

CHAPTER 8

Economic Efficiency Impacts

Hatred of costs can often be more intense than love of benefits.

—Lord Bertrand Russell (1872–1970)

INTRODUCTION

In Chapter 4 we presented procedures for planning-level estimation of the costs of construction, preservation, and fixed-facility operations, as well as other project costs typically borne by a transportation agency or operator. In Chapters 5 to 7 we discussed procedures for evaluating the monetary benefits of transportation investments from the perspectives of specific performance measures (travel time, safety, and vehicle operation). For a given transportation problem, there are typically several alternative decisions or actions, each with its unique set of costs and benefits. The combined monetary cost and benefit impact of each alternative can be represented by a performance measure known as *economic efficiency*, which is derived using the principles of economic analysis. Economic analysis is a decision-making tool that assesses the efficiency of investments from a monetary standpoint and incorporates the monetized costs and benefits associated with alternative decisions and actions. Across alternatives, differences in the amounts and timings of costs and benefits are likely to influence the relative attractiveness of such alternatives even if the initial investment requirements are not very different. Decisions to select the best of several alternative actions are encountered at every stage of the transportation project development process, and such choices are often made on the basis of economic considerations. As such, economic efficiency analysis (often referred to as *benefit–cost analysis*) can help guide transportation decision making in the various areas of design, construction, preservation, and operations.

8.1 INTEREST EQUATIONS AND EQUIVALENCIES

The fundamental principle underlying all engineering economic efficiency analyses is that the value of money is related directly to the time at which the value is considered. A given amount of money at the current time is not equivalent to the same amount at a past or future year, due to the combined forces of inflation and opportunity cost that erode the value of money over time. *Inflation* refers to the increase in prices of goods and services with time and is reflected by a decrease in the purchasing power of a given sum of money with time. *Opportunity cost* is the income that is foregone at a later time by not investing a given sum of money at a current period. In an engineering economic analysis of alternatives, all monetary amounts are in constant dollars. Inflation is not considered on the assumption that all costs and benefits of various alternatives are affected equally by inflation. If there is reason to believe that future component prices will be affected differently, appropriate adjustments should be made to reflect the differential impact of inflation (AASHTO, 1977).

8.1.1 Cash Flow Illustrations

The time stream of amounts of money that occur within a given period can be displayed either as a *cash flow table* or a *cash flow diagram*. On a cash flow table, there are two columns: one for time and the other for amount. On a cash flow diagram, time is represented on a horizontal axis, while vertical arrows depict the inflow or outflow of money at various points in time. The sign of the amount and the direction of the arrows in cash flow tables and figures, respectively, indicate the movement of the amount. A popular convention is to represent money “coming in” (i.e., returns or benefits) by positive signs and upward arrows pointing away from the horizontal time line in the cash flow table and diagram, respectively. This convention also stipulates that money “going out” (i.e., disbursements or costs) is represented by negative signs and downward arrows in the cash flow table and diagram, respectively. The entire *payment period* (often referred to as the *analysis period*, *planning horizon*, or *planning period*) is represented by the interval between the present time (often denoted by time = 0) and the end of the period (denoted by time = N). The planning period is typically divided into a number of equal periods called *compounding periods*. Each period is typically taken as one year.

8.1.2 The Concept of Interest

The amount by which a given sum of money differs from its future value is typically represented as *interest*.

Borrowed money to be paid back to a financial institution at a future time must comprise the initial amount (principal) plus interest. This reflects the fact that the value of the initial amount is not the same as the value of the amount at the time of payback. Interest is therefore described as the price of borrowing money, or simply, the time value of money, and the change in interest over time is referred to as the *interest rate*. The interest rate is used to determine the future value of a present sum or cash flow and the *discount rate* is the interest rate used in determining the present value of a future sum or cash flow.

A 10% annual interest rate indicates that for every dollar borrowed in the initial year, 10 cents must be paid as interest at the end of each year. Central banks typically control interest rates to remedy current or expected economic problems. For instance, in a sluggish economy, the U.S. Federal Reserve Board decreases interest rates to discourage saving and encourage individual spending and business investments; interest rates are increased when the economy is overheated. In a stable economy such as those of developed countries, interest rates are typically lower than those in economies with high inflation and a low certainty of investment returns.

8.1.3 Types of Compounding and Interest Rates

The interest rate associated with a borrowed sum of money could take many forms, such as being simple

or compound, discrete, continuous, fixed, or variable (Figure 8.1). In *simple interest* computations, the amount of interest at the end of each period is the same, as each of such amounts are a *fixed* percentage of the initial amount. The amount of compound interest in a given period is the interest charged on the total amount owed at the end of the preceding period (i.e., the sum of the principal and the previous period’s amount of interest). Therefore, amounts borrowed on compound interest involve higher payments for amortization. In the current business environment, interest is typically computed using compound interest rates.

Values of interest that are computed only at the end of each compounding period and with a constant interest rate are typically referred to as *fixed periodic rates*. In such instances, there is only one compounding period (e.g., a fixed annual rate refers to an interest rate with a one-year compounding period). In many cases, the compounding period is less than one year (quarterly, monthly, or weekly). In the financial environment, it is customary to quote interest rates on an annual basis followed by the compounding period if different from one year in length. For example, a case where the interest rate is 5% per interest period and the interest period is six months may be described as “10% compounded semiannually.” In this case, the annual rate of interest (10%) is referred to as the *nominal interest rate*. Close examination of

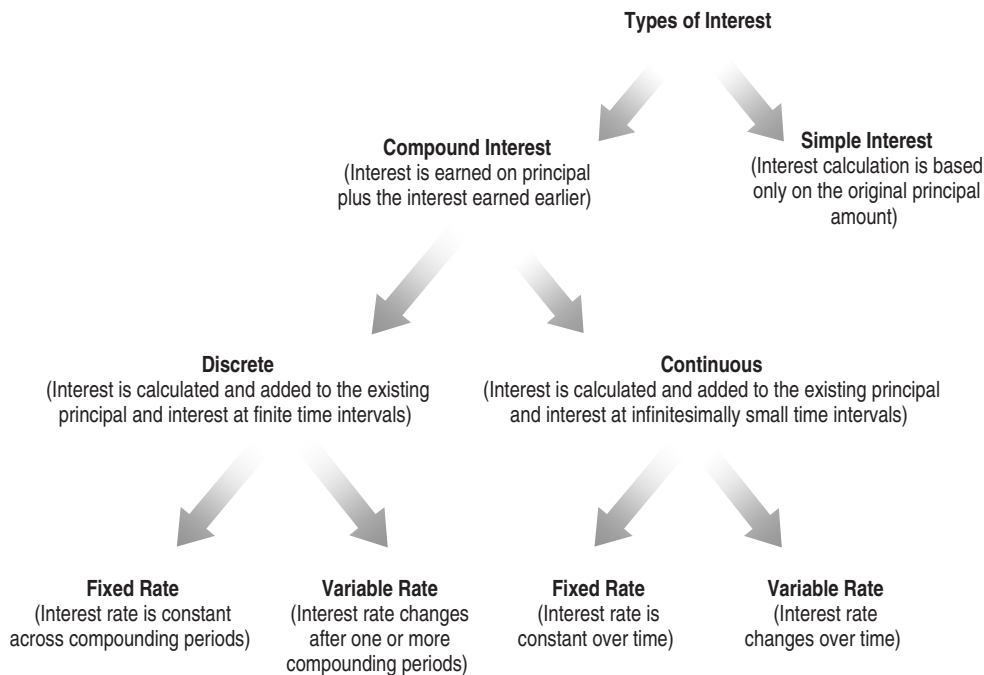


Figure 8.1 Types of interest rates.

this case would show that the actual annual rate on the principal is not 10% but a rate that exceeds 10% because compounding occurs twice during the year. The actual or exact rate of interest earned on the principal during one year is known as the *effective interest rate*, and can be computed using the equation $r_e = (1 + r_n/m)^m - 1$, where r_n is the nominal interest rate per year, r_e the effective interest rate (when compounding occurs m times during the year), and m the number of interest periods in a year (because $m > 0$, $r_e \geq r_n$). For example, if compounding occurs every four months (i.e., three times a year) and the nominal annual interest rate is 5%, then the effective annual interest rate is: $(1 + 0.05/3)^3 - 1 = 5.08\%$.

8.1.4 Interest Equations and Key Variables

Interest equations, also referred to as *equivalency equations*, are relationships between amounts of money that occur at different points in time and are used to estimate the worths of a single amount of money or a series of monetary amounts from one time period to another to reflect the time value of money. The key components of such relationships are the interest factors which are functions of the interest rate and the payment period. Interest factors are expressed as a formula (see Tables 8.1 and A8.1) or as a table of values derived from such a formula (see Table A8.3) and are provided separately for discrete compounding and continuous compounding of the interest rate. Table A8.1 presents interest factors for an annual series that follow an arithmetic gradient pattern for the case of discrete compounding. In some cases the analyst may be faced with nonuniform but systematic annual payments that will need conversion to a uniform annual series to facilitate the analysis. The functions for converting a few selected nonuniform series to their equivalent uniform annual series are presented in Table A8.2. Interest equations typically involve the following five key variables: P , the initial amount (at time = 0); F , the amount at a specified future period (at time = N); A , a periodic (typically at the end of each year) amount; i , the effective interest rate for the compounding period; and N , a specified number of compounding periods, or the *analysis period*.

(a) *Analysis Period* The analysis period, which is often referred to as the *project time horizon*, needs to be determined prior to the economic efficiency analysis of a project. The selection of an appropriate analysis period involves trade-off between two considerations (Dickey and Miller, 1984).

1. On one hand, a long analysis period often seems appropriate because transportation facilities are typically designed to provide service for generations, and it is often preferable to select an analysis period that is equal or close to the service life; otherwise, the often problematic issue of accounting for remaining service lives or residual values of the facility and of its user costs may arise.
2. On the other hand, when the analysis period selected is too long, the effect of discounted facility preservation and maintenance costs and user benefits over the analysis period would overwhelm the initial costs and may render initial cost amounts insignificant. Second, long analysis periods may be unrealistic in cases where the regional or national economy is prone to a high degree of uncertainty due to fluctuating economic trends (which invalidate the interest rate values used for the analysis), political upheavals (particularly in some developing countries), delays in starting or completing the project construction, and technological changes in facility rehabilitation and maintenance that change the benefits and costs associated with annual and periodic expenditure streams from previously established values used for the initial analysis.

As such, in selecting an analysis period, factors that need to be taken into account include the project type (and consequently, the length of service life), the variation in service lives of alternative investment options, the nature of the regional or national economy (developed vs. developing), the forecast uncertainties, the social discount rate and its stability, the rate of technological change, possible competing/complementary facilities, and the likelihood of construction or implementation time delays.

In cases where competing alternatives have different service lives, the best economic efficiency criterion to use is the equivalent uniform annual values of costs and benefits. It may also be possible to express the analysis period as a common multiple of the service lives of the investment alternatives (a similar replacement is assumed when each alternative reaches the end of its service life, this continuing until the end of the analysis period). Where the analysis period does not equal (or cannot be expressed as) a common multiple of the service lives of competing investment options, a replacement cycle to perpetuity may be assumed. Also, in the case where the same analysis period must be used for alternatives with different service lives, some alternatives would involve a residual value at the end of the analysis period, and such values would need to be translated into monetary values so that they can be considered fully in the economic efficiency analysis.

Table 8.1 Interest Equations for Discrete and Continuous Compounding^a

| Description | Cash Flow Diagram | Computational Formula | Factor Computation |
|---|-------------------|---|---|
| Finding the future compounded amount (F) at the end of a specified period given the initial amount (P) and interest rate | | $F = P \times \text{SPCAF}$ Single payment compound amount factor, SPCAF ($i\%$, N) | $\text{SPCAF} = (1 + i)^N$ $\text{SPCAF} = e^{Ni}$ |
| Finding the initial amount (P) that would yield a given future amount (F) at the end of a specified period given the interest rate | | $P = F \times \text{SPPWF}$ Single payment present worth factor, SPPWF ($i\%$, N) | $\text{SPPWF} = \frac{1}{(1 + i)^N}$ $\text{SPPWF} = \frac{1}{e^{Ni}}$ |
| Finding the uniform yearly amount (A) that would yield a given future amount (F) at the end of a specified period given the interest rate | | $A = F \times \text{SFDF}$ Sinking fund deposit factor, SFDF ($i\%$, N) | $\text{SFDF} = \frac{i}{(1 + i)^N - 1}$ $\text{SFDF} = \frac{e^i - 1}{e^{Ni} - 1}$ |
| Finding the future compounded amount (F) at the end of a specified period due to annual payments (A) given the interest rate | | $F = A \times \text{USCAF}$ Uniform series compound amount factor, USCAF ($i\%$, N) | $\text{USCAF} = \frac{(1 + i)^N - 1}{i}$ $\text{USCAF} = \frac{e^{Ni} - 1}{e^i - 1}$ |
| Finding the initial amount (P) that is equivalent to a series of uniform annual payments (A) given the interest rate | | $P = A \times \text{USPWF}$ Uniform series present worth factor, USPWF ($i\%$, N) | $\text{USPWF} = \frac{(1 + i)^N - 1}{i(1 + i)^N}$ $\text{USPWF} = \frac{1 - e^{-Ni}}{e^i - 1}$ |
| Finding the amount of uniform yearly payments (A) that would completely recover an initial amount (P) at the end of a specified period, given the interest rate | | $A = P \times \text{CRF}$ Capital recovery factor, CRF ($i\%$, N) | $\text{CRF} = \frac{i(1 + i)^N}{(1 + i)^N - 1}$ $\text{CRF} = \frac{e^i - 1}{1 - e^{-Ni}}$ |

^aIn the fourth column, upper and lower equations are for discrete and continuous compounding, respectively. For fixed discrete compounding yearly, i = nominal interest rate and N represents the number of years. When there is more than one compounding period per year, the equations and tables can be used as long as there is a cash flow at the end of each interest period. In that case, i represents the interest rate per period and N is the number of periods. When the compounding is more frequent than a year, but the cash flows are annual, the equation can be used with N as number of years and i as the effective annual interest rate. Interest factors may be computed using equations provided on this page or read from Table A8.3.

(b) *Interest Rate* Use of low interest rates for economic efficiency analysis tends to favor alternatives with high initial costs or with benefits occurring far off in the later years of the analysis period because benefits and costs that lie farther into the future receive more weight than do those that are more imminent. For a similar reason, using high interest rates tends to favor alternatives with low initial costs and/or have benefits that mostly occur early in the analysis period. As such, the value chosen as the interest rate has a profound influence on the outcome of economic efficiency evaluation.

The “real” (constant-dollar) interest rates used for economic efficiency impact evaluation of transportation investments typically range from 4 to 8%. The U.S. Office of Management and Budget recommends the use of a rate of 7% to represent the private-sector rate of return on capital investment. However, other agencies typically use lower rates to take cognizance of the social rate of time preference. A recent survey found that a 4% rate has been used for many years by the Army Corps of Engineers (which had the effect of favoring projects with long service lives or with net benefits occurring many years into the “future”), a 5% rate for several states, a 7% rate by the United Kingdom Department of Transport, and an 8% rate by the British Columbia Ministry of Transportation and Highways (Weisbrod, 2000).

(c) *Residual Value* In some economic efficiency studies, the analysis period is not equal to the service life of the facility. As such, there is some finite *residual value* of the facility remaining at the end of the analysis period, and such a “benefit,” or negative cost, needs to be taken into consideration in the analysis. In some cases, an agency may incur net residual costs at the end of the analysis period, often due to salvage or disposal expenses. The two fundamental components of residual value are the *remaining service life* and the *salvage value*.

The remaining service life (RSL) of a facility at the end of the analysis period is the additional time during which the facility can still provide acceptable levels of service. Failure to account for different RSL values across alternatives can result in bias in the evaluation. Salvage value is the value of recovered or recycled materials and assumes that the transportation facility (or component thereof) is removed from service or replaced at the end of the analysis period. A difference between RSL and salvage value is that the former is used for evaluation in cases where the transportation facility continues to operate at the end of the analysis period, whereas the latter is used when the end of the analysis period coincides with the termination of the facility.

Example 8.1 Five years from now, an airport authority intends to rehabilitate its runways at a cost of \$1.5 million. Ten years from now, the runways will be replaced. At that time, the salvage value of reclaimable materials will be \$0.75 million. Assuming an interest rate of 8%, find the combined present worth of these costs.

SOLUTION

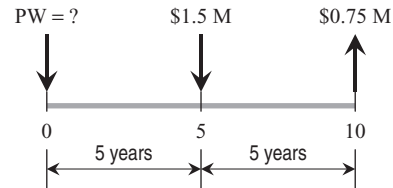


Figure EX8.1

$$PW = 1.5M \times SPPWF(8\%, 5) - 0.75M \times SPPWF(8\%, 10) = \$347,400.$$

Example 8.2 A major corridor investment is expected to yield \$50,000 per year in reduced crash costs, \$20,000 per year in reduced vehicle operating costs, and \$405,000 per year in reduced travel-time costs. What is the combined present worth of these benefits? Assume that the interest rate is 5%; the analysis period is 20 years; and salvage value is \$1.0 million

SOLUTION

$$PW = (50,000 + 20,000 + 405,000) \times USPWF(5\%, 20) + 1,000,000 \times SPPWF(5\%, 20) = \$6.30M$$

(d) *Interest Equations for Continuous Compounding of the Interest Rate* In some cases of economic evaluation, not only is the interest rate compounded several times within a year, but it is possible for the frequency of compounding of such rates (and consequently, their periods) to be so many that the number of compounding periods can be considered infinite. Consider the general case of an investment where i is the nominal interest rate per year and m is the number of interest periods in a year. This means that the interest rate per compounding period is given by i/m . The continuously compounded value of a single amount P after n years is given by

$$F = P \left(1 + \frac{i}{m} \right)^{mn} = P \left(1 + \frac{i}{m} \right)^{(m/i)in}$$

But $\lim_{m \rightarrow \infty} (1 + i/m)^{m/i} = e$. Therefore, $F = Pe^{in}$, where e^{in} is defined as the continuously compounded amount factor for the case of infinitely multiple compounding periods. Similarly, the continuously discounted value of a single future amount F , after n years, is given by $P = (F/e^{in})$. The factor $1/e^{in}$ is defined as the continuously discounted factor for the case of infinitely multiple discounting periods. For the case of an infinite number of compounding periods in a year, the effective annual interest rate is given by

$$\frac{F - P}{P} = \frac{P(e^i) - P}{P} = e^i - 1$$

Other special cases of interest equations are presented in Section 8.1.5.

8.1.5 Special Cases of Interest Equations

Most problems encountered in economic efficiency analysis for transportation decision making can be solved using the interest equations presented in Table 8.1. However, there are some variations of the problem, such as when periodic payments are being made to perpetuity, when there are infinite compounding periods in a year or when payments are not only compounded continuously but are being made with interest that is also continuously compounded. These special cases are discussed below.

(a) *Present Worth of Periodic Payments in Perpetuity*
 Consider the case of a transportation facility with a life cycle of N years, as shown in Figure 8.2. All postconstruction investments made during the life cycle can be compounded into a single amount, R . If it is assumed that the facility will be kept in service to perpetuity, then the life-cycle investment, R , will be repeated at every N -year period. The period N is assumed to be constant for this discussion, but N could be increasing or decreasing

with time, depending on the level of use and technological advances. Increasing levels of use would generally translate to decreasing values of N with time, and vice versa. Also, increasing quality of construction or preservation materials and other inputs would generally lead to increasing values of N . As with most transportation facilities, it is assumed that the initial investment (P) is not the same as the periodic investments (R), as the latter typically involves reconstruction, rehabilitation, and maintenance. A case in point is water port construction (where the initial investment includes right-of-way acquisition, geotechnical treatments, deck construction, dredging, etc., while recurring investments may involve dock structural rehabilitation and dredging). Another example is in highway construction, where the initial investment includes right-of-way acquisition, embankment construction, relocation of utilities, wetlands restoration, and other costs that are typically not found in the recurring investments of pavement resurfacing or reconstruction.

The present worth of all payments in perpetuity is given by

$$\begin{aligned} PW_{\infty} &= P + \frac{R}{(1+i)^N} + \frac{R}{(1+i)^{2N}} + \frac{R}{(1+i)^{3N}} + \dots \\ &= P + R \left[\frac{1}{(1+i)^N} + \frac{1}{(1+i)^{2N}} + \frac{1}{(1+i)^{3N}} + \dots \right] \\ &= P + R \left[\frac{1}{1 - 1/(1+i)^N} - 1 \right] = P + \frac{R}{(1+i)^N - 1} \end{aligned}$$

In cases where the facility already exists, P is a sunk cost, and the present worth is then equal to $R / ((1+i)^N - 1)$.

Example 8.3 A new airport runway will cost \$7.2 million to construct, including design, land acquisition, and other initial costs. It is expected that every 40

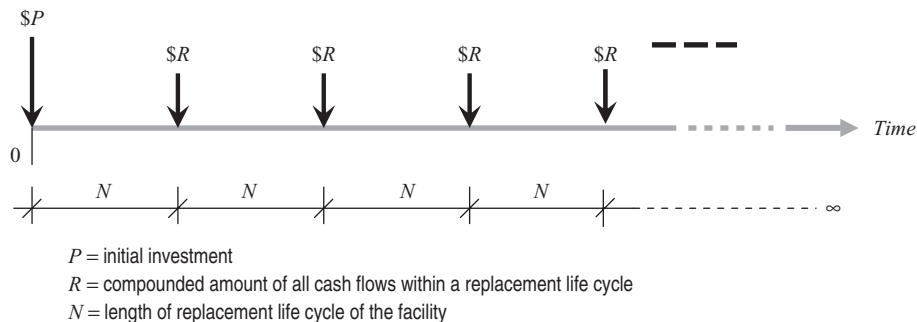


Figure 8.2 Present worth of periodic payments in perpetuity.

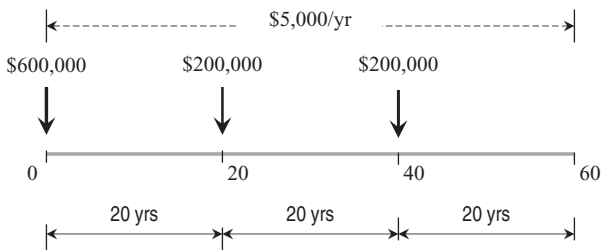
years, the runway will be reconstructed at a cost of \$3.1 million. Calculate the present worth of the initial and all reconstruction costs to perpetuity. Assume $i = 4\%$.

SOLUTION

$$PW_{\infty} = 7.2M + \frac{3.1}{(1 + 0.04)^{40} - 1} = \$7.46M$$

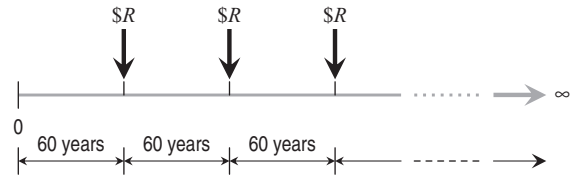
Example 8.4 After several decades of service, a railway bridge is slated for reconstruction. The estimated service life of the structure is 60 years. The reconstruction cost is \$600,000. During its replacement cycle, the bridge will require two rehabilitation events, each costing \$200,000, at the twentieth and fortieth years and the average annual cost of maintenance is \$5000. At the end of the replacement cycle, the bridge will again be reconstructed and the entire cycle is assumed to recur to perpetuity. What is the present worth of all bridge agency costs in perpetuity? Assume an interest rate of 5%.

SOLUTION All costs within the life cycle of the bridge are illustrated as follows:



$$\begin{aligned} R &= \text{compounded life-cycle cost} \\ &= 600,000SPCAF(5\%, 60) + 200,000SPCAF(5\%, 40) \\ &\quad + 200,000SPCAF(5\%, 20) \\ &\quad + 5000USCAF(5\%, 60) = \$14,914,087 \end{aligned}$$

Present worth of all costs in perpetuity ($PW_{5\%,\infty}$):



$$PW_{\infty} = \frac{R}{(1 + i)^N - 1} = \frac{14,914,087}{(1 + 0.05)^{60} - 1} = \$843,596$$

(b) *Present Worth of Continuously Compounded Payments with Continuously Compounded Interest* Another special case of economic evaluation involves exponentially increasing costs or benefits with continuously compounded interest (Figure 8.3). Consider a general case where the initial amount, R_0 , grows exponentially at a rate of r expressed as a percentage per year. Bringing all future streams to the present gives the present worth of $R_0 = R$, the present worth of $R_1 = Re^r/e^i$, the present worth of $R_2 = Re^{2r}/e^{2i}$, and the present worth of $R_n = Re^{nr}/e^{ni}$. Summing up the values of all present worth yields

$$\begin{aligned} PW_{CCP,CCI} &= R \left[1 + \frac{e^r}{e^i} + \frac{e^{2r}}{e^{2i}} + \dots + \frac{e^{nr}}{e^{ni}} \right] \\ &= R \left[\frac{e^{n(r-i)} - 1}{(r-i)} \right] \quad r < i \end{aligned}$$

the present worth of continuously compounded payments with continuously compounded interest.

Example 8.5 The average annual cost of operating the physical infrastructure of a small airport facility is currently \$100,000. Due to the growth in air traffic, the annual costs are compounded continuously at 3% per annum. What is the present worth of the operating costs

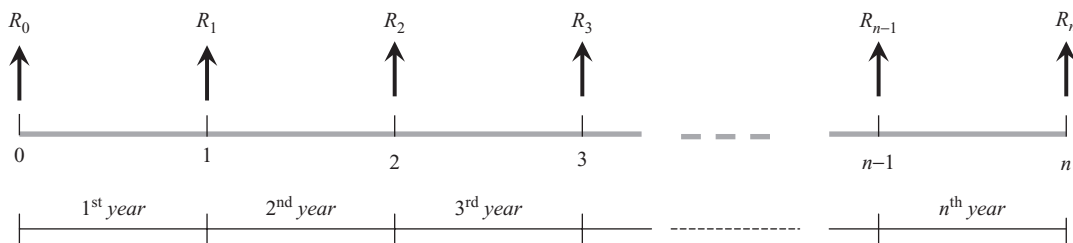


Figure 8.3 Present worth of continuously compounded payments with continuously compounded interest.

over a period of 10 years? Assume a 10% interest rate that is compounded continuously.

SOLUTION

$$PW_{CCP,CCI} = 100,000 \left[\frac{e^{10(0.03-0.1)} - 1}{0.03 - 0.1} \right] = \$719,160$$

8.2 CRITERIA FOR ECONOMIC EFFICIENCY IMPACT EVALUATION

After the present and future sums representing the benefits and/or costs of the relevant performance measures (expenses for facility construction preservation, and operation, and savings in safety, travel time, and vehicle operation) have been brought to their present worth or annualized, the question then is: How are they used to assess the economic efficiency of a proposed project? There are several criteria for doing this:

- Present worth of costs (PWC)
- Equivalent uniform annual cost (EUAC)
- Equivalent uniform annual return (EUAR)
- Net present value (NPV)
- Internal rate of return (IRR)
- Benefit–cost ratio (BCR)

These criteria are also sometimes referred to as *indicators* or *measures of economic efficiency*. The first two criteria are applicable only when all alternatives are associated with a similar level of benefits and cost minimization is therefore the sole evaluation criterion.

8.2.1 Present Worth of Costs

This method converts all costs of a transportation project into an equivalent single cost assumed to occur at the beginning of the analysis period.

Example 8.6 An airplane purchase is proposed by an airline. The initial cost of airplane type A is \$50 million, the average annual maintenance cost is \$0.25 million, and the salvage value will be \$8 million. For airplane type B, the initial cost is \$30 million, the average annual maintenance cost is \$0.75 million, and the salvage value will be \$2 million. Both types have a useful life of 15 years. Which alternative should be selected? Assume a 7% interest rate.

SOLUTION

$$PWC_A(\text{in millions}) = 50 + 0.25USPWF(7\%, 15) - 8SPPWF(7\%, 15) = \$49.38M$$

$$PWC_B(\text{in millions}) = 30 + 0.75USPWF(7\%, 15) - 2SPPWF(7\%, 15) = \$36.11M$$

Alternative B is more desirable because it has a lower present worth of life-cycle costs.

8.2.2 Equivalent Uniform Annual Cost

This method combines all of the costs of a transportation project into an equivalent annual cost over an analysis period of n years. This method is useful when alternatives have different analysis periods or when they have similar levels of effectiveness.

Example 8.7 Bus transit services in MetroCity can be performed satisfactorily using any one of two alternative bus types, A and B. Type A has an initial cost of \$100,000, an estimated life of six years, annual maintenance and operating costs of \$8000, and \$20,000 salvage value. Type B has an initial cost of \$75,000, an estimated life of five years, annual maintenance and operating costs of \$8000 for the first two years and \$12,000 for the remaining four years, and \$10,000 salvage value. Find the equivalent annual cost of each alternative, and decide which option is more desirable. Assume a 6% interest rate.

SOLUTION

$$\begin{aligned} EUAC_A(\text{thousands}) &= 100CRF(6\%, 6) + 8USPWF(6\%, 6) \\ &\quad \times CRF(6\%, 6) - 20SFDF(6\%, 6) \\ &= \$25.47 \end{aligned}$$

$$\begin{aligned} EUAC_B(\text{thousands}) &= 75CRF(6\%, 5) + 8USPWF(6\%, 2) \\ &\quad \times CRF(6\%, 6) + 12USPWF(6\%, 4) \\ &\quad \times SPPWF(6\%, 2) \times CRF(6\%, 6) \\ &\quad - 40 SFDF(6\%, 6) = \$22.57 \end{aligned}$$

Alternative B is more desirable because it has a lower value of equivalent uniform annual cost.

8.2.3 Equivalent Uniform Annual Return

The EUAR method combines all costs and benefits or returns associated with a transportation project into a single annual value of return (benefits less costs) over the analysis period. This method can be used when the alternatives have different levels of costs and different levels of benefits, or different analysis periods.

Example 8.8 Two alternative designs are proposed for renovating a water port. Alternative A involves an initial project cost of \$200 million, an estimated life of 25 years, a salvage value of \$22 million, annual maintenance and operating costs of \$15 million, and annual benefits of \$50 million in terms of monetized savings in inventory delay, safety and security, and vessel operations. Alternative B has an initial project

cost of \$175 million, an estimated life of 25 years, annual maintenance and operating costs of \$16 million, a salvage value of \$15 million, and annual benefits of \$40 million. Find the equivalent uniform annual return of each alternative and identify the alternative that should be undertaken. Assume a 4% interest rate.

SOLUTION

$$\begin{aligned} \text{EUAR}_A(\text{millions}) &= 50 - 200\text{CRF}(4\%, 25) \\ &\quad - 15 + 22\text{SFDF}(4\%, 25) = \$22.73\text{M} \\ \text{EUAR}_B(\text{millions}) &= 40 - 175\text{CRF}(4\%, 25) \\ &\quad - 16 + 15\text{SFDF}(4\%, 25) = \$13.6\text{M} \end{aligned}$$

Alternative A is more desirable because it has a higher equivalent annual return.

8.2.4 Net Present Value

The NPV of an investment is the difference between the present worth of benefits and that of costs. NPV reflects the value of the project at the time of the base year of the analysis, which may be considered the year of decision making. NPV is often considered as the most appropriate of all economic efficiency indicators because it provides a magnitude of net benefits in monetary terms. If a project involves borrowing or obtaining equity capital, then the interest required to obtain the funds should be considered a cost. Among competing transportation projects or policies, the alternative with the highest NPV is considered the most “economically efficient.”

Example 8.9 For the problem in Example 8.8, determine the net present value for each alternative.

SOLUTION

$$\begin{aligned} \text{NPV}_A(\text{millions}) &= 50\text{USPWF}(4\%, 25) \\ &\quad - 200 - 15\text{USPWF}(4\%, 25) + 22\text{SPPWF}(4\%, 25) \\ &= \$355\text{M} \\ \text{NPV}_B(\text{millions}) &= 40\text{USPWF}(4\%, 25) \\ &\quad - 175 - 16\text{USPWF}(4\%, 25) + 15\text{SPPWF}(4\%, 25) \\ &= \$206\text{M} \end{aligned}$$

Alternative A is more desirable because it has a higher net present value.

8.2.5 Internal Rate of Return

Agencies that seek to invest money in a project ask themselves whether their investment will pay back a

net rate of return that is greater than some minimum acceptable rate or whether it will yield a net profit before within a given period of time. The smaller the acceptable rate of return, the longer investors are willing to wait to see a net profit, and vice versa. The *minimum attractive rate of return* (MARR) is the lowest rate of return that investors will accept before they invest, considering the likely investment risks or the opportunity to invest elsewhere for possibly greater returns. MARR is related (inversely) to the *payback period* (the time taken for an investment to pay back to the investors a particular outlay such as their initial investment).

An economic rate of return is defined as the *vestcharge*, that is, the interest rate at which the net present worth or equivalent uniform annual return is equal to zero. The internal rate of return (IRR) method determines the interest rate that is associated with a zero net present value (NPV) and is consequently associated with an equivalency of the present worth of benefits and present worth of costs. Then the IRR is compared to the minimum attractive rate of return (MARR). If the IRR exceeds the MARR, the investment is considered worthwhile. Considering the general case discussed earlier, the IRR value is found by equating the present worth of benefits to the present worth of costs, or by equating the equivalent uniform annual benefits with the equivalent uniform annual costs.

Example 8.10 An urban rail transit agency is considering the purchase of a new \$30,000 ticketing system that will reduce travel time. The estimated life of the system is 10 years, at which time the value of the system will be \$15,000. The expected travel-time savings per year is \$5000 per year, and the average annual maintenance and operating cost is \$2000. Is the project economically more desirable than the do-nothing alternative? The minimum attractive rate of return is 5%.

SOLUTION Equating the net cash flow on both sides, we have:

$$\begin{aligned} 5000\text{USPWF}(i\%, 10) + 15,000\text{SPPWF}(i\%, 10) \\ \approx 30,000 + 2000\text{USPWF}(i\%, 10) \end{aligned}$$

Solving this equation by trial and error yields $i = 6.25\% > 5\%$. It is, therefore, economically more efficient to undertake the project than the do-nothing alternative.

8.2.6 Benefit–Cost Ratio

The *benefit–cost ratio* (BCR) is a ratio of the equivalent uniform annual value (or net present value) of all benefits to that of all costs incurred over the analysis period. An

investment with a BCR exceeding 1 is considered to be economically feasible, and the alternative with the highest BCR value is considered the best alternative.

Example 8.11 For the port problem in Example 8.8, determine the benefit–cost ratio for each alternative.

SOLUTION

$$\begin{aligned} \text{BCR}_A &= \frac{\text{PWB}_A}{\text{PWC}_A} \\ &= \frac{50\text{USPWF}(4\%, 25) + 22\text{SPPWF}(4\%, 25)}{200 + 15\text{USPWF}(4\%, 25)} \\ &= 1.93 \\ \text{BCR}_B &= \frac{\text{PWB}_B}{\text{PWC}_B} \\ &= \frac{40\text{USPWF}(4\%, 25) + 15\text{SPPWF}(4\%, 25)}{175 + 16\text{USPWF}(4\%, 25)} \\ &= 1.48 \end{aligned}$$

Alternative A is economically more efficient because it has a higher benefit cost ratio.

Certain procedures recommend that maintenance costs be considered as negative benefits. Using such an approach, the maintenance costs appear as a negative value added to the numerator of a benefit–cost ratio function. On the other hand, certain agencies such as the U.S. Office of Management and Budget (OMB) recommend a different treatment whereby maintenance costs are recognized as an element of the total life-cycle cost and implicitly appear as costs added to the denominator of the overall costs in the benefit–cost ratio (BCR) function.

The U.S. Flood Control Act of 1936 was probably the first instance where reference was explicitly made to the BCR concept in public project evaluation. By definition, any project with a positive NPV will also have a B/C ratio exceeding 1. However, projects with relatively high levels of benefits and costs have a higher NPV than those with smaller benefits and costs but may have higher or lower B/C ratios. Because of its inherent ambiguities, the BCR method is generally not recommended for transportation evaluation unless all B/C ratios are accompanied with explicit values of benefits and costs.

8.2.7 Evaluation Methods Using Incremental Attributes

The foregoing discussion pertained to determining the values of an economic efficiency performance criterion

(benefit, cost, IRR, benefit–cost ratio, etc.) associated with each individual investment. The best investment is which yields the “most desired” value of the performance criterion. In public projects, benefits represent savings in user costs. Consequently, mutually exclusive projects require an incremental approach including pairwise comparisons. In the incremental approach, a particular investment, generally the least cost or do-nothing, is taken as the base case or base alternative. The approach in this pairwise comparison method is to determine if the incremental gain in benefit justifies the additional cost.

Example 8.12 Three alternative congestion mitigation projects are being considered for an urban freeway corridor. The costs associated with the alternatives are given in Table E8.12. Assume an interest rate of 5% and an analysis period of 20 years. Which of the alternatives would you recommend on the basis of economic efficiency?

Table E8.12 Project Cost Data

| Alternative Costs | A: Road Widening | B: HOV Facility | C: ITS (Ramp Metering and Incident Management) |
|--|------------------|-----------------|--|
| Initial cost (\$1000s) | 8352 | 8400 | 4500 |
| Annual maintenance and operation (\$1000s) | 20 | 563 | 1000 |
| Annual user cost (\$1000s) | 1670 | 1100 | 1750 |

SOLUTION Comparing B with A yields

$$\begin{aligned} \text{NPV}_{B-A}(\text{thousands}) &= \text{present worth of user cost savings} \\ &\quad - \text{present worth of additional costs} \\ &= (1,670 - 1,100)\text{USPWF}(5\%, 20) \\ &\quad - [(8400 - 8352) + (563 - 20)\text{USPWF}(5\%, 20)] \\ &= 7103 - (48 + 6766) = \$289 \end{aligned}$$

Therefore Alternative B is a better than alternative A. Comparing B with C yields

$$\begin{aligned}
 \text{NPV}_{B-C}(\text{thousands}) &= (1750 - 1100)\text{USPWF}(5\%, 20) \\
 &\quad - [(8400 - 4500) + (563 - 1000)\text{USPWF}(5\%, 20)] \\
 &= 8100 - 3900 + 5446 = \$9,646
 \end{aligned}$$

Alternative B is a better alternative than alternative C. Therefore, of the three alternatives, B is the most economically efficient.

8.2.8 General Discussion of Economic Efficiency Criteria

Each economic efficiency analysis method has its unique logic, merits, and demerits. The equivalent uniform annual cost and present value of costs methods are applicable only when all competing alternatives are associated with the same level of service, and therefore the monetary value of benefits are similar across all alternatives. The equivalent uniform annual return and net present value methods consider the benefits and are therefore appropriate where competing alternatives have significant and very different levels of service. NPV, which is expressed as a monetary value and not a rate, ratio, or index, provides a readily comprehensible magnitude of the net benefit of an investment. For this reason, NPV is the method recommended by many agencies. Like the NPV and EUAR, the IRR method considers both benefits and costs. Also, no assumption is needed about the interest rate, although the minimum attractive rate of return must be specified. The main disadvantage of the IRR method is that a unique solution may not always be guaranteed. Many multilateral agencies, including the World Bank, have used the IRR method for project appraisal. The benefit–cost ratio duly considers both benefits and costs but is susceptible to the problems of any ratio-based index: different values of BCR may be obtained depending on the definition, units, and dimensions of the benefits and costs. Most important, BCR does not provide any indication of the total extent of benefit. Another issue associated with economic efficiency is that of relative weights: most analyses proceed on the assumption that all monetary values have the same weight regardless of source. Some agencies, however, assign weights on monetary amounts depending on the source, such as agency costs and user costs (see Chapter 18).

8.3 PROCEDURE FOR ECONOMIC EFFICIENCY ANALYSIS

The overall framework for carrying out an economic efficiency analysis on the basis of benefit and cost considerations is illustrated as Figure 8.4, which is a

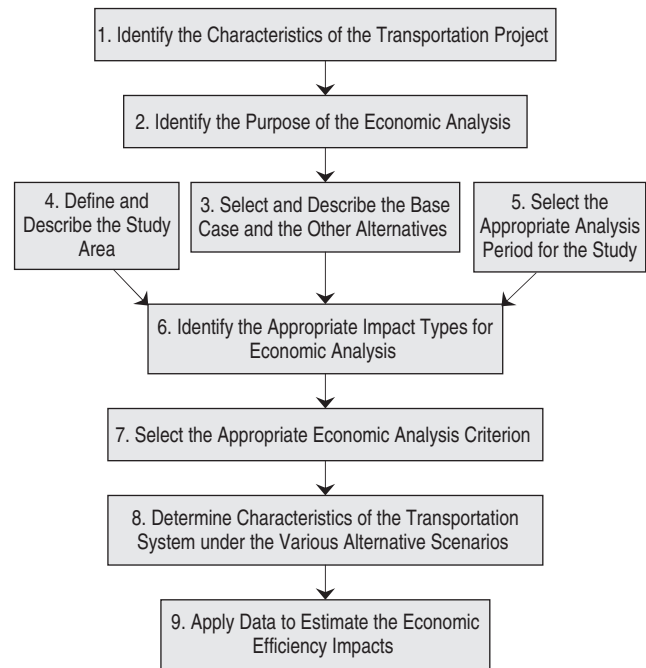


Figure 8.4 Framework for economic efficiency impact evaluation.

synthesis of the procedures presented by Booz Allen Hamilton (1999), Cambridge Systematics (2000), and Forkenbrock and Weisbrod (2001).

Step 1: Identify the Characteristics of the Transportation Project This step involves preliminary work for the analysis, such as identifying the scope of the project (passenger or freight or both), the mode (highway, transit, rail, etc.), the flow unit (trains, buses, trucks, airplanes, boats, etc.), the network feature (terminals or routes), and the scale (i.e., specific site location, specific strip or corridor, or an entire systemwide area such as city or county). The type of transportation improvement for each alternative is then identified. Examples include upgrading or expanding the existing facility, maintaining its services or providing a new facility, mode, or service. Finally, the purpose of the project should be identified, such as addressing an existing congestion problem, meeting expected future demand, and generating new economic development.

Step 2: Identify the Purpose of the Analysis There could be several reasons for an agency's efforts to evaluate the economic efficiency impacts of transportation projects. The specific purpose of the analysis is largely influenced by the type and purpose of the overall project itself. The purpose of the analysis, in turn, influences the framework and methodology for the analysis.

Step 3: Select the Base Case and Transportation Alternatives

(a) *Base Case* The base case should be chosen such that it allows a realistic representation of past, current, or possible future conditions. A base case can be “do-nothing” or the current condition, or the least-cost option under future conditions.

(b) *Alternative Scenario* The base case and alternative scenarios may differ by travel mode, facility type, service type facility location (in terms of setting, routing, or alignment), facility or service size (in terms of capacity and cost), area served or the expected change in the level or quality of service to be provided by the facility or service.

Step 4: Define and Describe the Study Area The factors that should be considered in selecting a primary study area for an economic efficiency analysis are as follows:

- *The area of jurisdiction for the sponsoring agency.* This could be the agency responsible for project funding, project spending (implementation), project evaluation, or a combination thereof. The study area could be a corridor, neighborhood, city, county, state, or province.
- *The area of direct project influence.* Whether the project involves a route/line or a specific terminal facility, the “area of direct influence” includes the area in which facility users or the community are affected.

Step 5: Select the Appropriate Analysis Period for Study Section 8.1.4 discussed factors that need to be taken into account in selecting the analysis period. The analysis period selected should primarily be long enough to distinguish between the costs and benefits between alternatives.

Step 6: Select the Appropriate Impact Types for the Evaluation Economic efficiency impacts are evaluated on the basis of the monetized equivalents of individual impact types, including (1) direct revenue (toll receipts and other out-of-pocket costs, etc.), and (2) impact types that can be monetized (e.g., travel time, vehicle operating costs, safety). The selection of impact types may also be influenced by other factors, such as the preferences of the sponsoring agency and the availability of data.

Step 7: Select the Appropriate Economic Efficiency Criterion for the Evaluation Factors that affect the selection of an appropriate economic efficiency criterion include the size of the project (and consequently, the levels of benefits and costs), the variation between the levels

of service (therefore, the benefits) of competing projects, the preferences of the sponsoring agency, the relationship between the facility service life, and the analysis period. In carrying out an economic efficiency analysis, it is vital to identify correctly the benefits and costs associated with each alternative so that the benefits and costs are not unduly over- or understated (Wohl and Hendrickson, 1984).

Step 8: Determine the Characteristics of the Transportation System under Various Alternative Scenarios

In this step, the analyst investigates how each alternative investment will affect users in terms of travel time, vehicle operating cost, and safety, besides other direct costs incurred and benefits accrued. Existing transportation planning software packages, such as QRS-II, Tranplan, EMME2, and MINUTP, may be used to construct transportation network simulations and analysis, and to forecast the demand and user impacts of changes to the transportation system. These packages consider both the supply and demand for transportation.

(a) *Supply-Side Modeling* The user impacts of transportation projects are typically analyzed using either a full simulation model or a sketch planning model that includes the affected project or corridor. Such a model should cover all travel modes of interest, and there should be data on the transportation system supply under alternative scenarios, such as capacity, projected vehicle volumes and trip distribution patterns, and system performance (i.e., the resulting travel times and costs for users of the affected travel modes, routes or links, and terminals or transfer points).

1. *Full simulation models* estimate traffic patterns, volumes, and travel times for each link and node of the network. Simulation models are applicable to situations where trip diversion and rerouting are components of user impacts and are also useful for separately estimating user impacts for various user categories (e.g., different types of businesses) that have significantly different origin–destination patterns.
2. *Sketch planning models* are generally used where there are relatively few routes or modal diversion alternatives to be considered, such as a transportation alternative that affects access in a downtown area. Typically capable of implementation using a simple spreadsheet, sketch planning models are used to estimate the volumes and travel time and cost impacts of localized transportation improvements.

Another aspect of user cost estimation under variations in supply-side conditions relates to work zones

associated with the construction or preservation activities of the facility.

(b) *Demand-Side Analysis* To complement data obtained from the supply-side analysis, information on travel demand (such as trip volumes by origin–destination combination) at the base year and at future years are necessary for each alternative transportation investment scenario. Using data from the supply and demand sides, the simulation or sketch planning model is used to estimate changes in volumes, travel times, travel costs, and volume–capacity ratios for the affected portions of the transportation system for each scenario. In Chapter 3, which deals with transportation demand and supply, we discuss how this step could be carried out.

Step 9: Apply the Data to Calculate the Economic Efficiency Impacts The calculation of project costs and user benefits involves the use of average values from available databases or models.

(a) *Project Costs* These costs typically consist of (1) right-of-way, rolling stock, and construction costs; (2) operating costs; and (3) maintenance costs. They are also typically referred to as *agency costs* and are discussed in Chapter 4.

(b) *Project User Benefits* The primary user benefit components are (1) savings in vehicle operating cost (2) travel-time savings, and (3) increased safety. Such benefits are realized over the entire project life and generally grow with the increasing travel volumes. In certain circumstances of the system operation, such as construction/rehabilitation work zones or congestion conditions, however, there may actually be an increase in certain user costs compared to normal operations, but these are typically short-lived.

The cost, time, and safety benefits for existing and diverted trips should be estimated directly. For induced trips (which would otherwise not occur), there is typically no relevant travel time or safety benefit. It is naturally expected that there would be some benefit for diverted and induced trips (trip makers enticed to switch to the new facility), which, nevertheless, does not exceed the benefits associated with the existing trips (otherwise, people would have switched in the preimplementation situation). Within these extremes, the exact magnitude of benefits for induced trips may vary depending on the nature of the improvement alternative relative to existing alternatives. When it is not possible to estimate accurately the benefits of induced trips, it is recommended that they be estimated to be roughly one-half of the per-trip benefit accruing for existing trips (Forkenbrock and Weisbrod, 2001; AASHTO, 2003). In economic terms, that is equivalent to a consumer surplus concept, in which

there is a linear demand response in terms of willingness to pay for increasing benefit levels.

8.4 SOFTWARE PACKAGES FOR ECONOMIC EFFICIENCY ANALYSIS

8.4.1 Surface Transportation Efficiency Analysis Model

FHWA's STEAM package assesses the economic efficiency of physical investments in multimodal urban transportation infrastructure as well as policy alternatives such as pricing and demand management measures (Cambridge systematics, 2000). Modes that can be analyzed are auto, carpool, truck, local bus, express bus, light rail, and heavy rail. The temporal scope of application includes total average weekday traffic, peak, or off-peak periods. Also, multiple trip purposes may be considered. The model is closely linked to outputs from the four-step urban transportation planning process: The study area is partitioned into traffic analysis zones and aggregated to districts where separate benefit and cost factors may be specified. Costs and benefits can be reported at the corridor or regional level. The benefit categories include reductions in vehicle operating costs, travel times, crash costs, emissions, energy consumption, and noise; and the agency costs include capital (infrastructure investment) and operating costs. The economic efficiency of each alternative is expressed as a net present worth or a benefit–cost ratio. Other quantitative impacts that can be considered include congestion, access to jobs, revenues and transfers (revenues) from fares, tolls, and fuel taxes, as well as the levels of risk in the results estimated (probability distributions for key outputs). Other related FHWA benefit–cost analysis tools include IMPACTS, SMITE, SCRITS, and SPASM. These software packages and their supporting documentation are available on-line at: <http://www.fhwa.dot.gov/steam/links.htm>.

8.4.2 MicroBenCost Model

In the United States, from the late 1970s through the early 1990s, the most widely accepted benefit–cost analysis model highways and transit was that presented in the 1977 AASHTO *Manual for User Benefit Analysis of Highway and Bus-Transit Improvements* (AASHTO, 1977). To facilitate use of the model, the Texas Transportation Institute and the National Cooperative Highway Research Program developed the MicroBencost computer software package (McFarland et al., 1993). The MicroBencost program compares the costs of an existing situation to that of a planned transportation improvement. It selects the best improvement alternative from several candidate projects and presents an objective ranking of projects in the order

of their potential benefits. MicroBencost can analyze a broad spectrum of projects, including new location or bypass facilities, pavement rehabilitation and bridge rehabilitation and replacement.

8.4.3 Highway Development and Management Standards Model

The highway development management (HDM) model was sponsored by the World Bank, and several other international organizations. HDM evaluates highway projects, standards, and programs mostly in developing countries and makes comparative economic evaluations of alternative construction and preservation scenarios either for a given road section or for an entire road network. The HDM model duly considers that the costs of construction, maintenance, and vehicle operation are functions of road characteristics such as vertical alignment, horizontal alignment, and road surface condition (University of Birmingham, 2005).

8.4.4 Highway Economic Requirements system

FHWA's HERS is an economic efficiency analysis tool that uses technical standards to identify highway deficiencies applies incremental benefit–cost analysis to select the most economically efficient portfolio of improvements for systemwide implementation, and predicts the system condition and user cost levels resulting from a given level of investment (FHWA, 2002a). The amounts and total costs of travel time, safety, and vehicle operation and the cost of emissions associated with each alternative improvement are used to assess the economic efficiency. In cases where funding is not available to achieve “optimal” spending levels, HERS prioritizes economically worthwhile improvement options according to relative merit (benefit–cost ratios are used) and then selects the best set of projects for systemwide implementation. HERS minimizes the expenditure of public funds while simultaneously maximizing highway user benefits given funding constraints or user-specified performance objectives. The HERS software is used not only for economic efficiency impact evaluation, but also for program development and needs assessments.

8.4.5 California DOT'S Cal-B/C System

The California life-cycle benefit–cost (Cal-B/C) analysis model carries out economic efficiency evaluation for planned highway and transit improvement projects. The model is capable of analyzing the impacts of lane additions, HOV lanes, passing/truck climbing lanes, and intersection improvements. Transit modes that can be analyzed include passenger rail, light rail, and bus transit. Cal-B/C calculates the savings in travel time, vehicle

operating cost, accident cost, and emissions. Performance measures include life-cycle costs and benefits, net present value, benefit–cost ratio, rate of return on investment, and project payback period. The model enables quick economic efficiency analysis, and comparison and ranking of roadway and transit alternatives that have similar benefits (Booz Allen Hamilton, 1999).

8.5 LIFE-CYCLE COST ANALYSIS

Life-cycle cost analysis (LCCA) is a special case of economic efficiency analysis where the streams of a facility's benefits and costs extend over an appreciable length of time such as one *life cycle* of the facility. LCCA can be used at the project level or the network level. Project-level LCCA focuses on a specific facility, such as a runway, bridge, or road segment, whereas network-level analysis considers an entire inventory of facilities. In both cases, LCCA helps in evaluating the overall long-term economic efficiency between competing alternative investment options by evaluating the benefits and costs of various alternative preservation and improvement strategies or funding levels over the life cycle(s), or part thereof, of a facility or facilities. The monetized costs and benefits associated with each alternative *activity profile* (planned actions over a facility life or part thereof) are determined and the alternative with the highest net present value is typically selected. The Federal Highway Administration (FHWA) and the Federal Transit Administration (FTA) encourage the use of LCCA in analyzing all major investment decisions. Studies conducted in the United States and abroad strongly suggest that cost-effective long-term investment decisions could be made at lower costs if LCCA were adopted properly (Darter et al., 1987; Peterson, 1985; Mouaket and Sinha, 1990; Al-Mansour and Sinha, 1994; FHWA, 2002b).

8.6 CASE STUDY: ECONOMIC EFFICIENCY IMPACT EVALUATION

As part of a major corridor expansion project, it is proposed to improve a 12-mile stretch of an existing 4 lane urban arterial highway. The improvement will involve lane and shoulder widening, median closings (full restriction of access between opposing lanes), and full control of access from local roads. Other details about the “do-nothing” and “improvement” scenarios are presented in Table 8.2. Also, the analysis period is 10 years, the interest rate is 5% per year, and the travel time value is \$14 for autos. Also assume that the traffic stream is comprised only of medium-size automobiles and that the delay due to congestion is negligible. For estimating the annual number

Table 8.2 Corridor Expansion Project Data

| | Do Nothing | Improvement |
|--|------------|--|
| Initial construction cost (\$) ^a | 0 | \$70,000,000 |
| Average annual preservation cost (\$/lane-mile) ^a | 25,000 | 1,500 |
| Residual value at end of 10 years (\$) ^a | 0 | 500,000 |
| Geometric features | | Median closings, full control of access from local roads |
| Average annual daily traffic, assume constant | 25,280 | 46,100 (expected) |
| Lane width (ft) | 6 | 8 |
| Shoulder width (ft) | 2 | 4 |
| Average operating speed (mph) | 50 | 65 |

^aAssume that all monetary amounts are in 2005 dollars.

of PDO and fatal-injury crashes, the following equations can be used:

$$N_{\text{PDO}} = 0.9211 + 0.8817 \ln(\text{length}) + 0.3812 \ln(\text{AADT}) - 0.1375\text{LW} - 0.0717\text{SW}$$

$$N_{\text{FI}} = -0.1211 + 0.0610 \ln(\text{length}) + 0.022 \ln(\text{AADT}) - 0.006\text{LW} - 0.006\text{SW}$$

Determine whether the improvement is economically feasible.

1. *Determine the vehicle operating cost impacts.* From Chapter 7, the VOC at urban roadways for four VOC components (tires, vehicle depreciation, maintenance, and fuel), combined, can be estimated as follows:

$$\text{VOC} = a_0 + a_1 v_s + a_2 v_s^2 \quad \text{for } v_s > 50 \text{ mph}$$

where speed is in mph and VOC is in cents per vehicle-mile, and a_0 , a_1 , and a_2 are coefficients that dependent on vehicle type. Using the parameters provided in Table 7.3 the VOC for each alternative can be found as follows:

Do-nothing alternative: Unit VOC,

$$U_{\text{DN}} = 33.5 - (0.058)(50) + (0.00029)(50)^2 = 31.33 \text{ cents per veh-mile}$$

$$\text{VMT}_{\text{DN}} = (25,280)(12 \text{ mi}) = 303,360 \text{ vehicle-miles}$$

Improvement alternative: Unit VOC,

$$U_{\text{IMP}} = 33.5 - (0.058)(65) + (0.00029)(65)^2 = 30.96 \text{ cents per veh-mile}$$

$$\text{VMT}_{\text{IMP}} = (46,100)(12 \text{ mi}) = 553,200 \text{ vehicle-miles}$$

Therefore, the annual VOC benefits to be derived by the improvement are

$$0.5(U_{\text{DN}} - U_{\text{IMP}})(\text{VMT}_{\text{DN}} + \text{VMT}_{\text{IMP}}) = (0.5)(31.33 - 30.96)(303,360 + 553,200)/100 = \$1584.60 \text{ per day} = \$578,392 \text{ per year}$$

2. *Determine the safety impacts.*

Do-nothing alternative: Estimated annual number of PDO crashes,

$$N_{\text{PDO}} = 0.9211 + 0.8817 \ln(12) + 0.3812 \ln(25,280) - (0.1375)(6) - (0.0717)(2) = 6.01$$

Therefore, the PDO crash rate,

$$U_{\text{PDO,DN}} = \frac{N_{\text{PDO}}}{\text{VMT}} = \frac{6.01}{(12)(25,280)(365)} = 5.43 \text{ per 100 million VMT}$$

The estimated annual number of fatal/injury crashes,

$$N_{\text{FI}} = -0.1211 + 0.0610 \ln(12) + 0.022 \ln(25,280) - (0.006)(6) - (0.006)(2) = 0.21$$

$$\text{VMT}_{\text{DN}} = (12)(25,280)(365) = 1.107 \text{ million/year}$$

Therefore, the fatal/injury crash rate,

$$U_{\text{FI,DN}} = 0.21/1.107 = 0.19 \text{ per million VMT}$$

Improvement alternative:

$$\begin{aligned} N_{\text{PDO}} &= 0.9211 + 0.8817 \ln(12) \\ &\quad + 3812 \ln(46,100) - (0.1375)(8) \\ &\quad - (0.0717)(4) = 5.82 \end{aligned}$$

$$\text{VMT}_{\text{IMP}} = (12)(46,100)(365) = 2.019 \text{ million VMT}$$

Therefore, the PDO crash rate,

$$\begin{aligned} U_{\text{PDO,IMP}} &= 5.82/2.019 \\ &= 2.88 \text{ per 100 million VMT} \\ N_{\text{FI}} &= -0.1211 + 0.0610 \ln(12) + 0.022 \ln(46,100) \\ &\quad - (0.006)(8) - (0.006)(4) \\ &= 0.195 \end{aligned}$$

Therefore, the fatal-injury crash rate, $U_{\text{FI,IMP}} = 0.195/2.019 = 0.097$ per 100 million VMT.

Safety savings of the improvement over do-nothing:

For PDOs:

$$\begin{aligned} \text{Benefits} &= 0.5(U_{\text{DN}} - U_{\text{IMP}})(\text{VMT}_{\text{DN}} + \text{VMT}_{\text{IMP}}) \\ &= (0.5)(5.43 - 2.88)(1.107 + 2.019) \\ &= 3.99 \text{ crashes} \end{aligned}$$

$$\begin{aligned} \text{Assuming average cost per PDO crash} \\ &= \$5936 \end{aligned}$$

Therefore,

$$\begin{aligned} \text{PDO safety benefits per year} &= (3.99)(\$5936) \\ &= \$23,662 \end{aligned}$$

For fatal/injury:

$$\begin{aligned} \text{Benefits} &= 0.5(U_{\text{DN}} - U_{\text{IMP}})(\text{VMT}_{\text{DN}} + \text{VMT}_{\text{IMP}}) \\ &= (0.5)(0.19 - 0.097)(1.107 + 2.019) \\ &= 0.15 \text{ crashes} \end{aligned}$$

$$\begin{aligned} \text{Assuming average cost of a fatal/injury crash} \\ &= \$999,958 \end{aligned}$$

Therefore,

$$\begin{aligned} \text{fatal-injury safety benefits per year} \\ &= (0.15)(\$999,958) = \$150,000 \end{aligned}$$

Total annual safety benefits

$$= \$23,662 + \$150,000 = \$173,662$$

3. *Determine the travel-time impacts.*

Do-nothing alternative:

$$\begin{aligned} \text{Average travel time, } U_{\text{DN}} &= \frac{\text{distance}}{\text{speed}} \\ &= \frac{12}{50} = 0.24 \text{ h per trip} \end{aligned}$$

Improvement alternative:

$$\begin{aligned} \text{Average travel time, } U_{\text{IMP}} \\ &= \frac{\text{distance}}{\text{speed}} = \frac{12}{65} = 0.185 \text{ h per trip} \end{aligned}$$

$$\begin{aligned} \text{Travel-time savings (hours) of the improvement} \\ \text{over do-nothing} &= 0.5(U_{\text{DN}} - U_{\text{IMP}})(V_{\text{DN}} + V_{\text{IMP}}) \\ &= (0.5)(0.24 - 0.185)(25,280 + 46,100) \\ &= 1963 \text{ h/day} = 716,495 \text{ h/year} \end{aligned}$$

Assuming the value of travel time = \$14/h, the annual travel-time benefits of the improvement scenario = $(716,477)(\$14) = \$10,030,930$.

4. *Calculate the net present value of the improvement scenario over the do-nothing scenario.*

Assuming no change in future year traffic volumes:

$$\begin{aligned} \text{NPV} &= \text{Present worth of benefits} - \text{Present worth of costs} \\ &= (578,392 + 173,662 + 10,030,930)\text{USPWF}(5\%, 10 \text{ yrs}) \\ &\quad - \{70,000,000 - [(25,000 - 1,500)12 \times 4] \\ &\quad \times \text{USPWF}(5\%, 10 \text{ yrs}) - 500,000 \text{SPPWF}(5\%, 10 \text{ yrs})\} \\ &= 83,266,202 - [70,000,000 - 8,710,416 - 306,950] \\ &= \$22 \text{ M.} \end{aligned}$$

Therefore, the improvement is economically feasible. If the increase in traffic is considered, the NPV will be ever higher.

8.7 FINAL COMMENTS ON ECONOMIC EFFICIENCY ANALYSIS

Consistent with the pursuit of social justice, equitable distribution of project benefits is increasingly being considered by many transportation agencies. In striving to ensure such distribution of benefits, transportation analysts may consider the concept of *Pareto efficiency*,

which states that an allocation of goods or services is Pareto efficient if no alternative allocation can better the condition for at least one person without negatively affecting anyone else. Boardman et al. (2001) discussed the concept of Pareto efficiency and its relationship to economic efficiency analysis, stating that the latter utilizes a decision rule with less conceptual appeal but greater feasibility than the actual Pareto efficiency rule. This decision rule is based on the well-known *Kaldor–Hicks criterion*, which states that a policy should be adopted if and only if those who would gain are capable of fully compensating those who would lose and yet remain better off. This criterion provides the basis for the *potential Pareto efficiency rule*, or more commonly, the *net benefits criterion*, which states that only policies that have positive net benefits should be adopted. Besides its feasibility, the potential Pareto efficiency rule is often considered justified for the following reasons: (1) by always choosing policies with positive net benefit, society maximizes aggregate wealth and therefore helps the relatively poor individuals in society; (2) different policies will have different gainers and losers, and costs and benefits tend to average out among individuals so that each individual is likely to realize positive net benefits from the full set of policies; and (3) it appropriately guards against unduly granting excessive weight to the preferences of organized groups (stakeholders) that have relatively large influence on political systems, and or granting insignificant weights to the perspectives of unorganized or uninfluential individuals in the society.

Another issue in economic analysis is the valuation of certain components of economic efficiency. Critics argue that economic efficiency debases the terms of public discourse by assigning monetary values to intangibles such as human life. An opposing school of thought contends that it is appropriate to place a value on intangibles, at least in the statistical sense, in order to assess properly the policies and projects that have a profound impact on these intangibles. In other areas of economic efficiency analysis, critics charge that such economic efficiency evaluations undermine democracy by imposing a single goal (efficiency) in the evaluation of public projects and policies. This would be true if public policy were determined strictly via benefit–cost analysis results compared to public policy being determined solely via democratic processes that give equal weight to all interests. In the real world, however, these extreme situations do not exist: Economic efficiency analyses rarely serve as a single decisive yardstick for policy making. Besides, it can be argued that by using economic efficiency analysis as one of the factors for decision making, less organized and less vocal constituencies who

have little electoral clout often have their interests better represented. In Chapter 18 we provide a methodology through which an analyst can incorporate a wide array of performance measures, including economic efficiency, socioeconomic impacts, and environmental effects to arrive at transportation decisions that achieve specified performance targets within established constraints.

SUMMARY

Decisions to select the best of several alternative actions are encountered at various stages of the transportation development process and economic efficiency (benefit–cost) is one of the most widely used and objective performance measures used to compare such alternatives. Economic efficiency analysis proceeds on the assumption that all significant benefits and costs can be expressed in monetary values. We reviewed the concepts of economic efficiency analysis, such as the fundamental principle that the value of money is related directly to the time at which the value is considered. Special cases of interest equations include the present worth of periodic payments in perpetuity and the effect of an infinite number of compounding periods in a year.

Criteria for economic efficiency evaluation, which utilize various forms of interest equations, include the equivalent uniform annual cost, present value of costs, equivalent uniform annual return, net present value, internal rate of return, and benefit–cost ratio. Each criterion has its unique logic, merits, and shortcomings, and agencies have their preferred choice of criteria.

The overall framework for carrying out economic efficiency analysis involves identification of the characteristics of the transportation project and the purpose of the analysis; selection of the base case and transportation alternatives; selection of the appropriate geographic study area, analysis period, impact types, and choice of an economic efficiency criterion; determination of the characteristics of the transportation system under various alternative scenarios; and application of the data to calculate the economic efficiency impacts. In reality, there is a great deal of variation associated with such input parameters, which consequently can make it difficult to predict outcomes with absolute certainty. A number of software packages are available for evaluating transportation alternatives on the basis of economic efficiency.

EXERCISES

- 8.1. List two possible transportation interventions (projects, practice, or policy changes) in your locality where the results of an economic efficiency analysis would be useful in deciding whether to

go ahead with that intervention. For each example, indicate (1) the motivation for assessing the economic efficiency and (2) the stage of the PDP with which the project/policy is concerned.

- 8.2. Explain the relationship between opportunity cost, interest rate, and inflation.
- 8.3. If you borrow \$4000, what single payment must you make after five years to repay the principal and interest at 10%? Alternatively, what uniform annual payment would be required?
- 8.4. Solve the following using a 7% interest rate compounded annually: (a) What is the amount that will be accumulated in a sinking fund at the end of 15 years if \$200 is deposited in the fund at the beginning of each of the 15 years? (b) Uniform deposits were made on January 1 of 1991, 1992, 1993, and 1994 into a fund intended to provide \$1000 on January 1 of 2005, 2006, and 2007. What were the size of these deposits?
- 8.5. (a) What annual expenditure for 12 years is equivalent to spending \$1000 at the end of the first year, \$1500 at the end of the fifth year, and \$2000 at the end of the ninth year if the interest rate is 8% per annum?
 (b) What single amount paid at the beginning of the first year is equivalent to the series of unequal payments in part (a), with an interest rate of 8%?
- 8.6. What uniform annual payment for 30 years is equivalent to spending \$10,000 at the present time (year 0), \$10,000 at the end of 10 years, \$10,000 at the end of 20 years, and \$2000 a year for 30 years? Assume an interest rate of 8%.
- 8.7. (a) What is the difference between effective and nominal interest rate?
 (b) What is the effective annual interest rate if the nominal interest rate is 12% and there are two compounding periods in the year?
 (c) What nominal annual interest rate, compounded quarterly, yields an effective annual interest rate of 22%?
 (d) What is the effective annual interest rate for a nominal annual interest rate of 10% compounded continuously?
- 8.8. Explain the following terms: (a) continuously compounded interest and (b) periodic payments to perpetuity.
- 8.9. The rate of growth of traffic on a newly constructed bridge is 3% per year. By the end of the first year,

500,000 vehicles will have traveled it. Determine the number of vehicles using the bridge in the tenth year of service. Assuming that a toll of \$0.75 is collected per vehicle, calculate the present worth of the total toll collections during the 10-year period. Assume an interest rate of 10% with continuous compounding. For simplicity, cash flows can be considered discrete, occurring at the end of each year.

- 8.10. Assuming that interest is compounded monthly, determine the present worth of the cash streams illustrated in Figure EX8.10.

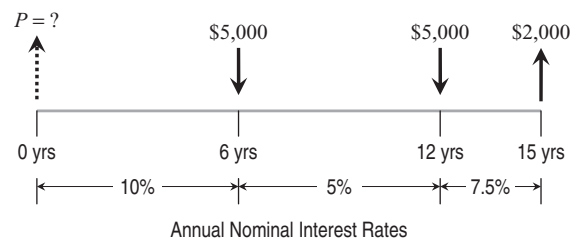


Figure EX8.10

- 8.11. A proposed transportation project has two alternatives with the costs and benefits shown in Table EX8.11. The user benefits refer to reductions in user costs for vehicle operation, delay, and crashes relative to a do-nothing alternative. Assuming a 10% interest rate per year and a MARR of 8%, and assuming that the do-nothing alternative is not feasible, indicate which alternative you would recommend, on the basis of any of the following economic efficiency criteria: (1) equivalent uniform annual return, (2) internal rate of return, or (3) benefit–cost ratio method.

Table EX8.11 Costs and Benefits of Alternatives

| | Alternative 1 | Alternative 2 |
|---------------------------------|----------------|---------------|
| Initial construction costs | \$50 million | \$20 million |
| Annual operating costs | \$0.5 million | \$1 million |
| Frequency of rehabilitation | Every 5 years | Every 3 years |
| Rehabilitation cost | \$1 million | \$1.2 million |
| Average annual maintenance cost | \$0.75 million | \$1.5 million |
| Annual user benefits | \$20 million | \$12 million |
| Service life | 15 years | 9 years |
| Salvage value | \$2.5 million | \$0.5 million |

8.12. Three alternative congestion mitigation strategies are being considered to enhance mobility at a congested bypass freeway. The prospective net cash flows for these alternatives are shown in Table EX8.12. The MARR is 15% per year. Using the NPV method and incremental analysis procedure, determine which is the best congestion mitigation alternative from the economic efficiency standpoint.

Table EX8.12 Cash Flows under Alternatives

| End of year | A Implement- ation Cost | B Implement- ation Cost | C Implement- ation Cost |
|-------------|----------------------------------|----------------------------------|----------------------------------|
| 0 | -\$6 million | -\$8 million | -\$7 million |
| 1 | +\$2 million | +\$3 million | +\$2.5 million |
| 2 | +\$3 million | +\$4.1 million | +\$3.4 million |
| 3 | +\$3.8 million | +\$4.5 million | +\$4.1 million |
| 4 | +\$4.1 million | +\$4.7 million | +\$4.4 million |
| 5 | +\$4.2 million | +\$4.8 million | +\$4.5 million |

8.13. The initial investment for constructing a median and guardrails for a four-lane rural highway is \$1,500,000. Maintaining these facilities is expected to cost \$2000 annually for the first five years of service and \$8000 for the next five years of service. It is expected that these facilities will be rehabilitated at the end of the tenth year at a cost of \$50,000, after which the maintenance costs are expected to decrease to \$5000 per year. What is the equivalent uniform annual cost over a 15-year period of service if the interest rate is 6% per year? Assume that zero salvage value is 20% of the initial construction cost. If the investment is scheduled five years from now, what amount should be set aside now to provide for these improvements?

8.14. An improvement in all the terminals on a transit network is scheduled every five years in perpetuity. Each improvement costs \$25,000. What is the present worth of all the costs of the improvement project if the interest rate is 8% per year?

8.15. At the current year (end of year 0), the user benefits from a road capacity enhancement project is \$10 million and the total project cost is \$100 million. What must be the minimum rate of growth of benefit (in percent per year) for the project to be feasible economically? Assume that the project life is 30 years and that the benefit grows continuously.

The interest rate is 5% per year compounded continuously.

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APPENDIX A8

Table A8.1 Interest Formulas for Arithmetic Gradient Series with Discrete Compounding

| Description | Cash Flow Diagram | Computational Formula | Factor Computation |
|--|-------------------|---|--|
| Finding the future compounded amount (F) at the end of a specified period due to linearly increasing annual payments (G), given the interest rate. | | $F = G \times \text{GSCAF}$ The gradient series compounded amount factor, GSCAF ($i\%$, N), may be computed as shown. | $\text{GSCAF} = \frac{(1+i)^N - 1}{i^2} - \frac{N}{i}$ |

Table A8.1 (continued)

| Description | Cash Flow Diagram | Computational Formula | Factor Computation |
|---|-------------------|---|---|
| <p>Finding the amount of linearly increasing annual payments (G) that would yield a future compounded amount (F) at the end of a specified period, given the interest rate.</p> | | <p>$G = F \times \text{GSSFDF}$ The gradient series sinking fund deposit factor, GSSFDF ($i\%$, N), may be computed as shown.</p> | $\text{GSSFDF} = \frac{i^2}{(1+i)^N - 1 - Ni}$ |
| <p>Finding the initial amount (P) that would be equivalent to specified linearly increasing annual payments (G), given the interest rate.</p> | | <p>$P = G \times \text{GSPWF}$ The gradient series present worth factor, GSPWF ($i\%$, N), may be read from Table A.8.2 or may be computed as shown.</p> | $\text{GSPWF} = \left[\frac{(1+i)^N - 1}{i^2} - \frac{N}{i} \right] / ((1+i)^N)$ |
| <p>Finding the amount of linearly increasing annual payments (G) that would completely recover an initial amount (P) at the end of a specified period, given the interest rate.</p> | | <p>$G = P \times \text{GSCRF}$ The gradient series capital recovery factor, GSCRF ($i\%$, N), may be computed as shown.</p> | $\text{GSCRF} = \frac{(1+i)^N \times i^2}{(1+i)^N - 1 - Ni}$ |

Table A8.2 Conversion Factors between Uniform Annual Series and Selected Nonuniform Series

| Type of Compounding | Direction of Conversion | Computational Formula |
|------------------------|--|--|
| Discrete compounding | Linear gradient series (<i>G</i>) to equivalent uniform series (<i>A</i>) ^a | $A = G \left[\frac{1}{i} - \frac{N}{(1+i)^N - 1} \right] = G \times \text{GSUAF}$ |
| | Geometric series (<i>M</i>) to equivalent uniform series (<i>A</i>) ^b | $A = M \frac{[(1+t)/(1+i)]^N - 1}{t-i} \frac{i(1+i)^N}{(1+i)^N - 1}$ |
| Continuous compounding | Linear gradient series (<i>G</i>) to equivalent uniform series (<i>A</i>) | $A = G \left(\frac{1}{e^i - 1} - \frac{N}{e^{Ni} - 1} \right)$ |
| | Geometric series (<i>M</i>) to equivalent uniform series (<i>A</i>) | $A = M \frac{(1+t)^N - e^{Ni}}{1+t-e^i} \frac{e^i - 1}{e^{Ni} - 1}$ |

^aGSUAF, the gradient series uniform amount factor, can also be read off from standard equivalency tables.

^bThe cash flow patterns are changing at a constant rate of *t*% per period. The initial cash flow in this series, *M*, occurs at the end of period 1. The cash flow at the end of period 2 is *M*(1 + *t*) and at the end of period *N* is *M*(1 + *t*)^{*N*-1}.

Table A8.3 Compound Interest Factors

| <i>N</i> | Single Payment | | Uniform Payment Series | | | | Arithmetic Gradient | |
|-----------|---|---|--|--|--|---|--|---|
| | Compound Amount Factor: Find <i>F</i> Given <i>P</i> <i>F/P</i> | Present Worth Factor: Find <i>P</i> Given <i>F</i> , <i>P/F</i> | Sinking Fund Factor: Find <i>A</i> Given <i>F</i> , <i>A/F</i> | Capital Recovery Factor: Find <i>A</i> Given <i>P</i> , <i>A/P</i> | Capital Amount Factor: Find <i>F</i> Given <i>A</i> , <i>F/A</i> | Present Worth Factor: Find <i>P</i> Given <i>A</i> , <i>P/A</i> | Gradient Uniform Series: Find <i>A</i> Given <i>G</i> , <i>A/G</i> | Gradient Present Worth: Find <i>P</i> Given <i>G</i> , <i>P/G</i> |
| | 2% | | | | | | | |
| 1 | 1.020 | 0.9804 | 1.0000 | 1.0200 | 1.000 | 0.980 | 0.000 | 0.000 |
| 2 | 1.040 | 0.9612 | 0.4950 | 0.5150 | 2.020 | 1.942 | 0.495 | 0.961 |
| 3 | 1.061 | 0.9423 | 0.3268 | 0.3468 | 3.060 | 2.884 | 0.987 | 2.846 |
| 4 | 1.082 | 0.9238 | 0.2426 | 0.2626 | 4.122 | 3.808 | 1.475 | 5.617 |
| 5 | 1.104 | 0.9057 | 0.1922 | 0.2122 | 5.204 | 4.713 | 1.960 | 9.240 |
| 6 | 1.126 | 0.8880 | 0.1585 | 0.1785 | 6.308 | 5.601 | 2.442 | 13.680 |
| 7 | 1.149 | 0.8706 | 0.1345 | 0.1545 | 7.434 | 6.472 | 2.921 | 18.903 |
| 8 | 1.172 | 0.8535 | 0.1165 | 0.1365 | 8.583 | 7.325 | 3.396 | 24.878 |
| 9 | 1.195 | 0.8368 | 0.1025 | 0.1225 | 9.755 | 8.162 | 3.868 | 31.572 |
| 10 | 1.219 | 0.8203 | 0.0913 | 0.1113 | 10.950 | 8.983 | 4.337 | 38.955 |
| 11 | 1.243 | 0.8043 | 0.0822 | 0.1022 | 12.169 | 9.787 | 4.802 | 46.998 |
| 12 | 1.268 | 0.7885 | 0.0746 | 0.0946 | 13.412 | 10.575 | 5.264 | 55.671 |
| 13 | 1.294 | 0.7730 | 0.0681 | 0.0881 | 14.680 | 11.348 | 5.723 | 64.948 |
| 14 | 1.319 | 0.7579 | 0.0626 | 0.0826 | 15.974 | 12.106 | 6.179 | 74.800 |
| 15 | 1.346 | 0.7430 | 0.0578 | 0.0778 | 17.293 | 12.849 | 6.631 | 85.202 |

Table A8.3 (continued)

| N | Single Payment | | Uniform Payment Series | | | | Arithmetic Gradient | |
|-----------|--|--|---|---|---|--|---|--|
| | Compound Amount Factor: Find F Given P F/P | Present Worth Factor: Find P Given F , P/F | Sinking Fund Factor: Find A Given F , A/F | Capital Recovery Factor: Find A Given P , A/P | Capital Amount Factor: Find F Given A , F/A | Present Worth Factor: Find P Given A , P/A | Gradient Uniform Series: Find A Given G , A/G | Gradient Present Worth: Find P Given G , P/G |
| 16 | 1.373 | 0.7284 | 0.0537 | 0.0737 | 18.639 | 13.578 | 7.080 | 96.129 |
| 17 | 1.400 | 0.7142 | 0.0500 | 0.0700 | 20.012 | 14.292 | 7.526 | 107.555 |
| 18 | 1.428 | 0.7002 | 0.0467 | 0.0667 | 21.412 | 14.992 | 7.968 | 119.458 |
| 19 | 1.457 | 0.6864 | 0.0438 | 0.0638 | 22.841 | 15.678 | 8.407 | 131.814 |
| 20 | 1.486 | 0.6730 | 0.0412 | 0.0612 | 24.297 | 16.351 | 8.843 | 144.600 |
| 21 | 1.516 | 0.6598 | 0.0388 | 0.0588 | 25.783 | 17.011 | 9.276 | 157.796 |
| 22 | 1.546 | 0.6468 | 0.0366 | 0.0566 | 27.299 | 17.658 | 9.705 | 171.379 |
| 23 | 1.577 | 0.6342 | 0.0347 | 0.0547 | 28.845 | 18.292 | 10.132 | 185.331 |
| 24 | 1.608 | 0.6217 | 0.0329 | 0.0529 | 30.422 | 18.914 | 10.555 | 199.630 |
| 25 | 1.641 | 0.6095 | 0.0312 | 0.0512 | 32.030 | 19.523 | 10.974 | 214.259 |
| 26 | 1.673 | 0.5976 | 0.0297 | 0.0497 | 33.671 | 20.121 | 11.391 | 229.199 |
| 27 | 1.707 | 0.5859 | 0.0283 | 0.0483 | 35.344 | 20.707 | 11.804 | 244.431 |
| 28 | 1.741 | 0.5744 | 0.0270 | 0.0470 | 37.051 | 21.281 | 12.214 | 259.939 |
| 29 | 1.776 | 0.5631 | 0.0258 | 0.0458 | 38.792 | 21.844 | 12.621 | 275.706 |
| 30 | 1.811 | 0.5521 | 0.0246 | 0.0446 | 40.568 | 22.396 | 13.025 | 291.716 |
| 36 | 2.040 | 0.4902 | 0.0192 | 0.0392 | 51.994 | 25.489 | 15.381 | 392.040 |
| 40 | 2.208 | 0.4529 | 0.0166 | 0.0366 | 60.402 | 27.355 | 16.889 | 461.993 |
| 48 | 2.587 | 0.3865 | 0.0126 | 0.0326 | 79.354 | 30.673 | 19.756 | 605.966 |
| 50 | 2.692 | 0.3715 | 0.0118 | 0.0318 | 84.579 | 31.424 | 20.442 | 642.361 |
| 52 | 2.800 | 0.3571 | 0.0111 | 0.0311 | 90.016 | 32.145 | 21.116 | 678.785 |
| 60 | 3.281 | 0.3048 | 0.0088 | 0.0288 | 114.052 | 34.761 | 23.696 | 823.698 |
| 70 | 4.000 | 0.2500 | 0.0067 | 0.0267 | 149.978 | 37.499 | 26.663 | 999.834 |
| 72 | 4.161 | 0.2403 | 0.0063 | 0.0263 | 158.057 | 37.984 | 27.223 | 1,034.056 |
| 80 | 4.875 | 0.2051 | 0.0052 | 0.0252 | 193.772 | 39.745 | 29.357 | 1,166.787 |
| 84 | 5.277 | 0.1895 | 0.0047 | 0.0247 | 213.867 | 40.526 | 30.362 | 1,230.419 |
| 90 | 5.943 | 0.1683 | 0.0040 | 0.0240 | 247.157 | 41.587 | 31.793 | 1,322.170 |
| 96 | 6.693 | 0.1494 | 0.0035 | 0.0235 | 284.647 | 42.529 | 33.137 | 1,409.297 |
| 100 | 7.245 | 0.1380 | 0.0032 | 0.0232 | 312.232 | 43.098 | 33.986 | 1,464.753 |
| 4% | | | | | | | | |
| 1 | 1.040 | 0.9615 | 1.0000 | 1.0400 | 1.000 | 0.962 | 0.000 | 0.000 |
| 2 | 1.082 | 0.9246 | 0.4902 | 0.5302 | 2.040 | 1.886 | 0.490 | 0.925 |
| 3 | 1.125 | 0.8890 | 0.3203 | 0.3603 | 3.122 | 2.775 | 0.974 | 2.703 |
| 4 | 1.170 | 0.8548 | 0.2355 | 0.2755 | 4.246 | 3.630 | 1.451 | 5.267 |
| 5 | 1.217 | 0.8219 | 0.1846 | 0.2246 | 5.416 | 4.452 | 1.922 | 8.555 |
| 6 | 1.265 | 0.7903 | 0.1508 | 0.1908 | 6.633 | 5.242 | 2.386 | 12.506 |
| 7 | 1.316 | 0.7599 | 0.1266 | 0.1666 | 7.898 | 6.002 | 2.843 | 17.066 |
| 8 | 1.369 | 0.7307 | 0.1085 | 0.1485 | 9.214 | 6.733 | 3.294 | 22.181 |
| 9 | 1.423 | 0.7026 | 0.0945 | 0.1345 | 10.583 | 7.435 | 3.739 | 27.801 |
| 10 | 1.480 | 0.6756 | 0.0833 | 0.1233 | 12.006 | 8.111 | 4.177 | 33.881 |

(continued overleaf)

Table A8.3 (continued)

| <i>N</i> | Single Payment | | Uniform Payment Series | | | | Arithmetic Gradient | |
|------------|---|---|--|--|--|---|--|---|
| | Compound Amount Factor: Find <i>F</i> Given <i>P</i> <i>F/P</i> | Present Worth Factor: Find <i>P</i> Given <i>F</i> , <i>P/F</i> | Sinking Fund Factor: Find <i>A</i> Given <i>F</i> , <i>A/F</i> | Capital Recovery Factor: Find <i>A</i> Given <i>P</i> , <i>A/P</i> | Capital Amount Factor: Find <i>F</i> Given <i>A</i> , <i>F/A</i> | Present Worth Factor: Find <i>P</i> Given <i>A</i> , <i>P/A</i> | Gradient Uniform Series: Find <i>A</i> Given <i>G</i> , <i>A/G</i> | Gradient Present Worth: Find <i>P</i> Given <i>G</i> , <i>P/G</i> |
| 11 | 1.539 | 0.6496 | 0.0741 | 0.1141 | 13.486 | 8.760 | 4.609 | 40.377 |
| 12 | 1.601 | 0.6246 | 0.0666 | 0.1066 | 15.026 | 9.385 | 5.034 | 47.248 |
| 13 | 1.665 | 0.6006 | 0.0601 | 0.1001 | 16.627 | 9.986 | 5.453 | 54.455 |
| 14 | 1.732 | 0.5775 | 0.0547 | 0.0947 | 18.292 | 10.563 | 5.866 | 61.962 |
| 15 | 1.801 | 0.5553 | 0.0499 | 0.0899 | 20.024 | 11.118 | 6.272 | 69.735 |
| 16 | 1.873 | 0.5339 | 0.0458 | 0.0858 | 21.825 | 11.652 | 6.672 | 77.744 |
| 17 | 1.948 | 0.5134 | 0.0422 | 0.0822 | 23.698 | 12.166 | 7.066 | 85.958 |
| 18 | 2.026 | 0.4936 | 0.0390 | 0.0790 | 25.645 | 12.659 | 7.453 | 94.350 |
| 19 | 2.107 | 0.4746 | 0.0361 | 0.0761 | 27.671 | 13.134 | 7.834 | 102.893 |
| 20 | 2.191 | 0.4564 | 0.0336 | 0.0736 | 29.778 | 13.590 | 8.209 | 111.565 |
| 21 | 2.279 | 0.4388 | 0.0313 | 0.0713 | 31.969 | 14.029 | 8.578 | 120.341 |
| 22 | 2.370 | 0.4220 | 0.0292 | 0.0692 | 34.248 | 14.451 | 8.941 | 129.202 |
| 23 | 2.465 | 0.4057 | 0.0273 | 0.0673 | 36.618 | 14.857 | 9.297 | 138.128 |
| 24 | 2.563 | 0.3901 | 0.0256 | 0.0656 | 39.083 | 15.247 | 9.648 | 147.101 |
| 25 | 2.666 | 0.3751 | 0.0240 | 0.0640 | 41.646 | 15.622 | 9.993 | 156.104 |
| 26 | 2.772 | 0.3607 | 0.0226 | 0.0626 | 44.312 | 15.983 | 10.331 | 165.121 |
| 27 | 2.883 | 0.3468 | 0.0212 | 0.0612 | 47.084 | 16.330 | 10.664 | 174.138 |
| 28 | 2.999 | 0.3335 | 0.0200 | 0.0600 | 49.968 | 16.663 | 10.991 | 183.142 |
| 29 | 3.119 | 0.3207 | 0.0189 | 0.0589 | 52.966 | 16.984 | 11.312 | 192.121 |
| 30 | 3.243 | 0.3083 | 0.0178 | 0.0578 | 56.085 | 17.292 | 11.627 | 201.062 |
| 31 | 3.373 | 0.2965 | 0.0169 | 0.0569 | 59.328 | 17.588 | 11.937 | 209.956 |
| 32 | 3.508 | 0.2851 | 0.0159 | 0.0559 | 62.701 | 17.874 | 12.241 | 218.792 |
| 33 | 3.648 | 0.2741 | 0.0151 | 0.0551 | 66.210 | 18.148 | 12.540 | 227.563 |
| 34 | 3.794 | 0.2636 | 0.0143 | 0.0543 | 69.858 | 18.411 | 12.832 | 236.261 |
| 35 | 3.946 | 0.2534 | 0.0136 | 0.0536 | 73.652 | 18.665 | 13.120 | 244.877 |
| 40 | 4.801 | 0.2083 | 0.0105 | 0.0505 | 95.026 | 19.793 | 14.477 | 286.530 |
| 45 | 5.841 | 0.1712 | 0.0083 | 0.0483 | 121.029 | 20.720 | 15.705 | 325.403 |
| 50 | 7.107 | 0.1407 | 0.0066 | 0.0466 | 152.667 | 21.482 | 16.812 | 361.164 |
| 55 | 8.646 | 0.1157 | 0.0052 | 0.0452 | 191.159 | 22.109 | 17.807 | 393.689 |
| 60 | 10.520 | 0.0951 | 0.0042 | 0.0442 | 237.991 | 22.623 | 18.697 | 422.997 |
| 65 | 12.799 | 0.0781 | 0.0034 | 0.0434 | 294.968 | 23.047 | 19.491 | 449.201 |
| 70 | 15.572 | 0.0642 | 0.0027 | 0.0427 | 364.290 | 23.395 | 20.196 | 472.479 |
| 75 | 18.945 | 0.0528 | 0.0022 | 0.0422 | 448.631 | 23.680 | 20.821 | 493.041 |
| 80 | 23.050 | 0.0434 | 0.0018 | 0.0418 | 551.245 | 23.915 | 21.372 | 511.116 |
| 85 | 28.044 | 0.0357 | 0.0015 | 0.0415 | 676.090 | 24.109 | 21.857 | 526.938 |
| 90 | 34.119 | 0.0293 | 0.0012 | 0.0412 | 827.983 | 24.267 | 22.283 | 540.737 |
| 95 | 41.511 | 0.0241 | 0.0010 | 0.0410 | 1,012.785 | 24.398 | 22.655 | 552.731 |
| 100 | 50.505 | 0.0198 | 0.0008 | 0.0408 | 1,237.624 | 24.505 | 22.980 | 563.125 |

Table A8.3 (continued)

| <i>N</i> | Single Payment | | Uniform Payment Series | | | | Arithmetic Gradient | |
|-----------|---|---|--|--|--|---|--|---|
| | Compound Amount Factor: Find <i>F</i> Given <i>P</i> <i>F/P</i> | Present Worth Factor: Find <i>P</i> Given <i>F</i> , <i>P/F</i> | Sinking Fund Factor: Find <i>A</i> Given <i>F</i> , <i>A/F</i> | Capital Recovery Factor: Find <i>A</i> Given <i>P</i> , <i>A/P</i> | Capital Amount Factor: Find <i>F</i> Given <i>A</i> , <i>F/A</i> | Present Worth Factor: Find <i>P</i> Given <i>A</i> , <i>P/A</i> | Gradient Uniform Series: Find <i>A</i> Given <i>G</i> , <i>A/G</i> | Gradient Present Worth: Find <i>P</i> Given <i>G</i> , <i>P/G</i> |
| | 5% | | | | | | | |
| 1 | 1.050 | 0.9524 | 1.0000 | 1.0500 | 1.000 | 0.952 | 0.000 | 0.000 |
| 2 | 1.103 | 0.9070 | 0.4878 | 0.5378 | 2.050 | 1.859 | 0.488 | 0.907 |
| 3 | 1.158 | 0.8638 | 0.3172 | 0.3672 | 3.153 | 2.723 | 0.967 | 2.635 |
| 4 | 1.216 | 0.8227 | 0.2320 | 0.2820 | 4.310 | 3.546 | 1.439 | 5.103 |
| 5 | 1.276 | 0.7835 | 0.1810 | 0.2310 | 5.526 | 4.329 | 1.903 | 8.237 |
| 6 | 1.340 | 0.7462 | 0.1470 | 0.1970 | 6.802 | 5.076 | 2.358 | 11.968 |
| 7 | 1.407 | 0.7107 | 0.1228 | 0.1728 | 8.142 | 5.786 | 2.805 | 16.232 |
| 8 | 1.477 | 0.6768 | 0.1047 | 0.1547 | 9.549 | 6.463 | 3.245 | 20.970 |
| 9 | 1.551 | 0.6446 | 0.0907 | 0.1407 | 11.027 | 7.108 | 3.676 | 26.127 |
| 10 | 1.629 | 0.6139 | 0.0795 | 0.1295 | 12.578 | 7.722 | 4.099 | 31.652 |
| 11 | 1.710 | 0.5847 | 0.0704 | 0.1204 | 14.207 | 8.306 | 4.514 | 37.499 |
| 12 | 1.796 | 0.5568 | 0.0628 | 0.1128 | 15.917 | 8.863 | 4.922 | 43.624 |
| 13 | 1.886 | 0.5303 | 0.0565 | 0.1065 | 17.713 | 9.394 | 5.322 | 49.988 |
| 14 | 1.980 | 0.5051 | 0.0510 | 0.1010 | 19.599 | 9.899 | 5.713 | 56.554 |
| 15 | 2.079 | 0.4810 | 0.0463 | 0.0963 | 21.579 | 10.380 | 6.097 | 63.288 |
| 16 | 2.183 | 0.4581 | 0.0423 | 0.0923 | 23.657 | 10.838 | 6.474 | 70.160 |
| 17 | 2.292 | 0.4363 | 0.0387 | 0.0887 | 25.840 | 11.274 | 6.842 | 77.140 |
| 18 | 2.407 | 0.4155 | 0.0355 | 0.0855 | 28.132 | 11.690 | 7.203 | 84.204 |
| 19 | 2.527 | 0.3957 | 0.0327 | 0.0827 | 30.539 | 12.085 | 7.557 | 91.328 |
| 20 | 2.653 | 0.3769 | 0.0302 | 0.0802 | 33.066 | 12.462 | 7.903 | 98.488 |
| 21 | 2.786 | 0.3589 | 0.0280 | 0.0780 | 35.719 | 12.821 | 8.242 | 105.667 |
| 22 | 2.925 | 0.3418 | 0.0260 | 0.0760 | 38.505 | 13.163 | 8.573 | 112.846 |
| 23 | 3.072 | 0.3256 | 0.0241 | 0.0741 | 41.430 | 13.489 | 8.897 | 120.009 |
| 24 | 3.225 | 0.3101 | 0.0225 | 0.0725 | 44.502 | 13.799 | 9.214 | 127.140 |
| 25 | 3.386 | 0.2953 | 0.0210 | 0.0710 | 47.727 | 14.094 | 9.524 | 134.228 |
| 26 | 3.556 | 0.2812 | 0.0196 | 0.0696 | 51.113 | 14.375 | 9.827 | 141.259 |
| 27 | 3.733 | 0.2678 | 0.0183 | 0.0683 | 54.669 | 14.643 | 10.122 | 148.223 |
| 28 | 3.920 | 0.2551 | 0.0171 | 0.0671 | 58.403 | 14.898 | 10.411 | 155.110 |
| 29 | 4.116 | 0.2429 | 0.0160 | 0.0660 | 62.323 | 15.141 | 10.694 | 161.913 |
| 30 | 4.322 | 0.2314 | 0.0151 | 0.0651 | 66.439 | 15.372 | 10.969 | 168.623 |
| 31 | 4.538 | 0.2204 | 0.0141 | 0.0641 | 70.761 | 15.593 | 11.238 | 175.233 |
| 32 | 4.765 | 0.2099 | 0.0133 | 0.0633 | 75.299 | 15.803 | 11.501 | 181.739 |
| 33 | 5.003 | 0.1999 | 0.0125 | 0.0625 | 80.064 | 16.003 | 11.757 | 188.135 |
| 34 | 5.253 | 0.1904 | 0.0118 | 0.0618 | 85.067 | 16.193 | 12.006 | 194.417 |
| 35 | 5.516 | 0.1813 | 0.0111 | 0.0611 | 90.320 | 16.374 | 12.250 | 200.581 |

(continued overleaf)

Table A8.3 (continued)

| N | Single Payment | | Uniform Payment Series | | | | Arithmetic Gradient | |
|-----------|--|---|--|--|--|---|--|---|
| | Compound Amount Factor: Find F Given P F/P | Present Worth Factor: Find P Given F, P/F | Sinking Fund Factor: Find A Given F, A/F | Capital Recovery Factor: Find A Given P, A/P | Capital Amount Factor: Find F Given A, F/A | Present Worth Factor: Find P Given A, P/A | Gradient Uniform Series: Find A Given G, A/G | Gradient Present Worth: Find P Given G, P/G |
| 40 | 7.040 | 0.1420 | 0.0083 | 0.0583 | 120.800 | 17.159 | 13.377 | 229.545 |
| 45 | 8.985 | 0.1113 | 0.0063 | 0.0563 | 159.700 | 17.774 | 14.364 | 255.315 |
| 50 | 11.467 | 0.0872 | 0.0048 | 0.0548 | 209.348 | 18.256 | 15.223 | 277.915 |
| 55 | 14.636 | 0.0683 | 0.0037 | 0.0537 | 272.713 | 18.633 | 15.966 | 297.510 |
| 60 | 18.679 | 0.0535 | 0.0028 | 0.0528 | 353.584 | 18.929 | 16.606 | 314.343 |
| 65 | 23.840 | 0.0419 | 0.0022 | 0.0522 | 456.798 | 19.161 | 17.154 | 328.691 |
| 70 | 30.426 | 0.0329 | 0.0017 | 0.0517 | 588.529 | 19.343 | 17.621 | 340.841 |
| 75 | 38.833 | 0.0258 | 0.0013 | 0.0513 | 756.654 | 19.485 | 18.018 | 351.072 |
| 80 | 49.561 | 0.0202 | 0.0010 | 0.0510 | 971.229 | 19.596 | 18.353 | 359.646 |
| 85 | 63.254 | 0.0158 | 0.0008 | 0.0508 | 1,245.087 | 19.684 | 18.635 | 366.801 |
| 90 | 80.730 | 0.0124 | 0.0006 | 0.0506 | 1,594.607 | 19.752 | 18.871 | 372.749 |
| 95 | 103.035 | 0.0097 | 0.0005 | 0.0505 | 2,040.694 | 19.806 | 19.069 | 377.677 |
| 100 | 131.501 | 0.0076 | 0.0004 | 0.0504 | 2,610.025 | 19.848 | 19.234 | 381.749 |
| 6% | | | | | | | | |
| 1 | 1.060 | 0.9434 | 1.0000 | 1.0600 | 1.000 | 0.943 | 0.000 | 0.000 |
| 2 | 1.124 | 0.8900 | 0.4854 | 0.5454 | 2.060 | 1.833 | 0.485 | 0.890 |
| 3 | 1.191 | 0.8396 | 0.3141 | 0.3741 | 3.184 | 2.673 | 0.961 | 2.569 |
| 4 | 1.262 | 0.7921 | 0.2286 | 0.2886 | 4.375 | 3.465 | 1.427 | 4.946 |
| 5 | 1.338 | 0.7473 | 0.1774 | 0.2374 | 5.637 | 4.212 | 1.884 | 7.935 |
| 6 | 1.419 | 0.7050 | 0.1434 | 0.2034 | 6.975 | 4.917 | 2.330 | 11.459 |
| 7 | 1.504 | 0.6651 | 0.1191 | 0.1791 | 8.394 | 5.582 | 2.768 | 15.450 |
| 8 | 1.594 | 0.6274 | 0.1010 | 0.1610 | 9.897 | 6.210 | 3.195 | 19.842 |
| 9 | 1.689 | 0.5919 | 0.0870 | 0.1470 | 11.491 | 6.802 | 3.613 | 24.577 |
| 10 | 1.791 | 0.5584 | 0.0759 | 0.1359 | 13.181 | 7.360 | 4.022 | 29.602 |
| 11 | 1.898 | 0.5268 | 0.0668 | 0.1268 | 14.972 | 7.887 | 4.421 | 34.870 |
| 12 | 2.012 | 0.4970 | 0.0593 | 0.1193 | 16.870 | 8.384 | 4.811 | 40.337 |
| 13 | 2.133 | 0.4688 | 0.0530 | 0.1130 | 18.882 | 8.853 | 5.192 | 45.963 |
| 14 | 2.261 | 0.4423 | 0.0476 | 0.1076 | 21.015 | 9.295 | 5.564 | 51.713 |
| 15 | 2.397 | 0.4173 | 0.0430 | 0.1030 | 23.276 | 9.712 | 5.926 | 57.555 |
| 16 | 2.540 | 0.3936 | 0.0390 | 0.0990 | 25.673 | 10.106 | 6.279 | 63.459 |
| 17 | 2.693 | 0.3714 | 0.0354 | 0.0954 | 28.213 | 10.477 | 6.624 | 69.401 |
| 18 | 2.854 | 0.3503 | 0.0324 | 0.0924 | 30.906 | 10.828 | 6.960 | 75.357 |
| 19 | 3.026 | 0.3305 | 0.0296 | 0.0896 | 33.760 | 11.158 | 7.287 | 81.306 |
| 20 | 3.207 | 0.3118 | 0.0272 | 0.0872 | 36.786 | 11.470 | 7.605 | 87.230 |
| 21 | 3.400 | 0.2942 | 0.0250 | 0.0850 | 39.993 | 11.764 | 7.915 | 93.114 |
| 22 | 3.604 | 0.2775 | 0.0230 | 0.0830 | 43.392 | 12.042 | 8.217 | 98.941 |
| 23 | 3.820 | 0.2618 | 0.0213 | 0.0813 | 46.996 | 12.303 | 8.510 | 104.701 |
| 24 | 4.049 | 0.2470 | 0.0197 | 0.0797 | 50.816 | 12.550 | 8.795 | 110.381 |
| 25 | 4.292 | 0.2330 | 0.0182 | 0.0782 | 54.865 | 12.783 | 9.072 | 115.973 |

Table A8.3 (continued)

| N | Single Payment | | Uniform Payment Series | | | | Arithmetic Gradient | |
|-----------|--|--|---|---|---|--|---|--|
| | Compound Amount Factor: Find F Given P F/P | Present Worth Factor: Find P Given F , P/F | Sinking Fund Factor: Find A Given F , A/F | Capital Recovery Factor: Find A Given P , A/P | Capital Amount Factor: Find F Given A , F/A | Present Worth Factor: Find P Given A , P/A | Gradient Uniform Series: Find A Given G , A/G | Gradient Present Worth: Find P Given G , P/G |
| 26 | 4.549 | 0.2198 | 0.0169 | 0.0769 | 59.156 | 13.003 | 9.341 | 121.468 |
| 27 | 4.822 | 0.2074 | 0.0157 | 0.0757 | 63.706 | 13.211 | 9.603 | 126.860 |
| 28 | 5.112 | 0.1956 | 0.0146 | 0.0746 | 68.528 | 13.406 | 9.857 | 132.142 |
| 29 | 5.418 | 0.1846 | 0.0136 | 0.0736 | 73.640 | 13.591 | 10.103 | 137.310 |
| 30 | 5.743 | 0.1741 | 0.0126 | 0.0726 | 79.058 | 13.765 | 10.342 | 142.359 |
| 31 | 6.088 | 0.1643 | 0.0118 | 0.0718 | 84.802 | 13.929 | 10.574 | 147.286 |
| 32 | 6.453 | 0.1550 | 0.0110 | 0.0710 | 90.890 | 14.084 | 10.799 | 152.090 |
| 33 | 6.841 | 0.1462 | 0.0103 | 0.0703 | 97.343 | 14.230 | 11.017 | 156.768 |
| 34 | 7.251 | 0.1379 | 0.0096 | 0.0696 | 104.184 | 14.368 | 11.228 | 161.319 |
| 35 | 7.686 | 0.1301 | 0.0090 | 0.0690 | 111.435 | 14.498 | 11.432 | 165.743 |
| 40 | 10.286 | 0.0972 | 0.0065 | 0.0665 | 154.762 | 15.046 | 12.359 | 185.957 |
| 45 | 13.765 | 0.0727 | 0.0047 | 0.0647 | 212.744 | 15.456 | 13.141 | 203.110 |
| 50 | 18.420 | 0.0543 | 0.0034 | 0.0634 | 290.336 | 15.762 | 13.796 | 217.457 |
| 55 | 24.650 | 0.0406 | 0.0025 | 0.0625 | 394.172 | 15.991 | 14.341 | 229.322 |
| 60 | 32.988 | 0.0303 | 0.0019 | 0.0619 | 533.128 | 16.161 | 14.791 | 239.043 |
| 65 | 44.145 | 0.0227 | 0.0014 | 0.0614 | 719.083 | 16.289 | 15.160 | 246.945 |
| 70 | 59.076 | 0.0169 | 0.0010 | 0.0610 | 967.932 | 16.385 | 15.461 | 253.327 |
| 75 | 79.057 | 0.0126 | 0.0008 | 0.0608 | 1,300.949 | 16.456 | 15.706 | 258.453 |
| 80 | 105.796 | 0.0095 | 0.0006 | 0.0606 | 1,746.600 | 16.509 | 15.903 | 262.549 |
| 85 | 141.579 | 0.0071 | 0.0004 | 0.0604 | 2,342.982 | 16.549 | 16.062 | 265.810 |
| 90 | 189.465 | 0.0053 | 0.0003 | 0.0603 | 3,141.075 | 16.579 | 16.189 | 268.395 |
| 95 | 253.546 | 0.0039 | 0.0002 | 0.0602 | 4,209.104 | 16.601 | 16.290 | 270.437 |
| 100 | 339.302 | 0.0029 | 0.0002 | 0.0602 | 5,638.368 | 16.618 | 16.371 | 272.047 |
| 7% | | | | | | | | |
| 1 | 1.070 | 0.9346 | 1.0000 | 1.0700 | 1.000 | 0.935 | 0.000 | 0.000 |
| 2 | 1.145 | 0.8734 | 0.4831 | 0.5531 | 2.070 | 1.808 | 0.483 | 0.873 |
| 3 | 1.225 | 0.8163 | 0.3111 | 0.3811 | 3.215 | 2.624 | 0.955 | 2.506 |
| 4 | 1.311 | 0.7629 | 0.2252 | 0.2952 | 4.440 | 3.387 | 1.416 | 4.795 |
| 5 | 1.403 | 0.7130 | 0.1739 | 0.2439 | 5.751 | 4.100 | 1.865 | 7.647 |
| 6 | 1.501 | 0.6663 | 0.1398 | 0.2098 | 7.153 | 4.767 | 2.303 | 10.978 |
| 7 | 1.606 | 0.6227 | 0.1156 | 0.1856 | 8.654 | 5.389 | 2.730 | 14.715 |
| 8 | 1.718 | 0.5820 | 0.0975 | 0.1675 | 10.260 | 5.971 | 3.147 | 18.789 |
| 9 | 1.838 | 0.5439 | 0.0835 | 0.1535 | 11.978 | 6.515 | 3.552 | 23.140 |
| 10 | 1.967 | 0.5083 | 0.0724 | 0.1424 | 13.816 | 7.024 | 3.946 | 27.716 |
| 11 | 2.105 | 0.4751 | 0.0634 | 0.1334 | 15.784 | 7.499 | 4.330 | 32.466 |
| 12 | 2.252 | 0.4440 | 0.0559 | 0.1259 | 17.888 | 7.943 | 4.703 | 37.351 |
| 13 | 2.410 | 0.4150 | 0.0497 | 0.1197 | 20.141 | 8.358 | 5.065 | 42.330 |
| 14 | 2.579 | 0.3878 | 0.0443 | 0.1143 | 22.550 | 8.745 | 5.417 | 47.372 |
| 15 | 2.759 | 0.3624 | 0.0398 | 0.1098 | 25.129 | 9.108 | 5.758 | 52.446 |

(continued overleaf)

Table A8.3 (continued)

| <i>N</i> | Single Payment | | Uniform Payment Series | | | | Arithmetic Gradient | |
|------------|---|---|--|--|--|---|--|---|
| | Compound Amount Factor: Find <i>F</i> Given <i>P</i> <i>F/P</i> | Present Worth Factor: Find <i>P</i> Given <i>F</i> , <i>P/F</i> | Sinking Fund Factor: Find <i>A</i> Given <i>F</i> , <i>A/F</i> | Capital Recovery Factor: Find <i>A</i> Given <i>P</i> , <i>A/P</i> | Capital Amount Factor: Find <i>F</i> Given <i>A</i> , <i>F/A</i> | Present Worth Factor: Find <i>P</i> Given <i>A</i> , <i>P/A</i> | Gradient Uniform Series: Find <i>A</i> Given <i>G</i> , <i>A/G</i> | Gradient Present Worth: Find <i>P</i> Given <i>G</i> , <i>P/G</i> |
| 16 | 2.952 | 0.3387 | 0.0359 | 0.1059 | 27.888 | 9.447 | 6.090 | 57.527 |
| 17 | 3.159 | 0.3166 | 0.0324 | 0.1024 | 30.840 | 9.763 | 6.411 | 62.592 |
| 18 | 3.380 | 0.2959 | 0.0294 | 0.0994 | 33.999 | 10.059 | 6.722 | 67.622 |
| 19 | 3.617 | 0.2765 | 0.0268 | 0.0968 | 37.379 | 10.336 | 7.024 | 72.599 |
| 20 | 3.870 | 0.2584 | 0.0244 | 0.0944 | 40.995 | 10.594 | 7.316 | 77.509 |
| 21 | 4.141 | 0.2415 | 0.0223 | 0.0923 | 44.865 | 10.836 | 7.599 | 82.339 |
| 22 | 4.430 | 0.2257 | 0.0204 | 0.0904 | 49.006 | 11.061 | 7.872 | 87.079 |
| 23 | 4.741 | 0.2109 | 0.0187 | 0.0887 | 53.436 | 11.272 | 8.137 | 91.720 |
| 24 | 5.072 | 0.1971 | 0.0172 | 0.0872 | 58.177 | 11.469 | 8.392 | 96.255 |
| 25 | 5.427 | 0.1842 | 0.0158 | 0.0858 | 63.249 | 11.654 | 8.639 | 100.676 |
| 26 | 5.807 | 0.1722 | 0.0146 | 0.0846 | 68.676 | 11.826 | 8.877 | 104.981 |
| 27 | 6.214 | 0.1609 | 0.0134 | 0.0834 | 74.484 | 11.987 | 9.107 | 109.166 |
| 28 | 6.649 | 0.1504 | 0.0124 | 0.0824 | 80.698 | 12.137 | 9.329 | 113.226 |
| 29 | 7.114 | 0.1406 | 0.0114 | 0.0814 | 87.347 | 12.278 | 9.543 | 117.162 |
| 30 | 7.612 | 0.1314 | 0.0106 | 0.0806 | 94.461 | 12.409 | 9.749 | 120.972 |
| 31 | 8.145 | 0.1228 | 0.0098 | 0.0798 | 102.073 | 12.532 | 9.947 | 124.655 |
| 32 | 8.715 | 0.1147 | 0.0091 | 0.0791 | 110.218 | 12.647 | 10.138 | 128.212 |
| 33 | 9.325 | 0.1072 | 0.0084 | 0.0784 | 118.933 | 12.754 | 10.322 | 131.643 |
| 34 | 9.978 | 0.1002 | 0.0078 | 0.0778 | 128.259 | 12.854 | 10.499 | 134.951 |
| 35 | 10.677 | 0.0937 | 0.0072 | 0.0772 | 138.237 | 12.948 | 10.669 | 138.135 |
| 40 | 14.974 | 0.0668 | 0.0050 | 0.0750 | 199.635 | 13.332 | 11.423 | 152.293 |
| 45 | 21.002 | 0.0476 | 0.0035 | 0.0735 | 285.749 | 13.606 | 12.036 | 163.756 |
| 50 | 29.457 | 0.0339 | 0.0025 | 0.0725 | 406.529 | 13.801 | 12.529 | 172.905 |
| 55 | 41.315 | 0.0242 | 0.0017 | 0.0717 | 575.929 | 13.940 | 12.921 | 180.124 |
| 60 | 57.946 | 0.0173 | 0.0012 | 0.0712 | 813.520 | 14.039 | 13.232 | 185.768 |
| 65 | 81.273 | 0.0123 | 0.0009 | 0.0709 | 1,146.755 | 14.110 | 13.476 | 190.145 |
| 70 | 113.989 | 0.0088 | 0.0006 | 0.0706 | 1,614.134 | 14.160 | 13.666 | 193.519 |
| 75 | 159.876 | 0.0063 | 0.0004 | 0.0704 | 2,269.657 | 14.196 | 13.814 | 196.104 |
| 80 | 224.234 | 0.0045 | 0.0003 | 0.0703 | 3,189.063 | 14.222 | 13.927 | 198.075 |
| 85 | 314.500 | 0.0032 | 0.0002 | 0.0702 | 4,478.576 | 14.240 | 14.015 | 199.572 |
| 90 | 441.103 | 0.0023 | 0.0002 | 0.0702 | 6,287.185 | 14.253 | 14.081 | 200.704 |
| 95 | 618.670 | 0.0016 | 0.0001 | 0.0701 | 8,823.854 | 14.263 | 14.132 | 201.558 |
| 100 | 867.716 | 0.0012 | 0.0001 | 0.0701 | 12,381.662 | 14.269 | 14.170 | 202.200 |
| 8% | | | | | | | | |
| 1 | 1.080 | 0.9259 | 1.0000 | 1.0800 | 1.000 | 0.926 | 0.000 | 0.000 |
| 2 | 1.166 | 0.8573 | 0.4808 | 0.5608 | 2.080 | 1.783 | 0.481 | 0.857 |
| 3 | 1.260 | 0.7938 | 0.3080 | 0.3880 | 3.246 | 2.577 | 0.949 | 2.445 |
| 4 | 1.360 | 0.7350 | 0.2219 | 0.3019 | 4.506 | 3.312 | 1.404 | 4.650 |
| 5 | 1.469 | 0.6806 | 0.1705 | 0.2505 | 5.867 | 3.993 | 1.846 | 7.372 |

Table A8.3 (continued)

| <i>N</i> | Single Payment | | Uniform Payment Series | | | | Arithmetic Gradient | |
|-----------|---|---|--|--|--|---|--|---|
| | Compound Amount Factor: Find <i>F</i> Given <i>P</i> <i>F/P</i> | Present Worth Factor: Find <i>P</i> Given <i>F</i> , <i>P/F</i> | Sinking Fund Factor: Find <i>A</i> Given <i>F</i> , <i>A/F</i> | Capital Recovery Factor: Find <i>A</i> Given <i>P</i> , <i>A/P</i> | Capital Amount Factor: Find <i>F</i> Given <i>A</i> , <i>F/A</i> | Present Worth Factor: Find <i>P</i> Given <i>A</i> , <i>P/A</i> | Gradient Uniform Series: Find <i>A</i> Given <i>G</i> , <i>A/G</i> | Gradient Present Worth: Find <i>P</i> Given <i>G</i> , <i>P/G</i> |
| 6 | 1.587 | 0.6302 | 0.1363 | 0.2163 | 7.336 | 4.623 | 2.276 | 10.523 |
| 7 | 1.714 | 0.5835 | 0.1121 | 0.1921 | 8.923 | 5.206 | 2.694 | 14.024 |
| 8 | 1.851 | 0.5403 | 0.0940 | 0.1740 | 10.637 | 5.747 | 3.099 | 17.806 |
| 9 | 1.999 | 0.5002 | 0.0801 | 0.1601 | 12.488 | 6.247 | 3.491 | 21.808 |
| 10 | 2.159 | 0.4632 | 0.0690 | 0.1490 | 14.487 | 6.710 | 3.871 | 25.977 |
| 11 | 2.332 | 0.4289 | 0.0601 | 0.1401 | 16.645 | 7.139 | 4.240 | 30.266 |
| 12 | 2.518 | 0.3971 | 0.0527 | 0.1327 | 18.977 | 7.536 | 4.596 | 34.634 |
| 13 | 2.720 | 0.3677 | 0.0465 | 0.1265 | 21.495 | 7.904 | 4.940 | 39.046 |
| 14 | 2.937 | 0.3405 | 0.0413 | 0.1213 | 24.215 | 8.244 | 5.273 | 43.472 |
| 15 | 3.172 | 0.3152 | 0.0368 | 0.1168 | 27.152 | 8.559 | 5.594 | 47.886 |
| 16 | 3.426 | 0.2919 | 0.0330 | 0.1130 | 30.324 | 8.851 | 5.905 | 52.264 |
| 17 | 3.700 | 0.2703 | 0.0296 | 0.1096 | 33.750 | 9.122 | 6.204 | 56.588 |
| 18 | 3.996 | 0.2502 | 0.0267 | 0.1067 | 37.450 | 9.372 | 6.492 | 60.843 |
| 19 | 4.316 | 0.2317 | 0.0241 | 0.1041 | 41.446 | 9.604 | 6.770 | 65.013 |
| 20 | 4.661 | 0.2145 | 0.0219 | 0.1019 | 45.762 | 9.818 | 7.037 | 69.090 |
| 21 | 5.034 | 0.1987 | 0.0198 | 0.0998 | 50.423 | 10.017 | 7.294 | 73.063 |
| 22 | 5.437 | 0.1839 | 0.0180 | 0.0980 | 55.457 | 10.201 | 7.541 | 76.926 |
| 23 | 5.871 | 0.1703 | 0.0164 | 0.0964 | 60.893 | 10.371 | 7.779 | 80.673 |
| 24 | 6.341 | 0.1577 | 0.0150 | 0.0950 | 66.765 | 10.529 | 8.007 | 84.300 |
| 25 | 6.848 | 0.1460 | 0.0137 | 0.0937 | 73.106 | 10.675 | 8.225 | 87.804 |
| 26 | 7.396 | 0.1352 | 0.0125 | 0.0925 | 79.954 | 10.810 | 8.435 | 91.184 |
| 27 | 7.988 | 0.1252 | 0.0114 | 0.0914 | 87.351 | 10.935 | 8.636 | 94.439 |
| 28 | 8.627 | 0.1159 | 0.0105 | 0.0905 | 95.339 | 11.051 | 8.829 | 97.569 |
| 29 | 9.317 | 0.1073 | 0.0096 | 0.0896 | 103.966 | 11.158 | 9.013 | 100.574 |
| 30 | 10.063 | 0.0994 | 0.0088 | 0.0888 | 113.283 | 11.258 | 9.190 | 103.456 |
| 31 | 10.868 | 0.0920 | 0.0081 | 0.0881 | 123.346 | 11.350 | 9.358 | 106.216 |
| 32 | 11.737 | 0.0852 | 0.0075 | 0.0875 | 134.214 | 11.435 | 9.520 | 108.857 |
| 33 | 12.676 | 0.0789 | 0.0069 | 0.0869 | 145.951 | 11.514 | 9.674 | 111.382 |
| 34 | 13.690 | 0.0730 | 0.0063 | 0.0863 | 158.627 | 11.587 | 9.821 | 113.792 |
| 35 | 14.785 | 0.0676 | 0.0058 | 0.0858 | 172.317 | 11.655 | 9.961 | 116.092 |
| 40 | 21.725 | 0.0460 | 0.0039 | 0.0839 | 259.057 | 11.925 | 10.570 | 126.042 |
| 45 | 31.920 | 0.0313 | 0.0026 | 0.0826 | 386.506 | 12.108 | 11.045 | 133.733 |
| 50 | 46.902 | 0.0213 | 0.0017 | 0.0817 | 573.770 | 12.233 | 11.411 | 139.593 |
| 55 | 68.914 | 0.0145 | 0.0012 | 0.0812 | 848.923 | 12.319 | 11.690 | 144.006 |
| 60 | 101.257 | 0.0099 | 0.0008 | 0.0808 | 1,253.213 | 12.377 | 11.902 | 147.300 |
| 65 | 148.780 | 0.0067 | 0.0005 | 0.0805 | 1,847.248 | 12.416 | 12.060 | 149.739 |
| 70 | 218.606 | 0.0046 | 0.0004 | 0.0804 | 2,720.080 | 12.443 | 12.178 | 151.533 |
| 75 | 321.205 | 0.0031 | 0.0002 | 0.0802 | 4,002.557 | 12.461 | 12.266 | 152.845 |
| 80 | 471.955 | 0.0021 | 0.0002 | 0.0802 | 5,886.935 | 12.474 | 12.330 | 153.800 |
| 85 | 693.456 | 0.0014 | 0.0001 | 0.0801 | 8,655.706 | 12.482 | 12.377 | 154.492 |

(continued overleaf)

Table A8.3 (continued)

| N | Single Payment | | Uniform Payment Series | | | | Arithmetic Gradient | |
|------------|---|---|--|--|--|---|--|---|
| | Compound Amount Factor: Find <i>F</i> Given <i>P</i> <i>F/P</i> | Present Worth Factor: Find <i>P</i> Given <i>F</i> , <i>P/F</i> | Sinking Fund Factor: Find <i>A</i> Given <i>F</i> , <i>A/F</i> | Capital Recovery Factor: Find <i>A</i> Given <i>P</i> , <i>A/P</i> | Capital Amount Factor: Find <i>F</i> Given <i>A</i> , <i>F/A</i> | Present Worth Factor: Find <i>P</i> Given <i>A</i> , <i>P/A</i> | Gradient Uniform Series: Find <i>A</i> Given <i>G</i> , <i>A/G</i> | Gradient Present Worth: Find <i>P</i> Given <i>G</i> , <i>P/G</i> |
| 90 | 1,018.915 | 0.0010 | 0.0001 | 0.0801 | 12,723.939 | 12.488 | 12.412 | 154.993 |
| 95 | 1,497.121 | 0.0007 | 0.0001 | 0.0801 | 18,701.507 | 12.492 | 12.437 | 155.352 |
| 100 | 2,199.761 | 0.0005 | 0.0000 | 0.0800 | 27,484.516 | 12.494 | 12.455 | 155.611 |
| 9% | | | | | | | | |
| 1 | 1.090 | 0.9174 | 1.0000 | 1.0900 | 1.000 | 0.917 | 0.000 | 0.000 |
| 2 | 1.188 | 0.8417 | 0.4785 | 0.5685 | 2.090 | 1.759 | 0.478 | 0.842 |
| 3 | 1.295 | 0.7722 | 0.3051 | 0.3951 | 3.278 | 2.531 | 0.943 | 2.386 |
| 4 | 1.412 | 0.7084 | 0.2187 | 0.3087 | 4.573 | 3.240 | 1.393 | 4.511 |
| 5 | 1.539 | 0.6499 | 0.1671 | 0.2571 | 5.985 | 3.890 | 1.828 | 7.111 |
| 6 | 1.677 | 0.5963 | 0.1329 | 0.2229 | 7.523 | 4.486 | 2.250 | 10.092 |
| 7 | 1.828 | 0.5470 | 0.1087 | 0.1987 | 9.200 | 5.033 | 2.657 | 13.375 |
| 8 | 1.993 | 0.5019 | 0.0907 | 0.1807 | 11.028 | 5.535 | 3.051 | 16.888 |
| 9 | 2.172 | 0.4604 | 0.0768 | 0.1668 | 13.021 | 5.995 | 3.431 | 20.571 |
| 10 | 2.367 | 0.4224 | 0.0658 | 0.1558 | 15.193 | 6.418 | 3.798 | 24.373 |
| 11 | 2.580 | 0.3875 | 0.0569 | 0.1469 | 17.560 | 6.805 | 4.151 | 28.248 |
| 12 | 2.813 | 0.3555 | 0.0497 | 0.1397 | 20.141 | 7.161 | 4.491 | 32.159 |
| 13 | 3.066 | 0.3262 | 0.0436 | 0.1336 | 22.953 | 7.487 | 4.818 | 36.073 |
| 14 | 3.342 | 0.2992 | 0.0384 | 0.1284 | 26.019 | 7.786 | 5.133 | 39.963 |
| 15 | 3.642 | 0.2745 | 0.0341 | 0.1241 | 29.361 | 8.061 | 5.435 | 43.807 |
| 16 | 3.970 | 0.2519 | 0.0303 | 0.1203 | 33.003 | 8.313 | 5.724 | 47.585 |
| 17 | 4.328 | 0.2311 | 0.0270 | 0.1170 | 36.974 | 8.544 | 6.002 | 51.282 |
| 18 | 4.717 | 0.2120 | 0.0242 | 0.1142 | 41.301 | 8.756 | 6.269 | 54.886 |
| 19 | 5.142 | 0.1945 | 0.0217 | 0.1117 | 46.018 | 8.950 | 6.524 | 58.387 |
| 20 | 5.604 | 0.1784 | 0.0195 | 0.1095 | 51.160 | 9.129 | 6.767 | 61.777 |
| 21 | 6.109 | 0.1637 | 0.0176 | 0.1076 | 56.765 | 9.292 | 7.001 | 65.051 |
| 22 | 6.659 | 0.1502 | 0.0159 | 0.1059 | 62.873 | 9.442 | 7.223 | 68.205 |
| 23 | 7.258 | 0.1378 | 0.0144 | 0.1044 | 69.532 | 9.580 | 7.436 | 71.236 |
| 24 | 7.911 | 0.1264 | 0.0130 | 0.1030 | 76.790 | 9.707 | 7.638 | 74.143 |
| 25 | 8.623 | 0.1160 | 0.0118 | 0.1018 | 84.701 | 9.823 | 7.832 | 76.926 |
| 26 | 9.399 | 0.1064 | 0.0107 | 0.1007 | 93.324 | 9.929 | 8.016 | 79.586 |
| 27 | 10.245 | 0.0976 | 0.0097 | 0.0997 | 102.723 | 10.027 | 8.191 | 82.124 |
| 28 | 11.167 | 0.0895 | 0.0089 | 0.0989 | 112.968 | 10.116 | 8.357 | 84.542 |
| 29 | 12.172 | 0.0822 | 0.0081 | 0.0981 | 124.135 | 10.198 | 8.515 | 86.842 |
| 30 | 13.268 | 0.0754 | 0.0073 | 0.0973 | 136.308 | 10.274 | 8.666 | 89.028 |
| 31 | 14.462 | 0.0691 | 0.0067 | 0.0967 | 149.575 | 10.343 | 8.808 | 91.102 |
| 32 | 15.763 | 0.0634 | 0.0061 | 0.0961 | 164.037 | 10.406 | 8.944 | 93.069 |
| 33 | 17.182 | 0.0582 | 0.0056 | 0.0956 | 179.800 | 10.464 | 9.072 | 94.931 |
| 34 | 18.728 | 0.0534 | 0.0051 | 0.0951 | 196.982 | 10.518 | 9.193 | 96.693 |
| 35 | 20.414 | 0.0490 | 0.0046 | 0.0946 | 215.711 | 10.567 | 9.308 | 98.359 |

Table A8.3 (continued)

| N | Single Payment | | Uniform Payment Series | | | | Arithmetic Gradient | |
|------------|--|--|---|---|---|--|---|--|
| | Compound Amount Factor: Find F Given P F/P | Present Worth Factor: Find P Given F , P/F | Sinking Fund Factor: Find A Given F , A/F | Capital Recovery Factor: Find A Given P , A/P | Capital Amount Factor: Find F Given A , F/A | Present Worth Factor: Find P Given A , P/A | Gradient Uniform Series: Find A Given G , A/G | Gradient Present Worth: Find P Given G , P/G |
| 40 | 31.409 | 0.0318 | 0.0030 | 0.0930 | 337.882 | 10.757 | 9.796 | 105.376 |
| 45 | 48.327 | 0.0207 | 0.0019 | 0.0919 | 525.859 | 10.881 | 10.160 | 110.556 |
| 50 | 74.358 | 0.0134 | 0.0012 | 0.0912 | 815.084 | 10.962 | 10.430 | 114.325 |
| 55 | 114.408 | 0.0087 | 0.0008 | 0.0908 | 1,260.092 | 11.014 | 10.626 | 117.036 |
| 60 | 176.031 | 0.0057 | 0.0005 | 0.0905 | 1,944.792 | 11.048 | 10.768 | 118.968 |
| 65 | 270.846 | 0.0037 | 0.0003 | 0.0903 | 2,998.288 | 11.070 | 10.870 | 120.334 |
| 70 | 416.730 | 0.0024 | 0.0002 | 0.0902 | 4,619.223 | 11.084 | 10.943 | 121.294 |
| 75 | 641.191 | 0.0016 | 0.0001 | 0.0901 | 7,113.232 | 11.094 | 10.994 | 121.965 |
| 80 | 986.552 | 0.0010 | 0.0001 | 0.0901 | 10,950.574 | 11.100 | 11.030 | 122.431 |
| 85 | 1,517.932 | 0.0007 | 0.0001 | 0.0901 | 16,854.800 | 11.104 | 11.055 | 122.753 |
| 90 | 2,335.527 | 0.0004 | 0.0000 | 0.0900 | 25,939.184 | 11.106 | 11.073 | 122.976 |
| 95 | 3,593.497 | 0.0003 | 0.0000 | 0.0900 | 39,916.635 | 11.108 | 11.085 | 123.129 |
| 100 | 5,529.041 | 0.0002 | 0.0000 | 0.0900 | 61,422.675 | 11.109 | 11.093 | 123.234 |
| 10% | | | | | | | | |
| 1 | 1.100 | 0.9091 | 1.0000 | 1.1000 | 1.000 | 0.909 | 0.000 | 0.000 |
| 2 | 1.210 | 0.8264 | 0.4762 | 0.5762 | 2.100 | 1.736 | 0.476 | 0.826 |
| 3 | 1.331 | 0.7513 | 0.3021 | 0.4021 | 3.310 | 2.487 | 0.937 | 2.329 |
| 4 | 1.464 | 0.6830 | 0.2155 | 0.3155 | 4.641 | 3.170 | 1.381 | 4.378 |
| 5 | 1.611 | 0.6209 | 0.1638 | 0.2638 | 6.105 | 3.791 | 1.810 | 6.862 |
| 6 | 1.772 | 0.5645 | 0.1296 | 0.2296 | 7.716 | 4.355 | 2.224 | 9.684 |
| 7 | 1.949 | 0.5132 | 0.1054 | 0.2054 | 9.487 | 4.868 | 2.622 | 12.763 |
| 8 | 2.144 | 0.4665 | 0.0874 | 0.1874 | 11.436 | 5.335 | 3.004 | 16.029 |
| 9 | 2.358 | 0.4241 | 0.0736 | 0.1736 | 13.579 | 5.759 | 3.372 | 19.421 |
| 10 | 2.594 | 0.3855 | 0.0627 | 0.1627 | 15.937 | 6.145 | 3.725 | 22.891 |
| 11 | 2.853 | 0.3505 | 0.0540 | 0.1540 | 18.531 | 6.495 | 4.064 | 26.396 |
| 12 | 3.138 | 0.3186 | 0.0468 | 0.1468 | 21.384 | 6.814 | 4.388 | 29.901 |
| 13 | 3.452 | 0.2897 | 0.0408 | 0.1408 | 24.523 | 7.103 | 4.699 | 33.377 |
| 14 | 3.797 | 0.2633 | 0.0357 | 0.1357 | 27.975 | 7.367 | 4.996 | 36.800 |
| 15 | 4.177 | 0.2394 | 0.0315 | 0.1315 | 31.772 | 7.606 | 5.279 | 40.152 |
| 16 | 4.595 | 0.2176 | 0.0278 | 0.1278 | 35.950 | 7.824 | 5.549 | 43.416 |
| 17 | 5.054 | 0.1978 | 0.0247 | 0.1247 | 40.545 | 8.022 | 5.807 | 46.582 |
| 18 | 5.560 | 0.1799 | 0.0219 | 0.1219 | 45.599 | 8.201 | 6.053 | 49.640 |
| 19 | 6.116 | 0.1635 | 0.0195 | 0.1195 | 51.159 | 8.365 | 6.286 | 52.583 |
| 20 | 6.727 | 0.1486 | 0.0175 | 0.1175 | 57.275 | 8.514 | 6.508 | 55.407 |
| 21 | 7.400 | 0.1351 | 0.0156 | 0.1156 | 64.002 | 8.649 | 6.719 | 58.110 |
| 22 | 8.140 | 0.1228 | 0.0140 | 0.1140 | 71.403 | 8.772 | 6.919 | 60.689 |
| 23 | 8.954 | 0.1117 | 0.0126 | 0.1126 | 79.543 | 8.883 | 7.108 | 63.146 |
| 24 | 9.850 | 0.1015 | 0.0113 | 0.1113 | 88.497 | 8.985 | 7.288 | 65.481 |
| 25 | 10.835 | 0.0923 | 0.0102 | 0.1102 | 98.347 | 9.077 | 7.458 | 67.696 |

(continued overleaf)

Table A8.3 (continued)

| <i>N</i> | Single Payment | | Uniform Payment Series | | | | Arithmetic Gradient | |
|------------|---|---|--|--|--|---|--|---|
| | Compound Amount Factor: Find <i>F</i> Given <i>P</i> <i>F/P</i> | Present Worth Factor: Find <i>P</i> Given <i>F</i> , <i>P/F</i> | Sinking Fund Factor: Find <i>A</i> Given <i>F</i> , <i>A/F</i> | Capital Recovery Factor: Find <i>A</i> Given <i>P</i> , <i>A/P</i> | Capital Amount Factor: Find <i>F</i> Given <i>A</i> , <i>F/A</i> | Present Worth Factor: Find <i>P</i> Given <i>A</i> , <i>P/A</i> | Gradient Uniform Series: Find <i>A</i> Given <i>G</i> , <i>A/G</i> | Gradient Present Worth: Find <i>P</i> Given <i>G</i> , <i>P/G</i> |
| 26 | 11.918 | 0.0839 | 0.0092 | 0.1092 | 109.182 | 9.161 | 7.619 | 69.794 |
| 27 | 13.110 | 0.0763 | 0.0083 | 0.1083 | 121.100 | 9.237 | 7.770 | 71.777 |
| 28 | 14.421 | 0.0693 | 0.0075 | 0.1075 | 134.210 | 9.307 | 7.914 | 73.650 |
| 29 | 15.863 | 0.0630 | 0.0067 | 0.1067 | 148.631 | 9.370 | 8.049 | 75.415 |
| 30 | 17.449 | 0.0573 | 0.0061 | 0.1061 | 164.494 | 9.427 | 8.176 | 77.077 |
| 31 | 19.194 | 0.0521 | 0.0055 | 0.1055 | 181.943 | 9.479 | 8.296 | 78.640 |
| 32 | 21.114 | 0.0474 | 0.0050 | 0.1050 | 201.138 | 9.526 | 8.409 | 80.108 |
| 33 | 23.225 | 0.0431 | 0.0045 | 0.1045 | 222.252 | 9.569 | 8.515 | 81.486 |
| 34 | 25.548 | 0.0391 | 0.0041 | 0.1041 | 245.477 | 9.609 | 8.615 | 82.777 |
| 35 | 28.102 | 0.0356 | 0.0037 | 0.1037 | 271.024 | 9.644 | 8.709 | 83.987 |
| 40 | 45.259 | 0.0221 | 0.0023 | 0.1023 | 442.593 | 9.779 | 9.096 | 88.953 |
| 45 | 72.890 | 0.0137 | 0.0014 | 0.1014 | 718.905 | 9.863 | 9.374 | 92.454 |
| 50 | 117.391 | 0.0085 | 0.0009 | 0.1009 | 1163.909 | 9.915 | 9.570 | 94.889 |
| 55 | 189.059 | 0.0053 | 0.0005 | 0.1005 | 1880.591 | 9.947 | 9.708 | 96.562 |
| 60 | 304.482 | 0.0033 | 0.0003 | 0.1003 | 3034.816 | 9.967 | 9.802 | 97.701 |
| 65 | 490.371 | 0.0020 | 0.0002 | 0.1002 | 4893.707 | 9.980 | 9.867 | 98.471 |
| 70 | 789.747 | 0.0013 | 0.0001 | 0.1001 | 7887.470 | 9.987 | 9.911 | 98.987 |
| 75 | 1,271.90 | 0.0008 | 0.0001 | 0.1001 | 12708.954 | 9.992 | 9.941 | 99.332 |
| 80 | 2,048.40 | 0.0005 | 0.0000 | 0.1000 | 20474.002 | 9.995 | 9.961 | 99.561 |
| 85 | 3,298.97 | 0.0003 | 0.0000 | 0.1000 | 32979.690 | 9.997 | 9.974 | 99.712 |
| 90 | 5,313.02 | 0.0002 | 0.0000 | 0.1000 | 53,120.226 | 9.998 | 9.983 | 99.812 |
| 95 | 8,556.68 | 0.0001 | 0.0000 | 0.1000 | 85,556.760 | 9.999 | 9.989 | 99.877 |
| 100 | 13,780.61 | 0.0001 | 0.0000 | 0.1000 | 137,796.123 | 9.999 | 9.993 | 99.920 |