_2} = \frac{V_1 - V_2}{R_2} = \frac{37.818\ V - 32.727\ V}{4\ \Omega} = \textbf{1.273 A}$$

Mathcad Solution: For this example, we will use Mathcad to work directly with the Kirchhoff's current law equations rather than taking the mathematical process down the line to more familiar forms. Simply define everything correctly, provide the **Guess** values, and insert **Given** where required. The process should be quite straightforward.

Note in Fig. 8.47 that the first equation comes from the fact that $I_1 + I_2 + I = 0$ while the second equation comes from $I_2 + I_3 = I$. Pay particular attention to the fact that the first equation is defined by Fig. 8.45 and the second by Fig. 8.46 because the direction of I_2 is different for each.

The results of $V_1 = 37.82$ V and $V_2 = 32.73$ V confirm the theoretical solution.
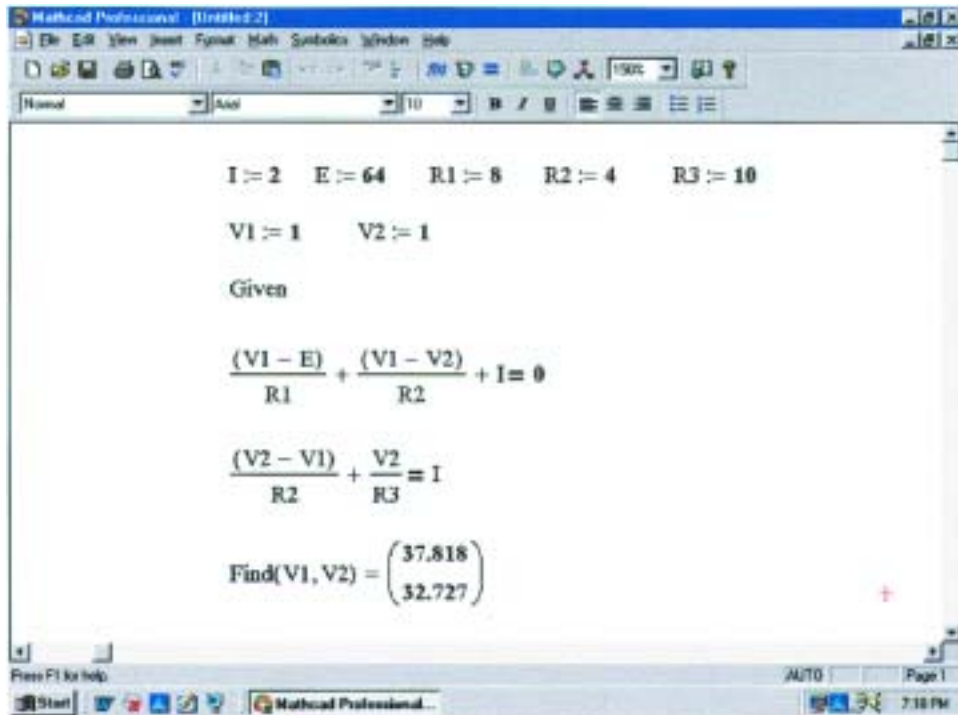
FIG. 8.47
Using Mathcad to verify the mathematical calculations of Example 8.20.

EXAMPLE 8.21 Determine the nodal voltages for the network of Fig. 8.48.
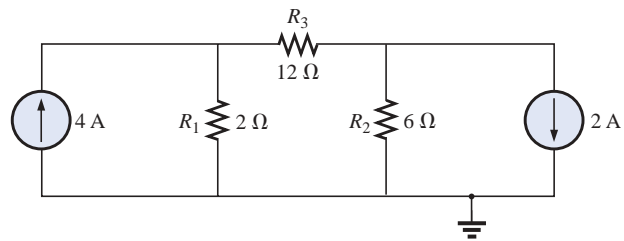
FIG. 8.48
Example 8.21.

Solution:

Steps 1 and 2: As indicated in Fig. 8.49.
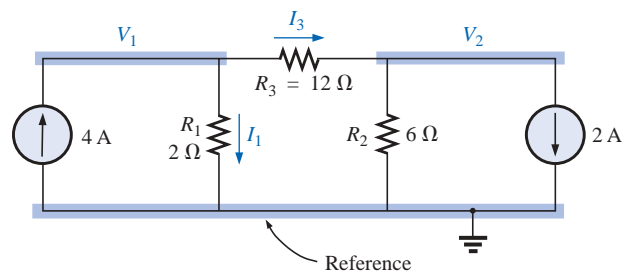
FIG. 8.49
Defining the nodes and applying Kirchhoff's current law to the node V_1.

Step 3: Included in Fig. 8.49 for the node V_1. Applying Kirchhoff's current law:

$$4 \text{ A} = I_1 + I_3$$

and

$$4 \text{ A} = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3} = \frac{V_1}{2 \text{ }\Omega} + \frac{V_1 - V_2}{12 \text{ }\Omega}$$

Expanding and rearranging:

$$V_1\left(\frac{1}{2 \text{ }\Omega} + \frac{1}{12 \text{ }\Omega}\right) - V_2\left(\frac{1}{12 \text{ }\Omega}\right) = 4 \text{ A}$$

For node V_2 the currents are defined as in Fig. 8.50.
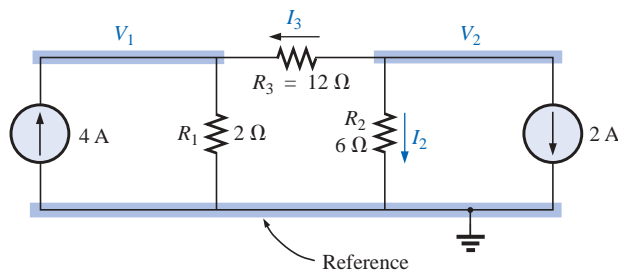
FIG. 8.50
Applying Kirchhoff's current law to the node V_2.

Applying Kirchhoff's current law:

$$0 = I_3 + I_2 + 2 \text{ A}$$

and

$$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_2} + 2 \text{ A} = 0 \longrightarrow \frac{V_2 - V_1}{12 \text{ }\Omega} + \frac{V_2}{6 \text{ }\Omega} + 2 \text{ A} = 0$$

Expanding and rearranging:

$$V_2\left(\frac{1}{12 \text{ }\Omega} + \frac{1}{6 \text{ }\Omega}\right) - V_1\left(\frac{1}{12 \text{ }\Omega}\right) = -2 \text{ A}$$

resulting in two equations and two unknowns (numbered for later reference):

$$\left.\begin{array}{c} V_1\left(\dfrac{1}{2 \text{ }\Omega} + \dfrac{1}{12 \text{ }\Omega}\right) - V_2\left(\dfrac{1}{12 \text{ }\Omega}\right) = +4 \text{ A} \\[3mm] V_2\left(\dfrac{1}{12 \text{ }\Omega} + \dfrac{1}{6 \text{ }\Omega}\right) - V_1\left(\dfrac{1}{12 \text{ }\Omega}\right) = -2 \text{ A} \end{array}\right\} \qquad \textbf{(8.3)}$$

producing

$$\left.\begin{array}{c} \dfrac{7}{12}V_1 - \dfrac{1}{12}V_2 = +4 \\[3mm] -\dfrac{1}{12}V_1 + \dfrac{3}{12}V_2 = -2 \end{array}\right| \quad \begin{array}{c} 7V_1 - V_2 = 48 \\[3mm] -1V_1 + 3V_2 = -24 \end{array}$$

and

$$V_1 = \frac{\begin{vmatrix} 48 & -1 \\ -24 & 3 \end{vmatrix}}{\begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{120}{20} = \textbf{+6 V}$$

$$V_2 = \frac{\begin{vmatrix} 7 & 48 \\ -1 & -24 \end{vmatrix}}{20} = \frac{-120}{20} = \textbf{-6 V}$$

Since V_1 is greater than V_2, the current through R_3 passes from V_1 to V_2. Its value is

$$I_{R_3} = \frac{V_1 - V_2}{R_3} = \frac{6 \text{ V} - (-6 \text{ V})}{12 \text{ }\Omega} = \frac{12 \text{ V}}{12 \text{ }\Omega} = \mathbf{1 \text{ A}}$$

The fact that V_1 is positive results in a current I_{R_1} from V_1 to ground equal to

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_1}{R_1} = \frac{6 \text{ V}}{2 \text{ }\Omega} = \mathbf{3 \text{ A}}$$

Finally, since V_2 is negative, the current I_{R_2} flows from ground to V_2 and is equal to

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_2}{R_2} = \frac{6 \text{ V}}{6 \text{ }\Omega} = \mathbf{1 \text{ A}}$$

Supernode

On occasion there will be independent voltage sources in the network to which nodal analysis is to be applied. In such cases we can convert the voltage source to a current source (if a series resistor is present) and proceed as before, or we can introduce the concept of a *supernode* and proceed as follows.

Start as before and assign a nodal voltage to each independent node of the network, including each independent voltage source as if it were a resistor or current source. Then mentally replace the independent voltage sources with short-circuit equivalents, and apply Kirchhoff's current law to the defined nodes of the network. Any node including the effect of elements tied only to *other* nodes is referred to as a *supernode* (since it has an additional number of terms). Finally, relate the defined nodes to the independent voltage sources of the network, and solve for the nodal voltages. The next example will clarify the definition of *supernode.*

EXAMPLE 8.22 Determine the nodal voltages V_1 and V_2 of Fig. 8.51 using the concept of a supernode.
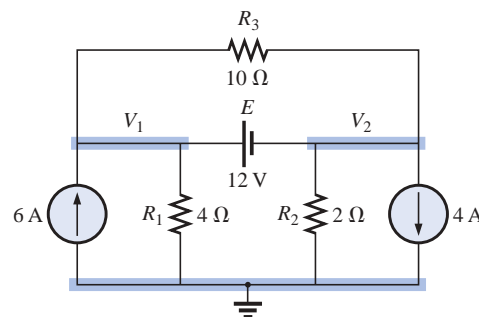
FIG. 8.51
Example 8.22.

Solution: Replacing the independent voltage source of 12 V with a short-circuit equivalent will result in the network of Fig. 8.52. Even though the mental application of a short-circuit equivalent is discussed above, it would be wise in the early stage of development to redraw the
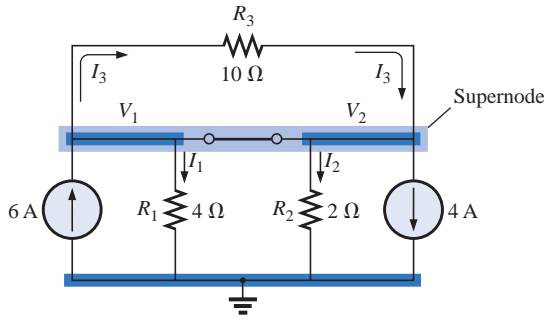
FIG. 8.52

Defining the supernode for the network of Fig. 8.51.

network as shown in Fig. 8.52. The result is a single supernode for which Kirchhoff's current law must be applied. Be sure to leave the other defined nodes in place and use them to define the currents from that region of the network. In particular, note that the current I_3 will leave the supernode at V_1 and then enter the same supernode at V_2. It must therefore appear twice when applying Kirchhoff's current law, as shown below:

$$\Sigma I_i = \Sigma I_o$$

$$6 \text{ A} + I_3 = I_1 + I_2 + 4 \text{ A} + I_3$$

or

$$I_1 + I_2 = 6 \text{ A} - 4 \text{ A} = 2 \text{ A}$$

Then

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = 2 \text{ A}$$

and

$$\frac{V_1}{4 \text{ } \Omega} + \frac{V_2}{2 \text{ } \Omega} = 2 \text{ A}$$

Relating the defined nodal voltages to the independent voltage source, we have

$$V_1 - V_2 = E = 12 \text{ V}$$

which results in two equations and two unknowns:

$$0.25V_1 + 0.5V_2 = 2$$
$$\underline{V_1 - \quad 1V_2 = 12}$$

Substituting:

$$V_1 = V_2 + 12$$

$$0.25(V_2 + 12) + 0.5V_2 = 2$$

and

$$0.75V_2 = 2 - 3 = -1$$

so that

$$V_2 = \frac{-1}{0.75} = -\textbf{1.333 V}$$

and $\quad V_1 = V_2 + 12 \text{ V} = -1.333 \text{ V} + 12 \text{ V} = \textbf{+10.667 V}$

The current of the network can then be determined as follows:

$$I_1\downarrow = \frac{V}{R_1} = \frac{10.667 \text{ V}}{4 \text{ } \Omega} = \textbf{2.667 A}$$

$$I_2\uparrow = \frac{V_2}{R_2} = \frac{1.333 \text{ V}}{2 \text{ } \Omega} = \textbf{0.667 A}$$

$$I_3 \atop \rightarrow = \frac{V_1 - V_2}{10 \text{ } \Omega} = \frac{10.667 \text{ V} - (-1.333 \text{ V})}{10 \text{ } \Omega} = \frac{12 \text{ V}}{10 \text{ } \Omega} = \textbf{1.2 A}$$

A careful examination of the network at the beginning of the analysis would have revealed that the voltage across the resistor R_3 must be 12 V and I_3 must be equal to 1.2 A.

8.10   NODAL ANALYSIS (FORMAT APPROACH)

A close examination of Eq. (8.3) appearing in Example 8.21 reveals that the subscripted voltage at the node in which Kirchhoff's current law is applied is multiplied by the sum of the conductances attached to that node. Note also that the other nodal voltages within the same equation are multiplied by the negative of the conductance between the two nodes. The current sources are represented to the right of the equals sign with a positive sign if they supply current to the node and with a negative sign if they draw current from the node.

These conclusions can be expanded to include networks with any number of nodes. This will allow us to write nodal equations rapidly and in a form that is convenient for the use of determinants. A major requirement, however, is that *all voltage sources must first be converted to current sources before the procedure is applied.* Note the parallelism between the following four steps of application and those required for mesh analysis in Section 8.8:

1. *Choose a reference node and assign a subscripted voltage label to the (N − 1) remaining nodes of the network.*
2. *The number of equations required for a complete solution is equal to the number of subscripted voltages (N − 1). Column 1 of each equation is formed by summing the conductances tied to the node of interest and multiplying the result by that subscripted nodal voltage.*
3. *We must now consider the mutual terms that, as noted in the preceding example, are always subtracted from the first column. It is possible to have more than one mutual term if the nodal voltage of current interest has an element in common with more than one other nodal voltage. This will be demonstrated in an example to follow. Each mutual term is the product of the mutual conductance and the other nodal voltage tied to that conductance.*
4. *The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is assigned a positive sign if it supplies current to a node and a negative sign if it draws current from the node.*
5. *Solve the resulting simultaneous equations for the desired voltages.*

Let us now consider a few examples.

EXAMPLE 8.23 Write the nodal equations for the network of Fig. 8.53.
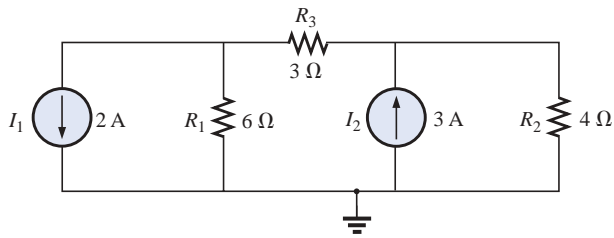
FIG. 8.53
Example 8.23.

Solution:

Step 1: The figure is redrawn with assigned subscripted voltages in Fig. 8.54.
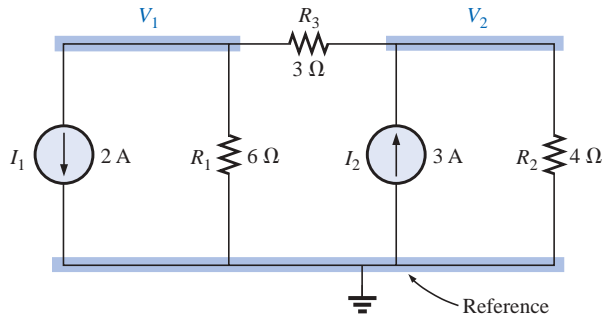
FIG. 8.54
Defining the nodes for the network of Fig. 8.53.

Steps 2 to 4:

Drawing current from node 1

$$V_1: \quad \underbrace{\left(\frac{1}{6\,\Omega} + \frac{1}{3\,\Omega} \right)}_{\substack{\text{Sum of} \\ \text{conductances} \\ \text{connected} \\ \text{to node 1}}} V_1 - \underbrace{\left(\frac{1}{3\,\Omega} \right)}_{\substack{\text{Mutual} \\ \text{conductance}}} V_2 = \overset{\downarrow}{-2}\,\text{A}$$

Supplying current to node 2

$$V_2: \quad \underbrace{\left(\frac{1}{4\,\Omega} + \frac{1}{3\,\Omega} \right)}_{\substack{\text{Sum of} \\ \text{conductances} \\ \text{connected} \\ \text{to node 2}}} V_2 - \underbrace{\left(\frac{1}{3\,\Omega} \right)}_{\substack{\text{Mutual} \\ \text{conductance}}} V_1 = \overset{\downarrow}{+3}\,\text{A}$$

and

$$\frac{1}{2}V_1 - \frac{1}{3}V_2 = -2$$

$$-\frac{1}{3}V_1 + \frac{7}{12}V_2 = 3$$

EXAMPLE 8.24 Find the voltage across the 3-Ω resistor of Fig. 8.55 by nodal analysis.
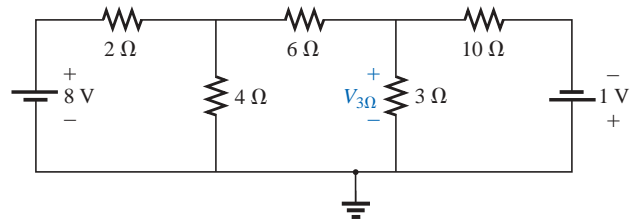
FIG. 8.55
Example 8.24.

Solution: Converting sources and choosing nodes (Fig. 8.56), we have
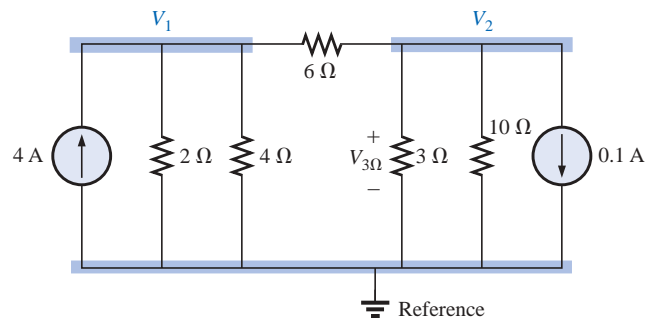
FIG. 8.56
Defining the nodes for the network of Fig. 8.55.

$$\left(\frac{1}{2\,\Omega} + \frac{1}{4\,\Omega} + \frac{1}{6\,\Omega}\right)V_1 - \left(\frac{1}{6\,\Omega}\right)V_2 = +4 \text{ A} \Big\}$$

$$\left(\frac{1}{10\,\Omega} + \frac{1}{3\,\Omega} + \frac{1}{6\,\Omega}\right)V_2 - \left(\frac{1}{6\,\Omega}\right)V_1 = -0.1 \text{ A}$$

$$\frac{11}{12}V_1 - \frac{1}{6}V_2 = 4$$

$$-\frac{1}{6}V_1 + \frac{3}{5}V_2 = -0.1$$

resulting in

$$11V_1 - 2V_2 = +48$$
$$-5V_1 + 18V_2 = -3$$

and

$$V_2 = V_{3\Omega} = \frac{\begin{vmatrix} 11 & 48 \\ -5 & -3 \end{vmatrix}}{\begin{vmatrix} 11 & -2 \\ -5 & 18 \end{vmatrix}} = \frac{-33 + 240}{198 - 10} = \frac{207}{188} = \textbf{1.101 V}$$

As demonstrated for mesh analysis, nodal analysis can also be a very useful technique for solving networks with only one source.

EXAMPLE 8.25 Using nodal analysis, determine the potential across the 4-Ω resistor in Fig. 8.57.

Solution 1: The reference and four subscripted voltage levels were chosen as shown in Fig. 8.58. A moment of reflection should reveal that for any difference in potential between V_1 and V_3, the current through and the potential drop across each 5-Ω resistor will be the same. Therefore, V_4 is simply a midvoltage level between V_1 and V_3 and is known if V_1 and V_3 are available. We will therefore not include it in a nodal voltage and will redraw the network as shown in Fig. 8.59. Understand, however, that V_4 can be included if desired, although four nodal voltages will result rather than the three to be obtained in the solution of this problem.

FIG. 8.57
Example 8.25.

$$V_1: \quad \left(\frac{1}{2\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{10\ \Omega}\right)V_1 - \left(\frac{1}{2\ \Omega}\right)V_2 - \left(\frac{1}{10\ \Omega}\right)V_3 = 0$$

$$V_2: \quad \left(\frac{1}{2\ \Omega} + \frac{1}{2\ \Omega}\right)V_2 - \left(\frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{2\ \Omega}\right)V_3 = 3\ \text{A}$$

$$V_3: \quad \left(\frac{1}{10\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{4\ \Omega}\right)V_3 - \left(\frac{1}{2\ \Omega}\right)V_2 - \left(\frac{1}{10\ \Omega}\right)V_1 = 0$$

which are rewritten as

$$1.1V_1 - 0.5V_2 - 0.1V_3 = 0$$
$$V_2 - 0.5V_1 - 0.5V_3 = 3$$
$$0.85V_3 - 0.5V_2 - 0.1V_1 = 0$$

For determinants,

$$\overset{c}{1.1V_1} - \overset{b}{0.5V_2} - \overset{a}{0.1V_3} = 0$$

$$\overset{b}{-0.5V_1} + 1V_2 - 0.5V_3 = 3$$

$$\overset{a}{-0.1V_1} - 0.5V_2 + 0.85V_3 = 0$$

Before continuing, note the symmetry about the major diagonal in the equation above. Recall a similar result for mesh analysis. Examples 8.23 and 8.24 also exhibit this property in the resulting equations. Keep this thought in mind as a check on future applications of nodal analysis.

$$V_3 = V_{4\Omega} = \frac{\begin{vmatrix} 1.1 & -0.5 & 0 \\ -0.5 & +1 & 3 \\ -0.1 & -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 1.1 & -0.5 & -0.1 \\ -0.5 & +1 & -0.5 \\ -0.1 & -0.5 & +0.85 \end{vmatrix}} = \textbf{4.645 V}$$

FIG. 8.58
Defining the nodes for the network of Fig. 8.57.
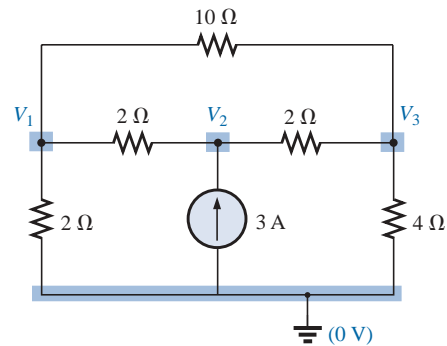
FIG. 8.59
Reducing the number of nodes for the network of Fig. 8.57 by combining the two 5-Ω resistors.

Mathcad Solution: By now the sequence of steps necessary to solve a series of equations using Mathcad should be quite familiar and less threatening than the first encounter. For this example, all the parameters were entered in the three simultaneous equations, avoiding the

need to define each parameter of the network. Simply provide a **Guess** at the three nodal voltages, apply the word **Given,** and enter the three equations properly as shown in Fig. 8.60. It does take some practice to ensure that the bracket is moved to the proper location before making an entry, but this is simply part of the rules set up to maintain control of the operations to be performed. Finally, request the desired nodal voltages using the correct format. The numerical results will appear, again confirming our theoretical solutions.
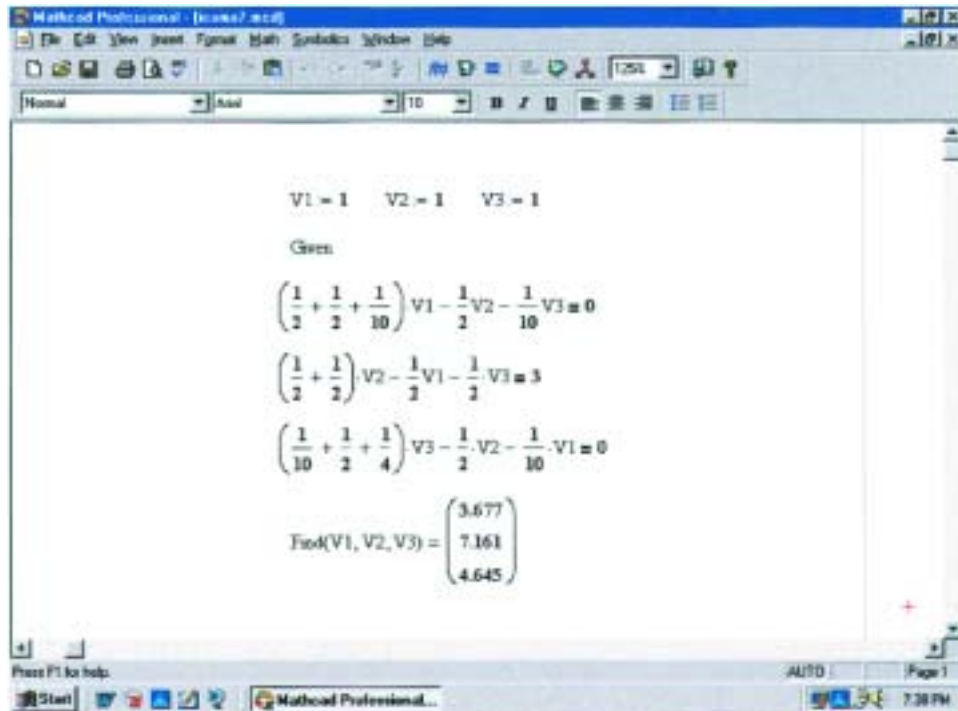
FIG. 8.60
Using Mathcad to verify the mathematical calculations of Example 8.25.

The next example has only one source applied to a ladder network.

EXAMPLE 8.26 Write the nodal equations and find the voltage across the 2-Ω resistor for the network of Fig. 8.61.
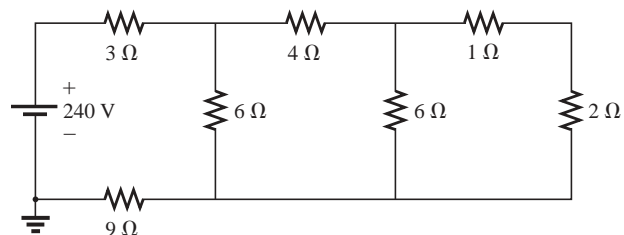
FIG. 8.61
Example 8.26.

Solution: The nodal voltages are chosen as shown in Fig. 8.62.
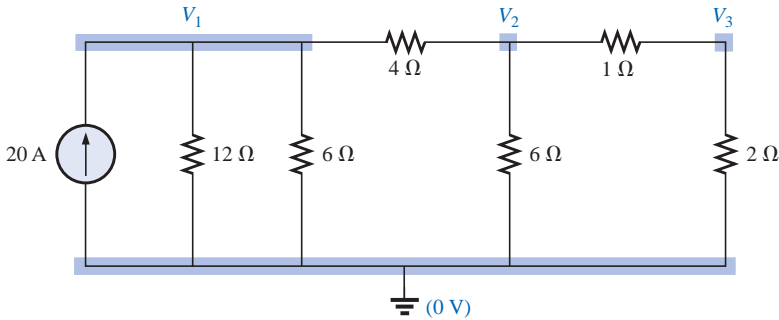
FIG. 8.62
*Converting the voltage source to a current source and defining the nodes for the
network of Fig. 8.61.*

$$V_1: \quad \left(\frac{1}{12\,\Omega} + \frac{1}{6\,\Omega} + \frac{1}{4\,\Omega}\right)V_1 - \left(\frac{1}{4\,\Omega}\right)V_2 + \quad 0 \quad = 20\text{ V}$$

$$V_2: \quad \left(\frac{1}{4\,\Omega} + \frac{1}{6\,\Omega} + \frac{1}{1\,\Omega}\right)V_2 - \left(\frac{1}{4\,\Omega}\right)V_1 - \left(\frac{1}{1\,\Omega}\right)V_3 = 0$$

$$V_3: \quad \left(\frac{1}{1\,\Omega} + \frac{1}{2\,\Omega}\right)V_3 - \left(\frac{1}{1\,\Omega}\right)V_2 + \quad 0 \quad = 0$$

and

$$0.5V_1 - 0.25V_2 + 0 \quad = 20$$

$$-0.25V_1 + \frac{17}{12}V_2 - 1V_3 = 0$$

$$0 - 1V_2 + 1.5V_3 = 0$$

Note the symmetry present about the major axis. Application of determinants reveals that

$$V_3 = V_{2\Omega} = \mathbf{10.667\ V}$$

8.11 BRIDGE NETWORKS

This section introduces the **bridge network,** a configuration that has a multitude of applications. In the chapters to follow, it will be employed in both dc and ac meters. In the electronics courses it will be encountered early in the discussion of rectifying circuits employed in converting a varying signal to one of a steady nature (such as dc). A number of other areas of application also require some knowledge of ac networks; these areas will be discussed later.

The bridge network may appear in one of the three forms as indicated in Fig. 8.63. The network of Fig. 8.63(c) is also called a *symmetrical lattice network* if $R_2 = R_3$ and $R_1 = R_4$. Figure 8.63(c) is an excellent example of how a planar network can be made to appear nonplanar. For the purposes of investigation, let us examine the network of Fig. 8.64 using mesh and nodal analysis.
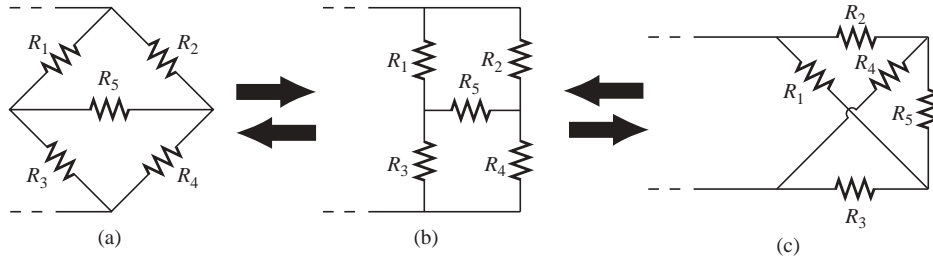
FIG. 8.63

Various formats for a bridge network.
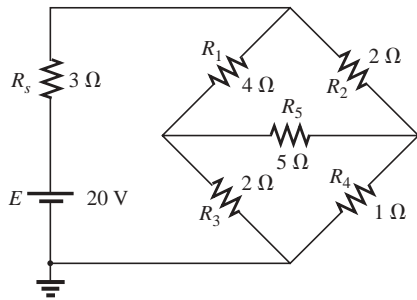
FIG. 8.64

Standard bridge configuration.
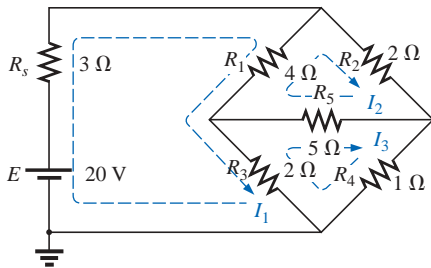
FIG. 8.65

Assigning the mesh currents to the network of Fig. 8.64.
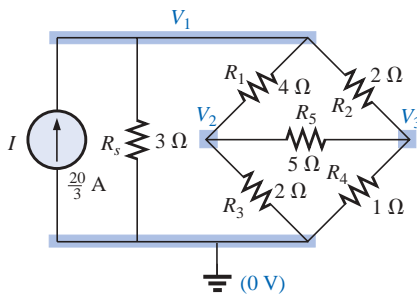
FIG. 8.66

Defining the nodal voltages for the network of Fig. 8.64.

Mesh analysis (Fig. 8.65) yields

$$(3\ \Omega + 4\ \Omega + 2\ \Omega)I_1 - (4\ \Omega)I_2 - (2\ \Omega)I_3 = 20\ \text{V}$$
$$(4\ \Omega + 5\ \Omega + 2\ \Omega)I_2 - (4\ \Omega)I_1 - (5\ \Omega)I_3 = 0$$
$$(2\ \Omega + 5\ \Omega + 1\ \Omega)I_3 - (2\ \Omega)I_1 - (5\ \Omega)I_2 = 0$$

and

$$9I_1 - 4I_2 - 2I_3 = 20$$
$$-4I_1 + 11I_2 - 5I_3 = 0$$
$$-2I_1 - 5I_2 + 8I_3 = 0$$

with the result that

$$I_1 = \mathbf{4\ A}$$
$$I_2 = \mathbf{2.667\ A}$$
$$I_3 = \mathbf{2.667\ A}$$

The net current through the 5-Ω resistor is

$$I_{5\Omega} = I_2 - I_3 = \mathbf{2.667\ A} - \mathbf{2.667\ A} = \mathbf{0\ A}$$

Nodal analysis (Fig. 8.66) yields

$$\left(\frac{1}{3\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{4\ \Omega}\right)V_2 - \left(\frac{1}{2\ \Omega}\right)V_3 = \frac{20}{3}\ \text{A}$$

$$\left(\frac{1}{4\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{5\ \Omega}\right)V_2 - \left(\frac{1}{4\ \Omega}\right)V_1 - \left(\frac{1}{5\ \Omega}\right)V_3 = 0$$

$$\left(\frac{1}{5\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{1\ \Omega}\right)V_3 - \left(\frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{5\ \Omega}\right)V_2 = 0$$

and

$$\left(\frac{1}{3\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{4\ \Omega}\right)V_2 - \left(\frac{1}{2\ \Omega}\right)V_3 = \frac{20}{3}\ \text{A}$$

$$-\left(\frac{1}{4\ \Omega}\right)V_1 + \left(\frac{1}{4\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{5\ \Omega}\right)V_2 - \left(\frac{1}{5\ \Omega}\right)V_3 = 0$$

$$-\left(\frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{5\ \Omega}\right)V_2 + \left(\frac{1}{5\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{1\ \Omega}\right)V_3 = 0$$

Note the symmetry of the solution.

With the TI-86 calculator, the top part of the determinant is determined by the following (take note of the calculations within parentheses):

det[[20/3,−1/4,−1/2][0,(1/4+1/2+1/5),−1/5][0,−1/5,(1/5+1/2+1/1)]] (ENTER)	10.5

CALC. 8.4

with the bottom of the determinant determined by:

det[[(1/3+1/4+1/2),−1/4,−1/2][−1/4,(1/4+1/2+1/5),−1/5][−1/2,−1/5,(1/5+1/2+1/1)]] (ENTER)　　1.312

CALC. 8.5

Finally,　　　　　　　　10.5/1.312 (ENTER)　8

CALC. 8.6

and　　　　　　$V_1 = \mathbf{8\ V}$

Similarly,　　$V_2 = \mathbf{2.667\ V}$　and　$V_3 = \mathbf{2.667\ V}$

and the voltage across the 5-Ω resistor is

$$V_{5\Omega} = V_2 - V_3 = \mathbf{2.667\ V} - \mathbf{2.667\ V} = \mathbf{0\ V}$$

Since $V_{5\Omega} = 0$ V, we can insert a short in place of the bridge arm without affecting the network behavior. (Certainly $V = IR = I \cdot (0) = 0$ V.) In Fig. 8.67, a short circuit has replaced the resistor R_5, and the voltage across R_4 is to be determined. The network is redrawn in Fig. 8.68, and

$$V_{1\Omega} = \frac{(2\ \Omega \parallel 1\ \Omega)20\ V}{(2\ \Omega \parallel 1\ \Omega) + (4\ \Omega \parallel 2\ \Omega) + 3\ \Omega} \quad \text{(voltage divider rule)}$$

$$= \frac{\frac{2}{3}(20\ V)}{\frac{2}{3} + \frac{8}{6} + 3} = \frac{\frac{2}{3}(20\ V)}{\frac{2}{3} + \frac{4}{3} + \frac{9}{3}}$$

$$= \frac{2(20\ V)}{2 + 4 + 9} = \frac{40\ V}{15} = \mathbf{2.667\ V}$$

FIG. 8.67

Substituting the short-circuit equivalent for the balance arm of a balanced bridge.

as obtained earlier.

We found through mesh analysis that $I_{5\Omega} = 0$ A, which has as its equivalent an open circuit as shown in Fig. 8.69(a). (Certainly $I = V/R = 0/(\infty\ \Omega) = 0$ A.) The voltage across the resistor R_4 will again be determined and compared with the result above.

The network is redrawn after combining series elements, as shown in Fig. 8.69(b), and

$$V_{3\Omega} = \frac{(6\ \Omega \parallel 3\ \Omega)(20\ V)}{6\ \Omega \parallel 3\ \Omega + 3\ \Omega} = \frac{2\ \Omega(20\ V)}{2\ \Omega + 3\ \Omega} = 8\ V$$

and　　　　$V_{1\Omega} = \frac{1\ \Omega(8\ V)}{1\ \Omega + 2\ \Omega} = \frac{8\ V}{3} = \mathbf{2.667\ V}$
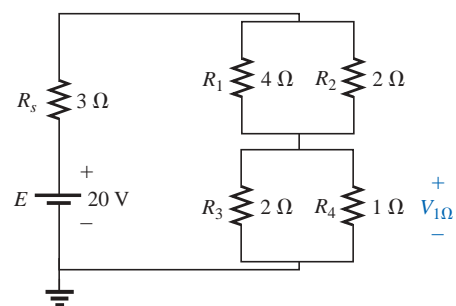
FIG. 8.68

Redrawing the network of Fig. 8.67.

as above.

The condition $V_{5\Omega} = 0$ V or $I_{5\Omega} = 0$ A exists only for a particular relationship between the resistors of the network. Let us now derive this relationship using the network of Fig. 8.70, in which it is indicated that $I = 0$ A and $V = 0$ V. Note that resistor R_s of the network of Fig. 8.69 will not appear in the following analysis.

The bridge network is said to be *balanced* when the condition of $I = 0$ A or $V = 0$ V exists.

If $V = 0$ V (short circuit between a and b), then

$$V_1 = V_2$$

FIG. 8.69

Substituting the open-circuit equivalent for the balance arm of a balanced bridge.
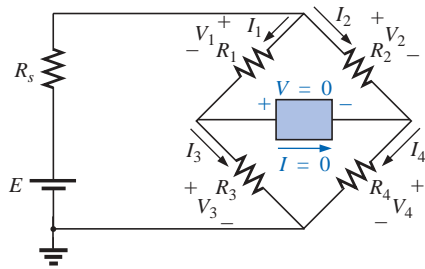
FIG. 8.70

Establishing the balance criteria for a bridge network.
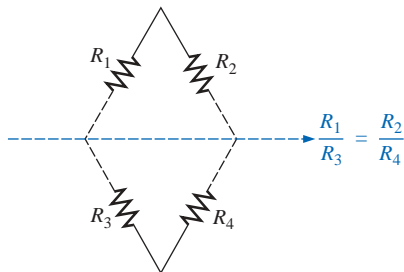
FIG. 8.71

A visual approach to remembering the balance condition.

and

$$I_1 R_1 = I_2 R_2$$

or

$$I_1 = \frac{I_2 R_2}{R_1}$$

In addition, when $V = 0$ V,

$$V_3 = V_4$$

and

$$I_3 R_3 = I_4 R_4$$

If we set $I = 0$ A, then $I_3 = I_1$ and $I_4 = I_2$, with the result that the above equation becomes

$$I_1 R_3 = I_2 R_4$$

Substituting for I_1 from above yields

$$\left(\frac{I_2 R_2}{R_1}\right) R_3 = I_2 R_4$$

or, rearranging, we have

$$\boxed{\frac{R_1}{R_3} = \frac{R_2}{R_4}} \tag{8.4}$$

This conclusion states that if the ratio of R_1 to R_3 is equal to that of R_2 to R_4, the bridge will be balanced, and $I = 0$ A or $V = 0$ V. A method of memorizing this form is indicated in Fig. 8.71.

For the example above, $R_1 = 4\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 2\ \Omega$, $R_4 = 1\ \Omega$, and

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \rightarrow \frac{4\ \Omega}{2\ \Omega} = \frac{2\ \Omega}{1\ \Omega} = 2$$

The emphasis in this section has been on the balanced situation. Understand that if the ratio is not satisfied, there will be a potential drop across the balance arm and a current through it. The methods just described (mesh and nodal analysis) will yield any and all potentials or currents desired, just as they did for the balanced situation.

8.12 Y-Δ (T-π) AND Δ-Y (π-T) CONVERSIONS

Circuit configurations are often encountered in which the resistors do not appear to be in series or parallel. Under these conditions, it may be necessary to convert the circuit from one form to another to solve for

any unknown quantities if mesh or nodal analysis is not applied. Two circuit configurations that often account for these difficulties are the **wye (Y)** and **delta (Δ) configurations,** depicted in Fig. 8.72(a). They are also referred to as the **tee (T)** and **pi (π),** respectively, as indicated in Fig. 8.72(b). Note that the pi is actually an inverted delta.

(a)   (b)
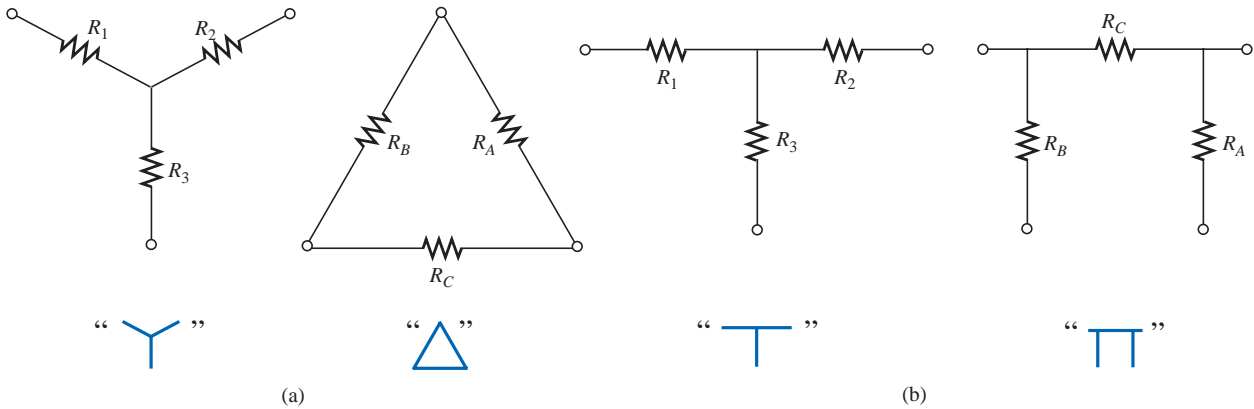
FIG. 8.72

The Y (T) and Δ (π) configurations.

The purpose of this section is to develop the equations for converting from Δ to Y, or vice versa. This type of conversion will normally lead to a network that can be solved using techniques such as those described in Chapter 7. In other words, in Fig. 8.73, with terminals *a, b,* and *c* held fast, if the wye (Y) configuration were desired *instead of* the inverted delta (Δ) configuration, all that would be necessary is a direct application of the equations to be derived. The phrase *instead of* is emphasized to ensure that it is understood that only one of these configurations is to appear at one time between the indicated terminals.

It is our purpose (referring to Fig. 8.73) to find some expression for R_1, R_2, and R_3 in terms of R_A, R_B, and R_C, and vice versa, that will ensure that the resistance between any two terminals of the Y configuration will be the same with the Δ configuration inserted in place of the Y configuration (and vice versa). If the two circuits are to be equivalent, the total resistance between any two terminals must be the same. Consider terminals *a-c* in the Δ-Y configurations of Fig. 8.74.
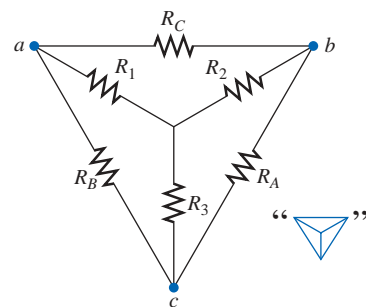
FIG. 8.73

Introducing the concept of Δ-Y or Y-Δ conversions.
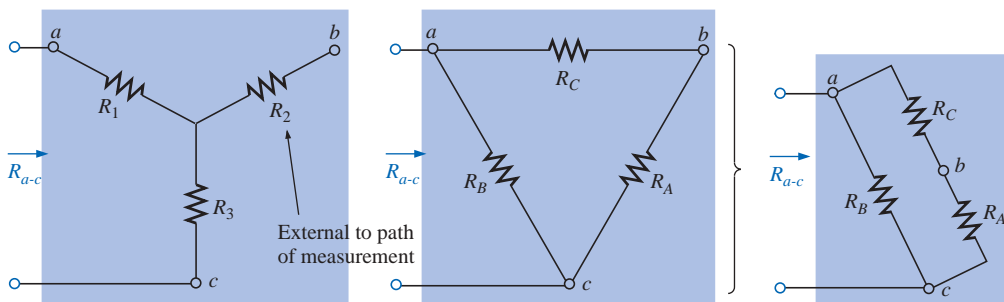
FIG. 8.74

Finding the resistance R_{a-c} for the Y and Δ configurations.

Let us first assume that we want to convert the Δ (R_A, R_B, R_C) to the Y (R_1, R_2, R_3). This requires that we have a relationship for R_1, R_2, and R_3 in terms of R_A, R_B, and R_C. If the resistance is to be the same between terminals a-c for both the Δ and the Y, the following must be true:

$$R_{a\text{-}c}\,(\text{Y}) = R_{a\text{-}c}\,(\Delta)$$

so that
$$R_{a\text{-}c} = R_1 + R_3 = \frac{R_B(R_A + R_C)}{R_B + (R_A + R_C)} \tag{8.5a}$$

Using the same approach for a-b and b-c, we obtain the following relationships:

$$R_{a\text{-}b} = R_1 + R_2 = \frac{R_C(R_A + R_B)}{R_C + (R_A + R_B)} \tag{8.5b}$$

and
$$R_{b\text{-}c} = R_2 + R_3 = \frac{R_A(R_B + R_C)}{R_A + (R_B + R_C)} \tag{8.5c}$$

Subtracting Eq. (8.5a) from Eq. (8.5b), we have

$$(R_1 + R_2) - (R_1 + R_3) = \left(\frac{R_C R_B + R_C R_A}{R_A + R_B + R_C}\right) - \left(\frac{R_B R_A + R_B R_C}{R_A + R_B + R_C}\right)$$

so that
$$R_2 - R_3 = \frac{R_A R_C - R_B R_A}{R_A + R_B + R_C} \tag{8.5d}$$

Subtracting Eq. (8.5d) from Eq. (8.5c) yields

$$(R_2 + R_3) - (R_2 - R_3) = \left(\frac{R_A R_B + R_A R_C}{R_A + R_B + R_C}\right) - \left(\frac{R_A R_C - R_B R_A}{R_A + R_B + R_C}\right)$$

so that
$$2R_3 = \frac{2R_B R_A}{R_A + R_B + R_C}$$

resulting in the following expression for R_3 in terms of R_A, R_B, and R_C:

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} \tag{8.6a}$$

Following the same procedure for R_1 and R_2, we have

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} \tag{8.6b}$$

and
$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} \tag{8.6c}$$

Note that each resistor of the Y is equal to the product of the resistors in the two closest branches of the Δ divided by the sum of the resistors in the Δ.

To obtain the relationships necessary to convert from a Y to a Δ, first divide Eq. (8.6a) by Eq. (8.6b):

$$\frac{R_3}{R_1} = \frac{(R_A R_B)/(R_A + R_B + R_C)}{(R_B R_C)/(R_A + R_B + R_C)} = \frac{R_A}{R_C}$$

or

$$R_A = \frac{R_C R_3}{R_1}$$

Then divide Eq. (8.6a) by Eq. (8.6c):

$$\frac{R_3}{R_2} = \frac{(R_A R_B)/(R_A + R_B + R_C)}{(R_A R_C)/(R_A + R_B + R_C)} = \frac{R_B}{R_C}$$

or

$$R_B = \frac{R_3 R_C}{R_2}$$

Substituting for R_A and R_B in Eq. (8.6c) yields

$$R_2 = \frac{(R_C R_3/R_1)R_C}{(R_3 R_C/R_2) + (R_C R_3/R_1) + R_C}$$

$$= \frac{(R_3/R_1)R_C}{(R_3/R_2) + (R_3/R_1) + 1}$$

Placing these over a common denominator, we obtain

$$R_2 = \frac{(R_3 R_C/R_1)}{(R_1 R_2 + R_1 R_3 + R_2 R_3)/(R_1 R_2)}$$

$$= \frac{R_2 R_3 R_C}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

and

$$\boxed{R_C = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}} \qquad \text{(8.7a)}$$

We follow the same procedure for R_B and R_A:

$$\boxed{R_A = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}} \qquad \text{(8.7b)}$$

and

$$\boxed{R_B = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}} \qquad \text{(8.7c)}$$

Note that the value of each resistor of the Δ is equal to the sum of the possible product combinations of the resistances of the Y divided by the resistance of the Y farthest from the resistor to be determined.

Let us consider what would occur if all the values of a Δ or Y were the same. If $R_A = R_B = R_C$, Equation (8.6a) would become (using R_A only) the following:

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{R_A R_A}{R_A + R_A + R_A} = \frac{R_A^2}{3R_A} = \frac{R_A}{3}$$

and, following the same procedure,

$$R_1 = \frac{R_A}{3} \qquad R_2 = \frac{R_A}{3}$$

In general, therefore,

$$R_Y = \frac{R_\Delta}{3}$$ (8.8a)

or $$R_\Delta = 3R_Y$$ (8.8b)

which indicates that *for a* Y *of three equal resistors, the value of each resistor of the* Δ *is equal to three times the value of any resistor of the* Y. If only two elements of a Y or a Δ are the same, the corresponding Δ or Y of each will also have two equal elements. The converting of equations will be left as an exercise for the reader.

The Y and the Δ will often appear as shown in Fig. 8.75. They are then referred to as a **tee (T)** and a **pi (π)** network, respectively. The equations used to convert from one form to the other are exactly the same as those developed for the Y and Δ transformation.

(a)   (b)
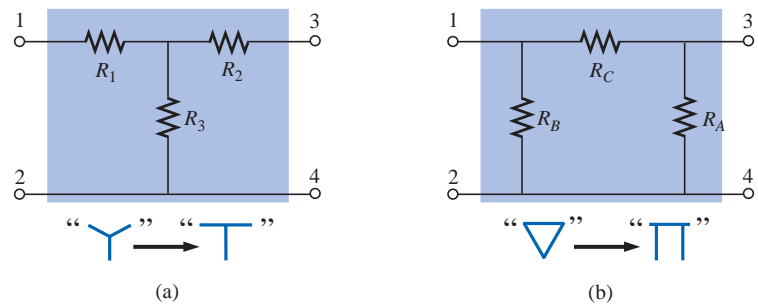
FIG. 8.75
The relationship between the Y and T configurations and the Δ and π configurations.

EXAMPLE 8.27 Convert the Δ of Fig. 8.76 to a Y.
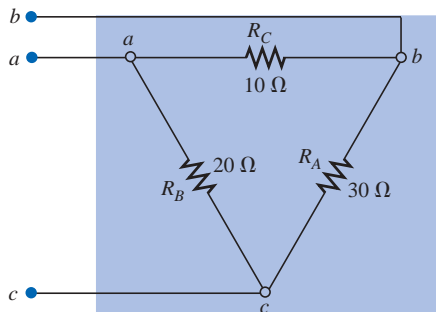
FIG. 8.76
Example 8.27.
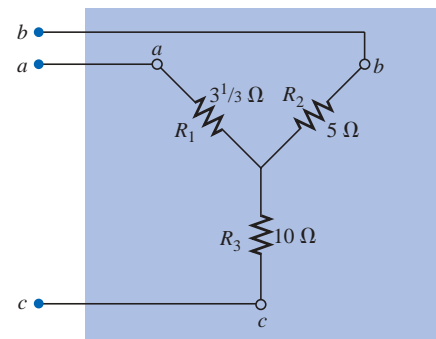
FIG. 8.77
The Y equivalent for the Δ of Fig. 8.76.

Solution:

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(20\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = \frac{200\ \Omega}{60} = 3\frac{1}{3}\ \Omega$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(30\ \Omega)(10\ \Omega)}{60\ \Omega} = \frac{300\ \Omega}{60} = \mathbf{5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(20\ \Omega)(30\ \Omega)}{60\ \Omega} = \frac{600\ \Omega}{60} = \mathbf{10\ \Omega}$$

The equivalent network is shown in Fig. 8.77 (page 298).
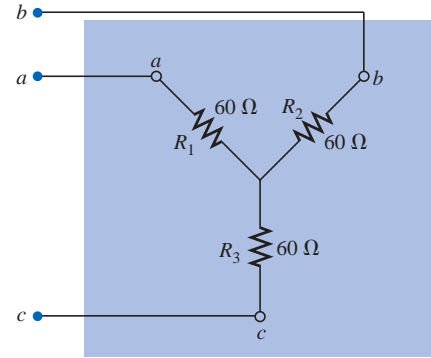
FIG. 8.78
Example 8.28.

EXAMPLE 8.28 Convert the Y of Fig. 8.78 to a Δ.

Solution:

$$R_A = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

$$= \frac{(60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega)}{60\ \Omega}$$

$$= \frac{3600\ \Omega + 3600\ \Omega + 3600\ \Omega}{60} = \frac{10,800\ \Omega}{60}$$

$$R_A = \mathbf{180\ \Omega}$$

However, the three resistors for the Y are equal, permitting the use of Eq. (8.8) and yielding

$$R_\Delta = 3R_Y = 3(60\ \Omega) = 180\ \Omega$$

and
$$R_B = R_C = \mathbf{180\ \Omega}$$

The equivalent network is shown in Fig. 8.79.
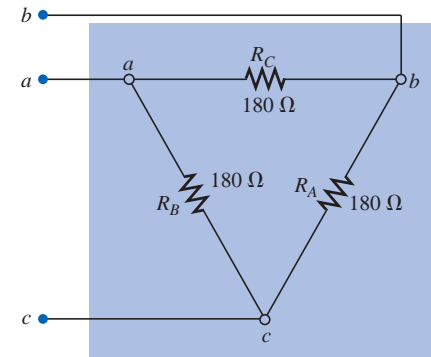
FIG. 8.79
The Δ equivalent for the Y of Fig. 8.78.

EXAMPLE 8.29 Find the total resistance of the network of Fig. 8.80, where $R_A = 3\ \Omega$, $R_B = 3\ \Omega$, and $R_C = 6\ \Omega$.

Solution:

Two resistors of the Δ were equal; therefore, two resistors of the Y will be equal.

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$

Replacing the Δ by the Y, as shown in Fig. 8.81, yields

$$R_T = 0.75\ \Omega + \frac{(4\ \Omega + 1.5\ \Omega)(2\ \Omega + 1.5\ \Omega)}{(4\ \Omega + 1.5\ \Omega) + (2\ \Omega + 1.5\ \Omega)}$$

$$= 0.75\ \Omega + \frac{(5.5\ \Omega)(3.5\ \Omega)}{5.5\ \Omega + 3.5\ \Omega}$$

$$= 0.75\ \Omega + 2.139\ \Omega$$

$$R_T = \mathbf{2.889\ \Omega}$$
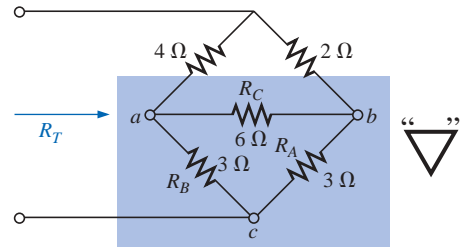
FIG. 8.80
Example 8.29.
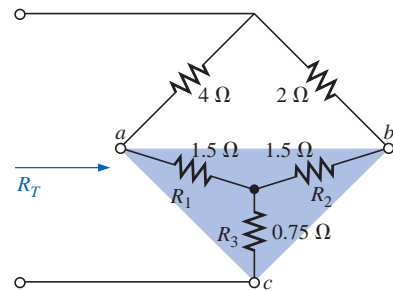
FIG. 8.81
Substituting the Y equivalent for the bottom Δ of Fig. 8.80.
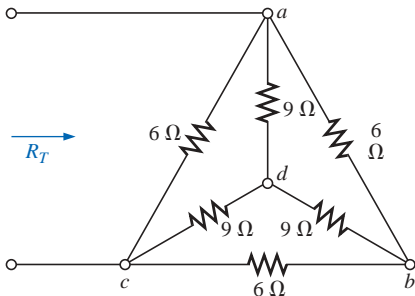
FIG. 8.82
Example 8.30.

EXAMPLE 8.30 Find the total resistance of the network of Fig. 8.82.

Solutions: Since all the resistors of the Δ or Y are the same, Equations (8.8a) and (8.8b) can be used to convert either form to the other.

a. *Converting the Δ to a Y.* Note: When this is done, the resulting d' of the new Y will be the same as the point d shown in the original figure, only because both systems are "balanced." That is, the resistance in each branch of each system has the same value:

$$R_Y = \frac{R_\Delta}{3} = \frac{6\ \Omega}{3} = 2\ \Omega \qquad \text{(Fig. 8.83)}$$
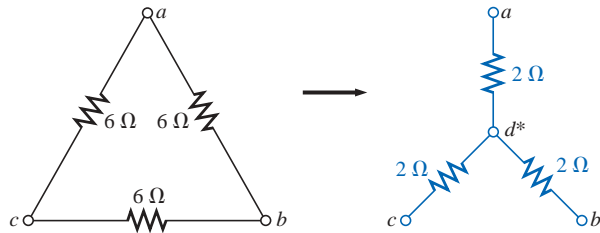
FIG. 8.83
Converting the Δ configuration of Fig. 8.82 to a Y configuration.
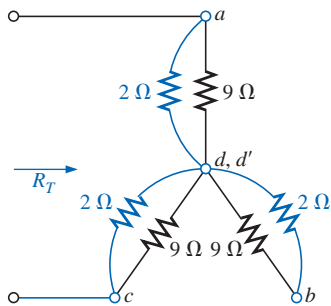
FIG. 8.84
Substituting the Y configuration for the converted Δ into the network of Fig. 8.82.

The network then appears as shown in Fig. 8.84.

$$R_T = 2\left[\frac{(2\ \Omega)(9\ \Omega)}{2\ \Omega + 9\ \Omega}\right] = \mathbf{3.2727\ \Omega}$$

b. *Converting the Y to a Δ:*

$$R_\Delta = 3R_Y = (3)(9\ \Omega) = 27\ \Omega \qquad \text{(Fig. 8.85)}$$

$$R'_T = \frac{(6\ \Omega)(27\ \Omega)}{6\ \Omega + 27\ \Omega} = \frac{162\ \Omega}{33} = 4.9091\ \Omega$$

$$R_T = \frac{R'_T\,(R'_T + R'_T)}{R'_T + (R'_T + R'_T)} = \frac{R'_T 2R'_T}{3R'_T} = \frac{2R'_T}{3}$$

$$= \frac{2(4.9091\ \Omega)}{3} = \mathbf{3.2727\ \Omega}$$

which checks with the previous solution.
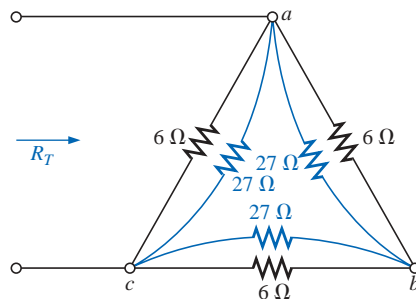
FIG. 8.85
Substituting the converted Y configuration into the network of Fig. 8.82.

PROBLEMS

SECTION 8.2 Current Sources

1. Find the voltage V_{ab} (with polarity) across the ideal current source of Fig. 8.93.
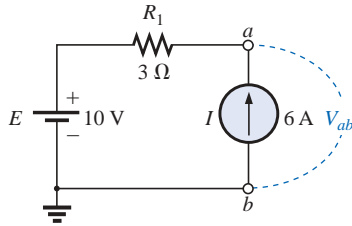
FIG. 8.93
Problem 1.

2. **a.** Determine V for the current source of Fig. 8.94(a) with an internal resistance of 10 kΩ.
   b. The source of part (a) is approximated by an ideal current source in Fig. 8.94(b) since the source resistance is much larger than the applied load. Determine the resulting voltage V for Fig. 8.94(b), and compare it to that obtained in part (a). Is the use of the ideal current source a good approximation?

(a)   (b)
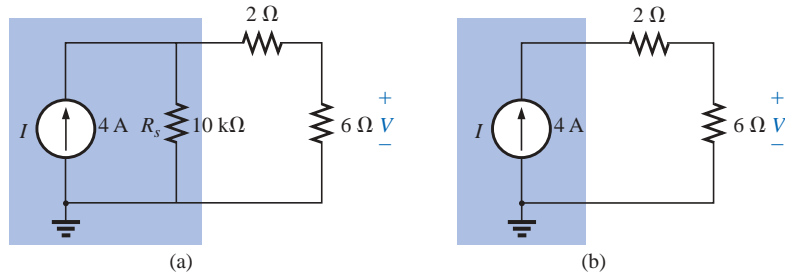
FIG. 8.94
Problem 2.

3. For the network of Fig. 8.95:
   a. Find the currents I_1 and I_s.
   b. Find the voltages V_s and V_3.

4. Find the voltage V_3 and the current I_2 for the network of Fig. 8.96.
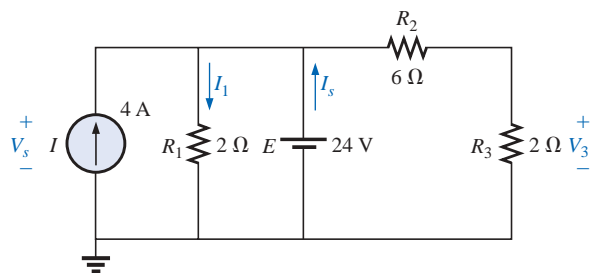
FIG. 8.95
Problem 3.
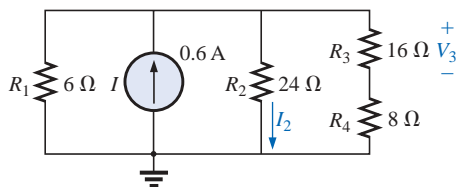
FIG. 8.96
Problem 4.

SECTION 8.3 Source Conversions

5. Convert the voltage sources of Fig. 8.97 to current sources.

(a)   (b)
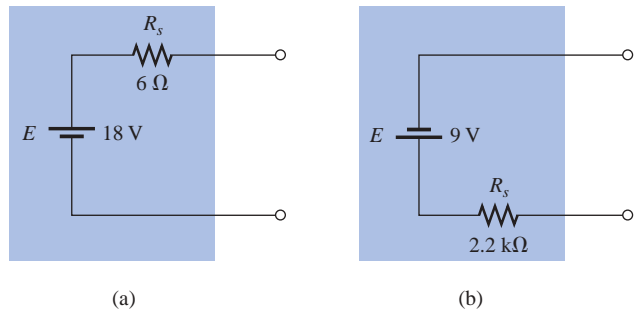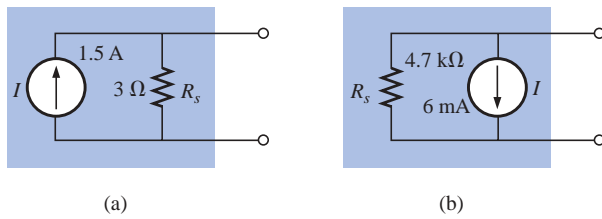
FIG. 8.97
Problem 5.

6. Convert the current sources of Fig. 8.98 to voltage sources.

(a)   (b)

FIG. 8.98
Problem 6.

7. For the network of Fig. 8.99:
  a. Find the current through the 2-Ω resistor.
  b. Convert the current source and 4-Ω resistor to a voltage source, and again solve for the current in the 2-Ω resistor. Compare the results.

8. For the configuration of Fig. 8.100:
  a. Convert the current source and 6.8-Ω resistor to a voltage source.
  b. Find the magnitude and direction of the current I_1.
  c. Find the voltage V_{ab} and the polarity of points a and b.
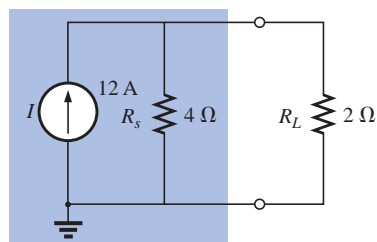
FIG. 8.99
Problem 7.
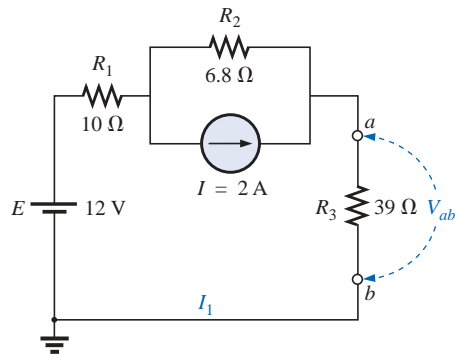
FIG. 8.100
Problem 8.

SECTION 8.4   Current Sources in Parallel

9. Find the voltage V_2 and the current I_1 for the network of Fig. 8.101.

10. a. Convert the voltage sources of Fig. 8.102 to current sources.
   b. Find the voltage V_{ab} and the polarity of points a and b.
   c. Find the magnitude and direction of the current I.
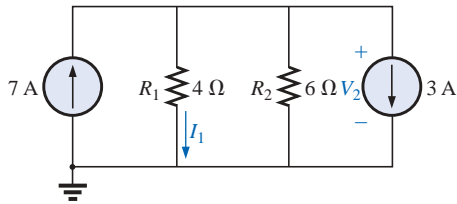
FIG. 8.101
Problem 9.
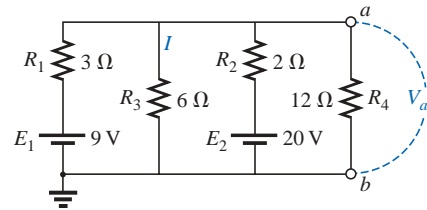
FIG. 8.102
Problem 10.

11. For the network of Fig. 8.103:
   a. Convert the voltage source to a current source.
   b. Reduce the network to a single current source, and determine the voltage V_1.
   c. Using the results of part (b), determine V_2.
   d. Calculate the current I_2.
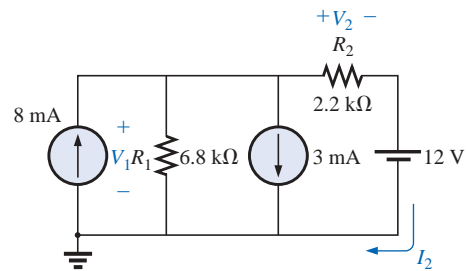
FIG. 8.103
Problem 11.

SECTION 8.6   Branch-Current Analysis

12. Using branch-current analysis, find the magnitude and direction of the current through each resistor for the networks of Fig. 8.104.

(a)

(b)
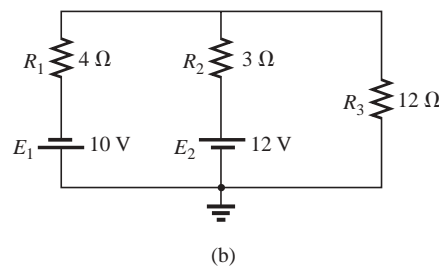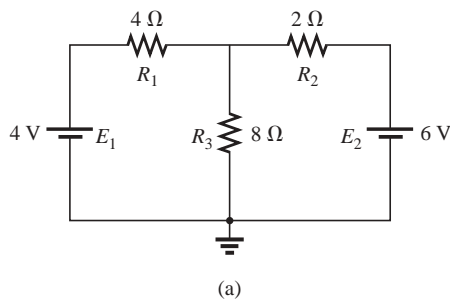
FIG. 8.104
Problems 12, 17, 25, and 54.

*13. Using branch-current analysis, find the current through each resistor for the networks of Fig. 8.105. The resistors are all standard values.
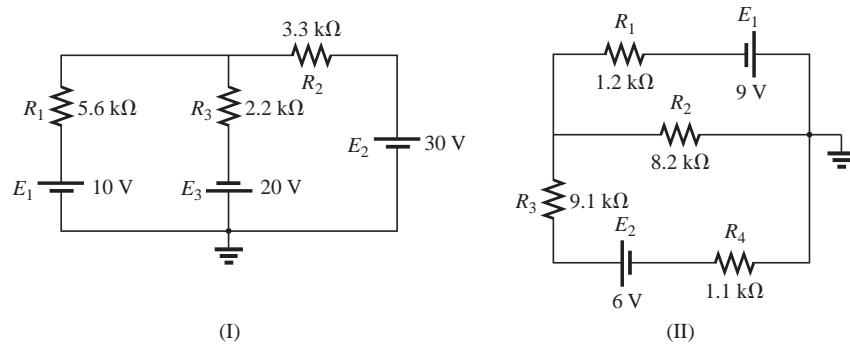
FIG. 8.105
Problems 13, 18, and 26.

*14. For the networks of Fig. 8.106, determine the current I_2 using branch-current analysis, and then find the voltage V_{ab}.
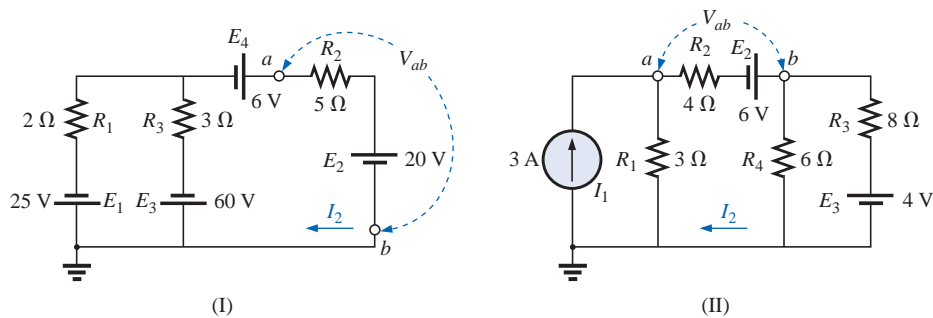
FIG. 8.106
Problems 14, 19, and 27.
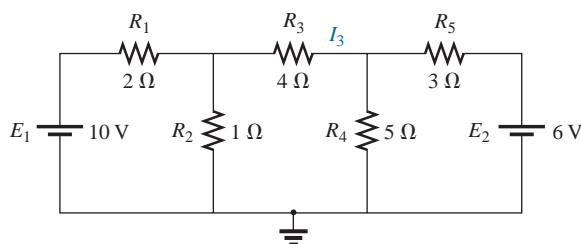
FIG. 8.107
Problems 15, 20, and 28.

*15. For the network of Fig. 8.107:
   a. Write the equations necessary to solve for the branch currents.
   b. By substitution of Kirchhoff's current law, reduce the set to three equations.
   c. Rewrite the equations in a format that can be solved using third-order determinants.
   d. Solve for the branch current through the resistor R_3.

*16. For the transistor configuration of Fig. 8.108:
  a. Solve for the currents I_B, I_C, and I_E using the fact that $V_{BE} = 0.7$ V and $V_{CE} = 8$ V.
  b. Find the voltages V_B, V_C, and V_E with respect to ground.
  c. What is the ratio of output current I_C to input current I_B? [*Note:* In transistor analysis this ratio is referred to as the *dc beta* of the transistor (β_{dc}).]
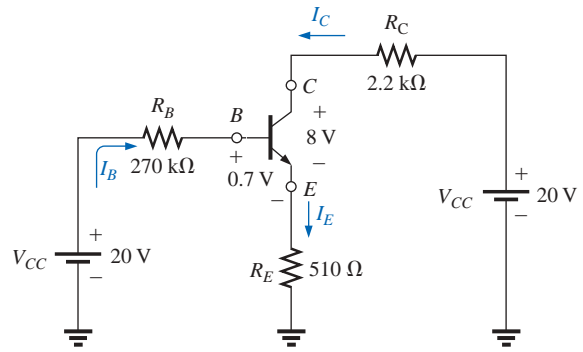
FIG. 8.108
Problem 16.

SECTION 8.7   Mesh Analysis (General Approach)

17. Find the current through each resistor for the networks of Fig. 8.104.

18. Find the current through each resistor for the networks of Fig. 8.105.

19. Find the mesh currents and the voltage V_{ab} for each network of Fig. 8.106. Use clockwise mesh currents.

20. a. Find the current I_3 for the network of Fig. 8.107 using mesh analysis.
  b. Based on the results of part (a), how would you compare the application of mesh analysis to the branch-current method?

*21. Using mesh analysis, determine the current through the 5-Ω resistor for each network of Fig. 8.109. Then determine the voltage V_a.

(a)   (b)
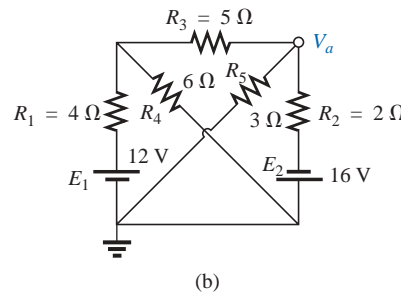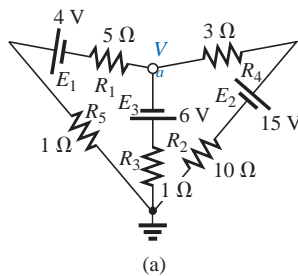
FIG. 8.109
Problems 21 and 29.

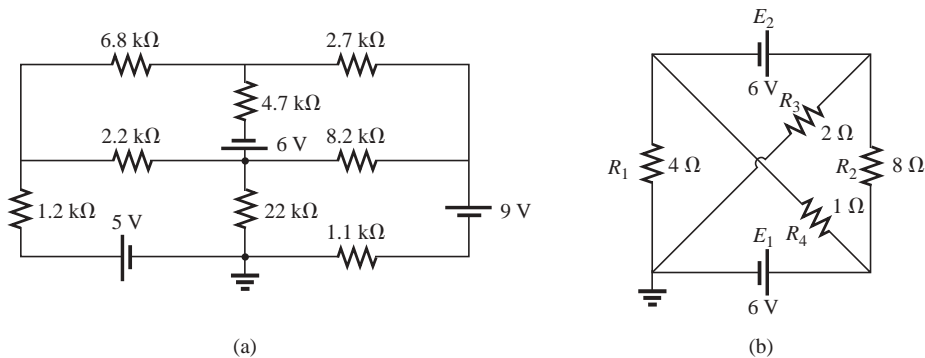***22.** Write the mesh equations for each of the networks of Fig. 8.110, and, using determinants, solve for the loop currents in each network. Use clockwise mesh currents.

(I)   (II)
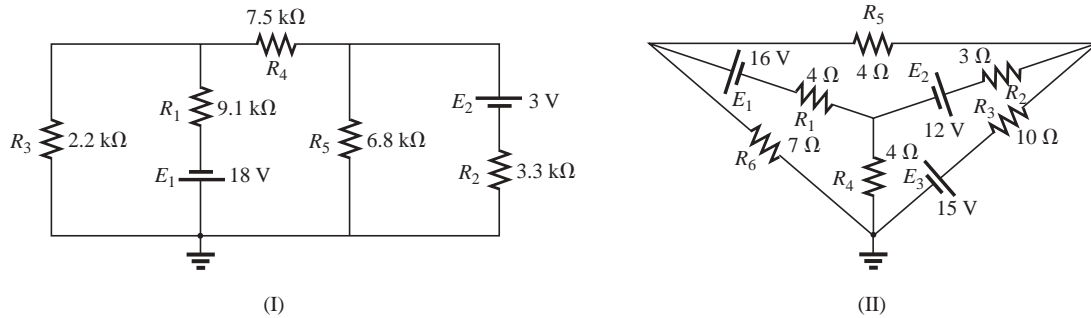
FIG. 8.110
Problems 22, 30, and 34.

*23. Write the mesh equations for each of the networks of Fig. 8.111, and, using determinants, solve for the loop currents in each network.

(a)   (b)

FIG. 8.111
Problems 23, 31, and 55.

*24. Using the supermesh approach, find the current through each element of the networks of Fig. 8.112.

(a)   (b)
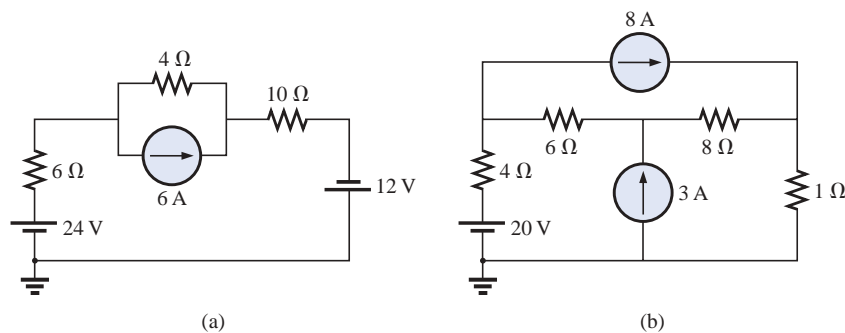
FIG. 8.112
Problem 24.

SECTION 8.8 Mesh Analysis (Format Approach)

25. Using the format approach, write the mesh equations for the networks of Fig. 8.104. Is symmetry present? Using determinants, solve for the mesh currents.

26. **a.** Using the format approach, write the mesh equations for the networks of Fig. 8.105.
   b. Using determinants, solve for the mesh currents.
   c. Determine the magnitude and direction of the current through each resistor.

27. **a.** Using the format approach, write the mesh equations for the networks of Fig. 8.106.
   b. Using determinants, solve for the mesh currents.
   c. Determine the magnitude and direction of the current through each resistor.

28. Using mesh analysis, determine the current I_3 for the network of Fig. 8.107, and compare your answer to the solution of Problem 15.

29. Using mesh analysis, determine $I_{5\Omega}$ and V_a for the network of Fig. 8.109(b).

30. Using mesh analysis, determine the mesh currents for the networks of Fig. 8.110.

31. Using mesh analysis, determine the mesh currents for the networks of Fig. 8.111.

SECTION 8.9 Nodal Analysis (General Approach)

32. Write the nodal equations for the networks of Fig. 8.113, and, using determinants, solve for the nodal voltages. Is symmetry present?
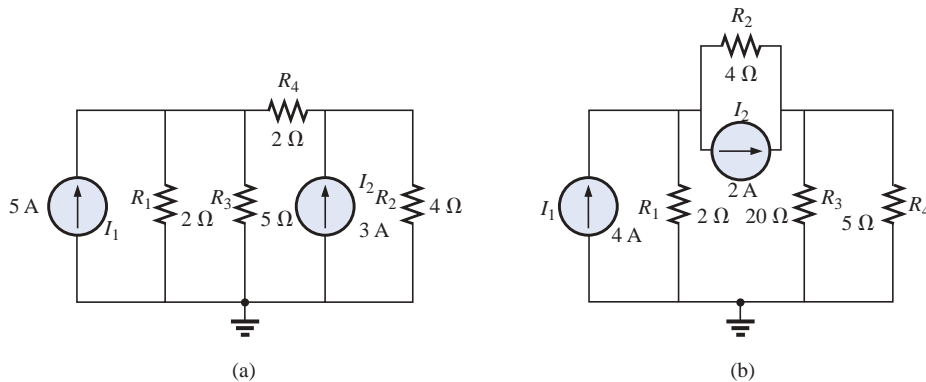
FIG. 8.113
Problems 32 and 38.

33. a. Write the nodal equations for the networks of Fig. 8.114.
   b. Using determinants, solve for the nodal voltages.
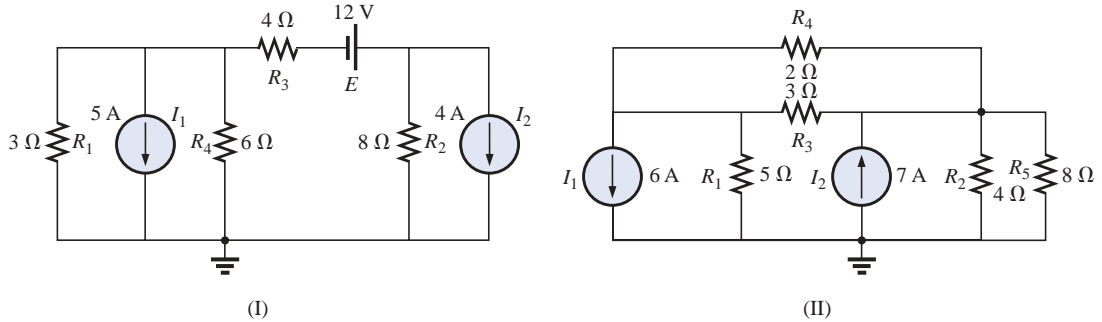   **c.** Determine the magnitude and polarity of the voltage across each resistor.

(I)

(II)

FIG. 8.114
Problems 33 and 39.

(I)

34. a. Write the nodal equations for the networks of Fig. 8.110.
   b. Using determinants, solve for the nodal voltages.
   c. Determine the magnitude and polarity of the voltage across each resistor.

*35. For the networks of Fig. 8.115, write the nodal equations and solve for the nodal voltages.

(II)
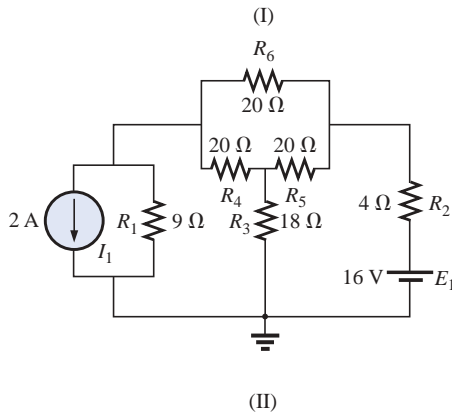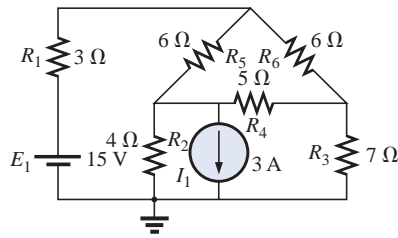
FIG. 8.115
Problems 35 and 40.

36. a. Determine the nodal voltages for the networks of Fig. 8.116.
   b. Find the voltage across each current source.

(I)

(II)
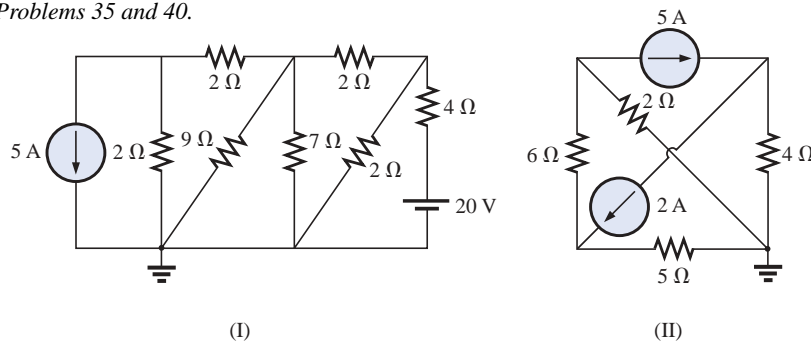
FIG. 8.116
Problems 36 and 41.

*37. Using the supernode approach, determine the nodal volt-ages for the networks of Fig. 8.117.
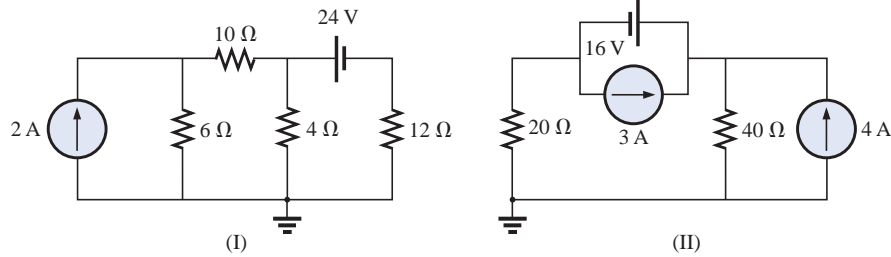

FIG. 8.117
Problems 37 and 56.

SECTION 8.10 Nodal Analysis (Format Approach)

38. Using the format approach, write the nodal equations for the networks of Fig. 8.113. Is symmetry present? Using determinants, solve for the nodal voltages.

39. a. Write the nodal equations for the networks of Fig. 8.114.
   b. Solve for the nodal voltages.
   c. Find the magnitude and polarity of the voltage across each resistor.

40. a. Write the nodal equations for the networks of Fig. 8.115.
   b. Solve for the nodal voltages.
   c. Find the magnitude and polarity of the voltage across each resistor.

41. Determine the nodal voltages for the networks of Fig. 8.116. Then determine the voltage across each current source.

SECTION 8.11 Bridge Networks

42. For the bridge network of Fig. 8.118:
   a. Write the mesh equations using the format approach.
   b. Determine the current through R_5.
   c. Is the bridge balanced?
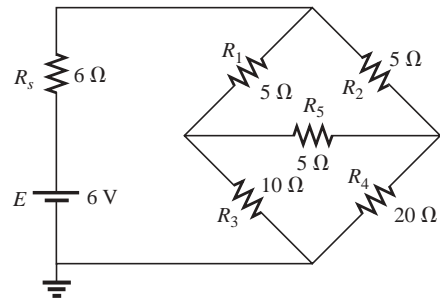   **d.** Is Equation (8.4) satisfied?

43. For the network of Fig. 8.118:
   a. Write the nodal equations using the format approach.
   b. Determine the voltage across R_5.
   c. Is the bridge balanced?
   d. Is Equation (8.4) satisfied?

44. For the bridge of Fig. 8.119:
   a. Write the mesh equations using the format approach.
   b. Determine the current through R_5.
   c. Is the bridge balanced?
   d. Is Equation (8.4) satisfied?
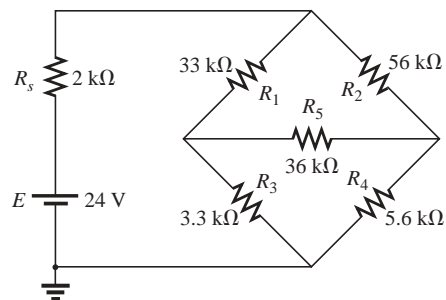

FIG. 8.118
Problems 42 and 43.

FIG. 8.119
Problems 44 and 45.
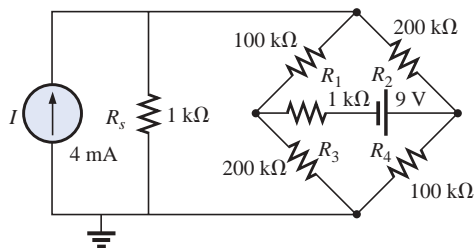
45. For the bridge network of Fig. 8.119:
   a. Write the nodal equations using the format approach.
   b. Determine the current across R_5.
   c. Is the bridge balanced?
   d. Is Equation (8.4) satisfied?

46. Write the nodal equations for the bridge configuration of Fig. 8.120. Use the format approach.

FIG. 8.120
Problem 46.

*47. Determine the current through the source resistor R_s of each network of Fig. 8.121 using either mesh or nodal analysis. Discuss why you chose one method over the other.
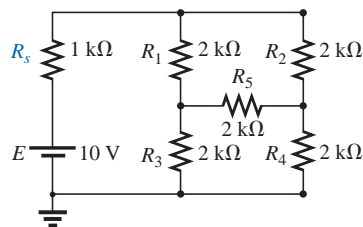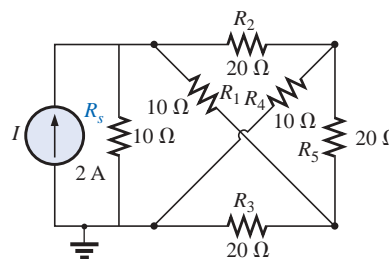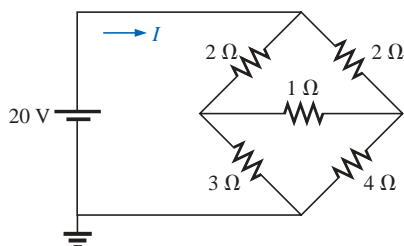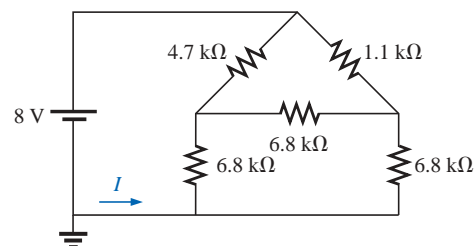

(a)   (b)

FIG. 8.121
Problem 47.

SECTION 8.12   Y-Δ (T-π) and Δ-Y (π-T) Conversions

48. Using a Δ-Y or Y-Δ conversion, find the current I in each of the networks of Fig. 8.122.

(a)   (b)

FIG. 8.122
Problem 48.

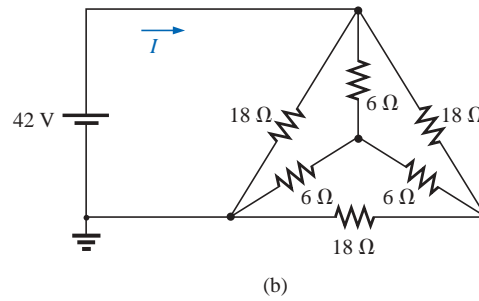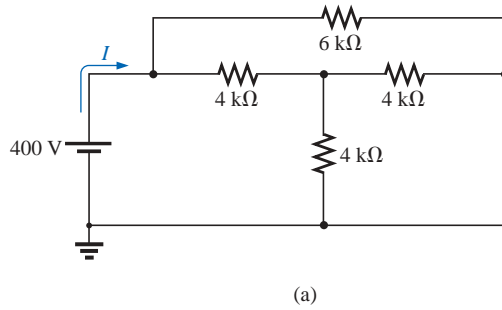***49.** Repeat Problem 48 for the networks of Fig. 8.123.

(a)

(b)

FIG. 8.123
Problem 49.

*50. Determine the current I for the network of Fig. 8.124.

*51. a. Replace the T configuration of Fig. 8.125 (composed of 6-kΩ resistors) with a π configuration.
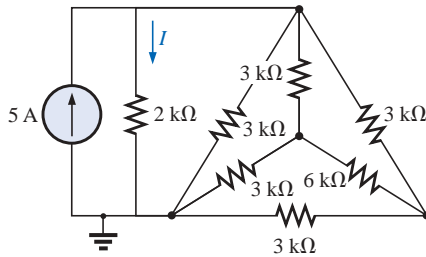   **b.** Solve for the source current I_{s_1}.

FIG. 8.124
Problem 50.

FIG. 8.125
Problem 51.

*52. a. Replace the π configuration of Fig. 8.126 (composed of 3-kΩ resistors) with a T configuration.
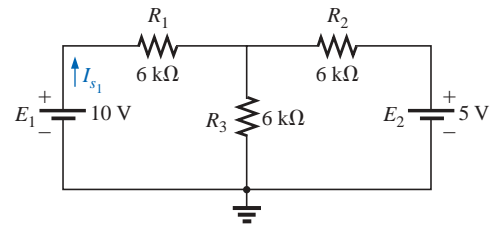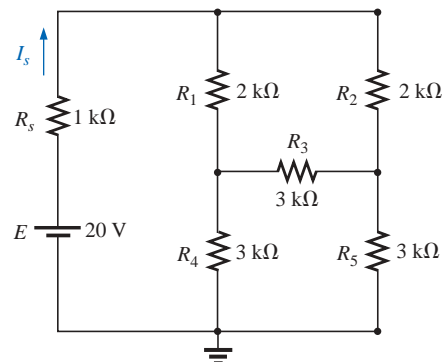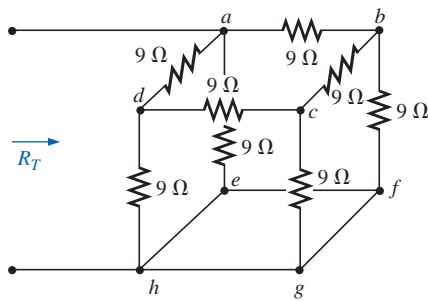   **b.** Solve for the source current I_s.

FIG. 8.126
Problem 52.

FIG. 8.127
Problem 53.

*53. Using Y-Δ or Δ-Y conversions, determine the total resistance of the network of Fig. 8.127.

SECTION 8.14   Computer Analysis

PSpice or Electronics Workbench

54. Using schematics, find the current through each element of Fig. 8.104.

*55. Using schematics, find the mesh currents for the network of Fig. 8.111(a).

*56. Using schematics, determine the nodal voltages for the network of Fig. 8.117(II).

Programming Language (C++, QBASIC, Pascal, etc.)

57. Given two simultaneous equations, write a program to solve for the unknown variables.

*58. Using mesh analysis and determinants, write a program to solve for both mesh currents of the network of Fig. 8.26 (for any component values).

*59. Using nodal analysis and determinants, write a program to solve for the nodal voltages of the network of Fig. 8.44 (for any component values).

GLOSSARY

Branch-current method   A technique for determining the branch currents of a multiloop network.

Bridge network   A network configuration typically having a diamond appearance in which no two elements are in series or parallel.

Current sources   Sources that supply a fixed current to a network and have a terminal voltage dependent on the network to which they are applied.

Delta (Δ), pi (π) configuration   A network structure that consists of three branches and has the appearance of the Greek letter delta (Δ) or pi (π).

Determinants method   A mathematical technique for finding the unknown variables of two or more simultaneous linear equations.

Mesh analysis   A technique for determining the mesh (loop) currents of a network that results in a reduced set of equations compared to the branch-current method.

Mesh (loop) current   A labeled current assigned to each distinct closed loop of a network that can, individually or in combination with other mesh currents, define all of the branch currents of a network.

Nodal analysis   A technique for determining the nodal voltages of a network.

Node   A junction of two or more branches in a network.

Wye (Y), tee (T) configuration   A network structure that consists of three branches and has the appearance of the capital letter Y or T.

Cycle   A portion of a waveform contained in one period of time.

Effective value   The equivalent dc value of any alternating voltage or current.

Electrodynamometer meters   Instruments that can measure both ac and dc quantities without a change in internal circuitry.

Frequency (f)   The number of cycles of a periodic waveform that occur in 1 second.

Frequency counter   An instrument that will provide a digital display of the frequency or period of a periodic time-varying signal.

Instantaneous value   The magnitude of a waveform at any instant of time, denoted by lowercase letters.

Oscilloscope   An instrument that will display, through the use of a cathode-ray tube, the characteristics of a time-varying signal.

Peak amplitude   The maximum value of a waveform as measured from its average, or mean, value, denoted by uppercase letters.

Peak-to-peak value   The magnitude of the total swing of a signal from positive to negative peaks. The sum of the absolute values of the positive and negative peak values.

Peak value   The maximum value of a waveform, denoted by uppercase letters.

Period (T)   The time interval between successive repetitions of a periodic waveform.

Periodic waveform   A waveform that continually repeats itself after a defined time interval.

Phase relationship   An indication of which of two waveforms leads or lags the other, and by how many degrees or radians.

Radian (rad)   A unit of measure used to define a particular segment of a circle. One radian is approximately equal to 57.3°; 2π rad are equal to 360°.

Root-mean-square (rms) value   The root-mean-square or effective value of a waveform.

Sinusoidal ac waveform   An alternating waveform of unique characteristics that oscillates with equal amplitude above and below a given axis.

VOM   A multimeter with the capability to measure resistance and both ac and dc levels of current and voltage.

Waveform   The path traced by a quantity, plotted as a function of some variable such as position, time, degrees, temperature, and so on.