ACI 207.2R-95

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Effect of Restraint, Volume Change, and Reinforcement on Cracking of Mass Concrete

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This report presents a discussion of the effects of heat generation and volume change on the design and behavior of reinforced mass concrete elements and structures. Particular emphasis is placed on the effects of restraint on cracking and the effects of controlled placing temperatures, concrete strength requirements, and type and fineness of cement on volume change. Formulas are presented for determining the amounts of reinforcing steel needed to control the size and spacing of cracks to specified limits under varying conditions of restraint and volume change.

Keywords: adiabatic conditions; age; cement types; concrete dams; concrete slabs; cooling; cracking (fracturing); crack propagation; crack width and spacing; creep properties; drying shrinkage; foundations; heat of hydration; heat transfer; machine bases; mass concrete; modulus of elasticity; moisture content; placing; portland cement physical properties; portland cements; pozzolans; reinforced concrete; reinforcing steels; restraints; shrinkage; stresses; structural design; temperature; temperature rise (in concrete); tensile strength; thermal expansion; volume change; walls.

CONTENTS

ACI 207.2R-95 supersedes ACI 207.2R-90 and became effective January 1, 1995.

The 1995 revisions consisted of many minor editorial and typographical corrections throughout, as well as some additional explanatory information.

CHAPTER 1—INTRODUCTION

1.1—Scope

This report is primarily concerned with limiting the width of cracks in structural members that occur principally from restraint of thermal contraction. A detailed discussion of the effects of heat generation and volume changes on the design and behavior of mass reinforced concrete elements and structures is presented. It is written primarily to provide guidance for the selection of concrete materials, mix requirements, reinforcement requirements, and construction procedures necessary to control the size and spacing of cracks. Particular emphasis is placed on the effect of restraint to volume change in both preventing and causing cracking and the need for controlling peak concrete temperature. The quality of concrete for resistance to weathering is not emphasized in recommending reduced cements contents; however, it should be understood that the concrete should be sufficiently durable to resist expected service conditions. The report can be applied to any concrete structure with a potential for unacceptable cracking; however, its general application is to massive concrete members 18 in. or more in thickness.

1.2—Definition

Mass concrete is defined in ACI 116R as: “Any volume of concrete with dimensions large enough to require that measures be taken to cope with the generation of heat and attendant volume change to minimize cracking.” Reinforced mass concrete in this report refers to concrete in which reinforcement is utilized to limit crack widths that may be caused by external forces or by volume change due to thermal changes, autogenous changes and drying shrinkage.

1.3—Approaches to control of cracking

All concrete elements and structures are subject to volume change in varying degrees, dependent upon the makeup, configuration, and environment of the concrete. Uniform volume change will not produce cracking if the element or structure is relatively free to change volume in all directions. This is rarely the case for massive concrete members since size alone usually causes nonuniform change and there is often sufficient restraint either internally or externally to produce cracking.

The measures used to control cracking depend to a large extent on the economics of the situation and the seriousness of cracking if not controlled. Cracks are objectionable where their size and spacing compromise the appearance, serviceability, function, or strength of the structure.

While cracks should be controlled to the minimum practicable width in all structures, the economics of achieving this goal must be considered. The change in volume can be minimized by such measures as reducing cement content, replacing part of the cement with pozzolans, precooling, postcooling, insulating to control the rate of heat absorbed or lost, and by other temperature control measures outlined in ACI 207.1R and ACI 207.4R. Restraint is modified by joints intended to handle contraction or expansion and also by the rate at which volume change takes place. Construction joints may also be used to reduce the number of uncontrolled cracks that may otherwise be expected. By appropriate consideration of the preceding measures, it is usually possible to control cracking or at least to minimize the crack widths. The subject of crack control in mass concrete is also discussed in Chapter 7 of ACI 224R and in Reference 1. The topic of evaluation and repair of cracks in concrete is covered in detail in ACI 224.1R.

In the design of reinforced concrete structures, cracking is presumed in the proportioning of reinforcement. For this reason, the designer does not normally distinguish between tension cracks due to volume change and those due to flexure. Instead of employing many of the previously recommended measures to control volume change, the designer may choose to add sufficient reinforcement to distribute the cracking so that one large crack is replaced by many smaller cracks of acceptably small widths. The selection of the necessary amount and spacing of reinforcement to accomplish this depends on the extent of the volume change to be expected, the spacing or number of cracks which would occur without the reinforcement, and the ability of reinforcement to distribute cracks.

The degree to which the designer will either reduce volume changes or use reinforcement for control of cracks in a given structure depends largely on the massiveness of the structure itself and on the magnitude of forces restraining volume change. No clear-cut line can be drawn to establish the extent to which measures should be taken to control the change in volume. Design strength requirements, placing restrictions, and the environment itself are sometimes so severe that it is impractical to prevent cracking by measures to minimize volume change. On the other hand, the designer normally has a wide range of choices when selecting design strengths and structural dimensions.

In many cases, the cost of increased structural dimensions required by the selection of lower strength concrete (within the limits of durability requirements) is more than repaid by the savings in reinforcing steel, reduced placing costs, and the savings in material cost of the concrete itself (see Section 6.5, Example 6.1.).

CHAPTER 2—VOLUME CHANGE

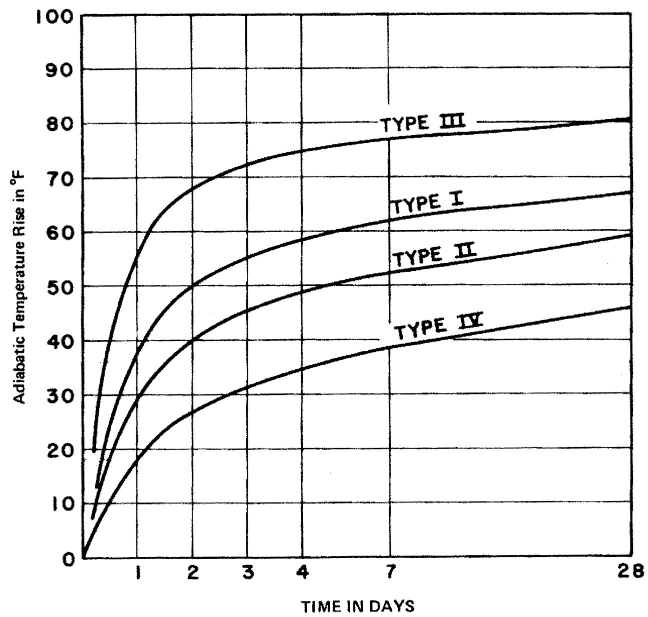
The thermal behavior of mass concrete has been thoroughly discussed in Chapter 5 of ACI 207.1R. This chapter's purpose is to offer some practical guidance in the magnitude of volume change that can be expected in reinforced concrete structures or elements. Such structures utilize cements with higher heat generation, smaller aggregate, more water, and less temperature control than normally used or recommended for mass concrete in dams.

In reinforced concrete elements, the primary concern is with these volume changes resulting from thermal and moisture changes. Other volume changes, which are not considered in this document, are alkali-aggregate expansion, autogenous shrinkage, and changes due to expansive cement. Autogenous shrinkage is the volume change due to the chemical process that occurs during hydration.

The change in temperature to be considered in the design of reinforced concrete elements is the difference between the peak temperature of the concrete attained during early hydration (normally within the first week following placement) and the minimum temperature to which the element will be subjected under service conditions. The initial hydration temperature rise produces little, if any, stress in the concrete. At this early age, the modulus of elasticity of concrete is so small that compressive stresses induced by the rise in temperature are insignificant even in zones of full restraint and, in addition, are relaxed by a high rate of early creep. By assuming a condition of no initial stress, a slightly conservative and realistic analysis results.

2.1—Heat generation

The rate and magnitude of heat generation of the concrete depends on the amount per unit volume of cement and pozzolan (if any), the compound composition and fineness of cement, and on the temperature during hydration of the cement. The hydration temperature is affected in turn by the amount of heat lost or gained as governed by the size of the member and exposure conditions. Thus, it can be seen that the exact temperature of the concrete at any given time depends on many variables.



Cement Type	Fineness ASTM C 115 cm^2/gm	28-Day Heat of Hydration Calories per gm
I	1790	87
II	1890	76
III	2030	105
IV	1910	60

Fig. 2.1—Temperature rise of mass concrete containing 376 lb of various types of cement per cubic yard of concrete

Fig. 2.1 shows curves for adiabatic temperature rise versus time for mass concrete placed at 73 F and containing 376 lb/yd^3 of various types of cement. These curves are typical of cements produced prior to 1960. The same cement types today may vary widely from those because of increased fineness and strengths. Current ASTM specifications only limit the heat of hydration directly of Type IV cements or of Type II cements if the purchaser specifically requests heat-of-hydration tests. Heat-of-hydration tests present a fairly accurate picture of the total heat-generating characteristics of cements at 28 days because of the relative insensitivity with age of the total heat generating capacity of cement at temperatures above 70 F. At early ages, however, cement is highly sensitive to temperature and therefore heat-of-solution tests, which are performed under relatively constant temperatures, do not reflect the early-age adiabatic temperature rise. The use of an isothermal calorimeter for measuring heat of hydration can provide data on the rate of heat output at early ages.[2] More accurate results for a specific cement, mix proportions, aggregate initial placing temperature, and a set of environmental conditions can be determined by adiabatic temperature-rise tests carefully performed in the laboratory under conditions that represent those that will occur in the field.

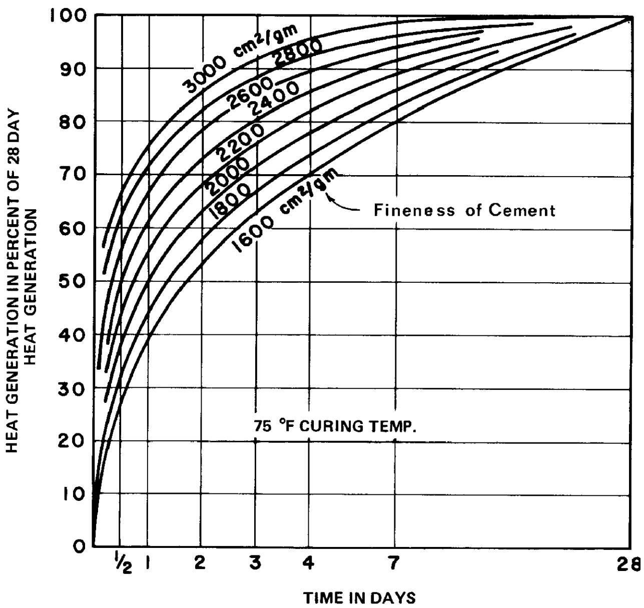


Fig. 2.2—Rate of heat generation as affected by Wagner fineness of cement (ASTM C 115) for cement paste cured at 75 F

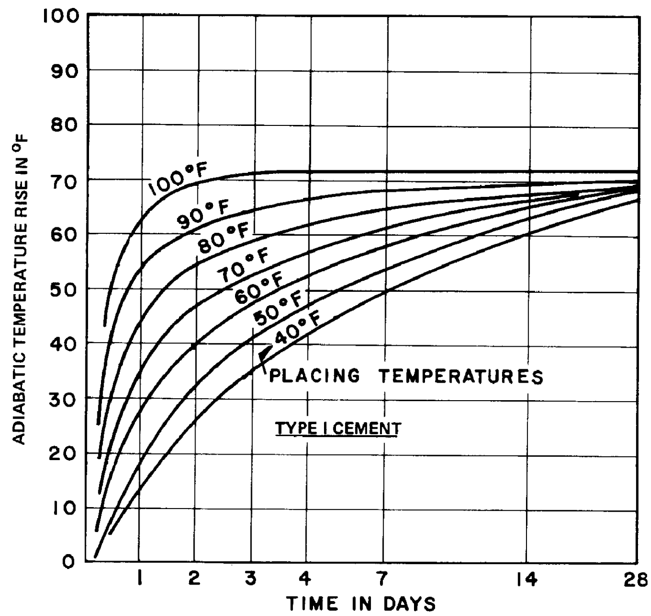


Fig. 2.3—Effect of placing temperature and time on adiabatic temperature rise of mass concrete containing 376 lb/yd³ of Type I cement

The fineness of cement affects the rate of heat generation more than it affects the total heat generation, in much the same fashion as placing temperature. The rate of heat generation as effected by cement fineness and placing temperature is shown in Fig. 2.2 and 2.3, respectively. These two figures are based on extrapolation of data from a study of the heats of hydration of cements by Verbeck and Foster.[3]

There are no maximum limitations on cement fineness in current specifications. By varying both fineness and chemical composition of the various types of cement, it is possible to vary widely the rate and total adiabatic temperature rise of the typical types shown in Fig. 2.1. It is therefore essential that both the fineness and chemical composition of the cement in question be considered in estimating the temperature rise of massive concrete members.

For a given fineness, the chemical composition of cement has a relatively constant effect on the generation of heat beyond the first 24 hr. As shown in Fig. 2.1, the concrete temperature rise for all four cement types is similar between 1 and 28 days. The 28-day adiabatic temperature rise in degrees F may be calculated by

$$H_a = \frac{1.8 h_g \overline{w}_c}{0.22(150)(27)} \qquad (2.1)$$

Where 0.22 in cal/gm-deg C and 150 in lb/ft^3 are the specific heat and density, respectively, of the concrete. 1.8 is the conversion factor from Celsius to Fahrenheit, 27 is the conversion factor from yd^3 to ft^3. hg in cal/gm is the 28-day measured heat generation of the cement by heat of hydration as per ASTM C 186, and $\overline{w}_c$ is the weight of cement in lb per yd^3 of concrete. For a concrete mix containing 376 lb of cement per yd^3 of concrete: H_a = 0.76 in degrees Fahrenheit. For low and medium cement contents, the total quantity of heat generated at any age is directly proportional to the quantity of cement in the concrete mix.

However, for high cement-content structural mixtures, the amount of cement may be sufficiently high to increase the very early age heat to a point where the elevated temperature in turn causes a more rapid rate of heat generation. When fly ash or other pozzolans used, the total quantity of heat generated is directly proportional to an equivalent cement content C_{eq}, which is the total quantity of cement plus a percentage to total pozzolan content. The contribution of pozzolans to heat generation as equivalent cement varies with age of concrete, type of pozzolan, the fineness of the pozzolan compared to the cement and pozzolan themselves. It is best determined by testing the combined portions of pozzolan and cement for fineness and heat of hydration and treating the blend in the same fashion as a type of cement.

In general, the relative contribution of the pozzolan to heat generation increases with age of concrete, fineness of pozzolan compared to cement, and with lower heat-generating cements. The early-age heat contribution of fly ash may conservatively be estimated to range between 15 and 35 percent of the heat contribution from same weight of cement. Generally, the low percentages correspond to combined finenesses of fly ash and cement as low as two-thirds to three-fourths that of the cement alone, while the higher percentages correspond to fineness equal to or greater than the cement alone.

The rate of heat generation as affected by initial temperature, member size, and environment is difficult to assess because of the complex variables involved. However, for large concrete members, it is advisable to compute their temperature history, taking into account the measured values of heat generation, concrete placement temperatures, and ambient temperature. The problem may be simplified somewhat if we

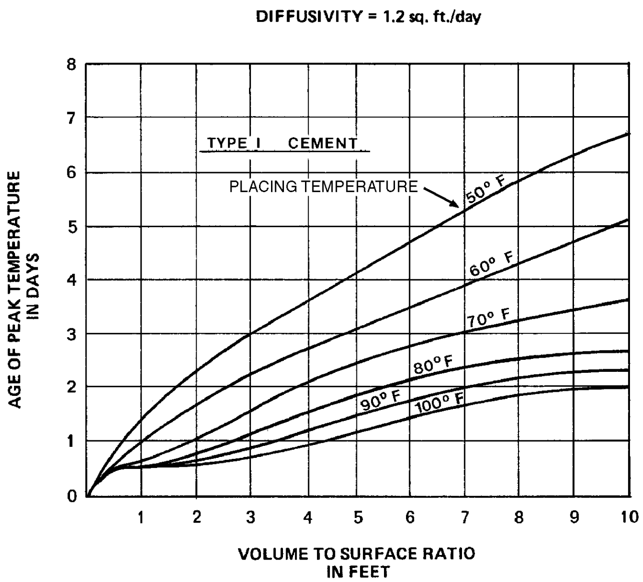


Fig. 2.4—Effect of placing temperature and surface exposure on age at peak temperature for Type I cement in concrete. Air temperature = placing temperature

assume that the placing temperature and ambient air temperature are identical. We can then make a correction for the actual difference, considering the size or volume-to-exposed surface ratio (*V/S*) of the member in question. The *V/S* ratio actually represents the average distance through which heat is dissipated from the concrete.

Usually, peak concrete temperatures for concrete structures may occur at any time during the first week. Fig. 2.4 shows the effect of placing temperature and member *V/S* on the age at which peak concrete temperatures occur for concrete containing Type I cement. Time would be shortened or lengthened for cements of higher or lower heat-generating characteristics.

For comparative purposes, the early-age heat generation of a Type III cement is approximately equivalent to a Type I cement at a 20 F higher placing temperature. In a similar fashion, the heat-generating characteristic of Types II and IV cement correspond closely to that of Type I cement at 10 and 20 F lower placing temperatures, respectively. Fig. 2.4 shows that for *V/S* less than 3 ft, peak temperature will be reached within 1 day under normal placing temperature (80 F or higher).

Fig. 2.5 gives the approximate maximum temperature rise for concrete members containing 4 bags (376 lb) of Type I cement per yd^3 for placing temperatures ranging from 50 to 100 F, assuming ambient air temperatures equal to placing temperatures. Corrections are required for different types and quantities of cementitious materials. A correction for the difference in air and placing temperatures can be made using Fig. 2.6 by estimating the time of peak temperatures from Fig. 2.4. The effect of water-reducing, set-retarding agents on the temperature rise of concrete is usually confined to the first 12 to 16 hr after mixing, during which time these agents have the greatest effect on the chemical reaction. Their presence does not alter appreciably the total heat generated in the concrete after the first 24 hr and no corrections are applied

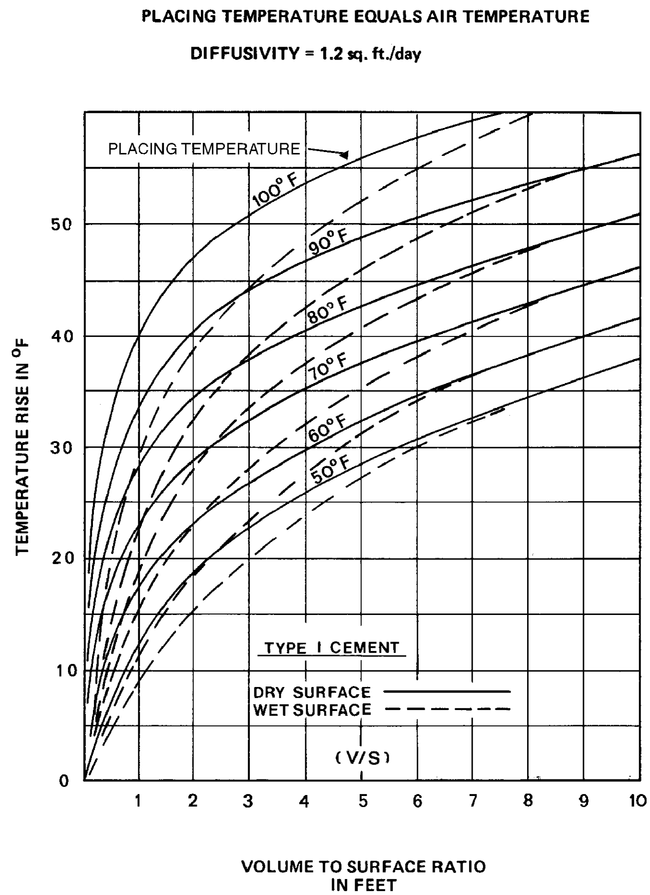


Fig. 2.5—Temperature rise of concrete members containing 376 lbs of cement per cubic yard for different placing temperatures

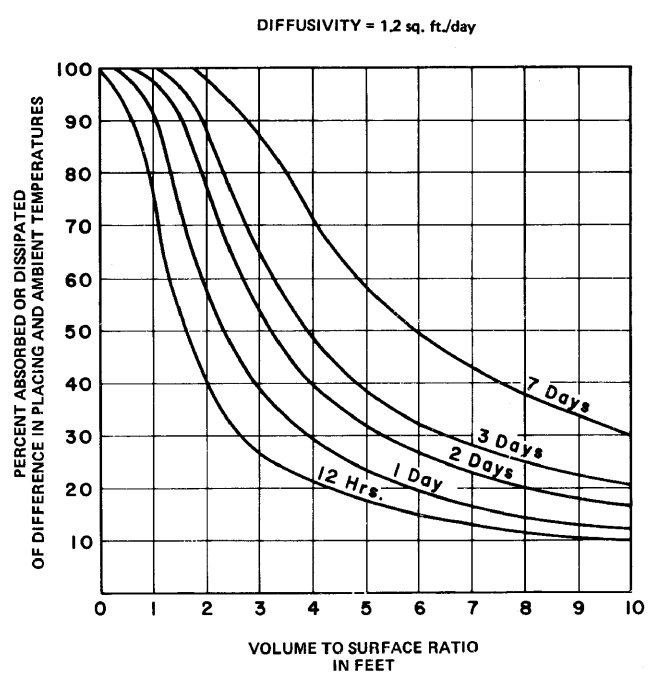


Fig. 2.6—Heat flow between air and concrete for difference between placing temperature and ambient air temperature

herein for the use of these agents.

A diffusivity of 1.2 ft^2/day has been assumed in the preparation of Fig. 2.4 through 2.6. A concrete of higher or lower diffusivity will, respectively, decrease or increase the volume-to-exposed surface ratio, and can be accounted for by multiplying the actual *V/S* by 1.2 divided by the actual concrete diffusivity.

2.2—Moisture contents and drying shrinkage

For tensile stress considerations, the volume change resulting from drying shrinkage is similar to volume change from temperature except that the loss of moisture from hardened concrete is extremely slow compared with the loss of heat. Drying shrinkage therefore depends on the length of moisture migration path and often affects the concrete near a surface. When the length of moisture migration or *V/S* is small, drying shrinkage adds to the stresses induced by external restraint and should be considered in the design of the reinforcement. When the *V/S* is large, the restraint to drying shrinkage is entirely internal and the result is tension on the surface or an extensive pattern of surface cracks extending only a short distance into the concrete. When surface cracks of this nature do occur, they are small and reinforcement is not particularly effective in altering the size or spacing of these cracks. Reinforcement is also not a solution for surface cracks in fresh concrete which are referred to as plastic cracking (see ACI 116R).

A 24 in. thick slab will lose approximately 30 percent of its evaporable water in 24 months of continuous exposure with both faces exposed to 50 percent relative humidity.[4] If we assume a total drying shrinkage potential at the exposed faces of 300 millionths, then the average drying shrinkage for a 24 in. slab under this exposure would be 90 millionths in 24 months. Concrete is not usually exposed to drying conditions this severe.

Drying shrinkage is affected by the size and type of aggregate used. "In general, concretes low in shrinkage often contain quartz, limestone, dolomite, granite, or feldspar, whereas those high in shrinkage often contain sandstone, slate, basalt, trap rock, or other aggregates which shrink considerably of themselves or have low rigidity to the compressive stresses developed by the shrinkage of paste."[5] In this discussion, an aggregate low in shrinkage qualities is assumed. Drying shrinkage may vary widely from the values used herein depending on many factors which are discussed in more detail in ACI 224R.

2.2.1 *Equivalent temperature change*—In the design of reinforcement for exterior restraint to volume change, it is more convenient to design only for temperature change rather than for temperature and shrinkage volume changes; therefore, it is desirable to express drying shrinkage in terms of equivalent change in concrete temperature T_{DS}. Creep can be expected to reduce significantly the stresses induced by drying shrinkage because of the long period required for full drying shrinkage to develop. We have therefore assumed an equivalent drying shrinkage of 150 millionths and an expansion coefficient of 5×10^{-6} per deg F as a basis in establishing the following formula for equivalent temperature drop. While the rate of drying and heat dissipation differ, their average path lengths (*V/S*) are the same. There is, however, a limitation on the length of moisture migration path affecting external restraint and its impact on total volume change. This limit has been assumed as 15 in. maximum in determining equivalent temperature change

$$T_{DS} = \left(30 - \frac{2V}{S}\right)\left(\frac{W_u - 125}{100}\right) \qquad (2.2)$$

where

T_{DS} = equivalent temperature change due to drying shrinkage, in deg F
W_u = water content of fresh concrete, lb/yd^3, but not less than 225 lb/yd^3
V = total volume, in.^{3}
S = area of the exposed surface, in.^{2}

2.3—Ambient, placement, and minimum service temperatures

In many structures, the most important temperature considerations are the average air temperatures during and immediately following the placement of concrete, and the minimum average temperature in the concrete that can be expected during the life of the structure. The temperature rise due to hydration may be small, particularly in thin exposed members, regardless of the type or amount of cement used in the mix, if placing and cooling conditions are right. On the other hand, the same member could have a high temperature rise if placed at high temperature in insulated forms.

2.4—Placement temperature

Specifications usually limit the maximum and minimum placing temperatures of concrete. ACI 305R recommends limiting the initial concrete placement temperature to between 75 and 100 F. The temperature of concrete placed during hot weather may exceed the mean daily ambient air temperature by 5 to 10 F unless measures are taken to cool the concrete or the coarse aggregate. Corrections should be made for the difference in air temperature and placing temperature, using Fig. 2.6. For example, if the temperature of the concrete, when placed, is 60 F during the first 24 hr, a concrete section having a *V/S* of 2 ft would absorb 60 percent of the difference, or 12 F. The maximum placing temperature in summer should be the highest average summer temperature for a given locality, but not more than 100 F.

Minimum concrete temperature recommendations at placing are given in ACI 306R, Table 3.1. These minimums establish the lowest placing temperature to be considered. Placing temperatures for spring and fall can reasonably be considered to be about halfway between the summer and winter placing temperatures.

2.5—Minimum temperature in service

The minimum expected final temperatures of concrete elements are as varied as their prolonged exposure conditions. Primary concern is for the final or operating exposure conditions, since cracks which may form or open during colder construction conditions may be expected to close during operating conditions, provided steel stresses remain in the elastic range during construction conditions. Minimum concrete temperatures can be conservatively taken as the average minimum exposure temperature occurring during a period of approximately 1 week. The mass temperature of earth or rock against concrete walls or slabs forms a heat source, which affects the average temperature of concrete members, depending upon the cooling path or *V/S* of the concrete. This heat source can be assumed to effect a constant temperature at some point 8 to 10 ft from the exposed concrete face.

The minimum temperature of concrete against earth or rock mass, T_{min}, can be approximated by

$$T_{min} = T_A + \frac{2(T_M - T_A)}{3}\sqrt{\frac{V/S}{96}} \qquad (2.3)$$

where

T_A = average minimum ambient air temperature over a prolonged exposure period of one week.
T_M = temperature of earth or rock mass; approximately 40 to 60 F, depending on climate
V/S = volume to exposed surface ratio, in.

2.6—Heat dissipation and cooling

Means of determining the dissipation of heat from bodies of mass concrete are discussed in ACI 207.1R and can readily be applied to massive reinforced structures. Reinforced elements or structures do not generally require the same degree of accuracy in determining peak temperatures as unreinforced mass concrete. In unreinforced mass concrete, peak temperatures are determined for the purpose of preventing cracking. In reinforced concrete, cracking is presumed to occur and the consequences of overestimating or underestimating the net temperature rise is usually minor compared to the overall volume change consideration. Sufficient accuracy is normally obtained by use of charts or graphs such as Fig. 2.5 to quickly estimate the net temperature rise for concrete members cooling in a constant temperature environment equal to the placing temperature, and by use of Fig. 2.6 to account for the difference in the actual and assumed cooling environment.

Fig. 2.5 gives the maximum temperature rise for concrete containing 376 lb of Type I portland cement per cubic yard of concrete in terms of *V/S* of the member. *V/S* actually represents the average distance through which heat is dissipated from the concrete. This distance will always be less than the minimum distance between faces. In determining the *V/S* consider only the surface area exposed to air or cast against forms. The insulating effect of formwork must be considered in the calculation of volume of the member. Steel forms are poor insulators; without insulation, they offer little resistance to heat dissipation from the concrete. The thickness of wood forms or insulation in the direction of principal heat flow must be considered in terms of their affecting the rate of heat dissipation (see ACI 306R). Each inch of wood has an equivalent insulating value of about 20 in. of concrete but can, for convenience, be assumed equivalent to 2 ft of additional concrete. Any faces farther apart than 20 times the thickness of the member can be ignored as contributing to heat flow. Therefore, for a long retaining wall, the end surfaces are normally ignored.

The *V/S* can best be determined by multiplying the calculated volume-to-exposed surface ratio of the member, excluding the insulating effect of forms by the ratio of the minimum flow path including forms divided by the minimum flow path excluding forms. For slabs, *V/S* should not exceed three-fourths of the slab thickness. While multiple lift slabs are not generally classed as reinforced slabs, *V/S* should not exceed the height of lift if ample time is provided for cooling lifts.

The temperature rise for other types of cement and for mixes containing differing quantities of cement or cement plus pozzolan from 376 lb can be proportioned as per Section 2.1.

Fig. 2.6 accounts for the difference in placing temperatures and ambient air temperatures. The *V/S* for Fig. 2.6 should be identical to those used with Fig. 2.5. In all previous temperature determinations the placing temperature has been assumed equal to ambient air temperature. This may not be the case if cooling measures have been taken during the hot-weather period or heating measures have been taken during cold weather. When the placing temperature of concrete is lower than the average ambient air temperature, heat will be absorbed by the concrete and only a proportion of the original temperature difference will be effective in lowering the peak temperature of the concrete. When the placing temperature is higher, the opposite effect is obtained. As an example, assume for an ambient air temperature of 75 F that the placing temperature of a 4 ft thick wall 12 ft high is 60 F instead of 75 F. The *V/S* would be 3.4 ft, assuming 1 in. wooden forms. The age for peak temperature would be 2.3 days from Fig. 2.4. From Fig. 2.6, 50 percent of the heat difference will be absorbed or 7.5 F; therefore, the base temperature or the effective placing temperature for determining temperature rise will be 68 F. In contrast, if no cooling methods are used, the actual placing temperature of the concrete will be 85 F, the age of peak temperature would be 1 day, and the base temperature or effective placing temperature for determining temperature rise will be 81 F.

2.7—Summary and examples

The maximum effective temperature change constitutes the summation of three basic temperature determinations. They are: (1) the difference between effective placing temperature and the temperature of final or operating exposure conditions, (2) the temperature rise of the concrete due to hydration, and (3) the equivalent temperature change to compensate for drying shrinkage. Measures for making these determinations have been previously discussed; therefore, the following example problems employ most of the calculations required in determining the maximum effective temperature change.

Example 2.1—A 2 ft wide retaining wall with rock base and backfill on one side; 20 ft high by 100 ft long placed in two 10-ft lifts, wood forms; summer placing with concrete cooled to 60 F; concrete mix designed for a specified strength of 3000 psi or average strength of 3700 psi at 90 days contains 215 lb of Type II cement (adiabatic curve same as Fig. 2.1), 225 lb of fly ash, and 235 lbs of water per yd^3. The insulating effect of 1 in. thick wood forms on each face would be to effectively increase the thickness by 2(20)/12 = 3.34 ft (assuming 1 in.-thick wood form is equivalent to 20 in. concrete).

1. Determine the *V/S*

$$V/S = \left[\frac{2(10)}{2(10)+2}\right]\left(\frac{2+3.34}{2}\right) = 2.43 \text{ ft}$$

2. Determine the difference between effective placing temperature and final exposure temperature:

a. Establish ambient air temperature for summer placement based on locality. Assume 75 F average temperature.

b. Concrete peaks at 2 days from Fig. 2.4. Using Fig. 2.6, the heat absorbed for *V/S* = 2.4 is approximately 60 percent.

c. Net effective placing temperature T_{pk} = 60 + 0.6(15) = 69 F.

d. Establish minimum exposure temperature for 1-week duration. Assume 20 F.

e. For final exposure conditions *V/S* equals approximately 24 in., since heat flow is restricted to one direction by the backfill. For two faces exposed, *V/S* would equal approximately 12 in.

f. $T_{min} = 20 \text{ F} + {}^2/_3(60\text{-}20)\sqrt{24/96} = 33.5$ F, say 34 F.

g. Difference = 69 – 34 = 35 F.

3. Determine the temperature rise:

a. From Fig. 2.5, the temperature rise for Type I cement for dry surface exposure and an effective placing temperature of 69 F and *V/S* of 2.4 ft = 30 F.

b. From Fig. 2.1, correction for Type II cement peaking at 2 days = T_c = (40/50)(30) = 24 F.

c. Correction for mix. C_{eq} = 215 + 225/4 = 272 lb, T_{C+F} = 24 F (272)/(376) = 17.4 F, say 18 F.

d. Temperature of the concrete at the end of 2 days = 69 + 18 = 87 F.

4. Determine the equivalent temperature for drying shrinkage. Since *V/S* for final exposure conditions is greater than 15 in., no additional temperature considerations are required for external restraint considerations.

5. The maximum effective temperature change T_E = 35 + 18 = 53 F.

Example 2.2—Same wall as Example 2.1, except that no cooling measures were taken and the concrete mix contains 470 lb/yd^3 of a Type I cement, having a turbidimeter fineness of 2000 cm^2/gm and 28-day heat of solution of 94 cal/gm.

1. a. With no cooling measures the placing temperature could be as much as 10 F above the ambient temperature of 75 F or T_p = 85 F.

b. From Fig. 2.4, the concrete peaks at three-fourths of a day for 85 F placing temperature. From Fig. 2.6, 36 percent of the difference in placing and air temperature is dissipated: 0.36 (85-75) = 4 F.

c. Effective placing temperature = 85 – 4 = 81 F.

d. Minimum temperature of the concrete against rock = 34 F.

e. Difference = 81 – 34 = 47 F.

2. a. The temperature rise from Fig. 2.5 for dry exposure, *V/S* of 2.4, and T_p of 81 F is 37 F.

b. Correction for fineness and heat of solution of cement.

From Fig. 2.2, the difference in fineness for 2000 versus 1800 at three-fourths of a day (18 hr) = 45/38 = 1.18.

From Eq. (2.1), the temperature difference due to heat of solution: H_a = 0.76 (94 – 87) = 5 F. Note that 87 cal/gm is the 28-day heat of hydration for Type I cement with a fineness of 1790 as shown in Fig. 2.1.

From Fig. 2.1, the adiabatic rise for Type I cement at 18 hr = 30 F.

Combining the preceding two corrections, the adiabatic rise of the cement at 18 hr would be 1.18 (30 + 5) = 41 F.

Temperature rise for 376 lb/yd^3 of cement = 41(37)/30 = 51 F.

c. Correction for cement content = 470(51)/376 = 64 F.

3. No addition for drying shrinkage.

4. The peak temperature of the concrete at 18 hr: 81 + 64 = 145 F.

5. The drop in temperature affecting volume change: 145 – 34 = 111 F.

In comparing the preceding two examples, the effect of mix difference and cooling measures combined for a difference in peak temperature of 145 – 87 = 58 F. This constitutes a volume change in Example 2.2 of about twice (.209 percent) that in Example 2.1 for the same wall.

CHAPTER 3—PROPERTIES

3.1—General

This chapter discusses the principal properties of massive concrete that affect the control of cracking and provides guidance to evaluate those properties.

3.2—Strength requirements

The dimensions of normal structural concrete are usually determined by structural requirements utilizing 28-day strength concrete of 3000 psi or more. When these dimensions are based on normal code stress limitations for concrete, the spacing of cracks will be primarily influenced by flexure, and the resultant steel stresses induced by volume change will normally be small in comparison with flexural stresses. Under these conditions, volume control measures do not have the significance that they have when concrete stresses in the elastic range are low and crack spacing is controlled primarily by volume change.

The dimensions of massive reinforced concrete sections are often set by criteria totally unrelated to the strength of concrete. Such criteria often are based on stability requirements where weight rather than strength is of primary importance; on arbitrary requirements for water tightness per ft of water pressure; on stiffness requirements for the support of large pieces of vibrating machinery where the mass itself is of primary importance; or on shielding requirements, as found in nuclear power plants. Once these dimensions are established they are then investigated using an assumed concrete strength to determine the reinforcement requirements to sustain the imposed loadings. In slabs, the design is almost always controlled by flexure. In walls, the reinforcement requirements are usually controlled by flexure or by minimum requirements as load-bearing partitions. Shear rarely controls except in the case of cantilevered retaining walls or structural frames involving beams and columns.

In flexure, the strength of massive reinforced sections is controlled almost entirely by the reinforcing steel. The effect of concrete strength on structural capacity is dependent on the quantity of reinforcing steel (steel ratio) and the eccentricity of applied loads. If the eccentricity of the loading with respect to member depth *e/d* is greater than 2, Fig. 3.1 shows the relationship of required concrete strength to structural capacity for steel ratios up to 0.005 using 3000 psi as the base for strength comparison. For steel ratios less than 0.005, there is no significant increase in structural capacity with higher strength concretes within the eccentricity limits of the chart. Most massive concrete walls and slabs will fall within the chart limits.

The principal reason for consideration of the effects of lower concrete strengths concerns the early loading of massive sections and the preeminent need in massive concrete to control the heat of hydration of the concrete. If design loading is not to take place until the concrete is 90 or 180 days old, there is no difficulty using pozzolans in designing low-heat-generating concrete of 3000 psi at those ages. Such concrete may, however, have significantly lower early strengths for sustaining construction loadings and could present a practical scheduling problem, requiring more time prior to form stripping and lift joint surface preparation. Normally, the designer investigates only those construction loads which exceed operational live loads and usually applies a lower load factor for these loads because of their temporary nature. From Fig. 3.1 it can readily be seen that for members subject to pure bending ($e/d = \infty$), less than 13 percent loss of capacity will be experienced in loading a member containing 0.5 percent steel when it has a compressive strength of only 1000 psi. Note that while structural capacity is relatively unaffected by the 1000-psi strength, short-term load and creep deflection will be significantly larger than for 3000-psi concrete. This is usually not significant for construction loadings, particularly since members with this low steel ratio have enough excess depth to offset the increase in deflection due to lower modulus of elasticity.

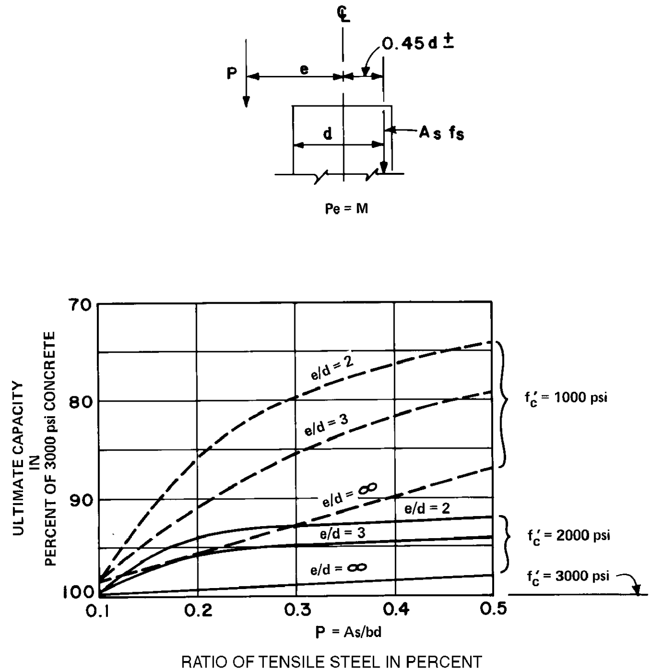


Fig. 3.1—Effect of concrete strength on ultimate capacity; fy = *60,000 psi*

Most massive reinforced concrete members subjected to flexural stress will have steel ratios in the range of 0.0015 to 0.002 in the tensile face. Fig. 3.1 shows that in this range, reinforced concrete in flexure is capable of sustaining up to 85 percent of the structural capacity of 3000-psi concrete with concrete strengths as low as 1000 psi. Construction loading rarely controls design. The decrease in load factors normally applied for temporary construction loads will more than account for the 15 percent loss in capacity associated with the lower strength concrete at the time of loading. Therefore, for massive reinforced sections within these limits a simple restriction of limiting imposed flexural loads until the concrete achieves a minimum compressive strength of 1000 psi should be adequate.

From the preceding, it should be obvious that massive reinforced concrete with low reinforcement ratios can tolerate substantially higher percentages of below-strength concrete than can normal structural concrete with high reinforcement ratios. From Fig. 3.1 a minimum strength of 2000 psi results in less than an 8.5 percent loss in ultimate capacity compared with 3000 psi strength.

As previously mentioned, shear strength may control the thickness of a cantilevered retaining wall. The strength of concrete in shear is approximately proportional to $\sqrt{f_c'}$ and, therefore, the loss in shear strength for a given reduction in compressive strength has a greater impact on design than the loss in flexural strength. The design loading for a wall sized on the basis of shear strength is the load of the backfill; rarely will construction schedules allow the lower lifts to attain 90 to 180-day strengths before the backfill must be completed. Since the shear at the base of the wall upon completion of the backfill controls, a design based on 2000 psi will require an approximately 22 percent wider base. For tapered walls, this

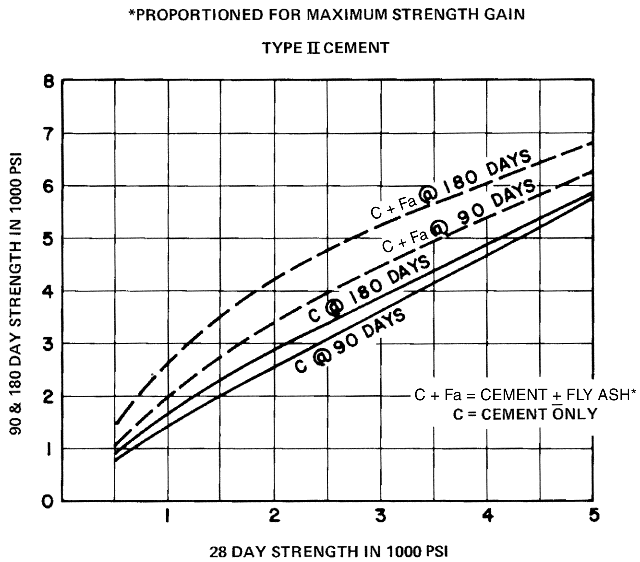


Fig. 3.2—Comparison of 28, 90, and 180-day compressive strength

would mean only an 11 percent increase in total volume. The 22 percent increase in base wall thickness would allow a 30 to 35 percent reduction in flexural reinforcement requirements (using strength design), which would directly offset the cost of the added concrete volume, possibly resulting in a lower overall cost for the wall. By restricting the placing of backfill against any lift until it has obtained a minimum strength of 1000 psi and restricting completion of backfill until the first lift has attained 2000 psi, a reasonable schedule for backfill with respect to concrete construction can be established. A 2000 psi strength requirement at 28 days complies with these types of construction requirements and will provide sufficient strength for durability under most exposure conditions particularly if 90 day strengths exceed 3000 psi.

3.3—Tensile strength

In conventional reinforced concrete design it is assumed that concrete has no tensile strength and a design compressive strength appreciably below average test strength is utilized. Neither approach is acceptable in determining the reinforcing steel requirement for volume-change crack control. The actual tensile strength is one of the most important considerations and should be determined to correspond in time to the critical volume change. Since compressive strength is normally specified, it is desirable to relate tensile and compressive strength.

Tensile strength of the concrete will be affected by the type of aggregates used. A restrained concrete of equal water-cement ratios (*w/c*) made from crushed coarse aggregate will withstand a larger drop in temperature without cracking than concrete made from rounded coarse aggregate. For a given compressive strength, however, the type of aggregate does not appreciably affect tensile strength. The age at which concrete attains its compressive strength does affect the tensile-compressive strength relationship such that the older the concrete, the larger the tensile strength for a given compressive strength.

The most commonly used test to determine the tensile strength of concrete is the splitting tensile test. This test tends to force the failure to occur within a narrow band of the specimen rather than occurring in the weakest section. If the failure does not occur away from the center section, the calculations will indicate a higher than actual strength. The tensile strength for normal weight concrete is usually taken as 6.7 $\sqrt{f_c'}$ and drying has little effect on the relationship.

Direct tensile tests made by attaching steel base plates with epoxy resins indicate approximately 25 percent lower strengths. Such tests are significantly affected by drying.[6]

If the concrete surface has been subjected to drying, a somewhat lower tensile strength than 6.7 $\sqrt{f_c'}$ should be used to predict cracks initiating at the surface. Where drying shrinkage has relatively little influence on section cracking, a tensile strength of 6 $\sqrt{f_c'}$ appears reasonable. The design tensile strength of concrete has a direct relationship to the calculated amount of reinforcing needed to restrict the size of cracks. Under these conditions, a minimum tensile strength of 4 $\sqrt{f_c'}$ is recommended where drying shrinkage may be considered significant.

In the preceding expressions it is more appropriate to use the probable compressive strength at critical cracking rather than the specified strength. For normal structural concrete it is therefore recommended that at least 700 psi be added to the specified strength in the design of concrete mixes. For massive reinforced sections (as described in Section 3.2) it is recommended that mixes be designed for the specified strength. The strength of concrete that controls the critical volume change for proportioning crack-control reinforcement may occur either during the first 7 days following placement or after a period of 3 to 6 months, depending primarily upon peak temperatures. If the cracking potential occurring upon initial cooling exceeds the cracking potential occurring during the seasonal temperature drop, the critical volume change will occur during the first week.

When the critical volume change is seasonal, some allowance should be made for the strength gain beyond 28 days at the time of cracking, particularly where fly ash is utilized. The strength gain from 28 days to 90 and 180 days of age as a percentage of the 28-day strength varies with the 28-day strength, depending on the cement and the proportions of fly ash or other pozzolans used. For concrete mixes properly proportioned for maximum strength gain, Fig. 3.2 gives a typical comparison for mixes with and without fly ash that use Type II cement.

When the critical volume change occurs during the first week, it is probably prudent to use 7-day standard-cured strengths in proportioning crack-control reinforcement. The 7-day strength of concrete normally ranges from 60 to 70 percent of 28-day strengths for standard cured specimens of Types II and I cements, respectively. Slightly lower strengths may be encountered when fly ash or other pozzolans are utilized. In-place strengths will vary depending on section mass and curing temperatures.

3.4—Modulus of elasticity

Unless more accurate determinations are made, the elastic modulus in tension and compression for hardened concrete may be assumed equal to $w_c^{1.5}\,33\sqrt{f_c'}$ (in psi) which for normal weight concrete $57{,}000\sqrt{f_c'}$. It also should be based on probable strength as discussed in Section 3.3. The modulus of elasticity in mass concrete can depart significantly from these values, and should be based on actual test results whenever possible.

3.5—Creep

Creep is related to a number of factors, including elastic modulus at the time of loading, age, and length of time under load. Although creep plays a large part in relieving thermally induced stresses in massive concrete, it plays a lesser role in thinner concrete sections where temperature changes occur over a relatively short time period. Its primary effect as noted in Section 2.2, is the relief of drying shrinkage stresses in small elements. In general, when maximum temperature changes occur over a relatively short time period, creep can only slightly modify temperature stresses.

3.6—Thermal properties of concrete

The thermal properties of concrete are coefficient of expansion, conductivity, specific heat, and diffusivity.

The relationship of diffusivity, conductivity, and specific heat is defined by

$$h^2 = \frac{K}{C_h \cdot w_c} \tag{3.1}$$

where

h^2 = diffusivity, ft²/hr
K = conductivity, Btu/ft·hr·F
C_h = specific heat, Btu/lb·F
w_c = weight of concrete, lb/ft³

These thermal properties have a significant effect on the change in concrete volume that may be expected and should be determined in the laboratory using job materials in advance of design, if possible. ACI 207.1R and ACI 207.4R discuss these properties in detail and present a broad range of measured values.

Where laboratory tests are not available, it is recommended that the thermal coefficient of expansion C_T be assumed as 5 x 10^{-6} in./in./F for calcareous aggregate, 6 x 10^{-6} in./in./F for silicious aggregate concrete, and 7 x 10^{-6} in./in./F for quartzite aggregate.

CHAPTER 4—RESTRAINT

4.1—General

To restrain an action is to check, suppress, curb, limit, or restrict its occurrence to some degree. The degree of restrain, K_R, is the ratio of actual stress resulting from volume change to the stress which would result if completely restrained. Numerically, the strain is equal to the product of the degree of restraint existing at the point in question and the change in unit length which would occur if the concrete were not restrained.

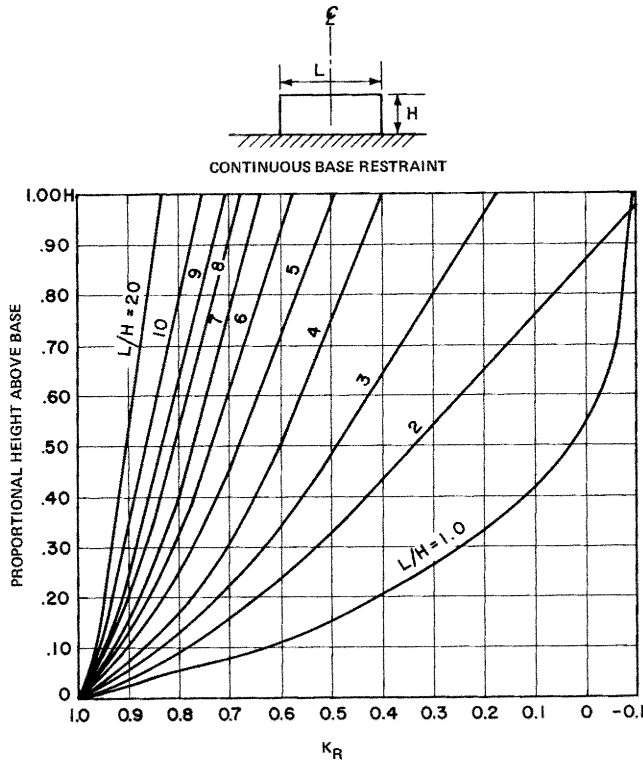


Fig. 4.1—Degree of tensile restraint at center section

All concrete elements are restrained to some degree by volume because there is always some restraint provided either by the supporting elements or by different parts of the element itself. Restrained volume change can induce tensile, compressive, or flexural stresses in the elements, depending on the type of restraint and whether the change in volume is an increase or decrease. We are normally not concerned with restraint conditions that induce compressive stresses in concrete because of the ability of concrete to withstand compression. We are primarily concerned with restraint conditions which induce tensile stresses in concrete which can lead to cracking.

In the following discussion, the types of restraint to be considered are external restraint (continuous and discontinuous) and internal restraint. Both types are interrelated and usually exist to some degree in all concrete elements.

4.2—Continuous external restraint

Continuous restraint exists along the contact surface of concrete and any material against which the concrete has been cast. The degree of restraint depends primarily on the relative dimensions, strength, and modulus of elasticity of the concrete and restraining material.

4.2.1 *Stress distribution*—By definition, the stress at any point in an uncracked concrete member is proportional to the strain in the concrete. The horizontal stress in a member continuously restrained at its base and subject to an otherwise uniform horizontal length change varies from point to point in accordance with the variation in degree of restraint throughout the member. The distribution of restraint varies with the length-to-height ratio (L/H) of the member. The case of concrete placed without time lapses for lifts is shown graphically in Fig. 4.1, which was derived from test data re-

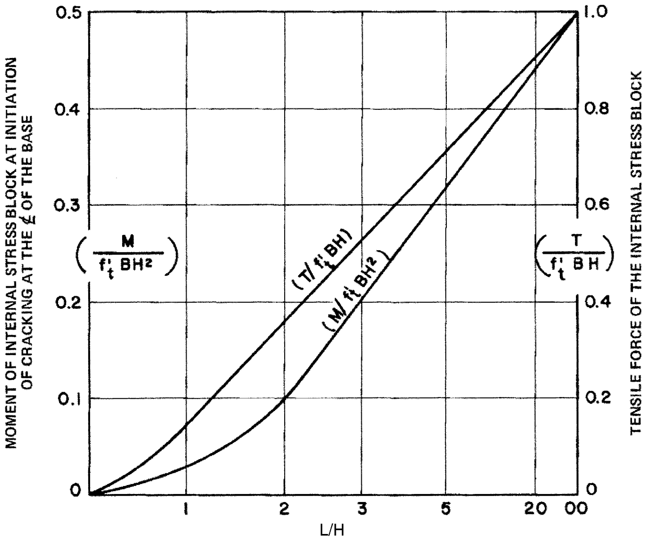


Fig. 4.2—Internal forces at initiation of cracks at restrained base

ported in 1940 by Carlson and Reading.[4,7]

For *L/H* equal to or greater than 2.5, restraint K_R at any point at a height *h* above the base may be approximated by

$$K_R = [(L/H-2)/(L/H+1)]^{h/H} \quad (4.1)$$

For *L/H* less than 2.5, restraint K_R at any point may be approximated by

$$K_R = [(L/H-1)/(L/H+10)]^{h/H} \quad (4.2)$$

Using the degree of restraint K_R, from Fig. 4.1 or calculated from Eq. (4.1) or (4.2), the tensile stress at any point on the centerline due to a decrease in length can be calculated from

$$f_t = K_R \Delta_c E_c \quad (4.3)$$

where

K_R = degree of restraint expressed as a ratio with 1.0 = 100 percent
Δ_c = contraction if there were no restraint
E_c = sustained modulus of elasticity of the concrete at the time when Δ_c occurred and for the duration involved

The stresses in concrete due to restraint decrease in direct proportion to the decrease in stiffness of the restraining foundation material. The multiplier to be used in determining K_R from Fig. 4.1 is given by

$$\text{Multiplier} = \frac{1}{1 + \dfrac{A_g E_c}{A_F E_F}}$$

where

A_g = gross area of concrete cross section
A_F = area of foundation or other element restraining shortening of element, generally taken as a plane surface at contact
E_F = modulus of elasticity of foundation or restraining element

For mass concrete on rock, the maximum effective restraining mass area A_F can be assumed at $2.5A_g$ and the values of the multipliers are then shown in the following table.

Multipliers for foundation rigidity

$\frac{E_F}{E_c}$	Multipliers
∞	1.0
2	0.83
1	0.71
0.5	0.56
0.2	0.33
0.1	0.20

4.2.2 *Cracking pattern*—When stress in the concrete due to restrained volume change reaches the tensile strength of the concrete, a crack will form. If a concrete member is subject to a uniform reduction in volume but is restrained at its base or at an edge, cracking will initiate at the base or restrained edge where the restraint is greatest and progress upward or outward until a point is reached where the stress is insufficient to continue the crack. After initial cracking, the tension caused by restraint in the region of the crack is transferred to the uncracked portion of the member, thereby increasing the tensile stresses above the crack. For *L/H* greater than about 2.5, Fig. 4.1 indicates that if there is enough tensile stress to initiate a crack, it should propagate to the full block height because of the stress-raising feature just mentioned. It has also been found from many tests that once begun, a crack will extend with less tensile stress than required to initiate it (see ACI 224R).

From the preceding discussion, unreinforced walls or slabs, fully restrained at their base and subject to sufficient volume change to produce full-section cracking, will ultimately attain full-section cracks spaced in the neighborhood of 1.0 to 2.0 times the height of the block. As each crack forms, the propagation of that crack to the full height of the block will cause a redistribution of base restraint such that each portion of the wall or slab will act as an individual section between cracks. Using Eq. (4.3) and K_R values from Fig. 4.1 or Eq. (4.1) or (4.2) to determine the stress distribution at the base centerline, the existing restraining force and moment at initiation of cracking can be determined from the internal stress block for various *L/H*, and is shown in Fig. 4.2. Since cracks do not immediately propagate to the full block height throughout the member, a driving force of continuing volume change must be present.

A propagating crack will increase the tensile stress at every section above the crack as it propagates. Throughout the section the stress increase is the same proportion as the proportional increase in stress that occurred at the present crack position in propagating the crack from its previous position. From Fig. 4.3, the maximum restraining force in the stress block, corresponding to maximum base shear, occurs with the volume reduction producing initial cracking. The maximum moment of the internal stress block, corresponding to maximum base restraint, does not occur until the crack propagates to a height of 0.2 to 0.3 times the height of section. At that point, the crack is free to propagate to its full height without a further reduction in volume. From Fig. 4.3 the maximum base restraint at the centerline of a block having an *L/H* of 2.5 is approximately $0.2f_t'BH.^2$ This may be assumed as the minimum base restraint capable of producing full-block cracking. The corresponding spacing of full-block cracking in unreinforced concrete would therefore be approximately 1.25*H*.

Prior to cracking, the stress in the reinforcement of nonflexural members subjected to shrinkage depends primarily on the differences in coefficients of expansion between steel and concrete. Where the coefficients are equal, the reinforcement becomes stressed as crack propagation reaches the steel. The tensile force of the cracked portion of the concrete is thus transferred to the steel without significantly affecting base restraint. The moment of the steel stressed throughout the height of the crack adds directly to the restraining moment of the internal stress block at the centerline between cracks. When the combined internal stress moment and steel stress moment equals $0.2f_t'BH^2$ then the combined restraint is sufficient to produce full block height cracking at the centerline between cracks.

For L/H values less than 2, Fig. 4.1 indicates negative restraint at the top. For decreasing volume, this would mean induced compression at the top. Therefore, full-section cracking is not likely to occur.

At any section, the summation of crack widths and extension of concrete must balance the change in concrete volume due to shrinkage. To control the width of cracks it is thus necessary to control their spacing, since extensibility of concrete is limited. If the change in volume requires a minimum crack spacing less than 2*H*, then reinforcement must be added to assure this spacing. From these postulations, if the required spacing is L' then the restraining moment of the reinforcing steel at the existing crack spacing of $2L'$ would be $0.2f_t'BH^2$ minus the restraining moment of Fig. 4.2 for $L/H = 2\,L'/H$.

A linear approximation of this difference can be determined by

$$M_{RH} = 0.2f_t'BH^2\left(1 - \frac{L'}{2H}\right) \tag{4.4}$$

where

M_{RH} = restraint moment required of reinforcing steel for full-height cracking
f_t' = tensile strength of concrete
H = height of block
B = width of block

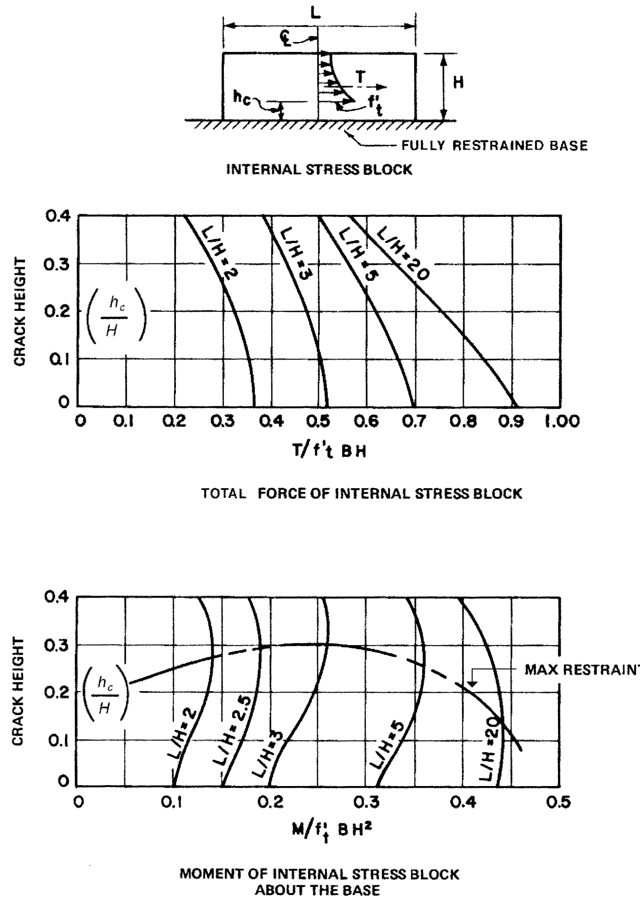


Fig. 4.3—Effect of crack propagation on internal forces

4.3—Discontinuous external or end restraint

When the contact surface of the concrete element under restraint and the supporting element is discontinuous, restraint to volume change remains concentrated at fixed locations. This is typical of all concrete elements spanning between supports. It is also typical for the central portions of members supported on materials of low tensile strength or of lower shear strength than concrete, which require substantial frictional drag at the ends to develop restraint.

4.3.1 *Stress distribution of members spanning between supports*—A member that is not vertically supported throughout its length is subject to flexural stress as well as stress due to length change. When a decrease in volume or length occurs in conjunction with flexural members spanning between supports, additional rotation of the cross sections must occur. If the supports themselves are also flexural members, a deflection will occur at the top of the supports and this deflection will induce moments at the ends of the member undergoing volume change. These flexural stresses will be in addition to the tensile stresses induced by the shear in the deflected supports (see Fig. 4.4). The end moments thus induced will increase tensile stresses in the bottom face and decrease tensile stresses in the top face of the member undergoing volume change. The magnitude of induced stress depends on the relative stiffnesses of the concrete element under restraint and the supporting members and may be de-

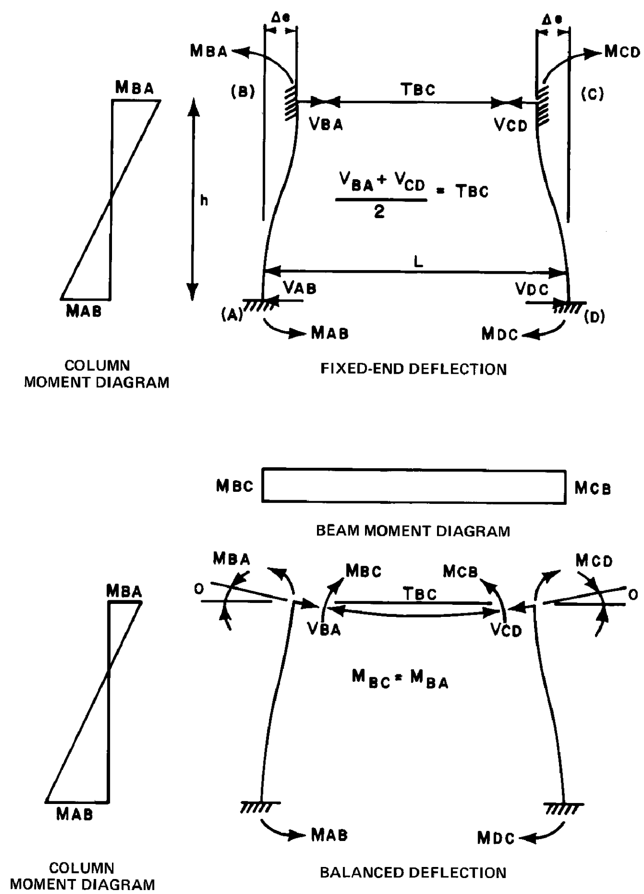


Fig. 4.4—Flexure of a simple frame induced by beam shortening

termined when the degree of restraint K_R has been determined for the support system. For members spanning two supports, the degree of restraint can be approximated by

$$K_R = \frac{1}{1 + \dfrac{A_B h^3}{4L I_c}} \qquad (4.5)$$

where L and A_B = the length and area, respectively, of the member undergoing volume change, and I_c and h = the average moment of inertia and height respectively of the two supporting end members.

The change in bottom face steel stress for members spanning flexural supports can be approximated by

$$\Delta f_s = \frac{K_R C_T T_E E_S}{2pnj}\left[\frac{h}{d}\left(\frac{K_f}{K_f + K_c}\right) + 4pnj\right] \qquad (4.6)$$

where

C_T = linear thermal coefficient as defined in Section 3.6
T_E = design temperature change including shrinkage effects
E_S = elastic modulus of steel
K_f = stiffness of beam or floor system undergoing volume change
K_c = average stiffness of vertical restraining elements subject to deflection by volume change

For complicated frames and members spanning continuously over more than two supports, the stress induced in the member from the change in volume should be determined by a frame analysis considering the effects of sideway, member elongations under direct load, and shear deflections of the support members.

If the supporting members are very stiff relative to the member undergoing volume change, the deflection at the top of the supporting members will be essentially a shear deflection and no end moments will be induced in the member. Under these conditions the change in steel stress throughout the member will simply be

$$\Delta f_s = 2K_R C_T T_E E_S \qquad (4.7)$$

A temperature gradient through a wall or slab with ends fixed or restrained against rotation will induce bending stresses throughout the member. When the restraint to rotation is sufficient to crack the member, cracking will be uniformly spaced throughout. Rotational stiffness is dependent on the moment of inertia of the cracked section. The ratio of the moments of inertia of cracked to uncracked sections in pure bending is $6jk^2$. Using this, the fixed-end moment for a cracked section would be

$$\text{FEM} = (T_1 - T_2)C_T E_c bd^2\left(\frac{jk^2}{2}\right) \qquad (4.8)$$

where T_1 - T_2 is the temperature difference across the member, and C_T = the expansion coefficient of the concrete.

4.3.2 *Stress distribution of vertically supported members*—The distribution of stresses due to volume change in members subject to a discontinuous shear restraint at the base, but vertically supported throughout its length, is dependent on the L/H of the member, which for all practical purposes is the same as Fig. 4.1 where L is the distance between points of effective shear transfer at the base. As the L/H approaches infinity, the distribution of stress approaches uniformity over the cross sectional area at any appreciable distance from the support.

For slabs placed on the subgrade material of little or no tensile strength and lower shear strength than the slab concrete, the distance between points of effective shear transfer depends on the frictional drag of the slab ends. A decrease in slab volume will curl the ends of the slab upward. Cracking will initiate at approximately the center of the base when the full depth of the member has a parabolic tensile stress distribution (see Fig. 4.5) with the stress at the base equal to the tensile strength of the concrete. The cracking moment for this internal stress distribution will be $f_t'BH^2/10$. (Fig. 4.6 shows internal restraint.) The balancing external restraining moment depends entirely on the weight of the concrete and

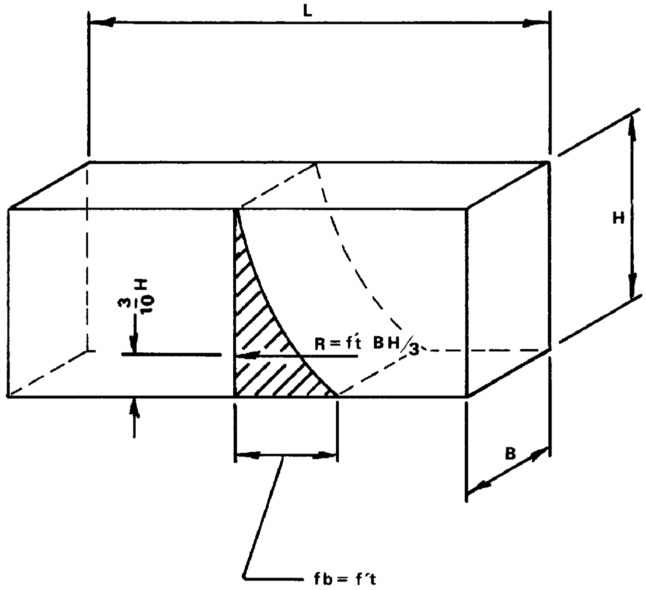


Fig. 4.5—Internal stress distribution of slabs on low-strength subgrade

the distribution of the base pressure. Assuming a parabolic base pressure distribution over two-thirds of the curling slab base, as shown in Fig. 4.7, the restraining moment will equal 0.075 $w_c\ BHL^2$, or

$$\frac{f_t'BH^2}{10} = 0.075\ w_c BHL^2$$

For ft' = 300 psi w_c = 144 lb/ft^3, and $L = 20\sqrt{H}$ (for L and H in ft).

When the overall slab length exceeds $20\sqrt{H}$, the distribution of stress in the central portion of the slab will approximately equal that of continuously restrained base having an L/H of $(L - 20\sqrt{H})/H$. When the spacing of cracks must be less than $20\sqrt{H}$, reinforcement must be provided. When the ratio of $(L - 20\sqrt{H})/H$ is less than 2, a minimum tensile force of $f_t'BH/3$ must be provided by the reinforcing steel to provide multiple cracks between the end sections. If the ratio of $(L - 20\sqrt{H})/H$ is greater than 2.5 the reinforcement must be capable of developing the full drag force of the end sections. This would be the full tensile force T of Fig. 4.2 for L/H corresponding to $(L - 20\sqrt{H})/H$. Thus the reinforcement requirements are

$$A_S = \frac{T}{f_S} \geq \frac{f_t'BH}{3f_s} \qquad (4.9)$$

where f_t' = tensile strength of concrete and f_s = allowable steel stress.

4.3.3 *Cracking pattern of vertically supported members*—When the stress of a member subject to discontinuous restraint or restrained at its ends exceeds the tensile strength of the concrete, a single crack will form between the points of restraint. Any additional cracking of the member must be

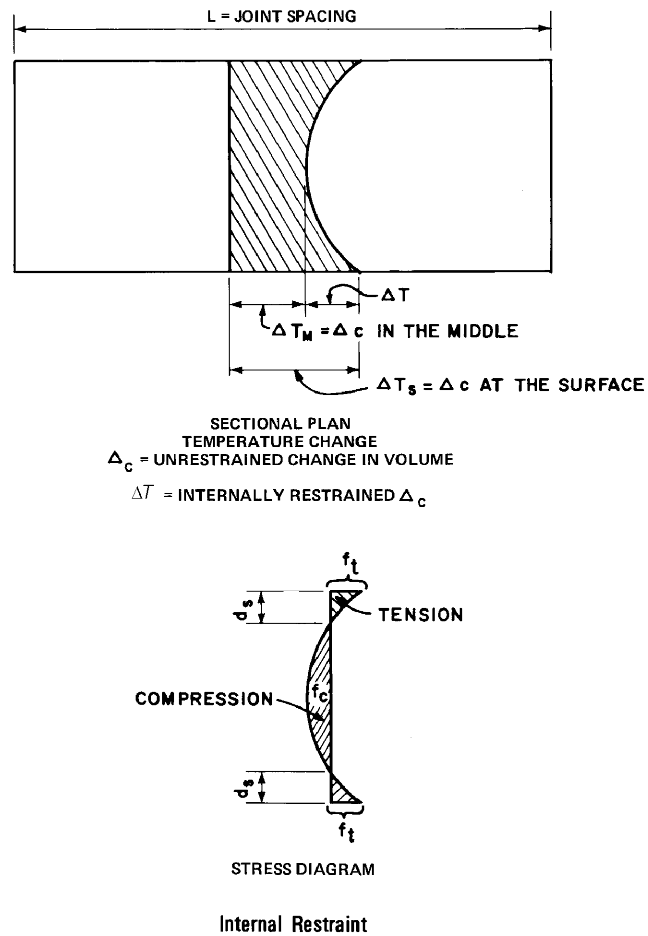


Fig. 4.6—Internal restraint

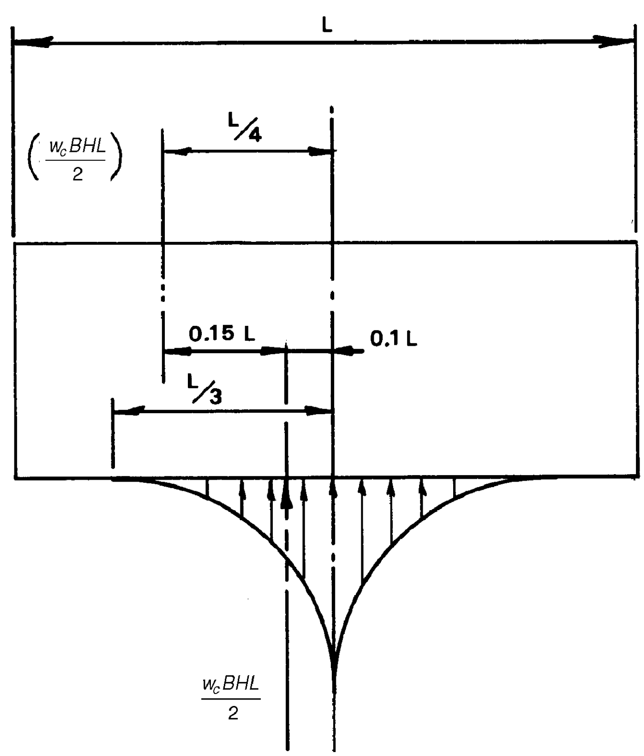


Fig. 4.7—Pressure distribution and restraining moments of curling slab

provided by enough reinforcing steel at a controlled stress level to equal the total restraint force induced at the member ends.

4.4—Internal restraint

Internal restraint exists in members with nonuniform volume change on a cross section. This occurs, for example, within walls, slabs, or masses with interior temperatures greater than surface temperatures or with differential drying shrinkage from outside to inside. It also occurs in slabs projecting through the walls of buildings with cold outside edges and warm interiors and in walls with the base or lower portions covered and the upper portions exposed to air.

Internal restraint depends on the differential volume change within a member. Its effects add algebraically to the effects of external restraint, except that their summation will never exceed the effects of 100 percent external restraint. Therefore, where high external restraint conditions exist the effects of internal restraint may be negligible.

4.4.1 *Stress distribution and cracking*—Internal restraint is similar to continuous edge restraint, except that the effective restraining plane is the plane of zero stress in the internal stress block and is dependent on the actual temperature gradient in the concrete (see Fig. 4.6). For section stability, the summation of tensile stress induced by the temperature or moisture gradient on a cross section must be balanced by an equal compressive force. This balance line locates the depth d_s of the internal stress block. If the depth of the tensile stress block d_s is large in comparison to the spacing of joints L, then the stress induced by volume change will not be significant. As an example, if the annual temperature range at the surface is four times the range in concrete, then a 100 ft thick dam would have a 15 ft deep tensile stress block using the distribution shown in Fig. 5.3.5 of ACI 207.1R. If we assume a 50 ft spacing of joints, the L/d_s ratio would be 3.3 and the degree of restraint at the surface would be 25 percent using Fig. 4.1 of this report and L/d_s as L/H. In contrast, from the same chart the daily cycle shows a penetration of only 2 to 2.5 ft. Using 2 ft as d_s, the degree of restraint at the surface would be approximately 85 percent and assuming a concrete tensile strength of 300 psi, a concrete modulus of 3×10^6 psi and a coefficient of thermal expansion of 5×10^{-6} in./in./F, cracking would occur at the face with a 24 F drop in surface temperature. For equal stress the annual temperature variation would have to be 82 F. Cracking from the daily temperature cycle is not usually significant in dams and large masses, particularly in moderate climates, because of the limited penetration or significance of such cracks. The 24 F drop in mean daily temperature corresponds to normal winter temperature fluctuations for moderate climates. See Chapter 5 of ACI 207.1R for a more complete discussion of surface cracking.

Temperatures on the opposite faces of a wall or slab may not be equal because of a difference in exposure conditions. The variation of temperatures through the slab or wall may be assumed to be parabolic or exponential.

Temperature distribution of this sort will curl the slab or wall if unrestrained, or induce bending stresses along the member if its ends are restrained as previously discussed in Section 4.3.1.

The plane of zero stress of the tensile stress block for projecting portions of concrete walls or slabs may be determined by a heat-flow analysis or by trial as just described. The proportion of cold volume to total volume is larger for members of this type than for dams or other large concrete masses. The penetration of the daily temperature cycle may therefore be assumed somewhat more than the 2 to 2.5 ft penetration previously mentioned for dams. Restraint at the free edge may also be determined for these cases from Fig. 4.1 by setting the depth of the tensile stress block d_s as a fixed plane 3 ft inside the exterior surface.

CHAPTER 5—CRACK WIDTHS

5.1—General

Reinforcement is utilized to restrict the size of cracks that would otherwise occur. Large-sized, randomly spaced cracks are objectionable and may indicate that the reinforcement transverse to the crack has yielded. This may be cause for concern, depending on the structure in question and the primary purpose of the reinforcement. Surface-crack widths are important from an esthetic viewpoint, are easy to measure, and are the subject of most limitations. While the width of a crack at the surface may initially be larger than the crack width at the reinforcement, the difference may be expected to decrease with time.

For water-retention elements, very narrow, just-visible cracks (0.002 in.) will probably leak, at least initially; however, nonmoving cracks up to 0.005 in. may heal in the presence of excess moisture and therefore would not be expected to leak continually. Any leakage may be expected to stain the exposed concrete face or create problems with surface coatings.

Most thermal cracks transverse to reinforcement do not appear to have significant impact on corrosion. (ACI 224R, ACI 224.1R).[8]

Fiber reinforcement is of some benefit in controlling cracks but may not be cost effective.

5.1.1 *Controlled cracking*—It has been common practice for many years to use expansion and contraction joints to reduce the size and number of uncontrolled cracks. In sidewalk and pavement construction, formed grooves have also been used to create planes of weakness and thereby induce cracking to coincide with the straight lines of the grooves. This concept has been expanded in the United Kingdom as a method of controlling cracks in massive walls and slabs. The British install plastic or metal bond breakers to induce cracks at specific locations. The British research indicates that a cross-sectional reduction of as little as 10 percent has proved successful in experiments, but 20 percent is recommended to assure full section cracking in practice.[9] The depth of surface grooves is obviously limited by any continuous reinforcement; therefore, some form of void must be cast into massive sections to achieve the needed section reduction. These voids can be formed with plastic pipes or deflatable duct tubes. Alternately, the reduction may be accomplished by using proprietary crack-inducing water barriers that have been designed to act as both bond breakers and water stops. The principal advantage of a crack-control system is that cracking can essentially be hidden by the formed grooves. Also, the crack size (width) loses its significance when there is a water barrier and the reinforcement crossing the crack is principally minimum steel that is not required for structural integrity.

5.2—Limitations

It is desirable to limit the width of cracks in massive structures to the minimum practical size, in keeping with the function of the structure. Reinforced mass concrete structures are generally designed in accordance with ACI 318. The crack-control provisions of ACI 318 develop reasonable details of reinforcement, in terms of bar size and spacing, for general conditions of flexure. The Commentary to the ACI Building Code says that the code limitations are based on crack widths of 0.016 in. for interior exposure and 0.013 in. for exterior exposure. The permissible crack widths versus exposure conditions in Table 4.1 of ACI 224R represent a historical viewpoint of "tolerable crack width." While they may not represent a current consensus, they do offer guidance to what has been considered acceptable. ACI 350R establishes minimum percentages of shrinkage and tem-perature reinforcement for sanitary engineering structures based on the spacing of construction joints from 20 to 60 ft. In addition, it restricts the working stress and z-value of Eq. (10-4) of ACI 318, based on the thickness of cover and type of exposure. For an 18 in. thick member with 2.5 in. cover, exposed to liquids, the crack width corresponding to the ACI 318 Commentary would be 0.011 in. for flexure and 0.009 in. for direct tension.

Limiting crack width by utilization of reinforcement becomes increasingly difficult as member size increases. The most effective means to control thermal cracking in any member is to restrict its peak hydration temperatures. This becomes increasingly important with increasing member size. For massive structures, the amount of reinforcement required to restrict crack width to less than 0.009 in. becomes impractical when any of the accepted formulas to predict crack width are used. Cracks of this width will allow some leakage; however, leakage will be minimum and controllable.

5.3—Calculations

A number of crack-width equations are proposed in the literature. ACI 318 adopts an expression based on one developed in a statistical study by Gergely and Lutz[10] reported in ACI SP-20.

$$w = 0.076\sqrt[3]{d_c A}\,\beta f_s 10^{-3} \tag{5.1}$$

where

w = maximum crack width at surface, in.
d_c = cover to center of bar, in.
A = average effective concrete area around a reinforcing bar ($2d_c$ x spacing), in.^{2}
ß = distance form neutral axis to the tensile face divided by distance from neutral axis to steel
f_s = calculated steel stress, ksi

In the preceding formula, the ß-ratio is taken as 1 for massive sections.

The maximum crack width for tension members is generally accepted as larger than the just-given expression for flexure. ACI 224R suggests the following to estimate maximum tensile crack width

$$w = 0.10 f_s \sqrt[3]{d_c A}\; 10^{-3} \tag{5.2}$$

The preceding expressions for maximum crack width for flexure and tension are based on applied loads without consideration for volume change. Any restraint of volume change will increase directly the actual crack width over that estimated by these formulas. Thus, any procedure which makes a reasonable estimation of expected volume change in its analysis will improve predictability. When the expected change in volume has been accounted for, Committee 207 believes the application of the Gergely and Lutz expression for crack width provides sufficient limitations in determining crack reinforcement without additional conservatism. Committee 207 has therefore chosen this expression to apply its procedures. The designer is always at liberty to chose a more conservative expression.

CHAPTER 6—APPLICATION

6.1—General

Determination of restraint, volume change, appropriate concrete properties, and crack widths have been discussed. They will now be combined for calculation of steel areas. Exterior loads that induce tensile stress in the concrete in addition to those induced by volume change must also be accounted for in steel area calculations.

6.2—Volume change plus flexure

For both normal structural and massive members, the change in stress f_s induced by a decrease in volume of flexural members (discussed in Section 4.3.1) should be added directly to the service-load stress, and crack width should be checked as per Sections 5.2 and 5.3.

For normal structural members, ACI 318 can be followed. This requires a value of z, a quantity limiting distribution of flexural reinforcement

$$z = f_s \sqrt[3]{d_c A} \tag{6.1}$$

where

f_s = calculated stress in reinforcement

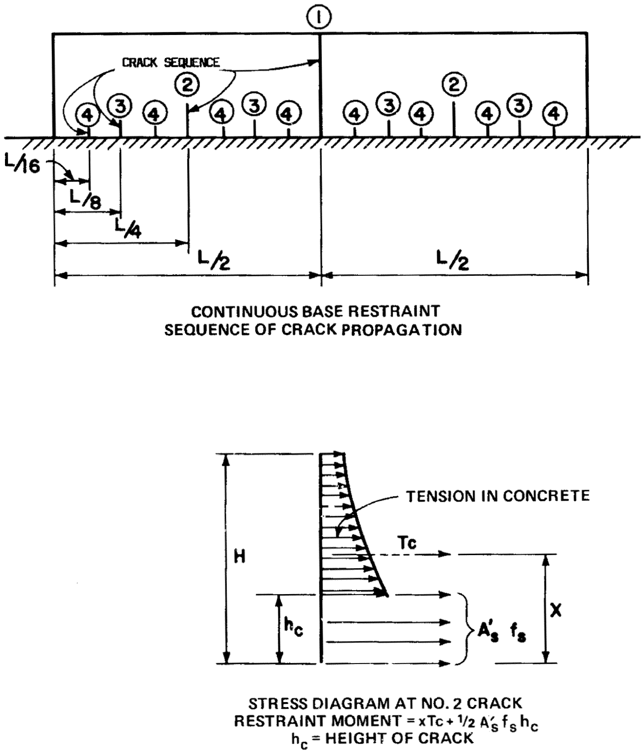


Fig. 6.1—Sequence of crack propagation and distribution of stress at No. 2 crack

d_c = thickness of concrete cover measured from the concrete surface at which cracks are being considered to the center of the nearest reinforcing bar

A = effective tension area of concrete surrounding a group of reinforcing bars and having the same centroid as that of reinforcement, divided by the number of bars

to be checked in lieu of crack width (notation as in ACI 318). The value of z should be limited to 175 for normal interior exposure, and 145 for normal exterior exposure.

For reinforced mass concrete, the combined stress should be limited by crack width based on Chapter 5. In addition, the minimum ratio of tensile-steel reinforcement for massive concrete members in flexure should be based on steel stress not to exceed 0.9 f_y, where f_y is the specified yield stress of steel in ksi.

6.3—Volume change without flexure

The spacing of cracks is largely dependent on the conditions of restraint when a decrease in volume occurs in a member not subject to flexure. Stress in the reinforcing steel can be determined using the Gergely-Lutz crack width formula with a ß of 1.0 by assuming a bar cover and spacing and calculating the stress in reinforcement f_s from

$$f_s = \frac{w \times 10^3}{0.076\sqrt[3]{d_c A}} \text{ (in ksi)} \tag{6.2}$$

where w is the permissible crack width.

6.3.1 *Continuous external restraint*—Members subject to continuous restraint at their bases or on one or more edges will crack under continuing volume change as described in Section 4.2.2. Cracks are not uniform and will vary in width throughout the height of the member.

Fig. 6.1 shows the sequence of cracking for a member subject to uniform volume change and continuous base restraint. As each new crack forms at approximately the midpoint of the uncracked portions of the base, the previously formed cracks will extend vertically. The maximum width of each crack will occur at vertical locations just above the top of the previously formed cracks. Below this point there are two more times the number of cracks to balance volume change. The concrete at the top of the partially extended crack is assumed stressed to f_t'. Therefore the summation of crack widths on any horizontal plane must approximately equal the total volume change ($K_R L C_T T_E$) minus concrete extensibility $L f_t'/E_c$.

The extensibility of concrete is affected significantly by creep; therefore, the time required for a given volume change to occur will directly affect the temperature drop T_E, producing cracking.

Hognestad[11] found that for the normal range of service-load stress for high-strength reinforcement, which is between 30 and 40 ksi, a mean value of the ratio of maximum crack width to average crack width was 1.5. If N is the number of cracks and w is the maximum crack width then the $N \cdot w/1.5$ will be the summation of crack widths in a given length and

$$\frac{N \cdot w}{1.5} = 12L(K_R C_T T_E - f_t'/E_c) \tag{6.3}$$

for L in ft. If the average crack spacing equals L', then $NL' = L$ and

$$L' = \frac{w}{18(K_R C_T T_E - f_t'/E_c)} \tag{6.4}$$

For most structures, the hydration heat effects are dissipated during the first week after placement. At this age, the extensibility or tensile strain capacity of the concrete is generally less than 100 microstrains and the effective temperature drop would constitute only hydration heat. For hot-weather placements, the maximum temperature drop will not occur until the concrete is 3 to 6 months old. At this age, creep and tensile strain capacity may be improved to provide more crack resistance. The age of critical volume change will be the age which requires the minimum average crack spacing L' from Eq. (6.4). For most parts of the United States, the critical volume change will occur for summer placement. A value for tensile strain capacity f_t'/E_c of 0.0001 for early-age cracking and 0.00015 for seasonal cracking is recommended.

It is necessary to calculate the required average crack spacing to determine the required restraining moment to be supplied by the reinforcing steel. Cracking throughout a member may or may not extend the full height of the member, depending on the L/H relationship (see Fig. 6.1). When cracks extend for just a portion of the height, only the reinforcing steel below the top of the crack is effective in contributing to the internal restraint moment. (From Fig. 6.1, the internal restraint moment between full-block cracks = $T_c x + A_s' f_s h_c/2$.) Even when some cracks do extend the full height, others extend only part way, so that the same situation applies between full-height cracks. For this reason, reinforcement is more effectively distributed if the wall is examined at several locations above the base to determine the average crack spacing required at each location corresponding to the degree of restraint K_R at each distance h from the base. The additional restraining moment $(A_s' f_s h_c)/2$ required of the reinforcing steel between the point h and the restrained base to produce the required crack spacing L' at h can be conservatively determined by substituting h for H in Eq. (4.4)

$$M_{Rh} = 0.20 f_t' B h^2 \left(1 - \frac{L'}{2h}\right) \tag{6.5}$$

The degree of restraint K_R to be used in the calculation of L' at h can be calculated as indicated in Section 4.2.1 or can be read directly from Fig. 4.1 as the proportional height above the base (h/H) corresponding to the actual L/H curves. It is conservative and usually convenient to assume the distance h as the free edge distance H and read K_R in Fig. 4.1 at the free edge using L/h as L/H.

In determining the volume change reinforcement required in each face of walls with continuous base restraint, calculations at lift intervals or at some arbitrary intervals above the base should be made as follows

$$A_b = 0.4 \frac{f_t' B h}{f_s N_H}\left(1 - \frac{L'}{2h}\right) \tag{6.6}$$

where

h = interval distance above the base being considered

N_H = total number of bars in the h distance above the base

A_b = area of bars required in each face of the wall

$A_s' h/N_H$ = A_b

As the distance h from the base increases, steel requirements will first increase and then decrease. Maximum steel requirements depend on base length, effective temperature drop and coefficient of thermal expansion. Fig. 6.2 gives the point of maximum steel requirements in terms of base length and design temperature for a coefficient of thermal expansion of 5×10^{-6} in./in./F. The same curve can be used for other expansion coefficients by using another design temperature equal to $C_T T_E / 5 \times 10^{-6}$. Fig. 6.2 also provides the point h above which only minimum steel is required. Recommendations for minimum steel requirements are given in Section 6.4. Only minimum steel is required where L' is greater than $2h$. Fig. 6.3, 6.4, and 6.5 give the maximum steel requirements in terms of crack width, effective temperature drop, and base length for concrete walls having a $C_T = 5 \times 10^{-6}$/F. These figures can be used to proportion steel requirements in place of the multiple calculations described above with only slightly higher total steel quantities being required.

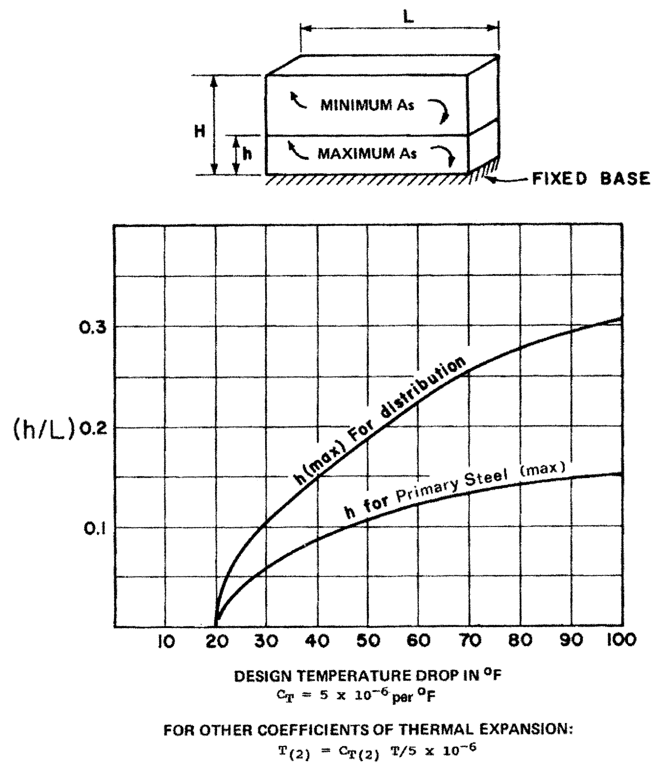


Fig. 6.2—Wall height requiring maximum temperature and shrinkage reinforcement as a ratio of base length

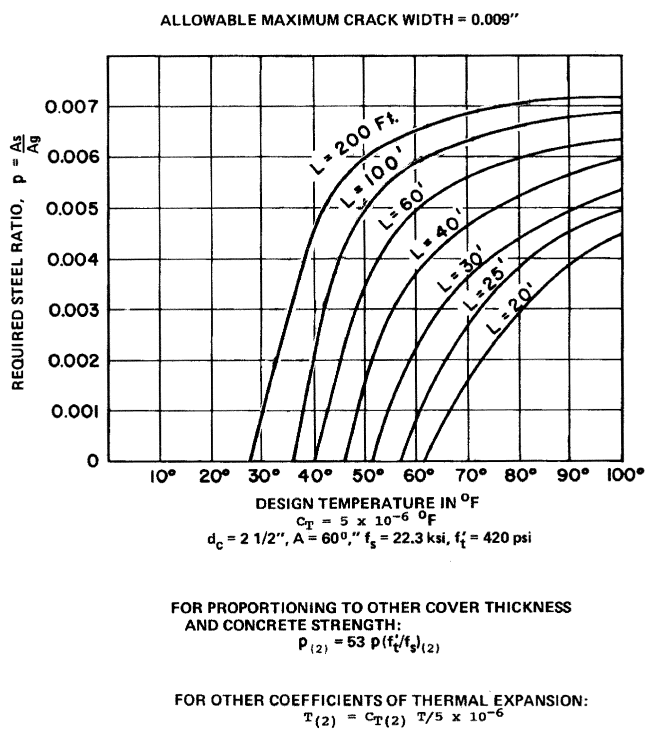


Fig. 6.3—Maximum temperature and shrinkage reinforcement for walls with fixed bases

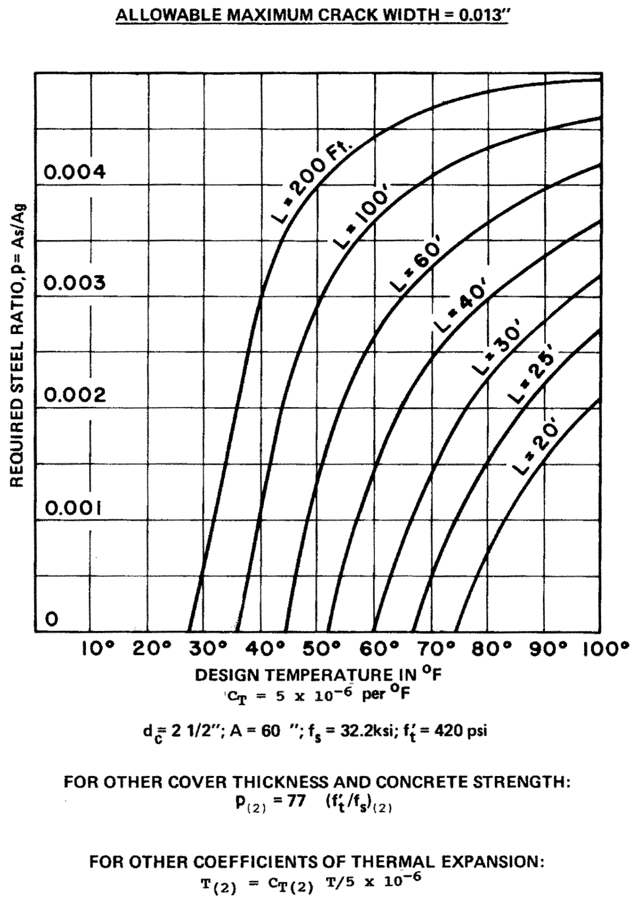


Fig. 6.4—Maximum temperature and shrinkage reinforcement for walls with fixed bases

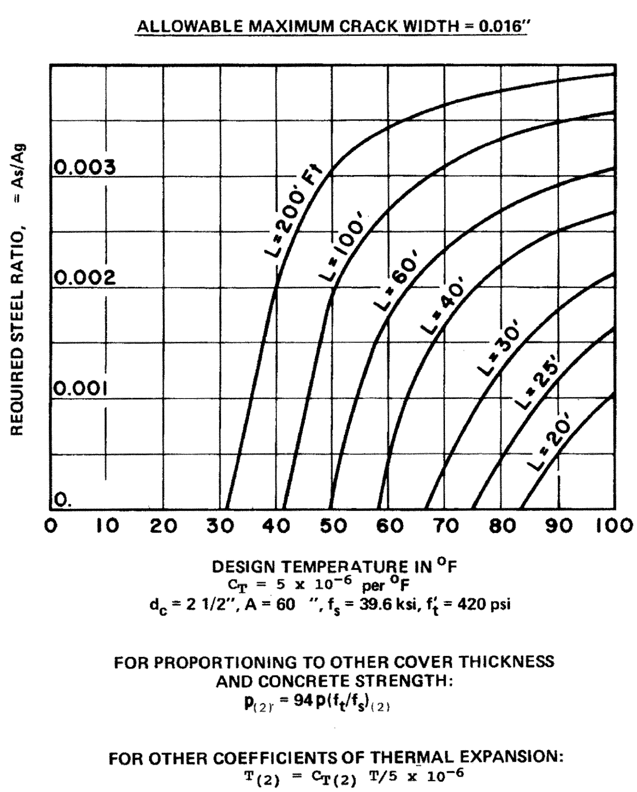


Fig. 6.5—Maximum temperature and shrinkage reinforcement for walls with fixed bases

The maximum height h over which these steel quantities are required can be determined from Fig. 6.2. Above h, only minimum steel is required. Requirements for concrete properties and cover distances other than noted can be proportioned as shown.

For slabs with continuous base restraint or walls with one side continuously restrained

$$A_b = 0.20\frac{f_t'}{f_s}\left(1 - \frac{L'}{2H}\right)\frac{BH}{N_B\left(\frac{H - d_c}{H}\right)} \qquad (6.7)$$

where N_B = total number of bars in the free face of the slab or wall.

In the case of relatively thick slabs, the amount of reinforcement required in the top face of the slab may be reduced by including the effect of the reinforcement in the sides. For this

$$A_b = 0.20\frac{f_t'}{f_s}\left(1 - \frac{L'}{2H}\right)\frac{BH}{N_B\left(\frac{H - d_c}{H}\right) + \frac{N_H}{2}} \qquad (6.8)$$

Only minimum steel is required where L' is greater than $2H$ (see Section 6.4).

In applying Eq. (6.7) and (6.8) to relatively large masses, the amount of reinforcement required will make it quite obvious that additional measures to control volume change should be used to control crack widths. Reinforcement is not practical in controlling the crack widths of very large externally restrained masses, and for these structures the principles of mass concrete construction described in ACI 207.1R must be followed to control cracking. The preceding formulas for crack spacing, however, can be utilized to establish a somewhat higher allowable temperature drop than normally used for mass concrete by acknowledging an acceptable crack. This can be seen in the design temperatures corresponding to zero steel requirements for the lengths of wall shown in Fig. 6.3 through 6.5.

Design temperatures in unreinforced sections should be kept approximately 10 F less than indicated for zero steel requirements because of the apparent sensitivity of crack widths to temperature in the cracking temperature range. Table 6.3.1 is based on this criteria.

When the expected temperature drop for the planned contraction joint spacing exceeds the design temperature limits

Table 6.3.1—Design temperature limits for unreinforced concrete walls (for limiting cracks to 0.009 in.)

Contraction joint spacing, ft	Coefficient of thermal expansion x 10^{-6}			
	4	5	6	7
100	30 F	24 F	20 F	17F
60	37 F	30 F	25 F	21 F
40	44 F	35 F	29 F	25 F
20	62 F	50 F	42 F	36 F

of Table 6.3.1, or when a larger spacing of contraction joints is desired, the utilization of crack control measures discussed in Section 5.1.1 in conjunction with these limits may be used to control the width of cracks in between contraction joints.

6.3.2 *Discontinuous external or end restraint*—Cracking will occur when the stress induced in the concrete by volume change exceeds the tensile strength of concrete as described in Section 4.3.3. When more than one crack is required to control crack widths, the total force in the reinforcing steel must equal the total restraint force induced at ends of the member. For members with continuous base support, this will require a minimum steel ratio of

$$p(min) = f_t'/f_s \tag{6.9}$$

More than one crack will be required when the permissible crack width is less than the volume change LC_TT_E.

For members spanning between supports, the change in stress of the bottom face reinforcement due to thermal cracking may be determined from Eq. (4.6) of Section 4.3.1.

When the volume change is less than the permissible crack width, no steel is required for volume change except as may be required as minimum (see Section 6.4).

6.3.3 *Internal restraint*—For relatively large masses, the spacing of surface cracks will be controlled by internal restraint as described in Section 4.4. These cracks, independent of external restraint conditions, are not deep enough to require more than nominal amounts of reinforcing near the surface to control crack widths. In the example given in Section 4.4, the surface tensile stress due to daily temperature fluctuations was more than the surface stress due to the annual change in temperature. The depth of tensile stress block for the daily temperature fluctuations was less than 2.5 ft in the example. If this is assumed as the maximum depth of the critical restraint plane for internal restraint, then a maximum surface crack spacing in large masses of approximately 5 ft can be expected. If LC_TT_E, using the maximum normal daily temperature fluctuation for T_E, is less than $w/12L'$, for L' in ft, then no surface reinforcement is required (note L' should not be taken as more than 5 ft). If $C_TT_E > w/12L'$, then the minimum steel requirements of Section 6.4 should probably be utilized.

When internal restraint results from exposure of projecting elements from warm interiors, such as slabs projecting through exterior walls or walls projecting out of the ground, determine the depth of the tensile stress block and restraint factor as outlined in Section 4.4.1. If the required average crack spacing is less than twice the depth of the tensile stress block determine the size of bars to be distributed throughout the tensile stress block by

$$A_b = \frac{1f_t'Bd_s}{3f_s N_H} \tag{6.10}$$

where N_H = the total number of bars distributed throughout d_s.

6.4—Recommendation for minimum reinforcement

The minimum requirements of ACI 318 should apply to all superstructure slabs and beams. The minimum total quantity of temperature and shrinkage reinforcement recommended for walls, slabs, and footings less than 48 in. thick, which have been investigated for crack control by the measures outlined herein, is 0.0015 times the cross sectional area A_g of the member. When shrinkage-compensating cement is used, the recommendation of ACI 223 for design of reinforcement should be followed. Not less than one-half nor more than two-thirds of the total quantity of reinforcement should be placed in any one face. For crack control the maximum bar spacing should be limited to 12 in. on center. For members more than 48 in. thick the minimum temperature and shrinkage requirements in each face should be limited by depth of cover d_c and bar spacing such that

$$A_s' = \frac{f_t'A}{f_s} \text{ or } \frac{A}{100} \text{ (as a limit for } f_t'/f_s\text{)} \tag{6.11}$$

The minimum bar size and spacing for members of this size should not be less than #6 bars at 12 in. on center.

No minimum temperature and shrinkage reinforcement is required for members 6 ft or more in thickness that are constructed by the principles and practices of ACI 207.1R to control the cracking of mass concrete provided the environmental conditions are such that cracking from internal restraint as discussed in Section 6.3.3 is not significant.

6.5—Design procedure

The basic procedure for problem solving is:

1. Determine the maximum effective temperature as outlined in Section 2.6.

2. Determine the restraint characteristics of the element or structure as outlined in Chapter 4.

3. Determine the physical properties of the concrete; tensile strength, elastic modulus, and coefficient of thermal expansion as outlined in Chapter 3.

4. Determine the allowable maximum crack width from Section 5.2 or by some other established criteria.

5. Determine the area of steel required to maintain cracking at the acceptable level.

a. For members subject to continuous external edge restraint determine the required average crack spacing for the height of slab or height intervals of 5 to 10 ft above the base of walls as per Section 6.3.1. Where the required crack spacing is less than the spacing of joints, provide reinforcement as per Section 6.3.2. In lieu of this the reinforcement in walls may be proportioned using Fig. 6.2 through 6.5. When the element or member is of sufficient size to require more than 1.5 in.^{2}/ft of temperature reinforcement or when its cost exceeds one-third the cost of concrete (excluding formwork), additional measures to control volume change should be applied as recommended in ACI 207.1R.

b. For discontinuous external or end restraint, reinforcement will be required if $w \le LC_TT_E$. If the member is subject to flexure, determine the change in steel stress as per

Section 6.2. If not, determine the steel requirements as per Section 6.3.2.

c. For members subject to internal restraint, provide reinforcement as per Section 6.3.3 if the required average crack spacing is less than twice the depth of the tensile stress block.

The following example problems illustrate this design procedure.

Example 6.1—Basement wall of power plant 30 ft high by 200 ft long is to be designed to retain backfill as a cantilevered wall for construction conditions. The wall is subject to ground water for its full height, with base slab on rock. It will be placed in 80 F ambient temperatures. Minimum final or operating air temperature will be 50 F. Assume the wall tapers from its maximum thickness at the base to 18 in. at the top. Maximum thickness at the base is controlled by shear and is 40 in. for 3000 psi concrete and 48 in. for 2000 psi concrete. Design for limited leakage by limiting crack width to 0.009 in. and determine required wall thickness and reinforcement for the following conditions:

a. Design for 3000 psi (3700 psi average strength) at 28 days and use the 470 lb/yd^3 mix of Example 2.2.

b. Same as (a) except contraction joints spaced 67 ± ft apart.

c. Design for 2000 psi at 28 days using mix of Example 2.1, no contraction joints and concrete cooled to 60 F placing temperature.

6.1(a)

Step 1.1—Volume-to-surface ratio (assume 10-ft lifts and wooden forms). Average thickness for first two lifts = 33 in. = 2.75 ft.

Wooden forms = 1.67 ft. of concrete

$$V/S = \left[\frac{2.75 + 2(1.67)}{2(10) + 2.75}\right]10 = 2.68 \text{ ft}$$

Step 1.2—Following Example 2.2 in Section 2.7, the effective placing temperature for 80 F concrete without cooling measures would be approximately 84 F.

Step 1.3—The minimum temperature T[inf/min] of concrete against earth, using Eq. (2.3), is 54 F.

Step 1.4—The temperature rise following Example 2.2 is 68 F.

Step 1.5—The design temperature equals 84 + 68 – 54 = 98 F.

Note: Seasonal temperature controls, since [5 (98) – 150] > [5 (68) – 100], as discussed in Section 6.3.1.

Step 2—Restraint (Fig. 4.1).

Step 3—Physical properties from Fig. 3.2; f_c' at 6 months = 4500 psi, tensile strength $f_c' = 6\sqrt{4500} = 402$ psi; and tensile strain capacity = 150 x 10^{-6} in./in., assume C_T = 5 x 10^{-6} in./in./F.

Step 4—Limiting crack width = 0.009 in.

Step 5(b)—f_s = 22 ksi for 2-1/2-in. cover and 12-in. spacing of bars from Eq. (6.2). Using Fig. 6.2 and 6.3, maximum temperature and shrinkage reinforcement is required for full height of wall for average thickness of 33 in.

Examples:—Eq. (6.4) at h = 5 ft

$$L' = \frac{0.009}{18(0.95 \times 5 \times 98 - 150) \times 10^{-6}} = 1.58 \text{ ft}$$

Eq. (6.6) at 5 ft

$$A_b = \frac{0.4(370)}{22,000}\left(\frac{38 \times 5 \times 12}{10}\right)\left(1 - \frac{1.58}{10}\right) = 1.40 \text{ in.}^2$$

From Fig. 6.3, ρ = 0.007 and $\rho_2 = 53\rho$ (402/22,000), or ρ_2 = 0.0068, since tensile strength is 402 psi and not 420 psi,∴ A_b = 0.0068B x 12/2 = 0.0408 B and at h = 5 ft, A_b = 0.0408 x 38 = 1.55 in.^{2}

h, ft	Average B, in.	K_R (L = 200 ft) Fig. 4.1	L' 6.4*	$L'/2h$	A_b 6.6*	With adjusted ρ A_b Fig. 6.3	Reinforcement
5	38	0.95	1.58	0.16	1.40	1.55	#10 at 10
10	34	0.86	1.84	0.09	1.35	1.39	
15	31	0.79	2.11	0.07	1.26	1.26	#9 at 10
20	27	0.74	2.35	0.06	1.11	1.10	
25	23	0.71	2.53	0.05	0.96	0.94	#9 at 12
30	20	0.66	2.88	0.05	0.83	0.82	#8 at 12

* Formula:
Concrete = 537 yd^3 @ $70 . . . . $37,600
Temperature reinforcement = 27 tons @ $800 21,600
Cost (excluding forms) $59,200

6.1(b)

Everything same as (a) except L = 67 ft. From Fig. 6.2; maximum steel required only for the first 20 ft.

h, ft	Average B, in.	K_R (L = 200 ft) Fig. 4.1	L' 6.4*	$L'/2h$	A_b 6.6*	With adjusted ρ A_b Fig. 6.3	Reinforcement
5	38	0.79	2.11	0.21	1.31*	1.44	#10 at 12
10	34	0.61	3.35	0.17	1.24*	1.28	
15	31	0.45	7.1	0.24	1.04*	1.17	#9 at 12
20	27	0.31	200	>1	0.24†	Minimum steel	#5 at 12
25	23	0.18	200	>1	0.21†		#4 at 12
30	20	0.07	200	>1	0.18†		

* Formula.
† A_b/min. = A_g x 0.0015/N_b (see Section 6.4).
Temperature reinforcement = 14.49 tons @ $80 . . . . $11,590

Note savings in reinforcing steel of $10,000 to be weighed against the cost of two joints and added construction time.

6.1(c)—For 2000 psi (2500 psi average strength) concrete, f_t at 6 months (using Fig. 3.2 and $C + F_a$) = 6 $\sqrt{4800}$ or 416 psi.

Steps 1.1-1.5—V/S for the first two lifts = 2.81.

For a 60 F placing temperature, the concrete peaks at 2 days from Fig. 2.4.

Approximately 12 F is absorbed using Fig. 2.6.

The temperature rise would be 19 F using Fig. 2.5. and accounting for cement type and quantity.

The design temperature equals 60 + 12 + 19 – 54 = 37 F.

From Fig. 6.3, ρ_2 = 53 (f_t/f_s) = 53 (0.003) x (416/22,000) = 0.0030

$$A_b = \rho_2 B \text{ x } 12/2 = 0.018\, B$$

Steps 2-4—Assume same as (a).

Step 5(b)—From Fig. 6.2 and 6.3 maximum steel ratio equals 0.003 for first 25 ft of wall.

h, ft	Average B, in.	K_R (L = 200 ft) Fig. 4.1	L' 6.4*	$L'/2h$	A_b (minimum steel)	With adjusted ρ A_b Fig. 6.3	Reinforcement
5	46	0.93	39 ft	1+	0.41	0.83	#6 at 12
10	41	0.86	55 ft	1+	0.37	0.74	
15	36	0.79	200 ft	1+	0.32	0.65	#5 at 12
20	31	0.73			0.28	0.56	
25	26	0.67			0.23	0.47	#4 at 12
30	23	0.61			0.21	0.41	

* Concrete = 612 yd^3 @ $70 . . . . $42,500
Temperature reinforcement = 6.5 tons @ $800 5,200
48,000
Savings in stress steel 3 tons for 8 in. additional depth . . . . - 2,400
Net cost (excluding forms) $45,600
Cost of cooling concrete asumed equal to savings in cement costs. Note Example c is $13,600 less than Example a for the same design requirements.

Example 6.2—Culvert roof 36 in. thick supporting 20 ft of fill, spanning 20 ft between 4 ft thick walls, 20 ft high by 100 ft long resting on a rock base, placed in 80 F ambient air, minimum final air temperature 20 F, no cooling of concrete, mix same as Example 2.1, stress steel #9 at 10 stressed to 24,000 psi in bottom face.

Step 1.1—The volume-to-surface ratio

$$V/S = \left[\frac{3(20)}{2(20+3)}\right]\left(\frac{3+2}{3}\right) = 2.2 \pm \text{ft}$$

Step 1.2—Effective placing temperature = 90 – 0.6(10) = 84 F (Using Fig. 2.6).

Step 1.3—Final temperature is

$$20 + \frac{2(60-20)}{3}\sqrt{\frac{36}{96}} = 36\text{ F}$$

Step 1.4—From Fig. 2.5, the temperature rise for a wet surface condition = 34 F.

For the same concrete placed at 69 F, the temperature rise for Example 2.1 was 30 F.

Considering adjustments for cement type and proportions, the actual rise for Example 2.1 was 18 F.

Therefore, the actual rise = 34(18)/30 = 20 F.

Step 1.5—Design temperature = 84 + 20 – 36 = 68 F.

Step 2—Restraint for end supports [Eq. (4.5)]

$$\frac{A_B h^3}{4L_b I_c} = \frac{(1)(3)(20)^3}{4(20)(1)(4)^3/12} = 56 \pm$$

$$\therefore K_R = \frac{1}{1+56} = 0.0175$$

Step 3—f_t' = 405 psi C_T = 5 x 10^{-6} in./in./F

Step 4—Assume w = 0.013 in., d_c = 2 in.∴ A_{dc} = 125 for #9 bars at 10 in. o.c.

$$\therefore f_s = \frac{0.013 \times 10^3}{0.076\sqrt[3]{125}} = 34.3\text{ksi allowable}$$

Step 5—Steel requirements [Eq. (4.6)]

$$p = \frac{1.20}{12(33)} = 0.003,\ n = 9,\ j = 0.94,\ h = 20\text{ ft}$$

$$K_f = (3)^3/20 = 1.35,\ K_c = (4)^3/20 = 3.2$$

$$\Delta f_s = \frac{0.0175(5\times10^{-6})(68)(29\times10^6)}{2(0.0039)(0.94)}$$

$$\left[\frac{20}{3}\left(\frac{1.35}{1.35+3.2}\right) + 4(0.0039)(0.94)\right] = 7000\ \text{psi}$$

Note: This is less than allowable of 34,300 psi; therefore no additional steel is required for volume change in the stress direction.

6.2(a)

For the roof slab of Example 6.2 find the temperature steel parallel to the wall. Assume 3-1/2 in. cover to center of temperature steel or f_s = 26,000 psi for bar at 12 in. spacing.

Note: Since the temperature rise of the slab is only 20 F the wall does not offer enough restraint to crack the slab therefore design the slab as an extension of the wall with a design temperature drop of 68 F.

Step 2—Restraint at 5 ft from the wall for L/h = 100/25 = 4, K_f = 0.40.

Step 5—From Eq. (6.4) **Note**: $K_R C_T T_E$ of 0.4(5)(68) is less than f_t'/E of 150.∴ No cracking will occur and only minimum steel is required

$$A_b = 0.0015(12)(32.5)/2 = 0.29\ \text{in.}^2/\text{ft}$$

Reinforcement = #5 at 12 in. each face.

Example 6.3—A 6 ft thick power-plant base slab supporting widely spaced walls. Construction joints but no contraction or expansion joints. Assumed placed in 75 F average ambient air temperature with final unheated interior of 50 F. Slab is designed for operating uplift conditions requiring #11 bars at 12 in. o.c. stressed to 24,000 psi.

a. Assume same concrete mix and conditions as Example 2.2.

b. Assume same concrete mix and conditions as Example 2.1.

6.3(a)

Step 1—The maximum *V/S* for a slab shall be 75 percent of the slab thickness. See paragraph 2.6. Therefore, *V/S* = 0.75 (6 ft) = 4.5 ft maximum.

a. Effective placing temperature using Fig. 2.6 and temperature peak of 1.5 days from Fig. 2.4.

$$T_{PE} = 85\text{ F} - 0.03(10) = 82\text{ F} \pm$$

b. Temperature rise using Fig. 2.5 for wet surface conditions.

For *V/S* = 4.5 at T_{PK} = 82 F; temperature rise = 41 F.

From Example 2.2: Adjustments for cement type = 41/30 = 1.38; adjustments for cement content 470/376 = 1.25.

∴ Net temperature rise = 1.38(1.25)(41) = 71 F.

c. Final temperature using Eq. (2.3)

$$T_F = 50 + (2/3(60-50))\sqrt{54/96} = 55\text{ F}$$

d. Design temperature drop

$$82 + 71 - 55 = 98 \text{ F}$$

Step 2—Restraint (Fig. 4.1)—Without contraction or expansion joints the length is unspecified therefore assume L/H is greater than 20 or $K_R = 0.9$ maximum.

Step 3—Physical properties, $f_t' = 6\sqrt{4600} = 405$ psi.

Step 4—Limiting crack width = 0.013 in. For bars at 12 in. o.c. and cover of 2$^1/_2$ in. the allowable steel stress from Eq. (6.2) is 32,200 psi.

Step 5—Steel requirements

$$L' = \frac{(0.013)}{18[0.9(5)(98) - 150]10^{-6}} = 2.5 \text{ ft} \qquad \text{(Eq. 6.4)}$$

$$A_b = \frac{0.20(405)}{32{,}000}\left(1 - \frac{2.5}{12}\right)\frac{12(72)}{1} \qquad \text{(Eq. 6.7)}$$

$$= 1.73 \text{ in.}$$

Check:

Δf_s for flexure (Eq. 4.7)

$\Delta f_s = 2(0.9)\ (5 \times 10^{-6})\ (98)\ (29 \times 10^6) = 25{,}600$ psi

$\Sigma f_s = 24{,}000 + 25{,}600 = 49{,}600$

Since combined stress is greater than the allowable, additional steel is needed, however, maximum steel requirements will be less than 1.56 + 1.73 = 3.29 in.^{2}/ft or #11 at 6 in. o.c. Assume final bar spacing of 7 in. o.c. for an allowable steel stress of 38,500 psi.

$$A_s = 1.56\left(\frac{24}{38.5 - 25.6}\right) = 2.90 \text{ in.}^2\text{/ft \#11 @ } 6 \therefore \text{OK}$$

6.3(b)

For Example b the design temperature would be 34 F and Δf_s = 8900 psi so that combined stress equals 32,900 psi which exceeds allowable of 32,200 psi by less than 3 percent; therefore, no additional steel is needed for temperature.

CHAPTER 7—REFERENCES

7.1—Recommended references

The documents of the various standards-producing organizations referred to in this document are listed below with their serial designations.

American Concrete Institute

116R Cement and Concrete Terminology—SP-19(85)
207.1R Mass Concrete
207.4R Cooling and Insulating Systems for Mass Concrete
223-83 Standard Practice for the Use of Shrinkage-Compensating Concrete
224.1R Causes, Evaluation, and Repair of Cracks in Concrete Structures
305R Hot Weather Concreting
306R Cold Weather Concreting
318 Building Code Requirements for Reinforced Concrete
350R Environmental Engineering Concrete Structures

ASTM

C 496 Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens
C 186 Standard Test Method for Heat of Hydration of Hydraulic Cement

7.2—Cited references

1. Carlson, Roy W.; Houghton, Donald L.; and Polivka, Milos, "Causes and Control of Cracking in Unreinforced Mass Concrete," ACI JOURNAL, *Proceedings* V. 76, No. 7, July 1979, pp. 821-837.

2. Milestone, N. B., and Rogers, D. E., "Use of an Isothermal Calorimeter for Determining Heats of Hydration at Early Ages," *World Cement Technology* (London), V. 12, No. 8, Oct. 1981, pp. 374-380.

3. Verbeck, George J., and Foster, Cecil W., "Long-Time Study of Cement Performance in Concrete. Chapter 6—The Heats of Hydration of the Cements," *Proceedings*, ASTM, V. 50, 1950, pp. 1235-1262.

4. Carlson, Roy W., "Drying Shrinkage of Large Concrete Members," ACI JOURNAL, *Proceedings* V. 33, No. 3, Jan.-Feb. 1937, pp. 327-336.

5. Troxell, George Earl, and Davis, Harmer E., *Composition and Properties of Concrete*, MacGraw-Hill Book Co., New York, 1956, p. 236.

6. Raphael, Jerome M., "Tensile Strength of Concrete," ACI JOURNAL, *Proceedings* V. 81, No. 2, Mar.-Apr. 1984, pp. 158-165.

7. "Control of Cracking in Mass Concrete Structures," *Engineering Monograph* No. 34, U.S. Bureau of Reclamation, Denver, 1965.

8. Darwin, David; Manning, David G.; Hognestad, Eivind; Beeby, Andrew W.; Rice, Paul F.; and Ghowrwal, Abdul, "Debate: Crack Width, Cover, and Corrosion," *Concrete International: Design & Construction*, V. 7, No. 5, May 1985, pp. 20-35.

9. Turton, C. D., "Practical Means of Control of Early Thermal Cracking in Reinforced Concrete Walls," Paper presented at the ACI Fall Convention, New Orleans, 1977.

10. Gergely, Peter, and Lutz, LeRoy A., "Maximum Crack Width in Reinforced Concrete Flexural Members," *Causes, Mechanism, and Control of Cracking in Concrete*, SP-20, American Concrete Institute, Detroit, 1968, pp. 87-117.

7.3—Additional references

1. Hognestad, Eivind, "High Strength Bars As Concrete Reinforcement, Part 2. Control of Flexural Cracking," *Journal*, PCA Research and Development Laboratories, V. 4, No. 1, Jan. 1962, pp. 46-63. Also, *Development Department Bulletin* D53, Portland Cement Association.

2. Concrete Manual, 8th Edition, U.S. Bureau of Reclamation, Denver, 1981, p. 17.

3. Tuthill, Lewis H., and Adams, Robert F., "Cracking Controlled in Massive, Reinforced Structural Concrete by

Application of Mass Concrete Practices," ACI JOURNAL, *Proceedings* V. 69, No. 8, Aug. 1972, pp. 481-491.

4. Houghton, D. L., "Determining Tensile Strain Capacity of Mass Concrete," ACI JOURNAL, *Proceedings* V. 73, No. 12, Dec. 1976, pp. 691-700.

APPENDIX

Notation

A = effective tension area of concrete surrounding a group of reinforcing bars and having the same centroid as that reinforcement, divided by the number of bars
A_B = area of a member subject to volume change
Ab = area of reinforcing bar
A_F = area of foundation or other element restraining shortening of element
A_g = gross area of concrete cross section
A_s = area of steel for a given width
A_s' = area of steel per ft of length for a given width
B, b = width of cross section
C = weight of portland cement per yd^3 of concrete, lb
C_{eq} = weight of portland cement plus a percentage of the weight of pozzolan per yd^3 of concrete, lb
C_h = specific heat, Btu/lb · F
C_T = linear thermal coefficient, 5 x 10^{-6} per F for limestone aggregate, 6 x 10^{-6} per F for siliceous river gravel aggregate
d = depth of member from compressive face to the centroid of the reinforcement
d_c = thickness of concrete cover measured from the concrete surface at which cracks are being considered to the center of the nearest reinforcing bar
d_s = assumed depth of tensile stress block for internal restraint considerations
e = eccentricity of a load with respect to the centroid of the section
E_c = modulus of elasticity of concrete
E_F = modulus of elasticity of foundation or restraining element
E_s = modulus of elasticity of steel
Fa = weight of fly ash per yd^2 of concrete, lb
f_c' = specified compressive strength of concrete, psi
f_s = calculated stress in reinforcement, psi
f_t = tensile stress, psi
f_t' = tensile strength of concrete, psi
f_y = design yield stress of steel
H = perpendicular distance from restrained edge to free edge. Where a slab is subject to edge restraint on two opposite edges, H is one-half the distance between edges. For slab on grade, H is the slab thickness in feet
H_a = adiabatic temperature rise of the concrete
h = height of vertical restraining element, column or wall, above fixed base or elemental height of a wall
h^2 = diffusivity in ft^2 per hour
h_c = elemental height of crack above base
h_g = 28 day heat generation of the cement by heat of hydration, cal/gm
I_c = moment of inertia of gross concrete section subjected to flexure by the restraining forces
j = ratio of distance between centroid of compression and centroid of tension to the depth d of a flexural member. $j = 1 - k/3$
K = conductivity, Btu/ft/hr/F
K_c = stiffness of vertical restraining element subjected to flexure by the restraining forces
K_f = stiffness of floor system being tensioned by restraint
K_R = degree of restraint. Ratio of actual stress resulting from volume change to the stress which would result if completely restrained. In most calculations, it is convenient to use the ratio of the difference in free length change and actual length change to the free length change
k = ratio of depth of compressive area to the depth d of flexural member using the straight line theory of stress distribution
L = distance between contraction or expansion joints in the direction of restraint or overall length of a member undergoing volume change
L' = calculated average distance between cracks
N = number of cracks
N_B = number of reinforcing bars in the free (unrestrained) face of a slab or wall
N_H = number of reinforcing bars spaced along the H face or faces perpendicular to the plane of restraint
n = ratio of modulus of elasticity of steel to that of concrete
p = area of steel divided by the appropriate area of concrete
M_{RH} = restraining moment to be supplied by the stress reinforcing steel for full height cracking
M_{Rh} = same as preceding for partial height
S = surface area of a concrete member exposed to air
T = tensile force, lb
T_A = average minimum ambient air temperature over a prolonged exposure period of 1 week
T_c = temperature generated by the total quantity of cementitious materials if all were portland cement
T_{C+F} = temperature generated by the mixture of portland cement and pozzolan
T_E = effective temperature change in members including an equivalent temperature change to compensate for drying shrinkage
T_{DS} = equivalent temperature drop to be used in lieu of drying shrinkage
T_M = temperature of earth or rock mass
T_{min} = minimum temperature of concrete against earth or rock mass, F
T_p = placing temperature of the fresh concrete
T_{PK} = effective placing temperature after accounting for heat gained from or lost to the air, F

T_1 = high temperature in a temperature gradient

T_2 = low temperature in a temperature gradient

V = volume of a concrete member

$\overline{W}_u$ = the water content of the fresh concrete lb per yd^3

$\overline{W}_c$ = weight of cement per cu yd of concrete, lb

w = maximum surface crack width, in.

w_c = weight of concrete, lb/ft^3 Section 4.3.2

x = distance between resultant tension force and the compression face, in.

z = quantity limiting distribution of flexural reinforcement, psi, see ACI 318

β = ratio of the distance from the neutral axis to the tension face of a flexural member to the distance from the neutral axis to the tension steel. Where flexure is not involved, R = 1

β = ratio of distance from neutral axis to the tensile face to the distance from neutral axis to steel

Δ_c = contraction of the concrete, in./in.

Metric conversions

1 in.	=	25.4 mm
1 ft	=	0.3048 m
1 $in.^2$	=	645.1 mm^2
1 ft^2	=	0.0929 m^2
1 in^3	=	16.39 x $10^3 mm^3$
1 ft^3	=	0.0283 m^3
1 yd^3	=	0.7646 m^3
1 lb	=	0.4536 kg
1 $lb/in.^2$ (psi)	=	6895 Pa
1 $kip/in.^2$ (ksi)	=	6.895 MPa
1 lb/ft^2	=	47.88 Pa
1 lb/ft^3 (pcf)	=	16.02 kg/m^3
1 lb/yd^3	=	0.5933 $kg/\cdot m^3$
1 Btu/lb·F	=	4.87 J/(kg·K)
1 Btu/lb·hr·F	=	1.731 W/m·K
1 in./in./F	=	1.8 mm/mm/C
Temperature		
t_C	=	$(t_F - 32)/1.8$
Difference in temperature		
t_C	=	$t_F/1.8$

This report was submitted to letter ballot of the committee and approved in accordance with ACI balloting procedures.