Evaluation of Strength Test Results of Concrete

Reported by ACI Committee 214

James E. Cook[*]
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Jerry Parnes
Secretary

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Alejandro Graf	Avi A. Mor	Roger E. Vaughan
Thomas M. Greene	Tarun R. Naik	Woodward L. Vogt

[*]Committee members who prepared this revision.

Statistical procedures provide tools of considerable value when evaluating the results of strength tests. Information derived from such procedures is also valuable in defining design criteria and specifications. This report discusses variations that occur in the strength of concrete and presents statistical procedures that are useful in the interpretation of these variations with respect to specified testing and criteria.

Keywords: coefficient of variation; quality control; standard deviation; strength.

CONTENTS

CHAPTER 1—INTRODUCTION

This document provides an introduction to the evaluation of concrete strength tests. The procedures described are applicable to the compressive-strength test results required by ACI 301, ACI 318, and other similar specifications and codes. The statistical concepts described are applicable for analysis of other common concrete test results including flexural strength, slump, air content, and density.

Most construction projects in the United States and Canada require routine sampling and fabrication of standard molded cylinders. These cylinders are usually cast from samples of concrete taken from the discharge of a truck or a batch of concrete and molded, cured, and tested under standardized procedures. The results represent the potential strength of the concrete rather than the actual strength of the concrete in the structure.

Inevitably, strength test results vary. Variations in measured strength may originate from any of the following sources:
- Batch-to-batch variations of the proportions and characteristics of the constituent materials in the concrete, the production, delivery, and handling process, and climatic conditions; and
- Variations in the sampling, specimen preparation, curing, and testing procedures (within-test).

Conclusions regarding the strength of concrete can only be derived from a series of tests. The characteristics of concrete strength can be estimated with reasonable accuracy only when an adequate number of tests are conducted, strictly in accordance with standard practices and test methods.

Statistical procedures provide tools of considerable value when evaluating the results of strength tests. Information derived from such procedures is also valuable in refining design criteria and specifications. This report discusses variations that occur in the strength of concrete and presents statistical procedures that are useful in the interpretation of these variations with respect to specified testing and acceptance criteria.

Table 2.1—Principal sources of strength variation

Variations due to the properties of concrete	Variations due to testing methods
•Changes in *w/cm* caused by: -Poor control of water -Excessive variation of moisture in aggregate or variable aggregate moisture measurements -Retempering	•Improper sampling procedures •Variations due to fabrication techniques: -Handling, storing, and curing of newly made cylinders -Poor quality, damaged, or distorted molds
•Variations in water requirement caused by: -Changes in aggregate grading, absorption, particle shape -Changes in cementitious and admixture properties -Changes in air content -Delivery time and temperature changes	•Changes in curing: -Temperature variation -Variable moisture control -Delays in bringing cylinders to the laboratory -Delays in beginning standard curing

•Variations in characteristics and proportions of ingredients:
  -Aggregates
  -Cementitious materials, including pozzolans
  -Admixtures
	•Poor testing procedures:
	 -Specimen preparation
	 -Test procedure
	 -Uncalibrated testing equipment

•Variations in mixing, transporting, placing, and consolidation	
•Variations in concrete temperature and curing	

For the statistical procedures described in this report to be valid, the data should be derived from samples obtained by means of a random sampling plan designed to reduce the possibility that selection will be exercised by the sampler. Random sampling means that each possible sample has an equal chance of being selected. To ensure this condition, the selection should be made by some objective mechanism such as a table of random numbers. If sample batches are selected on the basis of judgement by the sampler, biases are likely to be introduced that will invalidate the analysis using the procedures presented here. Natrella (1963) and ASTM D 3665 provide a discussion of random sampling and a useful short table of random numbers.

This report begins with a discussion of the sources of variability in concrete as produced, mixed, and transported, and the additional variability of samples obtained from the concrete as delivered and tested. The report then describes the statistical tools used to evaluate the variability of concrete and determine compliance with a given specification, including both random variation and variation due to assignable causes. Statistically based specifications are also reviewed.

CHAPTER 2—VARIATIONS IN STRENGTH
2.1—General

The magnitude of variations in the strength of concrete test specimens is a direct result of the degree of control exerted over the constituent materials, the concrete production and transportation process, and the sampling, specimen preparation, curing and testing procedures. Variability in strength can be traced to two fundamentally different sources: variability in strength-producing properties of the concrete mixture and ingredients, including batching and production, and variability in the measured strength caused by variations inherent in the testing process. Table 2.1 summarizes the principal sources of strength variation.

Variation in the measured characteristics may be either random or assignable depending on cause. Random variation is normal for any process; a stable process will show only random variation. Assignable causes represent systematic changes that are typically associated with a shift in some fundamental statistical characteristic, such as mean, standard deviation or coefficient of variation, or other statistical measure.

2.2—Properties of concrete

For a given set of raw materials, strength is governed to a large extent by the water-cementitious materials ratio (*w/cm*). The first criterion for producing concrete of consistent strength, therefore, is to keep tight control over the *w/cm*. Because the quantity of cementitious material can be measured reasonably accurately, maintaining a constant *w/cm* primarily requires strict control of the total quantity of water used.

The water requirement of concrete is strongly influenced by the source and characteristics of the aggregates, cement, and mineral and chemical admixtures used in the concrete, as well as the desired consistency, in the sense of workability and placeability. Water demand also varies with air content and can increase with temperature. Variations in water content can be caused by variations in constituent materials and variations in batching. A common source of variation is from water added on the job site to adjust the slump.

Water can be introduced into concrete in many ways—some of which may be intentional. The amount of water added at the batch plant and job site is relatively easy to record. Water from other sources, such as free moisture on aggregates, water left in the truck, or added but not recorded, can be difficult to determine. For a similar concrete mixture at the same temperature and air content, differences in slump from batch to batch can be attributed to changes in the total mixing water content among other factors.

The AASHTO Standard Test Method for Water Content of Freshly Mixed Concrete Using Microwave Oven Drying (TP 23) is one method of determining water content of fresh concrete. The accuracy of the test method is still under study. The test may be useful in detecting deviations in water content in fresh concrete at the construction site.

Variations in strength are also influenced by air content. The entrained air content influences both water requirement and strength. There is a direct relationship between strength and air content. The air content of a specific concrete mixture varies depending on variations in constituent materials, extent of mixing, and ambient site conditions. For good concrete control, the entrained air content should be monitored closely at the construction site.

The temperature of fresh concrete affects both the amount of water needed to achieve the proper consistency and the entrained air content. In addition, the concrete temperature during the first 24 hours of curing can have a significant effect on the later-age strengths of the concrete. Concrete cylinders that are not protected from temperatures outside the range specified in ASTM C 31 may not accurately reflect the potential strength of the concrete.

Admixtures can contribute to variability, because each admixture introduces another variable and source of variation. Batching and mixing of admixtures should be carefully controlled. Changes in water demand are also associated with variations in aggregate grading.

Construction practices will cause variations of the in-place strength due to inadequate mixing, improper consolidation, delays in placement, improper curing, and insufficient protection at early ages. These differences will not be reflected in specimens fabricated and stored under standard laboratory conditions. Construction practices can affect the strength results of cores, however, which may be drilled and tested when strength test results do not conform to project specifications.

2.3—Testing methods

Deviations from standard sampling and testing procedures will affect the measured or reported strength. Testing to determine compliance with contract specifications should be conducted strictly according to the methods specified in the appropriate contract documents, for example ASTM C 31 and ASTM C 39. Acceptance tests provide an estimate of the potential strength of the concrete, not necessarily the in-place strength. Deviations from standard moisture and temperature curing is often a reason for lower strength test results. A project can be penalized unnecessarily when variations from this source are excessive. Deviations from standard procedures often result in a lower measured strength. Field sampling, curing, and handling of specimens should be performed by ACI Certified Technicians, or equivalently trained, experienced, and certified personnel, and procedures should be carefully monitored. Provisions for maintaining specified curing conditions should be made. Specimens made from slowly hardening concrete should not be disturbed too soon (ASTM C 31).

The importance of using accurate, properly calibrated testing devices and using proper sample preparation procedures is essential, because test results can be no more accurate than the equipment and procedures used. Less variable test results do not necessarily indicate accurate test results, because a routinely applied, systematic error can provide results that are biased but uniform. Laboratory equipment and procedures should be calibrated and checked periodically; testing personnel should be trained and certified at the appropriate technical level and evaluated routinely.

CHAPTER 3—ANALYSIS OF STRENGTH DATA
3.1—Terminology

3.1.1 *Definitions*—In this chapter, the following terminology is adopted.

Concrete sample—a portion of concrete, taken at one time, from a single batch or single truckload of concrete.

Single cylinder strength or **individual strength**—the strength of a single cylinder; a single cylinder strength does not constitute a test result.

Companion cylinders—cylinders made from the same sample of concrete.

Strength test or **strength test result**—the average of two or more single-cylinder strengths of specimens made from the same concrete sample (companion cylinders) and tested at the same age.

Range or **within-test range**—the difference between the maximum and minimum strengths of individual concrete specimens comprising one strength test result.

Test record—a collection of strength test results of a single concrete mixture. Test records of similar concrete mixtures can be used to calculate the pooled standard deviation. Concrete mixtures are considered to be similar if their nominal strengths are within 6.9 MPa (1000 psi), represent similar materials, and are produced, delivered, and handled under similar conditions (ACI 318).
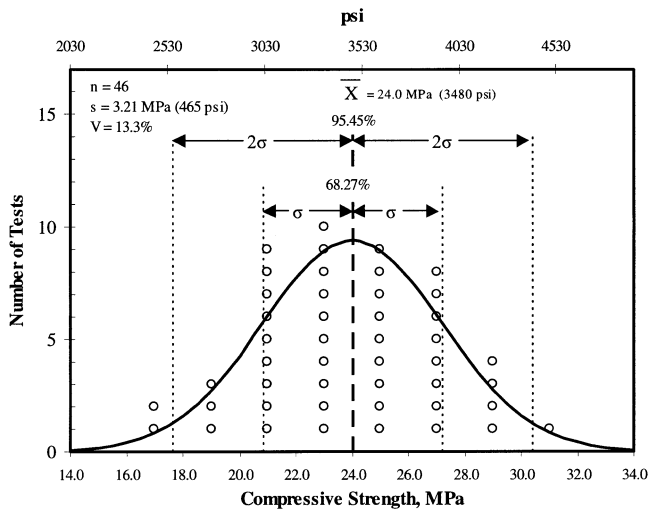
Fig. 3.1—Frequency distribution of strength data and corresponding assumed normal distribution.
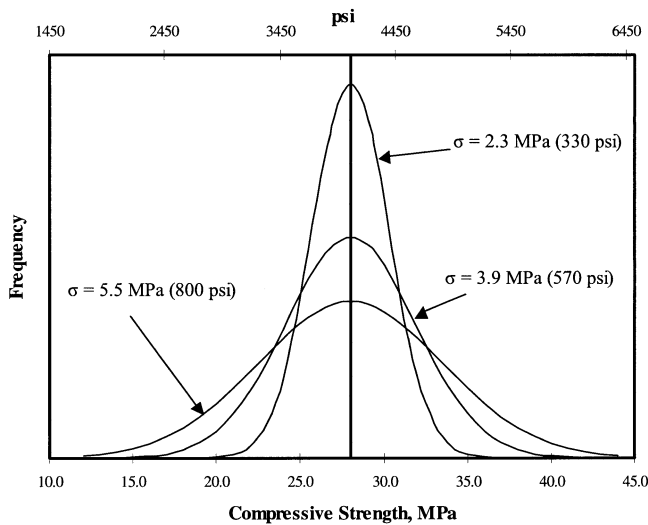

Fig. 3.2—Normal frequency curves for three different distributions with the same mean but different variability.

3.1.2 *Notation*

d_2 = factor for computing within-test standard deviation from average range (See Table 3.1.)

f'_{cr} = required average strength to ensure that no more than the permissible proportion of tests will fall below the specified compressive strength, used as the basis for selection of concrete proportions

f'_c = specified compressive strength

μ = population mean

n = number of tests in a record

$\overline{R}$ = within-test range

$\overline{R}_m$ = maximum average range, used in certain control charts

$\overline{R}$ = average range

σ = population standard deviation

σ_1 = population within-test standard deviation

σ_2 = population batch-to-batch standard deviation

s = sample standard deviation, an estimate of the population standard deviation, also termed $s_{overall}$

$\overline{s}$ = statistical average standard deviation, or "pooled" standard deviation

Table 3.1—Factors for computing within-test standard deviation from range

No. of specimens	d_2
2	1.128
3	1.693
4	2.059

Note: From Table 49, *ASTM Manual on Presentation of Data and Control Chart Analysis*, MNL 7.

s_1 = sample within-test standard deviation, also termed $s_{within\text{-}test}$

s_2 = sample batch-to-batch standard deviation, also termed $s_{producer}$

V = coefficient of variation

V_1 = within-test coefficient of variation

X_i = a strength test result

$\overline{X}$ = average of strength test results

z = a constant multiplier for standard deviation (s) that depends on the number of tests expected to fall below f'_c (See Table 4.3.)

3.2—General

A sufficient number of tests is needed to indicate accurately the variation in the concrete produced and to permit appropriate statistical procedures for interpreting the test results. Statistical procedures provide a sound basis for determining from such results the potential quality and strength of the concrete and for expressing results in the most useful form.

3.3—Statistical functions

A strength test result is defined as the average strength of all specimens of the same age, fabricated from a sample taken from a single batch of concrete. A strength test cannot be based on only one cylinder; a minimum of two cylinders is required for each test. Concrete tests for strength are typically treated as if they fall into a distribution pattern similar to the normal frequency distribution curve illustrated in Fig. 3.1. Cook (1989) reports that a skewed distribution may result for high-strength concrete where the limiting factor is the strength of the aggregate. If the data are not symmetrical about the mean, the data may be skewed. If the distribution is too peaked or too flat, kurtosis exists. Data exhibiting significant skewness or kurtosis are not normally distributed and any analysis presuming a normal distribution may be misleading rather than informative. Available data (Cook 1982) indicate that a normal distribution is appropriate under most cases when the strength of the concrete does not exceed 70 MPa (10,000 psi). Skewness and kurtosis should be considered for statistical evaluation of high-strength concrete. Cook (1989) provides simplified equations that can measure relative skewness and kurtosis for a particular set of data. In this document, strength test results are assumed to follow a normal distribution, unless otherwise noted.

When there is good control, the strength test values will tend to cluster near to the average value, that is, the histogram of test results is tall and narrow. As variation in strength results increases, the spread in the data increases and the normal distribution curve becomes lower and wider (Fig. 3.2). The normal distribution can be fully defined mathematically by two statistical parameters: the mean and standard deviation. These statistical parameters of the strength can be calculated as shown in Sections 3.3.1 and 3.3.2.

3.3.1 *Mean* $\overline{X}$— The average strength tests result $\overline{X}$ is calculated using Eq. (3-1).

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{1}{n}\sum X_i = \frac{1}{n}(X_1 + X_2 + X_3 + \ldots + X_n) \quad (3\text{-}1)$$

where X_i is the *i*-th strength test result, the average of at least two cylinder strength tests. X_2 is the second strength test result in the record, ΣX_i is the sum of all strength test results and *n* is the number of tests in the record.

3.3.2 *Standard deviation* s—The standard deviation is the most generally recognized measure of dispersion of the individual test data from their average. An estimate of the population standard deviation σ is the sample standard deviation *s*. The population consists of all possible data, often considered to be an infinite number of data points. The sample is a portion of the population, consisting of a finite amount of data. The sample standard deviation is obtained by Eq. (3-2a), or by its algebraic equivalent, Eq. (3-2b). The latter equation is preferable for computation purposes, because it is simpler and minimizes rounding errors. When using spreadsheet software, it is important to ensure that the sample standard deviation formula is used to calculate *s*.

$$s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}} = \quad (3\text{-}2a)$$

$$\sqrt{\frac{(X_1 - \overline{X})^2 + (X_2 - \overline{X})^2 + \ldots + (X_n - \overline{X})^2}{n-1}}$$

which is equivalent to

$$s = \sqrt{\frac{n\sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2}{n(n-1)}} = \sqrt{\frac{\sum_{i=1}^{n} X_i^2 - n\overline{X}^2}{n-1}} \quad (3\text{-}2b)$$

where *s* is the sample standard deviation, *n* is the number of strength test results in the record, $\overline{X}$ is the mean, or average, strength test result, and ΣX is the sum of the strength test results.

When considering two separate records of concrete mixtures with similar strength test results, it is frequently necessary to determine the statistical average standard deviation, also termed the pooled standard deviation. The statistical average standard deviation of two records is calculated as shown in Eq. (3-3).

$$\overline{s} = \sqrt{\frac{(n_A - 1)(s_A)^2 + (n_B - 1)(s_B)^2}{(n_A + n_B - 2)}} \quad (3\text{-}3)$$

where $\overline{s}$ is the statistical average standard deviation, or pooled standard deviation, determined from two records, s_A and s_B are the standard deviations of Record A and Record B, respectively, and n_A and n_B are the number of tests in Record A and Record B, respectively.

3.3.3 *Other statistical measures*—Several other derivative statistics are commonly used for comparison of different data sets or for estimation of dispersion in the absence of statistically valid sample sizes.

3.3.3.1 *Coefficient of variation* V—The sample standard deviation expressed as a percentage of the average strength is called the coefficient of variation

$$V = \frac{s}{\overline{X}} \times 100 \quad (3\text{-}4)$$

where *V* is the coefficient of variation, *s* is the sample standard deviation, and $\overline{X}$ is the average strength test result.

The coefficient of variation is less affected by the magnitude of the strength level (Cook 1989; Anderson 1985), and is therefore more useful than the standard deviation in comparing the degree of control for a wide range of compressive strengths. The coefficient of variation is typically used when comparing the dispersion of strength test results of records with average compressive strengths more than about 7 MPa [1000 psi] different.

3.3.3.2 *Range* R—Range is the statistic found by subtracting the lowest value in a data set from the highest value in that data set. In evaluation of concrete test results, the within-test range *R* of a strength test result is found by subtracting the lowest single cylinder strength from the highest single cylinder strength of the two or more cylinders used to comprise a strength test result. The average within-test range is used for estimating the within-test standard deviation. It is discussed in more detail in Section 3.4.1.

3.4—Strength variations

As noted in Chapters 1 and 2, variations in strength test results can be traced to two different sources:

1. Variations in testing methods; and

2. Variations in the properties or proportions of the constituent materials in the concrete mixture, variations in the production, delivery or handling procedures, and variations in climatic conditions.

It is possible to compute the variations attributable to each source using analysis of variance (ANOVA) techniques (Box, Hunter, and Hunter 1978) or with simpler techniques.

3.4.1 *Within-test variation*—Variability due to testing is estimated by the within-test variation based on differences in strengths of companion (replicate) cylinders comprising a strength test result. The within-test variation is affected by variations in sampling, molding, consolidating, transporting, curing, capping, and testing specimens. A single strength test result of a concrete mixture, however, does not provide sufficient data for statistical analysis. As with any statistical estimator, the confidence in the estimate is a function of the number of test results.

The within-test standard deviation is estimated from the average range $\overline{R}$ of at least 10, and preferably more, strength test results of a concrete mixture, tested at the same age, and the appropriate values of d_2 in Table 3.1 using Eq. (3-5). In Eq. (3-6), the within-test coefficient of variation, in percent, is determined from the within-test standard deviation and the average strength.

Table 3.2—Standards of concrete control*

	Overall variation				
Class of operation	Standard deviation for different control standards, MPa (psi)				
	Excellent	Very good	Good	Fair	Poor
General construction testing	Below 2.8 (below 400)	2.8 to 3.4 (400 to 500)	3.4 to 4.1 (500 to 600)	4.1 to 4.8 (600 to 700)	Above 4.8 (above 700)
Laboratory trial batches	Below 1.4 (below 200)	1.4 to 1.7 (200 to 250)	1.7 to 2.1 (250 to 300)	2.1 to 2.4 (300 to 350)	Above 2.4 (above 350)
	Within-test variation				
Class of operation	Coefficient of variation for different control standards, %				
	Excellent	Very good	Good	Fair	Poor
Field control testing	Below 3.0	3.0 to 4.0	4.0 to 5.0	5.0 to 6.0	Above 6.0
Laboratory trial batches	Below 2.0	2.0 to 3.0	3.0 to 4.0	4.0 to 5.0	Above 5.0

*$f_c' \leq 34.5$ MPa (5000 psi).

Table 3.3—Standards of concrete control*

	Overall variation				
Class of operation	Coefficient of variation for different control standards,%				
	Excellent	Very good	Good	Fair	Poor
General construction testing	Below 7.0	7.0 to 9.0	9.0 to 11.0	11.0 to 14.0	Above 14.0
Laboratory trial batches	Below 3.5	3.5 to 4.5	4.5 to 5.5	5.5 to 7.0	Above 7.0
	Within-test variation				
Class of operation	Coefficient of variation for different control standards, %				
	Excellent	Very good	Good	Fair	Poor
Field control testing	Below 3.0	3.0 to 4.0	4.0 to 5.0	5.0 to 6.0	Above 6.0
Laboratory trial batches	Below 2.0	2.0 to 3.0	3.0 to 4.0	4.0 to 5.0	Above 5.0

*$f_c' > 34.5$ MPa (5000 psi).

$$s_1 = \frac{1}{d_2}\overline{R} \qquad (3\text{-}5)$$

$$V_1 = \frac{s_1}{\overline{X}} \times 100 \qquad (3\text{-}6)$$

where s_1 is the sample within-test standard deviation, $\overline{R}$ is the average within-test range of at least 10 tests, d_2 is the factor for computing within-test standard deviation from the average range, V_1 is the sample within-test coefficient of variation, and $\overline{X}$ is the mean, or average, strength test result.

For example, if two cylinders were cast for each of 10 separate strength tests (the minimum number recommended), and the average within-test strength range was 1.75 MPa (254 psi), the estimated within-test standard deviation (d_2 = 1.128 for 2 cylinders) is 1.75/1.128 = 1.55 MPa (254/1.128 = 225 psi). The precision statement in ASTM C 39 indicates the within-test coefficient of variation for cylinder specimens made in the lab to be 2.37% and for cylinders made in the field to be 2.87%.

Consistent errors or bias in testing procedures will not necessarily be detected by comparing test results of cylinders from the same batch, however. Variations may be small with an improperly conducted test, if performed consistently.

3.4.2 *Batch-to-batch variations*—These variations reflect differences in strength from batch to batch, which can be attributed to variations in:

(a) Characteristics and properties of the ingredients; and
(b) Batching, mixing, and sampling.

Testing effects can inflate the apparent batch-to-batch variation slightly. The effects of testing on batch-to-batch variation are not usually revealed by analyzing test results from companion cylinders tested at the same age, because specimens from the same batch tend to be treated alike. Batch-to-batch variation can be estimated from strength test results of a concrete mixture if each test result represents a separate batch of concrete.

The overall variation, σ (for a population) or *s* (for a sample), has two component variations, the within-test, σ_1 (population) or s_1 (sample), and batch-to-batch, σ_2 (population) or s_2 (sample) variations. The sample variance—the square

of the sample standard deviation—is the sum of the sample within-test and sample batch-to-batch variances

$$s^2 = s_1^2 + s_2^2 \qquad (3\text{-}7)$$

from which the batch-to-batch standard deviation can be computed as

$$s_2 = \sqrt{s^2 - s_1^2} \qquad (3\text{-}8)$$

For example, if the overall sample standard deviation *s* from multiple batches is 3.40 MPa (493 psi), and the estimated within-test sample standard deviation s_1 is 1.91 MPa (277 psi), the batch-to-batch sample standard deviation s_2 can be estimated as 2.81 MPa (408 psi).

The within-test sample standard deviation estimates the variation attributable to sampling, specimen preparation, curing and testing, assuming proper testing methods are used. The batch-to-batch sample standard deviation estimates the variations attributable to constituent material suppliers, and the concrete producer. Values of the overall and the within-test sample standard deviations and coefficients of variation associated with different control standards are provided in Section 3.6 (Table 3.2 and 3.3).

3.5—Interpretation of statistical parameters

Once the statistical parameters have been computed, and with the assumption or verification that the results follow a normal frequency distribution curve, additional analysis of the test results is possible. Figure 3.3 indicates an approximate division of the area under the normal frequency distribution curve. For example, approximately 68% of the area (equivalent to 68% of the results) lies within ±1σ of the average, and 95% lies within ±2σ. This permits an estimate of the portion of the test results expected to fall within given multiples *z* of σ of the average or of any other specific value.

Agreement between the normal distribution and the actual distribution of the tests tends to increase as the number of tests increases. When only a small number of results are available, they may not fit the standard, bell-shaped pattern. Other causes of differences between the actual and the normal distribution are errors in sampling, testing, and recording.

Table 3.4—Expected percentages of individual tests lower than $f_c'^*$

Average strength μ	Expected percentage of low tests	Average strength μ	Expected percentage of low tests
$f_c' + 0.10\sigma$	46.0	$f_c' + 1.6\sigma$	5.5
$f_c' + 0.20\sigma$	42.1	$f_c' + 1.7\sigma$	4.5
$f_c' + 0.30\sigma$	38.2	$f_c' + 1.8\sigma$	3.6
$f_c' + 0.40\sigma$	34.5	$f_c' + 1.9\sigma$	2.9
$f_c' + 0.50\sigma$	30.9	$f_c' + 2.0\sigma$	2.3
$f_c' + 0.60\sigma$	27.4	$f_c' + 2.1\sigma$	1.8
$f_c' + 0.70\sigma$	24.2	$f_c' + 2.2\sigma$	1.4
$f_c' + 0.80\sigma$	21.2	$f_c' + 2.3\sigma$	1.1
$f_c' + 0.90\sigma$	18.4	$f_c' + 2.4\sigma$	0.8
$f_c' + 1.00\sigma$	15.9	$f_c' + 2.5\sigma$	0.6
$f_c' + 1.10\sigma$	13.6	$f_c' + 2.6\sigma$	0.45
$f_c' + 1.20\sigma$	11.5	$f_c' + 2.7\sigma$	0.35
$f_c' + 1.30\sigma$	9.7	$f_c' + 2.8\sigma$	0.25
$f_c' + 1.40\sigma$	8.1	$f_c' + 2.9\sigma$	0.19
$f_c' + 1.50\sigma$	6.7	$f_c' + 3.0\sigma$	0.13

*where μ exceeds f_c' by amount shown.

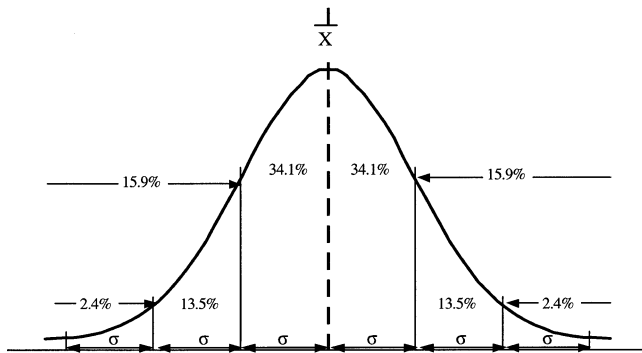Fig. 3.4—Cumulative distribution curves for different coefficients of variation.

Fig. 3.3—Approximate distribution of area under normal frequency distribution curve.

Failure to sample in a truly random manner, sampling from different populations, or the presence of skew or kurtosis in high-strength concretes (Cook 1989) are examples that would result in substantial differences between the actual and the normal distributions.

Table 3.4 was adapted from the normal cumulative distribution (the normal probability integral) and shows the probability of a fraction of tests falling below f_c' in terms of the average strength of the population of test results when the population average strength μ equals $f_c' + z\sigma$.

Cumulative distribution curves can also be plotted by accumulating the number of tests below any given strength for different coefficients of variation or standard deviations. The below-average half of the normal frequency distribution curve is shown for a variety of coefficients of variation in Fig. 3.4 and a variety of standard deviations in Fig. 3.5. By using the normal probability scale, the curves are plotted as a straight line and can be read in terms of frequencies for which test results will be greater than the indicated percentage of average strength of the population of strength test results (Fig 3.4) or compressive strength below average (Fig. 3.5). When lower coefficients of variation (or standard devia-

Fig. 3.5—Cumulative distribution curves for different standard deviations.

tions) are attained, the angle formed by the cumulative distribution curve and the 100% ordinate (Fig 3.4) or 0 standard deviation (Fig 3.5) decreases; the difference between the lowest and the highest probable strength is reduced, indicating the concrete test results are more consistent. These charts can be used to solve for probabilities graphically. Similar charts can be constructed to compare the performance of different concrete mixtures.

3.6—Standards of control

One of the primary purposes of statistical evaluation of concrete data is to identify sources of variability. This knowledge can then be used to help determine appropriate steps to maintain the desired level of control. Several different techniques can be used to detect variations in concrete production, materials processing and handling, and contractor and testing agency operations. One simple approach is to compare overall variability and within-test variability, using either

standard deviation or coefficient of variation, as appropriate, with previous performance.

Whether the standard deviation or the coefficient of variation is the appropriate measure of dispersion to use in any given situation depends upon which of the two measures is more nearly constant over the range of strengths of concern. Present information indicates that the standard deviation remains reasonably constant over a limited range of strengths; however, several studies show that the coefficient of variation is more nearly constant over a wider range of strengths, especially higher strengths (Cook 1982; Cook 1989). Comparison of level of control between compressive and flexural strengths is more easily conducted using the coefficient of variation. The coefficient of variation is also considered to be a more applicable statistic for within-test evaluations (Neville 1959; Metcalf 1970; Murdock 1953; Erntroy 1960; Rüsch 1964; and ASTM C 802). Either the standard deviation or the coefficient of variation can be used to evaluate the level of control of conventional-strength concrete mixtures, but for higher strengths, generally those in excess of 35 MPa (5000 psi), the coefficient of variation is preferred.

The standards of control given in Table 3.2 are appropriate for concrete having specified strengths up to 35 MPa (5000 psi), whereas Table 3.3 gives the appropriate standards of control for specified strengths over 35 MPa (5000 psi). As more high-strength test data become available, these standards of control may be modified. These standards of control were adopted based on examination and analysis of compressive strength data by ACI Committee 214 and ACI Committee 363. The strength tests were conducted using 150 x 300 mm (6 x 12 in.) cylinders, the standard size for acceptance testing in ASTM C 31. The standards of control are therefore applicable to these size specimens, tested at 28 days, and may be considered applicable with minor differences to other cylinder sizes, such as 100 x 200 mm (4 x 8 in.) cylinders, recognized in C 31. They are not applicable to strength tests on cubes or flexural strength test results.

CHAPTER 4—CRITERIA
4.1—General

The strength of concrete in a structure and the strength of test cylinders cast from a sample of that concrete are not necessarily the same. The strength of the cylinders obtained from that sample of concrete and used for contractual acceptance are to be cured and tested under tightly controlled conditions. The strengths of these cylinders are generally the primary evidence of the quality of concrete used in the structure. The engineer specifies the desired strength, the testing frequency, and the permitted tolerance in compressive strength.

Any specified quantity, including strength, should also have a tolerance. It is impractical to specify an absolute minimum strength, because there is always the possibility of even lower strengths simply due to random variation, even when control is good. The cylinders may not provide an accurate representation of the concrete in each portion of the structure. Strength-reduction factors are provided in design methodologies that allow for limited deviations from specified strengths without jeopardizing the safety of the structure. These methodologies evolved using probabilistic methods on the basis of construction practices, design procedures, and quality-control techniques used in the construction industry.

For a given mean strength, if a small percentage of the test results fall below the specified strength, the remaining test results will be greater than the specified strength. If the samples are selected randomly, there is only a small probability that the low strength results correspond to concrete located in a critical area. The consequences of a localized zone of low-strength concrete in a structure depend on many factors, including the probability of early overload; the location and magnitude of the low-quality zone in the structural element; the degree of reliance placed on strength in design; the initial cause of the low strength; and the implications, economic and otherwise, of loss of serviceability or structural failure.

There will always be a certain probability of tests falling below f_c'. ACI 318 and most other building codes and specifications establish tolerances for meeting the specified compressive strength acceptance criteria, analogous to the tolerances for other building materials.

To satisfy statistically based strength-performance requirements, the average strength of the concrete should be in excess of the specified compressive strength f_c'. The required average strength f_{cr}' which is the strength used in mixture proportioning, depends on the expected variability of test results as measured by the coefficient of variation or standard deviation, and on the allowable proportion of tests below the appropriate, specified acceptance criteria.

4.2—Data used to establish the minimum required average strength

To establish the required average strength f_{cr}' an estimate of the variability of the concrete to be supplied for construction is needed. The strength test record used to estimate the standard deviation or coefficient of variation should represent a group of at least 30 consecutive tests. The data used to estimate the variability should represent concrete produced to meet a specified strength close to that specified for the proposed work and similar in composition and production.

The requirement for 30 consecutive strength tests can be satisfied by using a test record of 30 consecutive batches of the same class of concrete or the statistical average of two test records totaling 30 or more tests. If the number of test results available is less than 30, a more conservative approach is needed. Test records with as few as 15 tests can be used to estimate the standard deviation; however, the calculated standard deviation should be increased by as much as 15% to account for the uncertainty in the estimate of the standard deviation. In the absence of sufficient information, a very conservative approach is required and the concrete is proportioned to produce relatively high average strengths.

In general, changes in materials and procedures will have a larger effect on the average strength level than the standard deviation or coefficient of variation. The data used to establish the variability should represent concrete produced to meet a specified strength close to that specified for the proposed work and similar in composition. Significant changes in composition are due to changes in type, brand or source of cementitious materials, admixtures, source of aggregates, and mixture proportions.

If only a small number of test results are available, the estimates of the standard deviation and coefficient of variation become less reliable. When the number of strength test results is between 15 and 30, the calculated standard deviation, multiplied by the appropriate modification factors obtained from Table 4.1, which was taken from ACI 318,

Table 4.1—Modification factors for standard deviation

Number of tests	Modification factors
Less than 15	See Table 4.2
15	1.16
20	1.08
25	1.03
30 or more	1.00

Table 4.2—Minimum required average strength without sufficient historical data

$f'_{cr} = f'_c + 6.9$ MPa (1000 psi)	when $f'_c < 20.7$ MPa (3000 psi)
$f'_{cr} = f'_c + 8.3$ MPa (1200 psi)	when $f'_c \geq 20.7$ MPa (3000 psi) and $f'_c \leq 34.5$ MPa (5000 psi)
$f'_{cr} = 1.10f'_c + 4.8$ MPa (700 psi)	when $f'_c > 34.5$ MPa (5000 psi)

provides a sufficiently conservative estimate to account for the uncertainty in the calculated standard deviation.

If previous information exists for concrete from the same plant meeting the similar requirements described above, that information can be used to establish a value of standard deviation s to be used in determining f'_{cr}.

Estimating the standard deviation using at least 30 tests is preferable. If it is necessary to use data from two test records to obtain at least 30 strength test results, the records should represent similar concrete mixtures containing similar materials and produced under similar quality control procedures and conditions, with a specified compressive strength f'_c that does not differ by more than 6.9 MPa (1000 psi) from the required strength f'_{cr}. In this case, the pooled standard deviation can be calculated using Eq. (3-3).

When the number of strength test results is less than 15, the calculated standard deviation is not sufficiently reliable. In these cases, the concrete is proportioned to produce relatively high average strengths as required in Table 4.2.

As a project progresses and more strength tests become available, all available strength tests should be analyzed to obtain a more reliable estimate of the standard deviation appropriate for that project. A revised value of f'_{cr}, which is typically lower, may be computed and used.

4.3—Criteria for strength requirements

The minimum required average strength f'_{cr} can be computed using Eq. (4-1a), (4-1b), or, equivalently, (4-2a) or (4-2b), Table 4.2, or Fig. 4.1 or 4.2, depending on whether the coefficient of variation or standard deviation is used. The value of f'_{cr} will be the same for a given set of strength test results regardless of whether the coefficient of variation or standard deviation is used.

$$f'_{cr} = f'_c /(1 - zV) \qquad (4\text{-}1a)$$

$$f'_{cr} = f'_c + zs \qquad (4\text{-}1b)$$

where z is selected to provide a sufficiently high probability of meeting the specified strength, assuming a normal distribution of strength test results. In most cases, f'_c is replaced by a specified acceptance criterion, such as $f'_c - 3.5$ MPa (500 psi) or $0.90f'_c$.
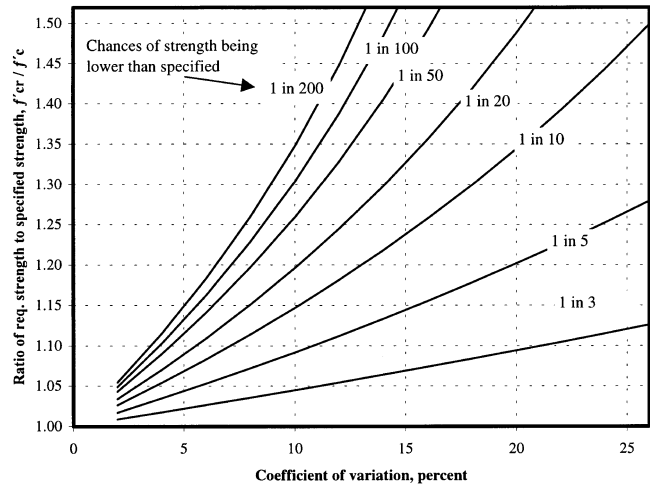

Fig. 4.1—Rates of required average strength f'_{cr} to specified strength f'_c for various coefficients of variation and chances of falling below specified strength.

Fig. 4.2—Excess of required average strength f'_{cr} to specified strength f'_c for various standard deviations and chances of falling below specified strength.

When a specification requires computation of the average of some number of tests, such as the average of three consecutive tests, the standard deviation or coefficient of variation of such an average will be lower than that computed using all individual test results. The standard deviation of an average is calculated by dividing the standard deviation of individual test results by the square root of the number of tests (n) in each average. For averages of consecutive tests, Eq. (4-1a) and (4-1b) become:

$$f'_{cr} = f'_c /(1 - zV/\sqrt{n}) \qquad (4\text{-}2a)$$

$$f'_{cr} = f'_c + zs\sqrt{n} \qquad (4\text{-}2b)$$

The value of n typically specified is 3; this value should not be confused with the number of strength test results used to estimate the mean or standard deviation of the record. Figure 4.3 shows that as the variability increases, f'_{cr} increases

Table 4.3—Probabilities associated with values of z

Percentages of tests within ± zσ	Chances of falling below lower limit	z
40	3 in 10 (30%)	0.52
50	2.5 in 10 (25%)	0.67
60	2 in 10 (20%)	0.84
68.27	1 in 6.3 (15.9%)	1.00
70	1.5 in 10 (15%)	1.04
80	*1 in 10 (10%)*	*1.28*
90	1 in 20 (5%)	1.65
95	1 in 40 (2.5%)	1.96
95.45	1 in 44 (2.3%)	2.00
98	*1 in 100 (1%)*	*2.33*
99	1 in 200 (0.5%)	2.58
99.73	1 in 741 (0.13%)	3.00

Note: Commonly used values in bold italic.

Fig. 4.3—Normal frequency curves for coefficients of variation of 10, 15, and 20%.

and thereby illustrates the economic value of good control. Table 4.3 provides values of z for various percentages of tests falling between the mean $+ z\sigma$ and the mean $-z\sigma$.

The amount by which the required average strength f_{cr}' should exceed the specified compressive strength f_c' depends on the acceptance criteria specified for a particular project. The following are criteria examples used to determine the required average strength for various specifications or elements of specifications. The numerical examples are presented in both SI and inch-pound units in a parallel format that have been hard converted and so are not exactly equivalent numerically.

4.3.1 *Criterion no. 1*—The engineer may specify a stated maximum percentage of individual, random strength tests results that will be permitted to fall below the specified compressive strength. This criterion is no longer used in the ACI 318 Building Code, but does occur from time to time in specifications based on allowable strength methods or in situations where the average strength is a fundamental part of the design methodology, such as in some pavement specifications. A typical requirement is to permit no more than 10% of the strength tests to fall below f_c'. The specified strength in these situations will generally be between 21 and 35 MPa (3000 and 5000 psi).

4.3.1.1 *Standard deviation method*—Assume sufficient data exist for which a standard deviation of 3.58 MPa (519 psi) has been calculated for a concrete mixture with a specified strength of 28 MPa. (An example is also given for a mixture with f_c' = 4000 psi; these are not equal strengths). From Table 4.3, 10% of the normal probability distribution lies more than 1.28 standard deviations below the mean. Using Eq. (4-1b)

$$f_{cr}' = f_c' + zs$$

$$f_{cr}' = 28 \text{ MPa} + 1.28 \times (3.58) \text{ MPa} = 32.6 \text{ MPa}$$

alternately, $f_{cr}' = 4000 \text{ psi} + 1.28 \times 519 \text{ psi} = 4660 \text{ psi}$

(maintaining appropriate significant figures).

Therefore, for a specified compressive strength of 28 MPa, the concrete mixture should be proportioned for an average strength of not less than 32.6 MPa so that, on average, no more

than 10% of the results will fall below f_c' (for a specified strength of 4000 psi, proportioned for not less than 4660 psi).

4.3.1.2 *Coefficient of variation method*—Assume sufficient data exist for which a coefficient of variation of 10.5% has been calculated for a concrete mixture with a specified strength of 28 MPa (or for a mixture with f_c' = 4000 psi). From Table 4.3, 10% of the normal probability distribution lies more than 1.28 standard deviations below the mean. Using Eq. (4-1a)

$$f_{cr}' = f_c'/(1 - zV)$$

$$f_{cr}' = 28 \text{ MPa} /[1 - (1.28 \times 10.5/100)] = 32.3 \text{ MPa}$$

alternately, $f_{cr}' = 4000 \text{ psi}/[1 - (1.28 \times 0.105)] = 4620 \text{ psi}$

(maintaining appropriate significant figures).

Therefore, for a specified compressive strength of 28 MPa, the concrete mixture should be proportioned for an average strength of not less than 32.3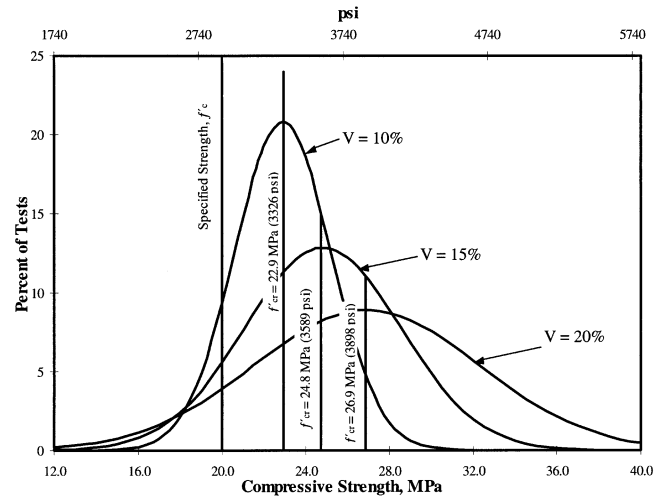 MPa so that, on average, no more than 10% of the results will fall below f_c' (for a specified strength of 4000 psi, proportioned for not less than 4620 psi).

4.3.2 *Criterion no. 2*—The engineer can specify a probability that an average of n consecutive strength tests will be below the specified compressive strength. For example, one of the acceptance criteria in ACI 318 stipulates that the average of any three consecutive strength test results should equal or exceed f_c'. The required average strength should be established such that nonconformance is anticipated no more often than 1 in 100 times (0.01).

4.3.2.1 *Standard deviation method*—Assume sufficient data exist for which a standard deviation of 3.58 MPa (519 psi) has been calculated for a concrete mixture with a specified strength of 28 MPa (or for a mixture with f_c' = 4000 psi). From Table 4.3, 1% of the normal probability distribution lies more than 2.33 standard deviations below the mean. Using Eq. (4-2b)

$$f_{cr}' = f_c' + zs/\sqrt{n}$$

$$f'_{cr} = 28 \text{ MPa} + [(2.33 \times 3.58 \text{ MPa})/\sqrt{3}] = 32.8 \text{ MPa}$$

alternately, $f'_{cr} = 4000 \text{ psi} + [(2.33 \times 519 \text{ psi})/\sqrt{3}] = 4700 \text{ psi}$

(maintaining appropriate significant figures).

Therefore, for a specified compressive strength of 28 MPa, the concrete mixture should be proportioned for an average strength of not less than 32.8 MPa so that, on average, no more than 1% of the moving average of three strength-test results will fall below f'_c (for a specified strength of 4000 psi, proportioned for not less than 4700 psi).

In ACI 318, Eq. (4-2b) is presented in slightly different form. The value 1.34 in ACI 318 is equivalent to the term $z/\sqrt{n} = 2.33/\sqrt{3} = 1.34$, because both z and n are already specified.

4.3.2.2 *Coefficient of variation method*—Assume sufficient data exist for which a coefficient of variation of 10.5% has been calculated for a concrete mixture with a specified strength of 28 MPa (or for a mixture with f'_c = 4000 psi). From Table 4.3, 1% of the normal probability distribution lies more than 2.33 standard deviations below the mean. Using Eq. (4-2a)

$$f'_{cr} = f'_c / [1 - (zV/\sqrt{n})]$$

$$f'_{cr} = 28 \text{ MPa}/[1 - (2.33 \times 10.5/100/\sqrt{3})] = 32.6 \text{ MPa}$$

alternately, $f'_{cr} = 4000 \text{ psi} /[1 - (2.33 \times 0.105/\sqrt{3})] = 4660 \text{ psi}$

(maintaining appropriate significant figures).

Therefore, for a specified compressive strength of 28 MPa, the concrete mixture should be proportioned for an average strength of not less than 32.6 MPa so that, on average, no more than 1% of the moving average of three consecutive strength-test results will fall below f'_c (for a specified strength of 4000 psi, proportioned for not less than 4660 psi).

4.3.3 *Criterion no. 3*—The engineer may specify a certain probability that a random individual strength test result will be no more than a certain amount below the specified compressive strength. For example, this criterion is used in ACI 318 by stipulating that no individual strength test result (where a test result is the average of at least two cylinders fabricated from the same batch) falls below f'_c by more than 3.5 MPa (500 psi). An alternative criterion is more appropriate for high-strength concrete. The acceptance criterion for high-strength concrete, 34.5 MPa (f'_c > 5000 psi), requires that no individual strength test result falls below 90% of f'_c. These two criteria are equivalent at 34.5 MPa (5000 psi). The minimum required average strength is established so that nonconformance of an individual, random test is anticipated no more often than 1 in 100 times in either case.

4.3.3.1 *Standard deviation method,* f'_c **£** *34.5 MPa (5000 psi)*—Assume sufficient data exist for which a standard deviation of 3.58 MPa (519 psi) has been calculated for a concrete mixture with a specified strength of 28 MPa (or for a mixture with f'_c = 4000 psi). From Table 4.3, 1% of the normal probability distribution lies more than 2.33 standard deviations below the mean. Using a modified form of Eq. (4-1b)

$$f'_{cr} = (f'_c - 3.5) + zs$$

$$f'_{cr} = (28 \text{ MPa} - 3.5 \text{ MPa}) + (2.33 \times 3.58 \text{ MPa}) = 32.8 \text{ MPa}$$

alternately, $f'_{cr} = (4000 \text{ psi} - 500 \text{ psi}) + (2.33 \times 519 \text{ psi}) = 4710 \text{ psi}$

(maintaining appropriate significant figures).

Therefore, for a specified compressive strength of 28 MPa, the concrete mixture should be proportioned for an average strength of not less than 32.8 MPa so that, on average, no more than 1% of the individual strength-test results will fall below f'_c – 3.5 MPa (for a specified strength of 4000 psi strength, proportioned for not less than 4710 psi).

4.3.3.2 *Standard deviation method,* f'_c *> 34.5 MPa (5000 psi)*—Assume sufficient data exist for which a standard deviation of 5.61 MPa (814 psi) has been calculated for a concrete mixture with a specified strength of 60 MPa (or for a mixture with f'_c = 9000 psi). From Table 4.3, 1% of the normal probability distribution lies more than 2.33 standard deviations below the mean. Using a modified form of Eq. (4-1b)

$$f'_{cr} = 0.90 \times f'_c + zs$$

$$f'_{cr} = (0.90 \times 60 \text{ MPa}) + (2.33 \times 5.61 \text{ MPa}) = 67.1 \text{ MPa}$$

alternately, $f'_{cr} = 0.90 \times 9000 \text{ psi} + 2.33 \times 814 \text{ psi} = 10,000 \text{ psi}$

(maintaining appropriate significant figures).

Therefore, for a specified compressive strength of 60 MPa the concrete mixture should be proportioned for an average strength of not less than 67.1 MPa so that, on average, no more than 1% of the individual strength-test results will fall below $0.90f'_c$ (for a specified 9000 psi strength, proportioned for not less than 10,000 psi).

4.3.3.3 *Coefficient of variation method,* f'_c **£** *34.5 MPa (5000 psi)*—Assume sufficient data exist for which a coefficient of variation of 10.5% has been calculated for a concrete mixture with a specified strength of 28 MPa (or for a mixture with f'_c = 4000 psi). From Table 4.3, 1% of the normal probability distribution lies more than 2.33 standard deviations below the mean. Using a modified form of Eq. (4-1a):

$$f'_{cr} = (f'_c - 3.5)/(1 - zV)$$

$$f'_{cr} = (28 \text{ MPa} - 3.5 \text{ MPa})/[1 - (2.33 \times 10.5/100)] = 32.4 \text{ MPa}$$

alternately, $f'_{cr} = (4000 \text{ psi} - 500 \text{ psi})/[1 - (2.33 \times 0.105)] = 4630 \text{ psi}$

(maintaining appropriate significant figures).

Therefore, for a specified compressive strength of 28 MPa, the concrete mixture should be proportioned for an average strength of not less than 32.4 MPa so that, on average, no more than 1% of the individual strength-test results will fall below f'_c – 3.5 MPa (for a specified strength of 4000 psi, proportioned for not less than 4630 psi).

4.3.3.4 *Coefficient of variation method,* f'_c *> 34.5 MPa (5000 psi)*—Assume sufficient data exist for which a coefficient of variation of 8.2% has been calculated for a concrete mixture with a specified strength of 60 MPa (or for a mixture with f'_c = 9000 psi). From Table 4.3, 1% of the normal prob-

Table 5.1—Probability of at least one event in *n* tests for selected single-event probabilities

n	Single event probability = 1.5%	Single event probability = 10%
1	1.5%	10.0%
5	7.3%	41.0%
7	10.0%	54.3%
10	14.0%	65.1%
20	26.1%	87.8%
50	53.0%	99.5%
100	77.9%	Approximately 100%

ability distribution lies more than 2.33 standard deviations below the mean. Using a modified form of Eq. (4-1a)

$$f'_{cr} = 0.90 \times f'_c/(1 - zV)$$

$$f'_{cr} = (0.90 \times 60 \text{ MPa})/[1 - (2.33 \times 8.2/100)] = 66.8 \text{ MPa}$$

alternately, $f'_{cr} = (0.90 \times 9000 \text{ psi})/[1 - (2.33 \times 0.082)] = 10{,}010 \text{ psi}$

(maintaining appropriate significant figures).

Therefore, for a specified compressive strength of 60 MPa, the concrete mixture should be proportioned for an average strength of not less than 66.8 MPa so that, on average, no more than 1% of the individual strength test results will fall below $0.90f'_c$ (for a specified 9000 psi strength, proportioned for not less than 10,010 psi).

4.3.4 *Multiple criteria*—In many instances, multiple criteria will be specified. ACI 318 and 318M require that concrete strengths conform to both individual test criteria and the moving average of three test criteria. Because both criteria are in effect, the required average compressive strength f'_{cr} should meet or exceed all requirements; that is, f'_{cr} should be the largest strength calculated using all relevant criteria. For example, assume sufficient data exist for which a coefficient of variation 8.2% has been calculated for a concrete mixture with a specified strength of 60 MPa (8700 psi). The required average strength for this concrete mixture should meet both of the following criteria:

1. Individual criterion (see 4.3.3.4): $f'_{cr} = 0.90 \times f'_c/(1 - 2.33V) = 66.8$ MPa (9690 psi).

2. Moving average criterion (see 4.3.2.2): $f'_{cr} = f'_c/(1 - 2.33V/\sqrt{3}) = 67.4$ MPa (9780 psi).

The moving average criterion governs, because 67.4 MPa > 66.8 MPa (9780 psi > 9690 psi) and f'_{cr} should be the largest strength calculated using all relevant criteria.

CHAPTER 5—EVALUATION OF DATA
5.1—General
Evaluation of strength data is required in many situations. Three commonly required applications are:
- Evaluation for mixture submittal purposes;
- Evaluation of level of control (typically called quality control); and
- Evaluation to determine compliance with specifications.

A major purpose of these evaluations is to identify departures from desired target values and, where possible, to assist with the formulation of an appropriate response. In all cases, the usefulness of the evaluation will be a function of the amount of test data and the statistical rigor of the analysis. Applications for routine quality control and compliance overlap considerably. Many of the evaluation tools or techniques used in one application are appropriate for use in the other.

Techniques appropriate for concrete mixture submittal evaluation were reviewed in Chapter 4. Techniques for routine quality control and compliance applications are provided and discussed in this chapter. Criteria for rejecting doubtful results, determination of an appropriate testing frequency, and guidelines for additional test procedures are also discussed.

It is informative to determine the likelihood of various outcomes when there is at most a 1% probability of a test less than $f'_c - 3.5$ MPa (500 psi) and, at most, a 1% probability that the moving average of three consecutive tests will be less than f'_c. The maximum probability that at least one event will occur in *n* independent tests may be estimated using Eq. (5-1) (Leming 1999)

$$Pr\{\text{at least 1 event} \mid n \text{ tests}\} = 1 - (1 - p)^n \qquad (5\text{-}1)$$

where *p* is the probability of a single event.

One value of interest for *p* is the single event probability of noncompliance with the strength criteria in ACI 318. Because *p* includes both possible cases ($f'_c - (3.5$ MPa [$f'_c - 500$ psi] and the moving average of three consecutive tests less than f'_c), *p* lies between 1.0 and 2.0%. In the absence of more details, the probability of a single test failing to meet the strength criteria of ACI 318 may be assumed to be 1.5%. Table 5.1 gives the probabilities of at least one occurrence of an event given various numbers of independent tests *n* when the single event probability *p* equals 1.5% (a test does not meet ACI 318 strength criteria) and 10% (a test falls below f'_c).

The probability is not trivial even for relatively small projects. For example, there is approximately a 10% probability of having at least one noncompliant test and slightly greater than 50% probability of having at least one test fall below f'_c for a project with only seven tests. There is a very high probability of such an occurrence on most projects, and a virtual certainty on large projects, even if the variation is due exclusively to random effects, and the minimum average strength was determined accurately using statistically valid methods. The probabilities are reduced somewhat for larger projects due to the effects of interference; however, the probabilities are still appreciable (Leming 1999).

5.2—Numbers of tests
For a particular project, a sufficient number of tests should be made to ensure accurate representation of the concrete. A test is defined as the average strength of at least two specimens of the same age fabricated from a sample taken from a single batch of concrete. The frequency of concrete tests can be established on the basis of time elapsed or volume placed. The engineer should establish the number of tests needed based on job conditions.

A project where all concrete operations are supervised by one engineer provides an excellent opportunity for control and for accurate estimates of the mean and standard deviation with a minimum of tests. Once operations are progressing smoothly, tests taken each day or shift, depending on the volume of concrete produced, can be sufficient to obtain data that reflect the variations of the concrete as delivered. The engineer can reduce the number of specimens required by the project specifications as the levels of control of the pro-

ducer, the laboratory, and the contractor are established. To avoid bias, all sampling for acceptance testing should be conducted using randomly selected batches of concrete.

For routine building construction, ACI 318 requires at least one test per day: one test every 115 m^3 (150 yd^3) or one test for every 460 m^2 (5000 ft^2) of the surface area of slabs and walls, but permits the engineer to waive testing on quantities less than 40 m^3 (50 yd^3). Testing should be conducted so that each of these criteria are satisfied. These testing frequencies generally result in testing concrete in one out of 10 to 20 trucks.

Testing more frequently than this can slow the construction process and should be specified only for compelling reasons. For example, more frequent testing is recommended for specialized or critical members or applications. For members where the structural performance is particularly sensitive to compressive strength, a testing frequency of one test for every 80 m^3 (100 yd^3) may be appropriate; one test for every 40 m^3 (50 yd^3) would be appropriate only for critical applications. Testing each load of concrete delivered for potential strength is rarely required.

In general, make a sufficient number of tests so that each different class of concrete placed during any one day will be represented by at least one test; a minimum of five tests should be conducted for each class of concrete on a given project. Guidelines for routine testing requirements can also be found in ACI 301, ACI 318, and ASTM C 94.

5.3—Rejection of doubtful specimens

The practice of arbitrary rejection of strength test results that appear too far out of line is not recommended because the normal distribution anticipates the possibility of such results. Discarding test results indiscriminately can seriously distort the strength distribution, making analysis of results less reliable. Occasionally, the strength of one cylinder from a group made from a sample deviates so far from the others as to be highly improbable. If questionable variations have been observed during fabrication, curing, or testing of a specimen, the specimen should be rejected on that basis alone.

ASTM E 178 provides criteria for rejecting the test result for one specimen in a set of specimens. In general, the result from a single specimen in a set of three or more specimens can be discarded if its deviation from a test mean is greater than three times the previously established within-test standard deviation (see Chapter 3), and should be accepted with suspicion if its deviation is greater than two times the within-test standard deviation. The test average should be computed from the remaining specimens. A test, that is, the average of all specimens of a single sample tested at the same age, should not be rejected unless it is very likely that the specimens are faulty. The test represents the best available estimate for the sample.

5.4—Additional test requirements

The potential compressive strength and variability of concrete is normally based on test results using a standard cylinder which has been sampled, molded, and cured initially in accordance with ASTM C 31, then moist cured at a controlled temperature (23 ± 2 C [73 ± 3 F]) until the specified test age, normally 28 days. When the nominal size of the coarse aggregate in the mixture exceeds 50 mm (2 in.), a larger test specimen is used, or the larger aggregate is removed by wet sieving. Analysis of concrete strength vari-

ability is based on these standard-sized specimens. Specimens of concrete made or cured under other than standard conditions provide additional information but should be analyzed and reported separately. Specimens that have not been produced, cured, or tested under standard conditions may or may not accurately reflect the potential concrete strength. Discrepancies and deviations from standard testing conditions should be noted on strength test reports.

The strength of concrete at later ages, such as 56, 91, or 182 days may be more relevant than the 28-day strength, particularly where a pozzolan or cement of slow strength gain is used or heat of hydration is a concern. Some elements or structures will not be loaded until the concrete has been allowed to mature for longer periods and advantage can be taken of strength gain after 28 days. Some concretes have been found to produce strengths at 28 days, which are less than 50% of their ultimate strength. Others, made with finely ground, Type III portland cements, may not gain appreciable strength after 28 days.

If design is based on strength at later ages, it may be necessary to correlate these later strengths with strength at 28 days because it is not always practical to use later-age specimens for concrete acceptance. This correlation should be established by field or laboratory tests before construction starts. If concrete batching plants are located in one place for long enough periods, establish this correlation for reference even though later-age concrete may not be immediately involved.

Many times, particularly in the early stages of a job, it is necessary to estimate the strength of concrete being produced before the 28-day strength results are available. Concrete cylinders should be made and tested from the same batch at seven days and, in some instances, at three days. Testing at very early ages using accelerated test procedures, such as found in ASTM C 684, can also be adopted. The 28-day strength can be estimated on the basis of a previously established correlation for the specific mixture using the method described in ASTM C 918. These early tests provide only an indication of acceptable performance; tests for the purposes of acceptance are still typically conducted at 28 days and are often the legal standard. A minimum of two cylinders are required for a valid test and more are sometimes specified.

Curing concrete test specimens at the construction site and under job conditions, that is, field or job-cured specimens, is sometimes recommended or required in such applications as fast-track construction or post-tensioning, because an acceptable in-place strength has to be attained, particularly at early ages, before the member can be safely loaded or stressed. Tests of job-cured specimens are highly desirable or necessary when determining the time of form removal, particularly in cold weather, and when establishing the strength of steam-cured concrete or concrete pipe and block. In addition, the adequacy of curing by the contractor can be evaluated only by monitoring strength gain on the job site. Do not confuse nor replace these special test requirements with the required standard control tests.

5.5—Basic quality-control charts

Quality-control charts have been used by manufacturing industries for many years as aids in reducing variability, increasing production efficiency, and identifying trends as early as practicable. Well-established methods for setting up such charts are outlined in convenient form in the ASTM
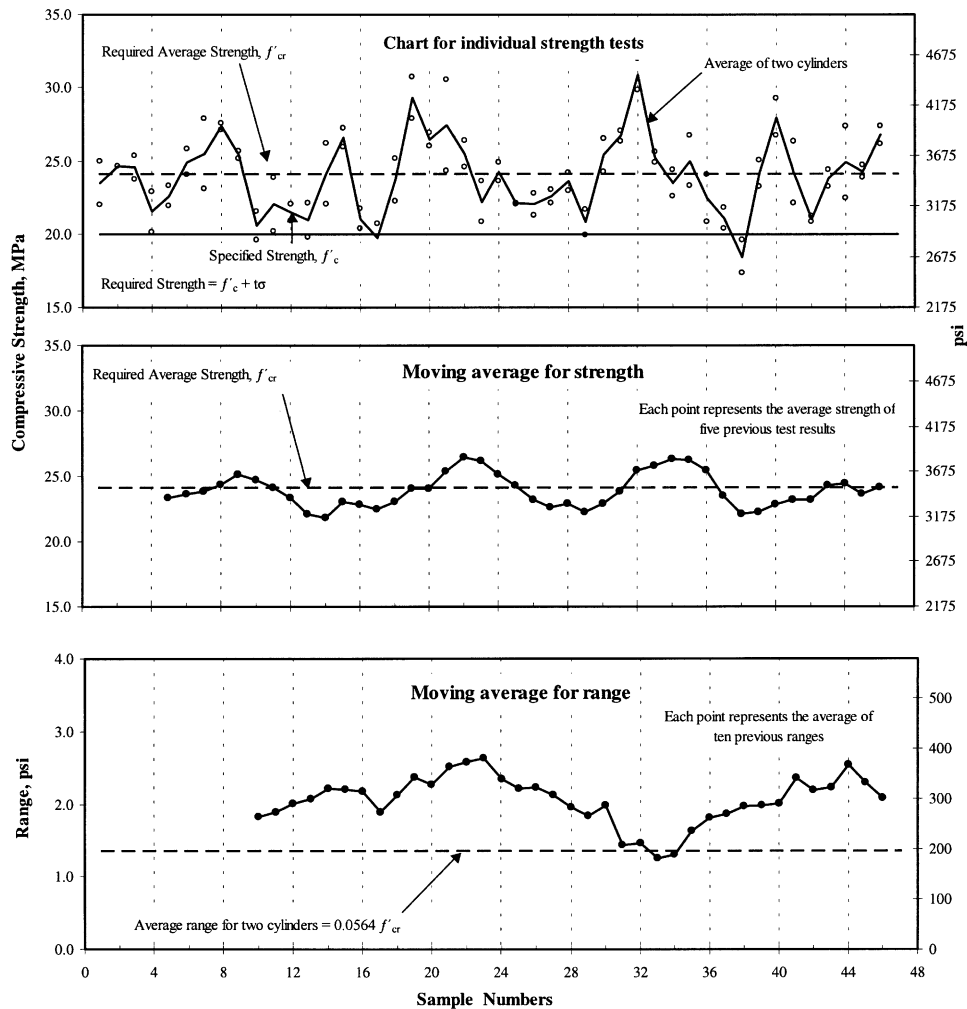
Fig. 5.1—Three simplified quality control charts (individual strength tests, moving average of five strength tests, and range of two cylinders in each test and moving average for range).

Manual on Presentation of Data and Control Chart Analysis, MNL 7. Trends become more readily apparent based on the pattern of previous results and limits established from ASTM MNL 7. Data that fall outside established limits indicate that something has affected the control of the process, and some type of action or interference with the existing process is typically required. In general, these action or process interference limit values are established using the guidelines published in this document, based on contract specifications or other values at which action should be taken. Frequently, the action or interference limits are equal to the acceptance criteria specified for a particular project.

Figure 5.1 illustrates three simplified charts prepared specifically for concrete control and are combined into one diagram. This technique permits evaluation of all charts simultaneously, which can ease analysis. While these charts may not contain all the features of formal control charts, they are useful to the engineer, architect, contractor, and supplier. Control charts are strongly recommended for concrete in continuous production over considerable periods.

5.5.1 *Simple strength chart*—The top chart in Fig. 5.1 shows the results of all strength tests plotted in succession based on casting date. The target for the average strength is established as indicated by Eq. (4-1a), Eq. (4-1b), or Table 4.2.

The chart often includes the specified strength and may include the acceptance criteria for individual tests. This chart is useful because it shows all of the available data but it can be difficult to detect meaningful shifts in a timely fashion.

5.5.2 *Moving average strength*—The middle chart in Fig. 5.1 shows the moving average of consecutive tests. This type of chart reduces the noise and scatter in the individual test chart. Trends in performance are more easily identified and will show the influence of effects, such as seasonal changes and changes in materials, more effectively. The chart often includes the acceptance criteria f_c' when the moving average of three tests is plotted.

The larger the number of tests averaged, the more powerful the chart is in helping identify trends. There is an obvious trade-off with timeliness, however. A trend should be identified as soon as possible so that appropriate corrective actions may be taken. Because the moving average of three consecutive strength tests is one of the compliance criteria of ACI 318, this parameter is frequently tracked in a control chart. Because tracking the moving average of three tests may not provide sufficient analytical power, the moving average of five consecutive strength tests is also frequently used. The number of tests averaged for this control chart and the appropriate interference limit can be varied to suit each job.

The concrete supplier with a large number of tests for a particular mixture can elect to track the moving average of 10 or 15 tests. A target value can be established based on f'_{cr}. While requiring significant amounts of data, any trends detected with this approach will necessarily be strong and shifts in average strength can be readily detected. The averages of 10 and 15 tests can also be used in mixture submittal documentation.

5.5.3 *Testing variability*

5.5.3.1 *Purpose*—The lower chart in Fig. 5.1 shows the moving average of the range, the maximum difference between companion cylinders comprising a single strength test, which is used to monitor the repeatability of testing. The laboratory has the responsibility of making accurate tests, and concrete will be penalized unnecessarily if tests show greater variations or lower average strength levels than actually exist. Because the range in strength between companion specimens from the same sample can be assumed to be the responsibility of the laboratory, a control chart for ranges should be maintained by the laboratory as a check on the uniformity of its operations. These changes will not reveal day-to-day differences in testing, curing, capping, and testing procedures or testing procedures that affect measured strength levels over long periods.

The average range of the previous 10 consecutive tests (sets of companion cylinders as discussed in Section 3.4.1) is typically plotted. The interference limits for this control chart are based on average strength and desired level of control.

5.5.3.2 *Calculation of acceptable testing variation*—Calculation of the acceptable range between companion cylinders of a test depends on the number of specimens in the group and the within-test variation, as discussed in Chapter 3. The following process can be used to establish interference limits for the moving average range control chart.

The expected value of the average range $\overline{R_m}$ can be determined by reformulating Eq. (3-5) as shown in Eq. (5-2).

$$\overline{R_m} = f'_{cr}\, V_1 d_2 \qquad (5\text{-}2)$$

The within-test coefficient of variation V_1 should not be greater than 5% for good control (Table 3.3). Therefore, the estimate of the corresponding average range will be

$$\overline{R_m} = (0.05 \times 1.128)\, f'_{cr} = 0.05640 f'_{cr} \qquad (5\text{-}3a)$$

for groups of two companion cylinders, or

$$\overline{R_m} = (0.05 \times 1.693)\, f'_{cr} = 0.08465 f'_{cr} \qquad (5\text{-}3b)$$

for groups of three companion cylinders. These interference limits are effective only after the average range, computed from companion cylinder strengths from at least 10 strength tests, has been calculated.

To be fully effective, maintain control charts on each project for the duration of the project. The testing laboratory should, as a minimum, maintain a control chart for average range and may also offer other control charts as a service to the engineer or architect. Concrete suppliers can track the moving average range on a mixture by mixture basis, because a single mixture can be used on multiple projects. Many suppliers track individual projects to obtain data for their own use.

5.6—Other evaluation techniques

A number of other techniques exist for evaluating series of data for quality-control purposes. As with basic control charts, these techniques were developed for general industrial applications but can be adapted for use with concrete properties. A complete description of these techniques is beyond the scope of this document, but the general outline of the cumulative sum (CUSUM) procedure, along with guidance on interpretation as applied to concrete properties, particularly compressive strength, is provided. A much more detailed description of analytical techniques and interpretation of the CUSUM technique can be found in Day (1991) and Dewar (1995); a simple example of this technique is provided in Appendix A.

5.6.1 *Overall variability and concrete supplier's variability*—In conventional practice, the mean compressive strength is estimated with as few as 10 tests, while at least 15 tests are needed to estimate the standard deviation. Changes in the mixture materials or proportions will have a larger effect on the average strength level than on the standard deviation. For these reasons, most control charts are based on averages of compressive strength. Monitoring the overall standard deviation can also provide insight into changes in the level of control or variability of production or raw materials for the concrete supplier.

An estimate of variation due to testing, the within-test standard deviation, can be obtained from the average range chart or by direct computation. As discussed in Chapter 3, the combined variation due to variation in raw materials and production, which can be termed the concrete supplier's or producer's variability, can be determined knowing the overall standard deviation and the within-test standard deviation. The producer's variability, as measured by the standard deviation, is the square root of the difference of the square of the overall standard deviation and the within-test standard deviation, as shown in Eq. (3-8), provided in slightly different form as Eq. (5-4).

$$s_{Producer} = \sqrt{s^2_{Overall} - s^2_{Within\ test}} \qquad (5\text{-}4)$$

The concrete supplier can directly track the variability of the production process. If the within-test standard deviation is reasonably consistent, as it is in a well-run testing program, the supplier can simply track overall standard deviation, which is easier. For a constant within-test variation, changes in the overall standard deviation can indicate changes in either the raw materials or the production of concrete and are, therefore, of value to the concrete supplier.

Control charts should incorporate a moving standard deviation of at least 10 and preferably 15 tests. With modern, computer-based spreadsheets this type of control chart is not difficult to implement. Due to the large number of tests required, the usefulness of this control chart to rapidly identify changes in the process is limited, however. Another technique (CUSUM), described Section 5.6.2, typically provides rapid identification of changes in various measured properties of concrete.

5.6.2 *CUSUM*—In both quality control and problem resolution there is a need to identify assignable causes in average strength level or in variability of strength. Early detection of

changes in the average strength level is useful so that causes may be identified and steps taken to avoid future problems or reduce costs. This requires being able to distinguish between random variations and variations due to assignable causes.

The cumulative sum (CUSUM) chart provides a method for detecting relatively small but real changes in average concrete strength or some other aspect of concrete performance. It can also help identify approximately when those shifts began and the approximate size of the shift. CUSUM will generally provide greater sensitivity in detecting a small, systemic change in average strength than the basic control charts discussed in this chapter and will detect these changes faster (Box, Hunter, and Hunter 1978; Day 1991; Dewar 1995; Day 1995).

There are limitations in using a CUSUM chart, particularly when data are highly variable, but the technique is only slightly more complicated than conventional strength analysis and is easily implemented either manually or using a spreadsheet or commercially available computer program. As with any single technique, the conclusions reached using a CUSUM chart should be confirmed by additional analysis or investigation before making critical decisions.

Although probably most commonly used to monitor compressive strength, it can be used with any number of variables. Day (1995) reports successfully using CUSUM charts to monitor a variety of concrete properties. He also notes that by monitoring multiple CUSUM charts and tracking a variety of properties simultaneously, the probability that a change will be missed or misdiagnosed is reduced. A review of the theory of the CUSUM technique and an example are provided in Appendix A.

CHAPTER 6—REFERENCES
6.1—Referenced standards and reports
The standards and reports listed below were the latest editions at the time this document was prepared. Because these documents are revised frequently, the reader is advised to contact the sponsoring group if it is desired to refer to the latest version.

American Concrete Institute
301 Specifications for Structural Concrete
318 Building Code Requirements for Structural Concrete and Commentary

ASTM
MNL7 Manual 7 on Presentation Data and Control Chart Analysis, 6th Edition
C 31 Practice for Making and Curing Concrete Test Specimens in the Field
C 39 Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens
C 94 Specification for ready-Mixed Concrete
C 684 Standard Test Method for Making, Accelerated Curing, and Testing Concrete Compression Test Specimens
C 802 Practice for Conducting an Interlaboratory Test Program to Determine the Precision of Test Methods for Construction Materials

C 918 Standard Test Method for Developing Early-Age Compression Test Values and Projecting Later-Age Strengths
D 3665 Standard Practice for Random Sampling of Construction Materials
E 178 Practice for Dealing with Outlying Observations

American Association of State Highway & Transportation Officials
TP 23 Edition 1A—Standard Test Method for Water Content of Freshly Mixed Concrete Using Microwave Oven Drying

British Standards Institution
BS 5703-3 Guide to data analysis and quality control using CUSUM techniques. CUSUM methods for process/quality control by measurement

These publications may be obtained from these organizations:

AASHTO
444 N. Capitol St. NW Ste 249
Washington, D.C. 20001
www.aashto.org

American Concrete Institute
38800 Country Club Dr.
Farmington Hills, Mich. 48331
www.concrete.org

ASTM
100 Barr Harbor Dr.
West Conshohocken, Penn. 19428
www.astm.org

British Standards Institution
389 Chiswick High Rd.
London W4 4AL UK
www.bsi.or.uk

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Day, K. W., 1995, *Concrete Mix Design, Quality Control and Specification*, E&FN Spon, 461 pp.

Day, K. W., 1991, "Computerized Concrete QC Using Spreadsheets and CUSUM Graphs," *ACI Compilation No. 13*, American Concrete Institute, Farmington Hills, Mich.

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Erntroy, H. C., 1960, "The Variation of Works Test Cubes," *Research Report* No. 10, Cement and Concrete Association, London, 28 pp.

Leming, M. L., 1999, "Probabilities of Low Strength Events in Concrete," *ACI Structures Journal*, V. 96, No. 3, May-June, pp. 369-376.

Metclaf, J. B., 1970, "The Specification of Concrete Strength, Part II, The Distribution of Strength of Concrete for Structures in Current Practice," *RRL Report* No. LR 300, Road Research Laboratory, Crawthorne, Berkshire, pp. 22.

Murdock, C. J., 1953, "The Control of Concrete Quality," *Proceedings,* Institution of Civil Engineers (London), V. 2, Part 1, July, pp. 426-453.

Natrella, M. G., 1963, "Experimental Statistics" *Handbook* No. 91 (reprinted 1966 with corrections), U.S. Department of Commerce, National Bureau of Standards (now National Institute of Standards and Technology, NIST, Gaithersburg, Md.).

Neville, A. M., 1959, "The Relation Between Standard Deviation and Mean Strength of Concrete Test Cubes," *Magazine of Concrete Research* (London), V. 11, No. 32, July, pp. 75-84.

Rüsch, H., 1964, "Statistical Quality Control of Concrete," *Materialprufung* (Dusseldorf), V.6, No. 11, Nov., pp. 387-394.

APPENDIX A—EXAMPLES OF CUSUM TECHNIQUE
A.1—Introduction

The cumulative sum, or CUSUM, chart can be used to help detect relatively small changes or shifts in average concrete strength or other concrete characteristics relatively quickly. This Appendix reviews the theory and demonstrates the application of the technique, which is only slightly more complicated than conventional strength analysis and is easily implemented either manually or using computer-based spreadsheets. As with any technique, the conclusions reached using a CUSUM chart should be confirmed by additional analysis or investigation before making critical decisions.

A.2—Theory

Deviations of individual test results from the mean have a normal distribution even if the parent distribution is not normal. Because the distribution of concrete compressive strength frequently approximates a normal distribution, the distribution of deviations from the mean strength is normal to a very good approximation. The average deviation from the mean is approximately zero for a stable process. Therefore, if ε_i is the difference between the average compressive strength and the *i*-th compressive strength test, or

$$\varepsilon_i = \bar{X} - X_i \qquad (A-1)$$

where $\bar{X}$ is the average compressive strength (established over a suitable time period), and X_i is the *i*-th compressive strength test, then:

$$\sum_{i=1}^{N} \varepsilon_i = \sum_{i=1}^{N} (\bar{X} - X_i) \approx 0 \qquad (A-2)$$

as long as the average strength does not change and the number of tests (*Nt*) is sufficiently large.

If a change occurs in some element of the concrete materials, production, handling, testing, in seasonal variation, or any other assignable cause, variation deviations of test results about the mean are no longer random and ε_i will no longer average 0. If the assignable cause is constant, the sum of ε_i will change in a linear fashion

$$\sum_{i=1}^{N} \varepsilon_i = \sum_{i=1}^{N} ((\bar{X} + \delta) - X_i) \approx (N - m)\delta \qquad (A-3)$$

where δ is the value of the change in the average strength, and *m* is the test in the sequence at which the change occurs. A positive δ means that there has been an increase in the average strength and the cumulative sum of the differences between the original average strength and the individual tests increases approximately linearly. If δ is negative, the average strength has decreased and the cumulative sum will decrease approximately linearly.

A shift in the average compressive strength can be detected by plotting the cumulative sum of the ε_i in sequence. A change in the slope of the CUSUM plot indicates a difference in the average strength from the assumed value. Once a trend is detected, further analysis of both the CUSUM chart and the concrete testing, handling, materials, production, or environment should be conducted to determine the likely source of the change.

A.3—Calculations

Previous data for a certain concrete mixture, produced to provide an f_c' of 30 MPa (4350 psi), indicate an average strength of 35.8 MPa (5190 psi). During a project, sequential compressive strength data become available. The CUSUM chart may be constructed from the data as shown in Table A.1. Sample calculations are shown for the first few entries.

The moving average of three tests (MA3) is also provided, because it is a commonly monitored quality-control variable. All data (compressive strengths, CUSUM, averages, and standard deviation) are reported to three significant figures.

Using these 19 test results only, the average compressive strength is 34.8 MPa (5050 psi) and the sample standard deviation is 2.41 MPa (350 psi). Based on only these 19 strength test results, the required average strength f_{cr}' is 33.5 MPa (f_{cr}' = 30 MPa plus the larger of (1.34 × 2.64) or (2.33 × 2.64 – 3.5), where 2.64 is the product of the standard deviation (2.41 MPa) and the interpolated modification factor from Table 4.1). It may be seen that:

1. The low standard deviation indicates apparent excellent control;

2. The average strength is greater than f_c', f_{cr}' but 1.0 MPa (150 psi) less than the average strength determined from the previous data;

3. There are no instances where a test is less than $f_c' - 3.5$ MPa (500 psi); and

Table A.1—Data for CUSUM example

No.	Test result, MPa	Difference, MPa	CUSUM, MPa	MA3, MPa
1	(37.1 + 36.9)/2 = 37.0 (average of two cylinders)	37.0 – 35.8 = 1.2	1.2	—
2	34.7	34.7 – 35.8 = –1.1	1.2 + (–1.1) = 0.1	—
3	32.8	32.8 – 35.8 = –3.0	0.1 + (–3.0) = –2.9	34.8
4	37.8	37.8 – 35.8 = 2.0	–2.9 + 2.0 = –0.9	35.1
5	35.2	–0.6	–1.5	35.3
6	36.5	0.7	–0.8	36.5
7	39.6	3.8	3.0	37.1
8	37.6	1.8	4.8	37.9
9	33.6	–2.2	2.6	36.9
10	33.6	–2.2	0.4	34.9
11	35.1	–0.7	–0.3	34.1
12	31.8	–4.0	–4.3	33.5
13	36.4	0.6	–3.7	34.4
14	32.5	–3.3	–7.0	33.6
15	31.0	–4.8	–11.8	33.3
16	31.7	–4.1	–15.9	31.7
17	37.0	1.2	–14.7	33.2
18	34.5	–1.3	–16.0	34.4
19	32.9	–2.9	–18.9	34.8

Notes: No. is the sequence number. Test is the average compressive strength, MPa; in this case, of at least two cylinder strengths. Difference is the difference, MPa, between the compressive strength test result and the previously determined average strength. CUSUM is the cumulative sum, MPa, of the differences. MA3 is the moving average of three consecutive compressive test results, MPa.

4. There are no instances where a moving average of a three result is less than f_c'.

All of these indicate satisfactory performance contractually and a process apparently in control.

Simple control charts (Fig. A.1 and A.2) do not indicate any significant problems, although the moving average does trend slightly lower for a period of time. The CUSUM chart, however, (Fig. A.3) clearly indicates that a shift has occurred. A decrease in the average strength level apparently originates no later than the 10th strength test.

A simple estimate of the decrease in strength level that occurred can be made from the slope of the CUSUM chart. The slope from Test No. 10 to Test No. 19 can be estimated as -18.9 (the cumulative sum of the differences at Test No. 19) divided by 9 (19-10 tests), or about 2.1 MPa (300 psi).

A.4—Analysis and comparison with conventional control charts

The preceding example demonstrates several of the potential advantages of the CUSUM chart method. No obvious indication of a change in the data is found in a simple plot of the strength data itself (Fig. A.1), because there is too much scatter due to random variation to easily detect trends or small changes.

Detection of trends or changes in variables can be provided by moving average charts, which improve trend detection by reducing the effect of random variation. Increasing the number of data points averaged increases the ease with which the trend is detected and improves the reliability of the trend identification, that is, the likelihood that the trend indicates a real change. Improvement comes at the price of having to wait for more data points. While the moving average of three
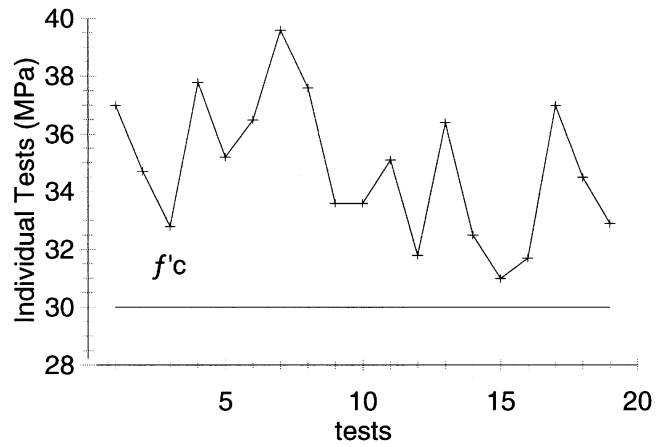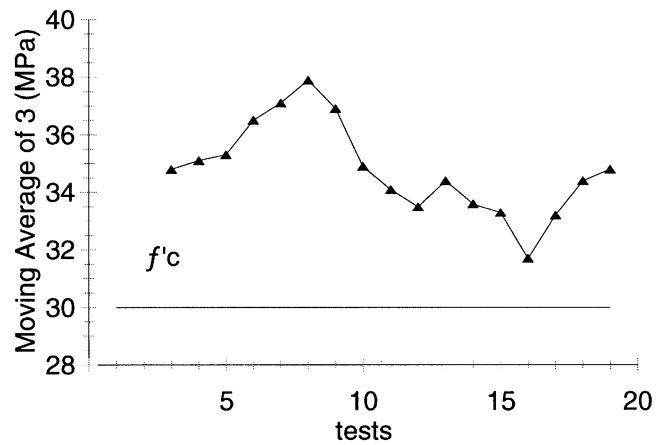

Fig. A.1—Individual test QC chart.

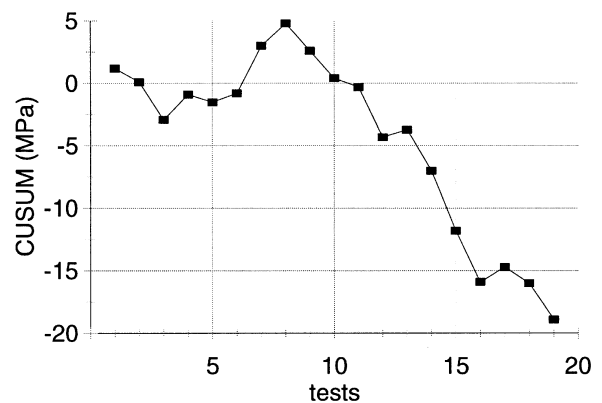*Fig. A.2—Moving average of three QC chart.*

Fig. A.3—CUSUM QC chart for compressive strength.

provides some improvement in trend detection, averaging over only three tests is not a strong indicator.

The moving average of three charts (Fig. A.2) does show a slightly lower trend in the data for a period of time. It, however, is not immediately apparent from Fig. A.2 that a significant change has occurred, or that if it has occurred, what the size of the change is or whether the trend has, in fact, been reversed near the end of the available data. Additional statistical evaluation might be initiated, but it is not clear that any corrective action is warranted and, in practice, none would probably be undertaken based on this analysis alone.

The primary advantages of the CUSUM chart are that small changes may be detected sooner than with the other methods described, and the timing and size of these changes may be estimated directly. As with any analytical tool, the CUSUM method has some limitations.

A.5—Management considerations of interference

A perfect technique to identify shifts in average strength will identify all real shifts without falsely classifying a random variation as a shift. Practical techniques balance the two types of errors:

- Type I (rejecting a true shift); and
- Type II (accepting a false shift) errors.

The probability of an error increases when analysis is based on fewer data points, or shorter runs, but analysis based on shorter runs is frequently preferable so that deviations can be corrected as soon as possible. Both types of errors have associated costs.

An unexpected decrease in average strength will typically prompt both corrective action to increase the average reported strength and an investigation to determine the source(s) of the decrease. There is a management cost associated with this investigation, and there may be a cost associated with at least a temporary increase in cost of the concrete as the average strength of the mixture is increased. These costs can be offset by the reduction in risk associated with a low strength on the job. If no real problem is detected or subsequent analysis indicates the original analysis was incorrect, the losses are real but may be small compared with the reduction in potential costs. Overreaction and overcorrection can also cause problems, however.

Interference with the process in the absence of an assignable cause can lead to several difficulties. Once a change in the average strength has been implemented, the change in the average affects the CUSUM chart as would any other assignable cause. Both the chart and the process should be "rezeroed" to the new average to account for the interference with the process. Multiple changes over a relatively short time can shift the true average sufficiently from the presumed average that the chart provides considerably less useful information. Over-correction should also be avoided due to the additional costs of unnecessary changes and to inducing more variation in the data than would have occurred in the absence of the interference.

The relative costs of these two errors versus the delay in identifying a real change usually mean that interference will occur more often than actually needed, but analyze each situation separately. Determining when to change the average strength is not always obvious.

A.6—Establishing limits for interference

Interference as early as possible is usually desirable. Analysis of the CUSUM chart can provide an estimate of δ using the slope and the approximate time when the change occurred using the intercept with 0. In general, the steeper the slope, the stronger the trend and the larger the change; the longer the trend, the more certain the change. In a fast-moving project, nine or 10 test results can easily accrue in a short period of time and early interference can be impractical. Where results between test number 10 and test number 19, in the example, accrue over several days, the strength of the trend makes it

unlikely that a change in the process would be delayed past, perhaps, test number 15, or only after about five data points.

In the example data used, the change is readily apparent due to the relatively low standard deviation and relatively large strength change (about 2.1 MPa [300 psi]). Smaller changes in average strength result in flatter slopes that take a longer time to identify. Identification in these situations is not as critical as larger changes resulting in steeper slopes that are easier to identify conclusively. When small changes in average strength occur combined with high variability, identification in a timely manner can be difficult. Both qualitative and statistically rigorous guidelines for interference depend on variability and the required level of certainty in the analysis. In practical problems, the costs of changes relative to the effect of the changes are be considered.

To determine the cause or causes of a change, the time at which the change occurred should be estimated relatively accurately. In the example data, it is not clear from Fig. A.3 if the change occurred at the 10th test, the 8th test, or some other test. Methods for determining when a change in the average strength has occurred and when a run is simply due to random variation based on statistically rigorous analysis are desirable; however, they are neither self evident nor as powerful as might be desired.

Day (1995) and Brown (1984) report the use of a truncated V-mask as described in BS 5703 to identify when a statistically significant change has occurred. The V-mask, which is a linear function of the process standard deviation, was developed for any CUSUM analysis. Day (1995), however, notes that the V-mask for concrete strength CUSUM analysis often requires many data points and that simple examination of the CUSUM chart is frequently adequate for an experienced concrete analyst, particularly if multiple measures of concrete mixture behavior are plotted simultaneously. Day reports that a real change in the concrete mixture can frequently be identified with only a few points on CUSUM charts, which plot several different measures of concrete mixture behavior.

Judgement is required in visual trend identification. When developing a spreadsheet analysis, it is possible to graph the data such that random changes appear to be significant. In Fig. A.4, data from only the first 12 tests are shown. There appears to be an upward trend in the data from Test no. 3 to Test no. 8, and a downward trend from Test no. 8 to Test no. 12. These trends are not statistically significant, however. The scale of the graph should be established appropriately.

A.7—Difficulties with CUSUM chart

There are several situations that can produce difficulties with CUSUM analysis. Some of the more common problems arising with interpretation are listed below:

1. The CUSUM graph is sensitive to the average strength value used in calculating the cumulative sum. An error in determining this value or the use of a target strength instead of the population average strength will result in a non-zero average for the cumulative sum. In Fig. A.5, the CUSUM graph is shown with three different initial estimates of the average compressive strength. One curve represents using 35.8 MPa (5190 psi) as the initial estimate of the average strength (except for the scale, this is the same as Fig. A.3). The other two curves represent the effects of errors of ± 2.0 MPa (290 psi) in the estimate. Small errors in estimating the
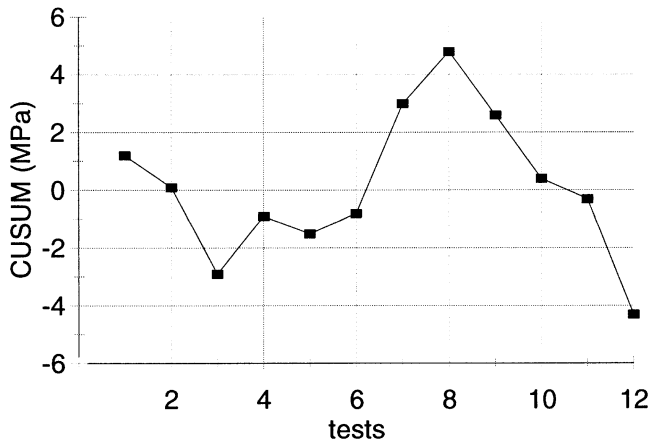
Fig. A.4—CUSUM chart with partial data.

Fig. A.5—CUSUM QC chart for compressive strength with three different initial average strengths.

average strength can compound rapidly creating misleading results.

2. A single aberration in the data can create what appears to be an offset in a trend. If a trend occurs with a single offset, it may often be ignored in the analysis.

3. A single concrete mixture will typically have a slightly different average strength for each different project. Different contractors can exercise different levels of control and different testing agencies will invariably provide test data with slightly different averages. A series of test results from different jobs and testing agencies that are intermixed may show random variation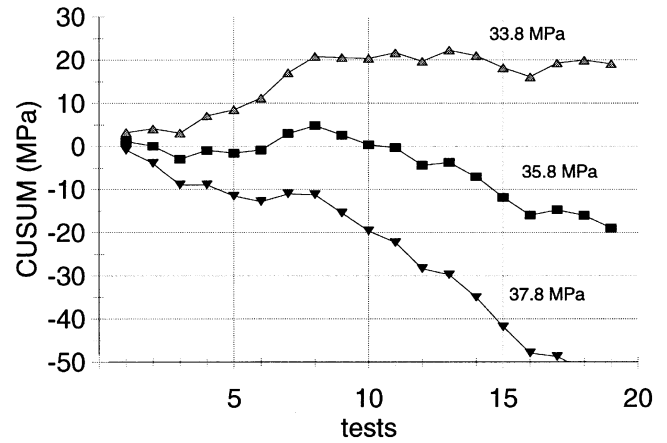. If the plotted data consist of runs of test results from different jobs and testing agencies, the differences in average of each set of data may produce a trend in the CUSUM chart. When a statistically significant trend has been found with multiple projects, the sample standard deviation should be calculated based on multiple data sets rather than one set. This will provide a more accurate, and typically smaller, estimate of the true standard deviation.

4. An estimate of the point at which the change in average strength occurred can be obtained from regression analysis of the CUSUM chart. The extra precision presumably obtained in such an analysis is rarely of practical value.