Chapter 33

Alternating Current Circuits

▲ *These large transformers are used to increase the voltage at a power plant for distribution of energy by electrical transmission to the power grid. Voltages can be changed relatively easily because power is distributed by alternating current rather than direct current. (Lester Lefkowitz/Getty Images)*

CHAPTER OUTLINE

In this chapter we describe alternating current (AC) circuits. Every time we turn on a television set, a stereo, or any of a multitude of other electrical appliances in a home, we are calling on alternating currents to provide the power to operate them. We begin our study by investigating the characteristics of simple series circuits that contain resistors, inductors, and capacitors and that are driven by a sinusoidal voltage. We shall find that the maximum alternating current in each element is proportional to the maximum alternating voltage across the element. In addition, when the applied voltage is sinusoidal, the current in each element is also sinusoidal, but not necessarily in phase with the applied voltage. The primary aim of this chapter can be summarized as follows: if an AC source applies an alternating voltage to a series circuit containing resistors, inductors, and capacitors, we want to know the amplitude and time characteristics of the alternating current. We conclude the chapter with two sections concerning transformers, power transmission, and electrical filters.

33.1 AC Sources

An AC circuit consists of circuit elements and a power source that provides an alternating voltage Δv. This time-varying voltage is described by

$$\Delta v = \Delta V_{\max} \sin \omega t$$

where $\Delta V_{\max}$ is the maximum output voltage of the AC source, or the **voltage amplitude.** There are various possibilities for AC sources, including generators, as discussed in Section 31.5, and electrical oscillators. In a home, each electrical outlet serves as an AC source.

From Equation 15.12, the angular frequency of the AC voltage is

$$\omega = 2\pi f = \frac{2\pi}{T}$$

where f is the frequency of the source and T is the period. The source determines the frequency of the current in any circuit connected to it. Because the output voltage of an AC source varies sinusoidally with time, the voltage is positive during one half of the cycle and negative during the other half, as in Figure 33.1. Likewise, the current in any circuit driven by an AC source is an alternating current that also varies sinusoidally with time. Commercial electric-power plants in the United States use a frequency of 60 Hz, which corresponds to an angular frequency of 377 rad/s.

PITFALL PREVENTION

33.1 Time-Varying Values

We will use lowercase symbols Δv and i to indicate the instantaneous values of time-varying voltages and currents. Capital letters represent fixed values of voltage and current, such as $\Delta V_{\max}$ and $I_{\max}$.

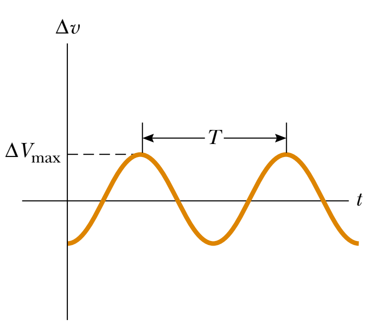
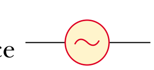
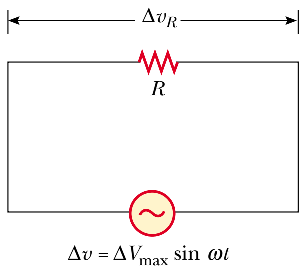


Figure 33.1 The voltage supplied by an AC source is sinusoidal with a period T.

33.2 Resistors in an AC Circuit

Consider a simple AC circuit consisting of a resistor and an AC source , as shown in Figure 33.2. At any instant, the algebraic sum of the voltages around a closed loop in a circuit must be zero (Kirchhoff's loop rule). Therefore, $\Delta v + \Delta v_R = 0$, so



Active Figure 33.2 A circuit consisting of a resistor of resistance R connected to an AC source, designated by the symbol ~ .

At the Active Figures link at http://www.pse6.com, you can adjust the resistance, the frequency, and the maximum voltage. The results can be studied with the graph and phasor diagram in Figure 33.3.

that the magnitude of the source voltage equals the magnitude of the voltage across the resistor:

$$\Delta v = \Delta v_R = \Delta V_{max} \sin \omega t \tag{33.1}$$

where Δv_R is the **instantaneous voltage across the resistor.** Therefore, from Equation 27.8, $R = \Delta V/I$, the instantaneous current in the resistor is

$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t \tag{33.2}$$

where I_{max} is the maximum current:

Maximum current in a resistor

$$I_{max} = \frac{\Delta V_{max}}{R}$$

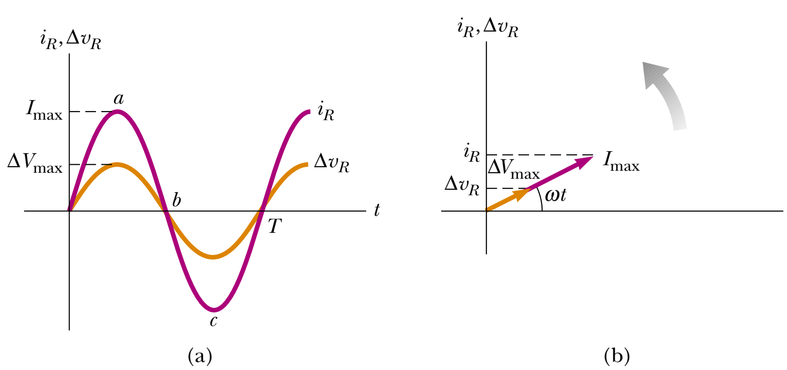
From Equations 33.1 and 33.2, we see that the instantaneous voltage across the resistor is

Voltage across a resistor

$$\Delta v_R = I_{max} R \sin \omega t \tag{33.3}$$

A plot of voltage and current versus time for this circuit is shown in Figure 33.3a. At point a, the current has a maximum value in one direction, arbitrarily called the positive direction. Between points a and b, the current is decreasing in magnitude but is still in the positive direction. At b, the current is momentarily zero; it then begins to increase in the negative direction between points b and c. At c, the current has reached its maximum value in the negative direction.

The current and voltage are in step with each other because they vary identically with time. Because i_R and Δv_R both vary as $\sin \omega t$ and reach their maximum values at the same time, as shown in Figure 33.3a, they are said to be **in phase,** similar to the way that two waves can be in phase, as discussed in our study of wave motion in



Active Figure 33.3 (a) Plots of the instantaneous current i_R and instantaneous voltage Δv_R across a resistor as functions of time. The current is in phase with the voltage, which means that the current is zero when the voltage is zero, maximum when the voltage is maximum, and minimum when the voltage is minimum. At time $t = T$, one cycle of the time-varying voltage and current has been completed. (b) Phasor diagram for the resistive circuit showing that the current is in phase with the voltage.

At the Active Figures link at http://www.pse6.com, you can adjust the resistance, the frequency, and the maximum voltage of the circuit in Figure 33.2. The results can be studied with the graph and phasor diagram in this figure.

Chapter 18. Thus, **for a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor.** For resistors in AC circuits, there are no new concepts to learn. Resistors behave essentially the same way in both DC and AC circuits. This will not be the case for capacitors and inductors.

To simplify our analysis of circuits containing two or more elements, we use graphical constructions called *phasor diagrams.* A **phasor** is a vector whose length is proportional to the maximum value of the variable it represents ($\Delta V_{\max}$ for voltage and $I_{\max}$ for current in the present discussion) and which rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable. The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents.

PITFALL PREVENTION

33.2 We've Seen This Idea Before

To help with this discussion of phasors, review Section 15.4, in which we represented the simple harmonic motion of a real object to the projection of uniform circular motion of an imaginary object onto a coordinate axis. Phasors are a direct analog to this discussion.

Figure 33.3b shows voltage and current phasors for the circuit of Figure 33.2 at some instant of time. The projections of the phasor arrows onto the vertical axis are determined by a sine function of the angle of the phasor with respect to the horizontal axis. For example, the projection of the current phasor in Figure 33.3b is $I_{\max} \sin \omega t$. Notice that this is the same expression as Equation 33.2. Thus, we can use the projections of phasors to represent current values that vary sinusoidally in time. We can do the same with time-varying voltages. The advantage of this approach is that the phase relationships among currents and voltages can be represented as vector additions of phasors, using our vector addition techniques from Chapter 3.

In the case of the single-loop resistive circuit of Figure 33.2, the current and voltage phasors lie along the same line, as in Figure 33.3b, because i_R and Δv_R are in phase. The current and voltage in circuits containing capacitors and inductors have different phase relationships.

Quick Quiz 33.1 Consider the voltage phasor in Figure 33.4, shown at three instants of time. Choose the part of the figure that represents the instant of time at which the instantaneous value of the voltage has the largest magnitude.

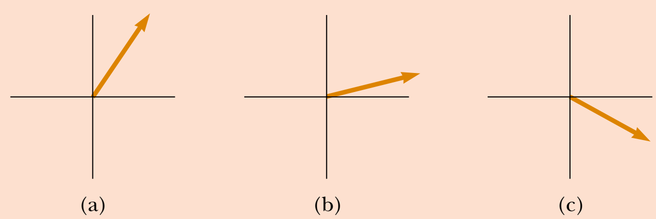


Figure 33.4 (Quick Quizzes 33.1 and 33.2) A voltage phasor is shown at three instants of time.

Quick Quiz 33.2 For the voltage phasor in Figure 33.4, choose the part of the figure that represents the instant of time at which the instantaneous value of the voltage has the smallest magnitude.

For the simple resistive circuit in Figure 33.2, note that **the average value of the current over one cycle is zero.** That is, the current is maintained in the positive direction for the same amount of time and at the same magnitude as it is maintained in the negative direction. However, the direction of the current has no effect on the behavior of the resistor. We can understand this by realizing that collisions between electrons and the fixed atoms of the resistor result in an increase in the resistor's temperature. Although this temperature increase depends on the magnitude of the current, it is independent of the direction of the current.

We can make this discussion quantitative by recalling that the rate at which energy is delivered to a resistor is the power $\mathcal{P} = i^2R$, where i is the instantaneous current in

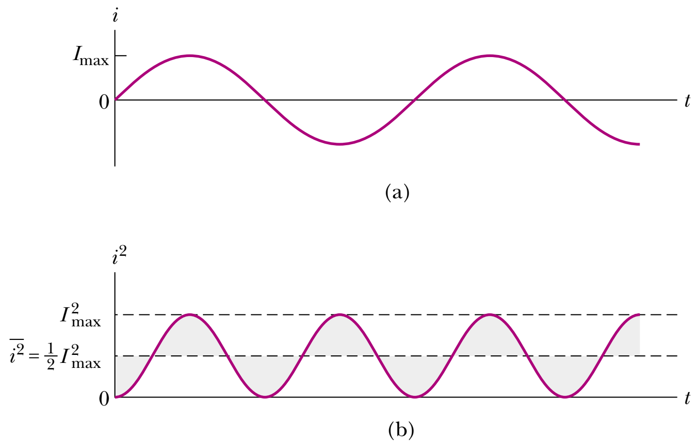


Figure 33.5 (a) Graph of the current in a resistor as a function of time. (b) Graph of the current squared in a resistor as a function of time. Notice that the gray shaded regions *under* the curve and *above* the dashed line for $I_{max}^2/2$ have the same area as the gray shaded regions *above* the curve and *below* the dashed line for $I_{max}^2/2$. Thus, the average value of i^2 is $I_{max}^2/2$.

the resistor. Because this rate is proportional to the square of the current, it makes no difference whether the current is direct or alternating—that is, whether the sign associated with the current is positive or negative. However, the temperature increase produced by an alternating current having a maximum value I_{max} is not the same as that produced by a direct current equal to I_{max}. This is because the alternating current is at this maximum value for only an instant during each cycle (Fig. 33.5a). What is of importance in an AC circuit is an average value of current, referred to as the **rms current.** As we learned in Section 21.1, the notation *rms* stands for *root-mean-square*, which in this case means the square root of the mean (average) value of the square of the current: $I_{rms} = \sqrt{\overline{i^2}}$. Because i^2 varies as $\sin^2 \omega t$ and because the average value of i^2 is $\frac{1}{2} I_{max}^2$ (see Fig. 33.5b), the rms current is[1]

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max} \quad (33.4)$$

rms current

This equation states that an alternating current whose maximum value is 2.00 A delivers to a resistor the same power as a direct current that has a value of (0.707)(2.00 A) = 1.41 A. Thus, the average power delivered to a resistor that carries an alternating current is

Average power delivered to a resistor

$$\mathcal{P}_{av} = I_{rms}^2 R$$

[1] That the square root of the average value of i^2 is equal to $I_{max}/\sqrt{2}$ can be shown as follows. The current in the circuit varies with time according to the expression $i = I_{max} \sin \omega t$, so $i^2 = I_{max}^2 \sin^2 \omega t$. Therefore, we can find the average value of i^2 by calculating the average value of $\sin^2 \omega t$. A graph of $\cos^2 \omega t$ versus time is identical to a graph of $\sin^2 \omega t$ versus time, except that the points are shifted on the time axis. Thus, the time average of $\sin^2 \omega t$ is equal to the time average of $\cos^2 \omega t$ when taken over one or more complete cycles. That is,

$$(\sin^2 \omega t)_{av} = (\cos^2 \omega t)_{av}$$

Using this fact and the trigonometric identity $\sin^2 \theta + \cos^2 \theta = 1$, we obtain

$$(\sin^2 \omega t)_{av} + (\cos^2 \omega t)_{av} = 2(\sin^2 \omega t)_{av} = 1$$

$$(\sin^2 \omega t)_{av} = \tfrac{1}{2}$$

When we substitute this result in the expression $i^2 = I_{max}^2 \sin^2 \omega t$, we obtain $(i^2)_{av} = \overline{i^2} = I_{rms}^2 = I_{max}^2/2$, or $I_{rms} = I_{max}/\sqrt{2}$. The factor $1/\sqrt{2}$ is valid only for sinusoidally varying currents. Other waveforms, such as sawtooth variations, have different factors.

Alternating voltage is also best discussed in terms of rms voltage, and the relationship is identical to that for current:

rms voltage

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}} = 0.707\,\Delta V_{\text{max}} \tag{33.5}$$

When we speak of measuring a 120-V alternating voltage from an electrical outlet, we are referring to an rms voltage of 120 V. A quick calculation using Equation 33.5 shows that such an alternating voltage has a maximum value of about 170 V. One reason we use rms values when discussing alternating currents and voltages in this chapter is that AC ammeters and voltmeters are designed to read rms values. Furthermore, with rms values, many of the equations we use have the same form as their direct current counterparts.

Quick Quiz 33.3 Which of the following statements might be true for a resistor connected to a sinusoidal AC source? (a) $\mathcal{P}_{\text{av}} = 0$ and $i_{\text{av}} = 0$ (b) $\mathcal{P}_{\text{av}} = 0$ and $i_{\text{av}} > 0$ (c) $\mathcal{P}_{\text{av}} > 0$ and $i_{\text{av}} = 0$ (d) $\mathcal{P}_{\text{av}} > 0$ and $i_{\text{av}} > 0$.

Example 33.1 What Is the rms Current?

The voltage output of an AC source is given by the expression $\Delta v = (200\text{ V})\sin\omega t$. Find the rms current in the circuit when this source is connected to a 100-Ω resistor.

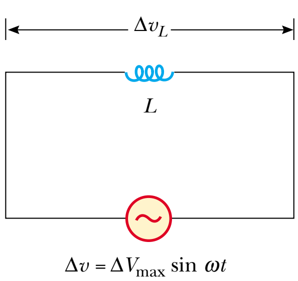
Solution Comparing this expression for voltage output with the general form $\Delta v = \Delta V_{\text{max}}\sin\omega t$, we see that $\Delta V_{\text{max}} = 200$ V. Thus, the rms voltage is

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}} = \frac{200\text{ V}}{\sqrt{2}} = 141\text{ V}$$

Therefore,

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{R} = \frac{141\text{ V}}{100\ \Omega} = \boxed{1.41\text{ A}}$$

33.3 Inductors in an AC Circuit



Active Figure 33.6 A circuit consisting of an inductor of inductance L connected to an AC source.

At the Active Figures link at http://www.pse6.com, you can adjust the inductance, the frequency, and the maximum voltage. The results can be studied with the graph and phasor diagram in Figure 33.7.

Now consider an AC circuit consisting only of an inductor connected to the terminals of an AC source, as shown in Figure 33.6. If $\Delta v_L = \boldsymbol{\varepsilon}_L = -L(di/dt)$ is the self-induced instantaneous voltage across the inductor (see Eq. 32.1), then Kirchhoff's loop rule applied to this circuit gives $\Delta v + \Delta v_L = 0$, or

$$\Delta v - L\frac{di}{dt} = 0$$

When we substitute $\Delta V_{\text{max}}\sin\omega t$ for Δv and rearrange, we obtain

$$\Delta v = L\frac{di}{dt} = \Delta V_{\text{max}}\sin\omega t \tag{33.6}$$

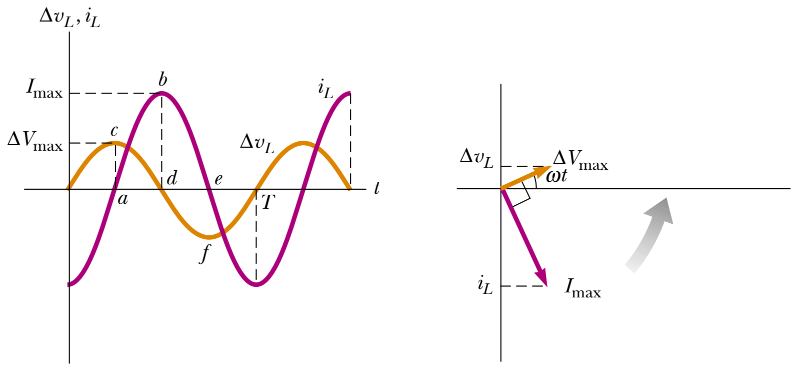
Solving this equation for di, we find that

$$di = \frac{\Delta V_{\text{max}}}{L}\sin\omega t\,dt$$

Integrating this expression[2] gives the instantaneous current i_L in the inductor as a function of time:

$$i_L = \frac{\Delta V_{\text{max}}}{L}\int\sin\omega t\,dt = -\frac{\Delta V_{\text{max}}}{\omega L}\cos\omega t \tag{33.7}$$

[2] We neglect the constant of integration here because it depends on the initial conditions, which are not important for this situation.



(a) (b)

Active Figure 33.7 (a) Plots of the instantaneous current i_L and instantaneous voltage Δv_L across an inductor as functions of time. The current lags behind the voltage by 90°. (b) Phasor diagram for the inductive circuit, showing that the current lags behind the voltage by 90°.

At the Active Figures link at http://www.pse6.com, you can adjust the inductance, the frequency, and the maximum voltage of the circuit in Figure 33.6. The results can be studied with the graph and phasor diagram in this figure.

When we use the trigonometric identity $\cos \omega t = -\sin(\omega t - \pi/2)$, we can express Equation 33.7 as

$$i_L = \frac{\Delta V_{max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad (33.8)$$

Current in an inductor

Comparing this result with Equation 33.6, we see that the instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are *out* of phase by $(\pi/2)$ rad $= 90°$.

A plot of voltage and current versus time is provided in Figure 33.7a. In general, inductors in an AC circuit produce a current that is out of phase with the AC voltage. For example, when the current i_L in the inductor is a maximum (point b in Figure 33.7a), it is momentarily not changing, so the voltage across the inductor is zero (point d). At points like a and e, the current is zero and the rate of change of current is at a maximum. Thus, the voltage across the inductor is also at a maximum (points c and f). Note that the voltage reaches its maximum value one quarter of a period before the current reaches its maximum value. Thus, we see that

for a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90° (one-quarter cycle in time).

As with the relationship between current and voltage for a resistor, we can represent this relationship for an inductor with a phasor diagram as in Figure 33.7b. Notice that the phasors are at 90° to one another, representing the 90° phase difference between current and voltage.

From Equation 33.7 we see that the current in an inductive circuit reaches its maximum value when $\cos \omega t = -1$:

$$I_{max} = \frac{\Delta V_{max}}{\omega L} \qquad (33.9)$$

Maximum current in an inductor

This looks similar to the relationship between current, voltage, and resistance in a DC circuit, $I = \Delta V/R$ (Eq. 27.8). In fact, because I_{max} has units of amperes and ΔV_{max} has units of volts, ωL must have units of ohms. Therefore, ωL has the same units as resistance and is related to current and voltage in the same way as resistance. It must behave in a manner similar to resistance, in the sense that it represents opposition to the flow of charge. Notice that because ωL depends on the applied frequency ω, the inductor *reacts* differently, in terms of offering resistance to current, for different frequencies. For this reason, we define ωL as the **inductive reactance:**

Inductive reactance

$$X_L \equiv \omega L \tag{33.10}$$

and we can write Equation 33.9 as

$$I_{\text{max}} = \frac{\Delta V_{\text{max}}}{X_L} \tag{33.11}$$

The expression for the rms current in an inductor is similar to Equation 33.9, with I_{max} replaced by I_{rms} and ΔV_{max} replaced by ΔV_{rms}.

Equation 33.10 indicates that, for a given applied voltage, the inductive reactance increases as the frequency increases. This is consistent with Faraday's law—the greater the rate of change of current in the inductor, the larger is the back emf. The larger back emf translates to an increase in the reactance and a decrease in the current.

Using Equations 33.6 and 33.11, we find that the instantaneous voltage across the inductor is

Voltage across an inductor

$$\Delta v_L = -L\frac{di}{dt} = -\Delta V_{\text{max}} \sin \omega t = -I_{\text{max}} X_L \sin \omega t \tag{33.12}$$

Quick Quiz 33.4 Consider the AC circuit in Figure 33.8. The frequency of the AC source is adjusted while its voltage amplitude is held constant. The lightbulb will glow the brightest at (a) high frequencies (b) low frequencies (c) The brightness will be the same at all frequencies.

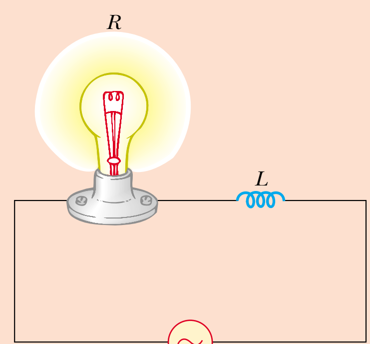


Figure 33.8 (Quick Quiz 33.4) At what frequencies will the bulb glow the brightest?

Example 33.2 A Purely Inductive AC Circuit

In a purely inductive AC circuit (see Fig. 33.6), $L = 25.0$ mH and the rms voltage is 150 V. Calculate the inductive reactance and rms current in the circuit if the frequency is 60.0 Hz.

Solution Equation 33.10 gives

$$X_L = \omega L = 2\pi f L = 2\pi(60.0 \text{ Hz})(25.0 \times 10^{-3} \text{ H})$$

$$= \boxed{9.42\ \Omega}$$

From an rms version of Equation 33.11, the rms current is

$$I_{\text{rms}} = \frac{\Delta V_{L,\text{rms}}}{X_L} = \frac{150 \text{ V}}{9.42\ \Omega} = \boxed{15.9 \text{ A}}$$

What If? **What if the frequency increases to 6.00 kHz? What happens to the rms current in the circuit?**

Answer If the frequency increases, the inductive reactance increases because the current is changing at a higher rate. The increase in inductive reactance results in a lower current.

Let us calculate the new inductive reactance:

$$X_L = 2\pi(6.00 \times 10^3 \text{ Hz})(25.0 \times 10^{-3} \text{ H}) = 942\ \Omega$$

The new current is

$$I_{\text{rms}} = \frac{150 \text{ V}}{942\ \Omega} = 0.159 \text{ A}$$

33.4 Capacitors in an AC Circuit

Figure 33.9 shows an AC circuit consisting of a capacitor connected across the terminals of an AC source. Kirchhoff's loop rule applied to this circuit gives $\Delta v + \Delta v_C = 0$, so that the magnitude of the source voltage is equal to the magnitude of the voltage across the capacitor:

$$\Delta v = \Delta v_C = \Delta V_{\max} \sin \omega t \tag{33.13}$$

where Δv_C is the instantaneous voltage across the capacitor. We know from the definition of capacitance that $C = q/\Delta v_C$; hence, Equation 33.13 gives

$$q = C\,\Delta V_{\max} \sin \omega t \tag{33.14}$$

where q is the instantaneous charge on the capacitor. Because $i = dq/dt$, differentiating Equation 33.14 with respect to time gives the instantaneous current in the circuit:

$$i_C = \frac{dq}{dt} = \omega C\,\Delta V_{\max} \cos \omega t \tag{33.15}$$

Using the trigonometric identity

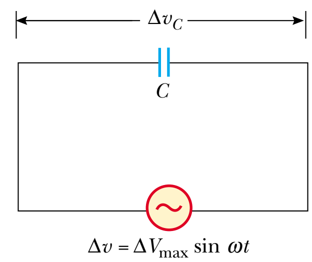
$$\cos \omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

we can express Equation 33.15 in the alternative form

Current in a capacitor

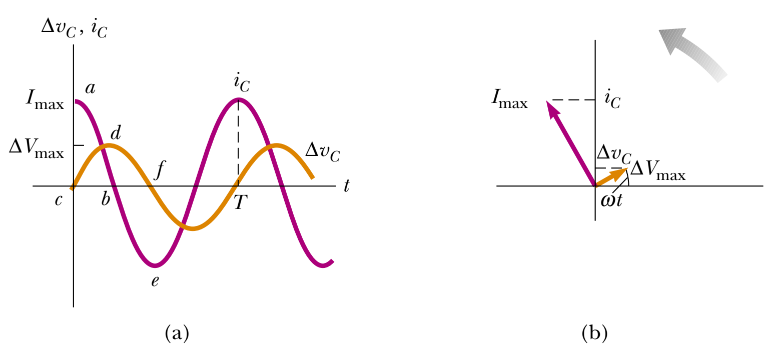
$$i_C = \omega C\,\Delta V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right) \tag{33.16}$$

Comparing this expression with Equation 33.13, we see that the current is $\pi/2$ rad $= 90°$ out of phase with the voltage across the capacitor. A plot of current and voltage versus time (Fig. 33.10a) shows that the current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value.



Active Figure 33.9 A circuit consisting of a capacitor of capacitance C connected to an AC source.

At the Active Figures link at http://www.pse6.com, you can adjust the capacitance, the frequency, and the maximum voltage. The results can be studied with the graph and phasor diagram in Figure 33.10.



Active Figure 33.10 (a) Plots of the instantaneous current i_C and instantaneous voltage Δv_C across a capacitor as functions of time. The voltage lags behind the current by 90°. (b) Phasor diagram for the capacitive circuit, showing that the current leads the voltage by 90°.

At the Active Figures link at http://www.pse6.com, you can adjust the capacitance, the frequency, and the maximum voltage of the circuit in Figure 33.9. The results can be studied with the graph and phasor diagram in this figure.

Looking more closely, consider a point such as b where the current is zero. This occurs when the capacitor has just reached its maximum charge, so the voltage across the capacitor is a maximum (point d). At points such as a and e, the current is a maximum, which occurs at those instants at which the charge on the capacitor has just gone to zero and it begins to charge up with the opposite polarity. Because the charge is zero, the voltage across the capacitor is zero (points c and f). Thus, the current and voltage are out of phase.

As with inductors, we can represent the current and voltage for a capacitor on a phasor diagram. The phasor diagram in Figure 33.10b shows that

> for a sinusoidally applied voltage, the current always leads the voltage across a capacitor by 90°.

From Equation 33.15, we see that the current in the circuit reaches its maximum value when $\cos \omega t = 1$:

$$I_{\max} = \omega C \Delta V_{\max} = \frac{\Delta V_{\max}}{(1/\omega C)} \tag{33.17}$$

As in the case with inductors, this looks like Equation 27.8, so that the denominator must play the role of resistance, with units of ohms. We give the combination $1/\omega C$ the symbol X_C, and because this function varies with frequency, we define it as the **capacitive reactance:**

Capacitive reactance

$$X_C \equiv \frac{1}{\omega C} \tag{33.18}$$

and we can write Equation 33.17 as

Maximum current in a capacitor

$$I_{\max} = \frac{\Delta V_{\max}}{X_C} \tag{33.19}$$

The rms current is given by an expression similar to Equation 33.19, with $I_{\max}$ replaced by I_{rms} and $\Delta V_{\max}$ replaced by ΔV_{rms}.

Combining Equations 33.13 and 33.19, we can express the instantaneous voltage across the capacitor as

Voltage across a capacitor

$$\Delta v_C = \Delta V_{\max} \sin \omega t = I_{\max} X_C \sin \omega t \tag{33.20}$$

Equations 33.18 and 33.19 indicate that as the frequency of the voltage source increases, the capacitive reactance decreases and therefore the maximum current increases. Again, note that the frequency of the current is determined by the frequency of the voltage source driving the circuit. As the frequency approaches zero, the capacitive reactance approaches infinity, and hence the current approaches zero. This makes sense because the circuit approaches direct current conditions as ω approaches zero, and the capacitor represents an open circuit.

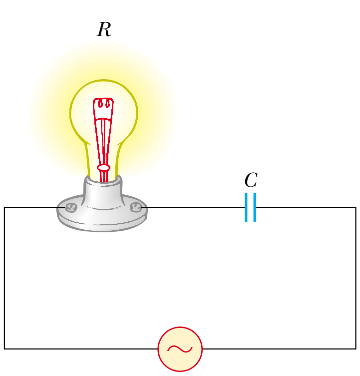


Figure 33.11 (Quick Quiz 33.5)

Quick Quiz 33.5 Consider the AC circuit in Figure 33.11. The frequency of the AC source is adjusted while its voltage amplitude is held constant. The lightbulb will glow the brightest at (a) high frequencies (b) low frequencies (c) The brightness will be same at all frequencies.

Quick Quiz 33.6 Consider the AC circuit in Figure 33.12. The frequency of the AC source is adjusted while its voltage amplitude is held constant. The lightbulb will glow the brightest at (a) high frequencies (b) low frequencies (c) The brightness will be same at all frequencies.

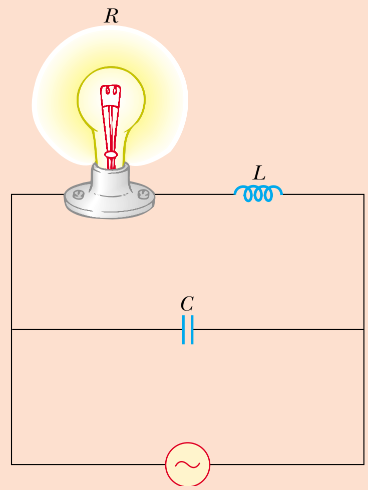


Figure 33.12 (Quick Quiz 33.6)

Example 33.3 A Purely Capacitive AC Circuit

An 8.00-μF capacitor is connected to the terminals of a 60.0-Hz AC source whose rms voltage is 150 V. Find the capacitive reactance and the rms current in the circuit.

Solution Using Equation 33.18 and the fact that $\omega = 2\pi f = 377\ \text{s}^{-1}$ gives

$$X_C = \frac{1}{\omega C} = \frac{1}{(377\ \text{s}^{-1})(8.00 \times 10^{-6}\ \text{F})} = \boxed{332\ \Omega}$$

Hence, from a modified Equation 33.19, the rms current is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_C} = \frac{150\ \text{V}}{332\ \Omega} = \boxed{0.452\ \text{A}}$$

What If? **What if the frequency is doubled? What happens to the rms current in the circuit?**

Answer If the frequency increases, the capacitive reactance decreases—just the opposite as in the case of an inductor. The decrease in capacitive reactance results in an increase in the current.

Let us calculate the new capacitive reactance:

$$X_C = \frac{1}{\omega C} = \frac{1}{2(377\ \text{s}^{-1})(8.00 \times 10^{-6}\ \text{F})} = 166\ \Omega$$

The new current is

$$I_{\text{rms}} = \frac{150\ \text{V}}{166\ \Omega} = 0.904\ \text{A}$$

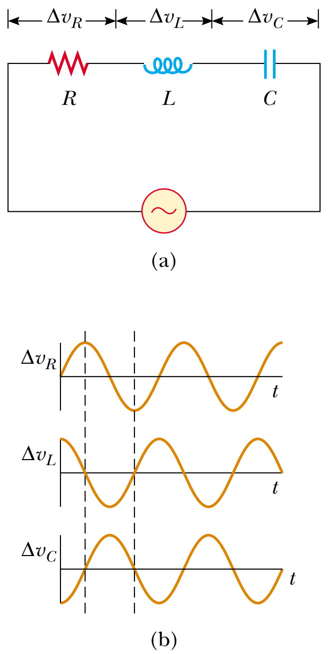
33.5 The *RLC* Series Circuit

Figure 33.13a shows a circuit that contains a resistor, an inductor, and a capacitor connected in series across an alternating voltage source. As before, we assume that the applied voltage varies sinusoidally with time. It is convenient to assume that the instantaneous applied voltage is given by

$$\Delta v = \Delta V_{\max} \sin \omega t$$

while the current varies as

$$i = I_{\max} \sin(\omega t - \phi)$$



Active Figure 33.13 (a) A series circuit consisting of a resistor, an inductor, and a capacitor connected to an AC source. (b) Phase relationships for instantaneous voltages in the series *RLC* circuit.

At the Active Figures link at http://www.pse6.com, you can adjust the resistance, the inductance, and the capacitance. The results can be studied with the graph in this figure and the phasor diagram in Figure 33.15.

where ϕ is some **phase angle** between the current and the applied voltage. Based on our discussions of phase in Sections 33.3 and 33.4, we expect that the current will generally not be in phase with the voltage in an *RLC* circuit. Our aim is to determine ϕ and I_{max}. Figure 33.13b shows the voltage versus time across each element in the circuit and their phase relationships.

First, we note that because the elements are in series, the current everywhere in the circuit must be the same at any instant. That is, **the current at all points in a series AC circuit has the same amplitude and phase.** Based on the preceding sections, we know that the voltage across each element has a different amplitude and phase. In particular, the voltage across the resistor is in phase with the current, the voltage across the inductor leads the current by 90°, and the voltage across the capacitor lags behind the current by 90°. Using these phase relationships, we can express the instantaneous voltages across the three circuit elements as

$$\Delta v_R = I_{max} R \sin \omega t = \Delta V_R \sin \omega t \tag{33.21}$$

$$\Delta v_L = I_{max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t \tag{33.22}$$

$$\Delta v_C = I_{max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t \tag{33.23}$$

where ΔV_R, ΔV_L, and ΔV_C are the maximum voltage values across the elements:

$$\Delta V_R = I_{max} R \qquad \Delta V_L = I_{max} X_L \qquad \Delta V_C = I_{max} X_C$$

At this point, we could proceed by noting that the instantaneous voltage Δv across the three elements equals the sum

$$\Delta v = \Delta v_R + \Delta v_L + \Delta v_C$$

Although this analytical approach is correct, it is simpler to obtain the sum by examining the phasor diagram, shown in Figure 33.14. Because the current at any instant is the same in all elements, we combine the three phasor pairs shown in Figure 33.14 to obtain Figure 33.15a, in which a single phasor I_{max} is used to represent the current in each element. Because phasors are rotating vectors, we can combine the three parts of Figure 33.14 by using vector addition. To obtain the vector sum of the three voltage phasors in Figure 33.15a, we redraw the phasor diagram as in Figure 33.15b. From this diagram, we see that the vector sum of the voltage amplitudes ΔV_R, ΔV_L, and ΔV_C equals a phasor whose length is the maximum applied voltage ΔV_{max}, and which makes an angle ϕ with the current phasor I_{max}. The voltage phasors ΔV_L and ΔV_C are in opposite directions along the same line, so we can construct the difference phasor $\Delta V_L - \Delta V_C$, which is perpendicular to the phasor ΔV_R. From either one of the right triangles

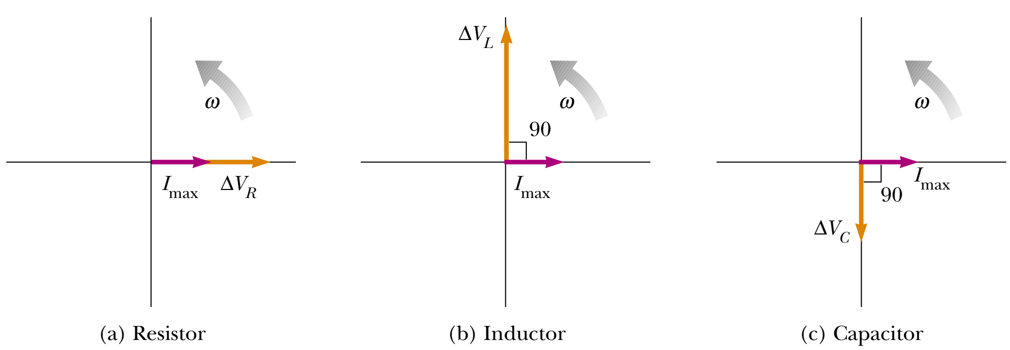
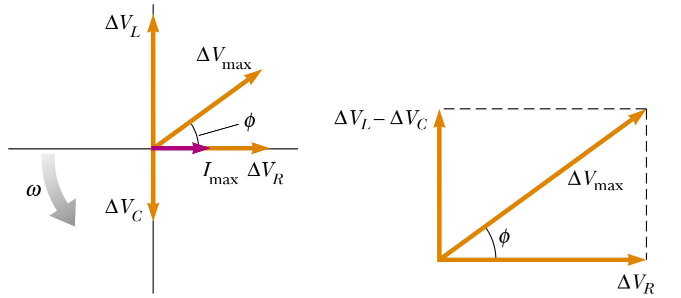


Figure 33.14 Phase relationships between the voltage and current phasors for (a) a resistor, (b) an inductor, and (c) a capacitor connected in series.



(a) (b)

Active Figure 33.15 (a) Phasor diagram for the series *RLC* circuit shown in Figure 33.13a. The phasor ΔV_R is in phase with the current phasor I_{max}, the phasor ΔV_L leads I_{max} by 90°, and the phasor ΔV_C lags I_{max} by 90°. The total voltage ΔV_{max} makes an angle ϕ with I_{max}. (b) Simplified version of the phasor diagram shown in part (a).

At the Active Figures link at* http://www.pse6.com, *you can adjust the resistance, the inductance, and the capacitance of the circuit in Figure 33.13a. The results can be studied with the graphs in Figure 33.13b and the phasor diagram in this figure.

in Figure 33.15b, we see that

$$\Delta V_{max} = \sqrt{\Delta V_R{}^2 + (\Delta V_L - \Delta V_C)^2} = \sqrt{(I_{max} R)^2 + (I_{max} X_L - I_{max} X_C)^2}$$

$$\Delta V_{max} = I_{max}\sqrt{R^2 + (X_L - X_C)^2} \qquad (33.24)$$

Therefore, we can express the maximum current as

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Maximum current in an *RLC* circuit

Once again, this has the same mathematical form as Equation 27.8. The denominator of the fraction plays the role of resistance and is called the **impedance** Z of the circuit:

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2} \qquad (33.25)$$

Impedance

where impedance also has units of ohms. Therefore, we can write Equation 33.24 in the form

$$\Delta V_{max} = I_{max} Z \qquad (33.26)$$

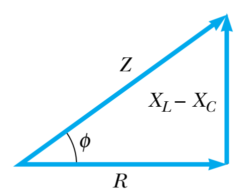


Figure 33.16 An impedance triangle for a series *RLC* circuit gives the relationship $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

We can regard Equation 33.26 as the AC equivalent of Equation 27.8. Note that the impedance and therefore the current in an AC circuit depend upon the resistance, the inductance, the capacitance, and the frequency (because the reactances are frequency-dependent).

By removing the common factor I_{max} from each phasor in Figure 33.15a, we can construct the *impedance triangle* shown in Figure 33.16. From this phasor diagram we find that the phase angle ϕ between the current and the voltage is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \qquad (33.27)$$

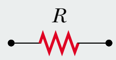
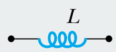
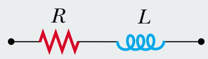
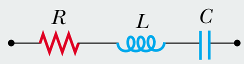
Phase angle

Also, from Figure 33.16, we see that $\cos\phi = R/Z$. When $X_L > X_C$ (which occurs at high frequencies), the phase angle is positive, signifying that the current lags behind the applied voltage, as in Figure 33.15a. We describe this situation by saying that the circuit is *more inductive than capacitive.* When $X_L < X_C$, the phase angle is negative, signifying that the current leads the applied voltage, and the circuit is *more capacitive than inductive.* When $X_L = X_C$, the phase angle is zero and the circuit is *purely resistive.*

Table 33.1 gives impedance values and phase angles for various series circuits containing different combinations of elements.

Table 33.1

Impedance Values and Phase Angles for Various Circuit-Element Combinations[a]

Circuit Elements	Impedance Z	Phase Angle ϕ
	R	$0°$
	X_C	$-90°$
	X_L	$+90°$
	$\sqrt{R^2 + X_C^{\,2}}$	Negative, between $-90°$ and $0°$
	$\sqrt{R^2 + X_L^{\,2}}$	Positive, between $0°$ and $90°$
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

[a] In each case, an AC voltage (not shown) is applied across the elements.

Quick Quiz 33.7 Label each part of Figure 33.17 as being $X_L > X_C$, $X_L = X_C$, or $X_L < X_C$.

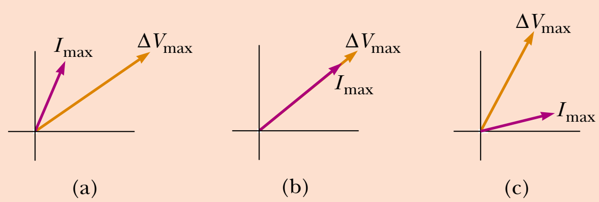


Figure 33.17 (Quick Quiz 33.7) Match the phasor diagrams to the relationships between the reactances.

Example 33.4 Finding L from a Phasor Diagram

In a series *RLC* circuit, the applied voltage has a maximum value of 120 V and oscillates at a frequency of 60.0 Hz. The circuit contains an inductor whose inductance can be varied, a 200-Ω resistor, and a 4.00-μF capacitor. What value of L should an engineer analyzing the circuit choose such that the voltage across the capacitor lags the applied voltage by 30.0°?

Solution The phase relationships for the voltages across the elements are shown in Figure 33.18. From the figure we see that the phase angle is $\phi = -60.0°$. (The phasors representing I_{max} and ΔV_R are in the same direction.) From Equation 33.27, we find that

$$X_L = X_C + R\tan\phi$$

Substituting Equations 33.10 and 33.18 (with $\omega = 2\pi f$) into this expression gives

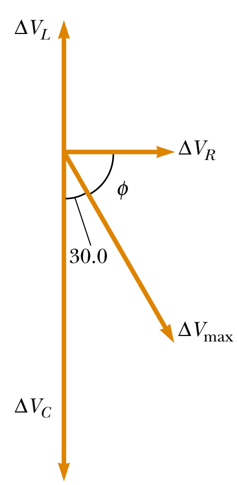


Figure 33.18 (Example 33.4) The phasor diagram for the given information.

$$2\pi f L = \frac{1}{2\pi f C} + R\tan\phi$$

$$L = \frac{1}{2\pi f}\left(\frac{1}{2\pi f C} + R\tan\phi\right)$$

Substituting the given values into the equation gives $L = \boxed{0.84\ \text{H.}}$

Example 33.5 Analyzing a Series *RLC* Circuit Interactive

A series *RLC* AC circuit has $R = 425\ \Omega$, $L = 1.25$ H, $C = 3.50\ \mu$F, $\omega = 377\ \text{s}^{-1}$, and $\Delta V_{\text{max}} = 150$ V.

(A) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit.

Solution The reactances are $X_L = \omega L = \boxed{471\ \Omega}$ and $X_C = 1/\omega C = \boxed{758\ \Omega.}$

The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{(425\ \Omega)^2 + (471\ \Omega - 758\ \Omega)^2} = \boxed{513\ \Omega}$$

(B) Find the maximum current in the circuit.

Solution

$$I_{\text{max}} = \frac{\Delta V_{\text{max}}}{Z} = \frac{150\ \text{V}}{513\ \Omega} = \boxed{0.292\ \text{A}}$$

(C) Find the phase angle between the current and voltage.

Solution

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{471\ \Omega - 758\ \Omega}{425\ \Omega}\right)$$
$$= \boxed{-34.0°}$$

Because the capacitive reactance is larger than the inductive reactance, the circuit is more capacitive than inductive. In this case, the phase angle ϕ is negative and the current leads the applied voltage.

(D) Find both the maximum voltage and the instantaneous voltage across each element.

Solution The maximum voltages are

$$\Delta V_R = I_{\text{max}} R = (0.292\ \text{A})(425\ \Omega) = \boxed{124\ \text{V}}$$

$$\Delta V_L = I_{\text{max}} X_L = (0.292\ \text{A})(471\ \Omega) = \boxed{138\ \text{V}}$$

$$\Delta V_C = I_{\text{max}} X_C = (0.292\ \text{A})(758\ \Omega) = \boxed{221\ \text{V}}$$

Using Equations 33.21, 33.22, and 33.23, we find that we can write the instantaneous voltages across the three elements as

$$\Delta v_R = \boxed{(124\ \text{V})\sin 377t}$$

$$\Delta v_L = \boxed{(138\ \text{V})\cos 377t}$$

$$\Delta v_C = \boxed{(-221\ \text{V})\cos 377t}$$

What If? What if you added up the maximum voltages across the three circuit elements? Is this a physically meaningful quantity?

Answer The sum of the maximum voltages across the elements is $\Delta V_R + \Delta V_L + \Delta V_C = 484$ V. Note that this sum is much greater than the maximum voltage of the source, 150 V. The sum of the maximum voltages is a meaningless quantity because when sinusoidally varying quantities are added, *both their amplitudes and their phases* must be taken into account. We know that the maximum voltages across the various elements occur at different times. That is, the voltages must be added in a way that takes account of the different phases.

At the Interactive Worked Example link at **http://www.pse6.com**, *you can investigate the* RLC *circuit for various values of the circuit elements.*

33.6 Power in an AC Circuit

Let us now take an energy approach to analyzing AC circuits, considering the transfer of energy from the AC source to the circuit. In Example 28.1 we found that the power delivered by a battery to a DC circuit is equal to the product of the current and the emf of the battery. Likewise, the instantaneous power delivered by an AC source to a circuit is the product of the source current and the applied voltage. For the *RLC* circuit shown in Figure 33.13a, we can express the instantaneous power $\mathcal{P}$ as

$$\begin{aligned}\mathcal{P} &= i\,\Delta v = I_{\max}\sin(\omega t - \phi)\,\Delta V_{\max}\sin\omega t \\ &= I_{\max}\,\Delta V_{\max}\sin\omega t\sin(\omega t - \phi)\end{aligned} \tag{33.28}$$

This result is a complicated function of time and therefore is not very useful from a practical viewpoint. What is generally of interest is the average power over one or more cycles. Such an average can be computed by first using the trigonometric identity $\sin(\omega t - \phi) = \sin\omega t\cos\phi - \cos\omega t\sin\phi$. Substituting this into Equation 33.28 gives

$$\mathcal{P} = I_{\max}\,\Delta V_{\max}\sin^2\omega t\cos\phi - I_{\max}\,\Delta V_{\max}\sin\omega t\cos\omega t\sin\phi \tag{33.29}$$

We now take the time average of $\mathcal{P}$ over one or more cycles, noting that $I_{\max}$, $\Delta V_{\max}$, ϕ, and ω are all constants. The time average of the first term on the right in Equation 33.29 involves the average value of $\sin^2\omega t$, which is $\frac{1}{2}$ (as shown in footnote 1). The time average of the second term on the right is identically zero because $\sin\omega t\cos\omega t = \frac{1}{2}\sin 2\omega t$, and the average value of $\sin 2\omega t$ is zero. Therefore, we can express the **average power** $\mathcal{P}_{av}$ as

$$\mathcal{P}_{av} = \tfrac{1}{2}I_{\max}\,\Delta V_{\max}\cos\phi \tag{33.30}$$

It is convenient to express the average power in terms of the rms current and rms voltage defined by Equations 33.4 and 33.5:

Average power delivered to an *RLC* circuit

$$\mathcal{P}_{av} = I_{rms}\,\Delta V_{rms}\cos\phi \tag{33.31}$$

where the quantity $\cos\phi$ is called the **power factor.** By inspecting Figure 33.15b, we see that the maximum voltage across the resistor is given by $\Delta V_R = \Delta V_{\max}\cos\phi = I_{\max}R$. Using Equation 33.5 and the fact that $\cos\phi = I_{\max}R/\Delta V_{\max}$, we find that we can express $\mathcal{P}_{av}$ as

$$\mathcal{P}_{av} = I_{rms}\,\Delta V_{rms}\cos\phi = I_{rms}\left(\frac{\Delta V_{\max}}{\sqrt{2}}\right)\frac{I_{\max}R}{\Delta V_{\max}} = I_{rms}\frac{I_{\max}R}{\sqrt{2}}$$

After making the substitution $I_{\max} = \sqrt{2}\,I_{rms}$ from Equation 33.4, we have

$$\mathcal{P}_{av} = I_{rms}^2\,R \tag{33.32}$$

In words, **the average power delivered by the source is converted to internal energy in the resistor,** just as in the case of a DC circuit. When the load is purely resistive, then $\phi = 0$, $\cos\phi = 1$, and from Equation 33.31 we see that

$$\mathcal{P}_{av} = I_{rms}\,\Delta V_{rms}$$

We find that **no power losses are associated with pure capacitors and pure inductors in an AC circuit.** To see why this is true, let us first analyze the power in an AC circuit containing only a source and a capacitor. When the current begins to increase in one direction in an AC circuit, charge begins to accumulate on the capacitor, and a voltage appears across it. When this voltage reaches its maximum value, the energy stored in the capacitor is $\frac{1}{2}C(\Delta V_{\max})^2$. However, this energy storage is only momentary. The capacitor is charged and discharged twice during each cycle: charge is delivered to the capacitor during two quarters of the cycle and is returned to the voltage source during the remaining two quarters. Therefore, **the average power supplied by the source is zero.** In other words, **no power losses occur in a capacitor in an AC circuit.**

Let us now consider the case of an inductor. When the current reaches its maximum value, the energy stored in the inductor is a maximum and is given by $\frac{1}{2}LI_{\max}^2$. When the current begins to decrease in the circuit, this stored energy is returned to the source as the inductor attempts to maintain the current in the circuit.

Equation 33.31 shows that the power delivered by an AC source to any circuit depends on the phase—a result that has many interesting applications. For example, a factory that uses large motors in machines, generators, or transformers has a large inductive load (because of all the windings). To deliver greater power to such devices in the factory without using excessively high voltages, technicians introduce capacitance in the circuits to shift the phase.

Quick Quiz 33.8 An AC source drives an *RLC* circuit with a fixed voltage amplitude. If the driving frequency is ω_1, the circuit is more capacitive than inductive and the phase angle is $-10°$. If the driving frequency is ω_2, the circuit is more inductive than capacitive and the phase angle is $+10°$. The largest amount of power is delivered to the circuit at (a) ω_1 (b) ω_2 (c) The same amount of power is delivered at both frequencies.

Example 33.6 Average Power in an *RLC* Series Circuit

Calculate the average power delivered to the series *RLC* circuit described in Example 33.5.

Solution First, let us calculate the rms voltage and rms current, using the values of ΔV_{max} and I_{max} from Example 33.5:

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}} = \frac{150\ \text{V}}{\sqrt{2}} = 106\ \text{V}$$

$$I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} = \frac{0.292\ \text{A}}{\sqrt{2}} = 0.206\ \text{A}$$

Because $\phi = -34.0°$, the power factor is $\cos(-34.0°) = 0.829$; hence, the average power delivered is

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}\,\Delta V_{\text{rms}} \cos\phi = (0.206\ \text{A})(106\ \text{V})(0.829)$$

$$= \boxed{18.1\ \text{W}}$$

We can obtain the same result using Equation 33.32.

33.7 Resonance in a Series *RLC* Circuit

A series *RLC* circuit is said to be **in resonance** when the current has its maximum value. In general, the rms current can be written

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} \tag{33.33}$$

where Z is the impedance. Substituting the expression for Z from Equation 33.25 into 33.33 gives

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}} \tag{33.34}$$

Because the impedance depends on the frequency of the source, the current in the *RLC* circuit also depends on the frequency. The frequency ω_0 at which $X_L - X_C = 0$ is called the **resonance frequency** of the circuit. To find ω_0, we use the condition $X_L = X_C$, from which we obtain $\omega_0 L = 1/\omega_0 C$, or

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{33.35}$$

Resonance frequency

This frequency also corresponds to the natural frequency of oscillation of an *LC* circuit (see Section 32.5). Therefore, the current in a series *RLC* circuit reaches its maximum value when the frequency of the applied voltage matches the natural oscillator frequency—which depends only on L and C. Furthermore, at this frequency the current is in phase with the applied voltage.

Quick Quiz 33.9 The impedance of a series RLC circuit at resonance is (a) larger than R (b) less than R (c) equal to R (d) impossible to determine.

A plot of rms current versus frequency for a series RLC circuit is shown in Figure 33.19a. The data assume a constant $\Delta V_{\text{rms}} = 5.0$ mV, that $L = 5.0\ \mu$H, and that $C = 2.0$ nF. The three curves correspond to three values of R. In each case, the current reaches its maximum value at the resonance frequency ω_0. Furthermore, the curves become narrower and taller as the resistance decreases.

By inspecting Equation 33.34, we must conclude that, when $R = 0$, the current becomes infinite at resonance. However, real circuits always have some resistance, which limits the value of the current to some finite value.

It is also interesting to calculate the average power as a function of frequency for a series RLC circuit. Using Equations 33.32, 33.33, and 33.25, we find that

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}^2 R = \frac{(\Delta V_{\text{rms}})^2}{Z^2} R = \frac{(\Delta V_{\text{rms}})^2 R}{R^2 + (X_L - X_C)^2} \tag{33.36}$$

Because $X_L = \omega L$, $X_C = 1/\omega C$, and $\omega_0{}^2 = 1/LC$, we can express the term $(X_L - X_C)^2$ as

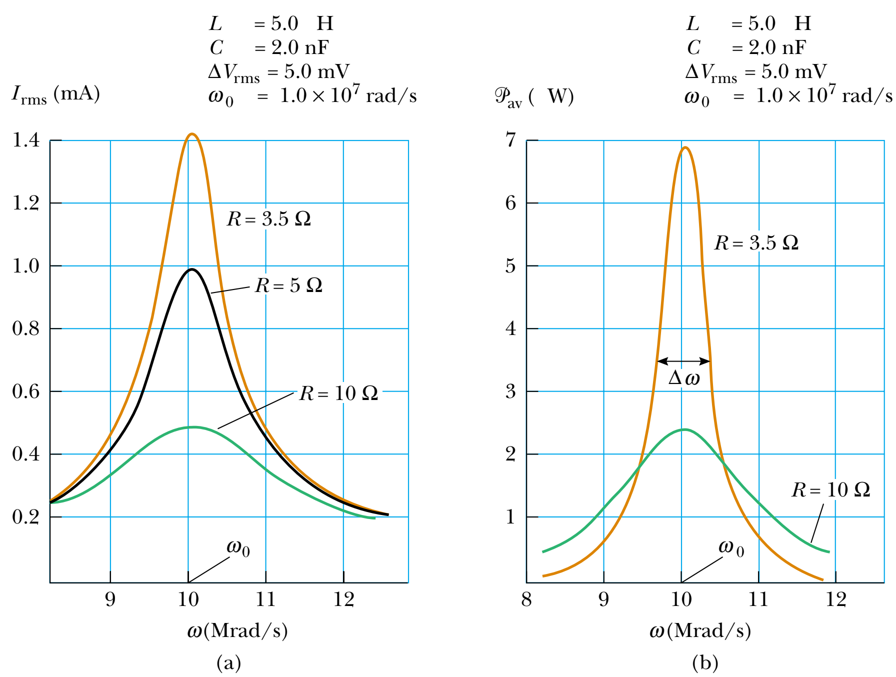
$$(X_L - X_C)^2 = \left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{L^2}{\omega^2}(\omega^2 - \omega_0{}^2)^2$$

Using this result in Equation 33.36 gives

Average power as a function of frequency in an *RLC* circuit

$$\mathcal{P}_{\text{av}} = \frac{(\Delta V_{\text{rms}})^2 R\omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega_0{}^2)^2} \tag{33.37}$$

This expression shows that **at resonance, when $\omega = \omega_0$, the average power is a maximum** and has the value $(\Delta V_{\text{rms}})^2/R$. Figure 33.19b is a plot of average power



At the Active Figures link at http://www.pse6.com, you can adjust the resistance, the inductance, and the capacitance of the circuit in Figure 33.13a. You can then determine the current and power for a given frequency or sweep through the frequencies to generate resonance curves.

Active Figure 33.19 (a) The rms current versus frequency for a series RLC circuit, for three values of R. The current reaches its maximum value at the resonance frequency ω_0. (b) Average power delivered to the circuit versus frequency for the series RLC circuit, for two values of R.

versus frequency for two values of R in a series RLC circuit. As the resistance is made smaller, the curve becomes sharper in the vicinity of the resonance frequency. This curve sharpness is usually described by a dimensionless parameter known as the **quality factor,**[3] denoted by Q:

Quality factor

$$Q = \frac{\omega_0}{\Delta\omega}$$

where $\Delta\omega$ is the width of the curve measured between the two values of ω for which $\mathcal{P}_{av}$ has half its maximum value, called the *half-power points* (see Fig. 33.19b.) It is left as a problem (Problem 72) to show that the width at the half-power points has the value $\Delta\omega = R/L$, so

$$Q = \frac{\omega_0 L}{R} \tag{33.38}$$

The curves plotted in Figure 33.20 show that a high-Q circuit responds to only a very narrow range of frequencies, whereas a low-Q circuit can detect a much broader range of frequencies. Typical values of Q in electronic circuits range from 10 to 100.

The receiving circuit of a radio is an important application of a resonant circuit. One tunes the radio to a particular station (which transmits an electromagnetic wave or signal of a specific frequency) by varying a capacitor, which changes the resonance frequency of the receiving circuit. When the resonance frequency of the circuit matches that of the incoming electromagnetic wave, the current in the receiving circuit increases. This signal caused by the incoming wave is then amplified and fed to a speaker. Because many signals are often present over a range of frequencies, it is important to design a high-Q circuit to eliminate unwanted signals. In this manner, stations whose frequencies are near but not equal to the resonance frequency give signals at the receiver that are negligibly small relative to the signal that matches the resonance frequency.

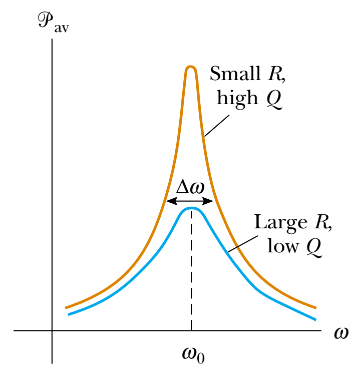


Figure 33.20 Average power versus frequency for a series RLC circuit. The width $\Delta\omega$ of each curve is measured between the two points where the power is half its maximum value. The power is a maximum at the resonance frequency ω_0.

Quick Quiz 33.10 An airport metal detector (see page 1003) is essentially a resonant circuit. The portal you step through is an inductor (a large loop of conducting wire) within the circuit. The frequency of the circuit is tuned to its resonance frequency when there is no metal in the inductor. Any metal on your body increases the effective inductance of the loop and changes the current in it. If you want the detector to detect a small metallic object, should the circuit have (a) a high quality factor or (b) a low quality factor?

Example 33.7 A Resonating Series *RLC* Circuit **Interactive**

Consider a series RLC circuit for which $R = 150\ \Omega$, $L = 20.0$ mH, $\Delta V_{rms} = 20.0$ V, and $\omega = 5\,000\ s^{-1}$. Determine the value of the capacitance for which the current is a maximum.

Solution The current has its maximum value at the resonance frequency ω_0, which should be made to match the "driving" frequency of $5\,000\ s^{-1}$:

$$\omega_0 = 5.00 \times 10^3\ s^{-1} = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{\omega_0{}^2 L} = \frac{1}{(5.00 \times 10^3\ s^{-1})^2(20.0 \times 10^{-3}\ H)}$$

$$= \boxed{2.00\ \mu F}$$

At the Interactive Worked Example link at **http://www.pse6.com**, *you can explore resonance in an* RLC *circuit.*

[3] The quality factor is also defined as the ratio $2\pi E/\Delta E$ where E is the energy stored in the oscillating system and ΔE is the energy decrease per cycle of oscillation due to the resistance.

33.8 The Transformer and Power Transmission

As discussed in Section 27.6, when electric power is transmitted over great distances, it is economical to use a high voltage and a low current to minimize the I^2R loss in the transmission lines. Consequently, 350-kV lines are common, and in many areas even higher-voltage (765-kV) lines are used. At the receiving end of such lines, the consumer requires power at a low voltage (for safety and for efficiency in design). Therefore, a device is required that can change the alternating voltage and current without causing appreciable changes in the power delivered. The AC transformer is that device.

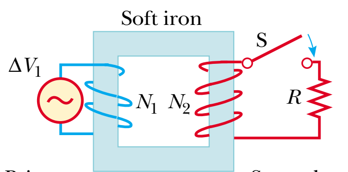


Figure 33.21 An ideal transformer consists of two coils wound on the same iron core. An alternating voltage ΔV_1 is applied to the primary coil, and the output voltage ΔV_2 is across the resistor of resistance R.

In its simplest form, the **AC transformer** consists of two coils of wire wound around a core of iron, as illustrated in Figure 33.21. (Compare this to Faraday's experiment in Figure 31.2.) The coil on the left, which is connected to the input alternating voltage source and has N_1 turns, is called the *primary winding* (or the *primary*). The coil on the right, consisting of N_2 turns and connected to a load resistor R, is called the *secondary winding* (or the *secondary*). The purpose of the iron core is to increase the magnetic flux through the coil and to provide a medium in which nearly all the magnetic field lines through one coil pass through the other coil. Eddy-current losses are reduced by using a laminated core. Iron is used as the core material because it is a soft ferromagnetic substance and hence reduces hysteresis losses. Transformation of energy to internal energy in the finite resistance of the coil wires is usually quite small. Typical transformers have power efficiencies from 90% to 99%. In the discussion that follows, we assume an *ideal transformer*, one in which the energy losses in the windings and core are zero.

First, let us consider what happens in the primary circuit. If we assume that the resistance of the primary is negligible relative to its inductive reactance, then the primary circuit is equivalent to a simple circuit consisting of an inductor connected to an AC source. Because the current is 90° out of phase with the voltage, the power factor $\cos\phi$ is zero, and hence the average power delivered from the source to the primary circuit is zero. Faraday's law states that the voltage ΔV_1 across the primary is

$$\Delta V_1 = -N_1 \frac{d\Phi_B}{dt} \tag{33.39}$$

where Φ_B is the magnetic flux through each turn. If we assume that all magnetic field lines remain within the iron core, the flux through each turn of the primary equals the flux through each turn of the secondary. Hence, the voltage across the secondary is

$$\Delta V_2 = -N_2 \frac{d\Phi_B}{dt} \tag{33.40}$$

Solving Equation 33.39 for $d\Phi_B/dt$ and substituting the result into Equation 33.40, we find that

$$\Delta V_2 = \frac{N_2}{N_1}\Delta V_1 \tag{33.41}$$

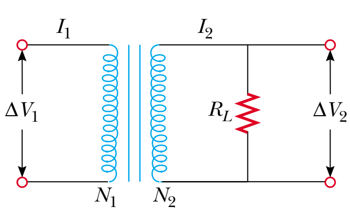


Figure 33.22 Circuit diagram for a transformer.

When $N_2 > N_1$, the output voltage ΔV_2 exceeds the input voltage ΔV_1. This setup is referred to as a *step-up transformer*. When $N_2 < N_1$, the output voltage is less than the input voltage, and we have a *step-down transformer*.

When the switch in the secondary circuit is closed, a current I_2 is induced in the secondary. If the load in the secondary circuit is a pure resistance, the induced current is in phase with the induced voltage. The power supplied to the secondary circuit must be provided by the AC source connected to the primary circuit, as shown in Figure 33.22. In an ideal transformer, where there are no losses, the power

$I_1\,\Delta V_1$ supplied by the source is equal to the power $I_2\,\Delta V_2$ in the secondary circuit. That is,

$$I_1\,\Delta V_1 = I_2\,\Delta V_2 \tag{33.42}$$

The value of the load resistance R_L determines the value of the secondary current because $I_2 = \Delta V_2/R_L$. Furthermore, the current in the primary is $I_1 = \Delta V_1/R_{\text{eq}}$, where

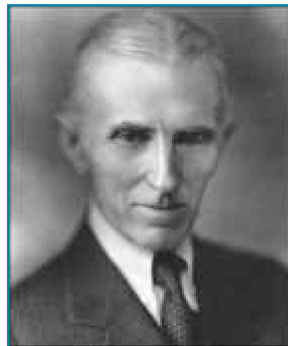
$$R_{\text{eq}} = \left(\frac{N_1}{N_2}\right)^2 R_L \tag{33.43}$$

is the equivalent resistance of the load resistance when viewed from the primary side. From this analysis we see that a transformer may be used to match resistances between the primary circuit and the load. In this manner, maximum power transfer can be achieved between a given power source and the load resistance. For example, a transformer connected between the 1-kΩ output of an audio amplifier and an 8-Ω speaker ensures that as much of the audio signal as possible is transferred into the speaker. In stereo terminology, this is called *impedance matching*.

We can now also understand why transformers are useful for transmitting power over long distances. Because the generator voltage is stepped up, the current in the transmission line is reduced, and hence I^2R losses are reduced. In practice, the voltage is stepped up to around 230 000 V at the generating station, stepped down to around 20 000 V at a distributing station, then to 4 000 V for delivery to residential areas, and finally to 120–240 V at the customer's site.

There is a practical upper limit to the voltages that can be used in transmission lines. Excessive voltages could ionize the air surrounding the transmission lines, which could result in a conducting path to ground or to other objects in the vicinity. This, of course, would present a serious hazard to any living creatures. For this reason, a long string of insulators is used to keep high-voltage wires away from their supporting metal towers. Other insulators are used to maintain separation between wires.

Many common household electronic devices require low voltages to operate properly. A small transformer that plugs directly into the wall, like the one illustrated in Figure 33.23, can provide the proper voltage. The photograph shows the two windings wrapped around a common iron core that is found inside all these little "black boxes." This particular transformer converts the 120-V AC in the wall socket to 12.5-V AC. (Can you determine the ratio of the numbers of turns in the two coils?) Some black boxes also make use of diodes to convert the alternating current to direct current. (See Section 33.9.)



Nikola Tesla

American Physicist (1856–1943)

Tesla was born in Croatia but spent most of his professional life as an inventor in the United States. He was a key figure in the development of alternating current electricity, high-voltage transformers, and the transport of electrical power using AC transmission lines. Tesla's viewpoint was at odds with the ideas of Thomas Edison, who committed himself to the use of direct current in power transmission. Tesla's AC approach won out. *(UPI/CORBIS)*

George Semple

Figure 33.23 The primary winding in this transformer is directly attached to the prongs of the plug. The secondary winding is connected to the power cord on the right, which runs to an electronic device. Many of these power-supply transformers also convert alternating current to direct current.

George Semple

This transformer is smaller than the one in the opening photograph for this chapter. In addition, it is a step-down transformer. It drops the voltage from 4 000 V to 240 V for delivery to a group of residences.

Example 33.8 The Economics of AC Power

An electricity-generating station needs to deliver energy at a rate of 20 MW to a city 1.0 km away.

(A) If the resistance of the wires is 2.0 Ω and the energy costs about 10¢/kWh, estimate what it costs the utility company for the energy converted to internal energy in the wires during one day. A common voltage for commercial power generators is 22 kV, but a step-up transformer is used to boost the voltage to 230 kV before transmission.

Solution Conceptualize by noting that the resistance of the wires is in series with the resistance representing the load (homes and businesses). Thus, there will be a voltage drop in the wires, which means that some of the transmitted energy is converted to internal energy in the wires and never reaches the load. Because this is an estimate, let us categorize this as a problem in which the power factor is equal to 1. To analyze the problem, we begin by calculating I_{rms} from Equation 33.31:

$$I_{\text{rms}} = \frac{\mathcal{P}_{\text{av}}}{\Delta V_{\text{rms}}} = \frac{20 \times 10^6 \text{ W}}{230 \times 10^3 \text{ V}} = 87 \text{ A}$$

Now, we determine the rate at which energy is delivered to the resistance in the wires from Equation 33.32:

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}^2 R = (87 \text{ A})^2 (2.0 \ \Omega) = 15 \text{ kW}$$

Over the course of a day, the energy loss due to the resistance of the wires is (15 kW)(24 h) = 360 kWh, at a cost of $\boxed{\$36.}$

(B) Repeat the calculation for the situation in which the power plant delivers the energy at its original voltage of 22 kV.

Solution Again using Equation 33.31, we find

$$I_{\text{rms}} = \frac{\mathcal{P}_{\text{av}}}{\Delta V_{\text{rms}}} = \frac{20 \times 10^6 \text{ W}}{22 \times 10^3 \text{ V}} = 910 \text{ A}$$

and, from Equation 33.32,

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}^2 R = (910 \text{ A})^2 (2.0 \ \Omega) = 1.7 \times 10^3 \text{ kW}$$

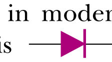
$$\text{Cost per day} = (1.7 \times 10^3 \text{ kW})(24 \text{ h})(\$0.10/\text{kWh})$$

$$= \boxed{\$4\,100}$$

To finalize the example, note the tremendous savings that are possible through the use of transformers and high-voltage transmission lines. This, in combination with the efficiency of using alternating current to operate motors, led to the universal adoption of alternating current instead of direct current for commercial power grids.

33.9 Rectifiers and Filters

Portable electronic devices such as radios and compact disc (CD) players are often powered by direct current supplied by batteries. Many devices come with AC–DC converters such as that in Figure 33.23. Such a converter contains a transformer that steps the voltage down from 120 V to typically 9 V and a circuit that converts alternating current to direct current. The process of converting alternating current to direct current is called **rectification,** and the converting device is called a **rectifier.**

The most important element in a rectifier circuit is a **diode,** a circuit element that conducts current in one direction but not the other. Most diodes used in modern electronics are semiconductor devices. The circuit symbol for a diode is , where the arrow indicates the direction of the current in the diode. A diode has low resistance to current in one direction (the direction of the arrow) and high resistance to current in the opposite direction. We can understand how a diode rectifies a current by considering Figure 33.24a, which shows a diode and a resistor connected to the secondary of a transformer. The transformer reduces the voltage from 120-V AC to the lower voltage that is needed for the device having a resistance R (the load resistance). Because the diode conducts current in only one direction, the alternating current in the load resistor is reduced to the form shown by the solid curve in Figure 33.24b. The diode conducts current only when the side of the symbol containing the arrowhead has a positive potential relative to the other side. In this situation, the diode acts as a *half-wave rectifier* because current is present in the circuit during only half of each cycle.

When a capacitor is added to the circuit, as shown by the dashed lines and the capacitor symbol in Figure 33.24a, the circuit is a simple DC power supply. The time variation in the current in the load resistor (the dashed curve in Fig. 33.24b) is close to

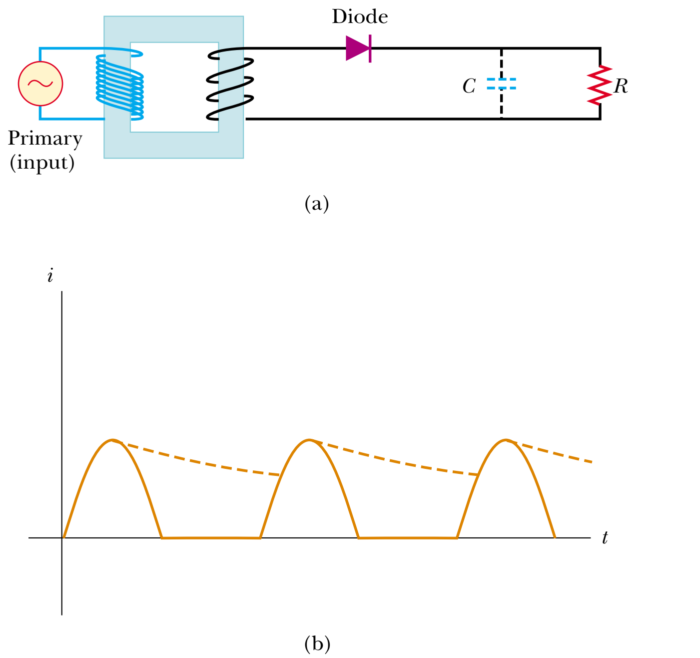
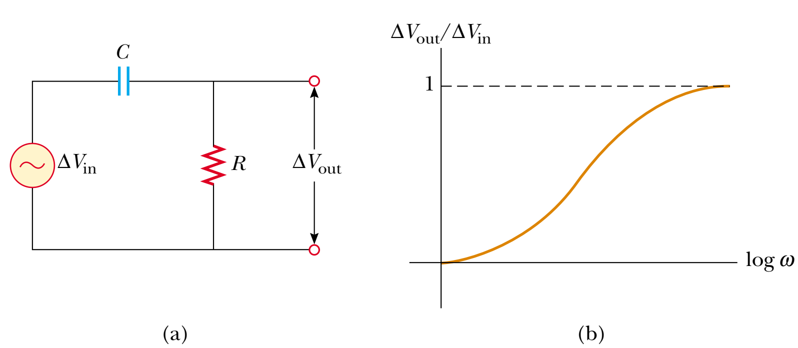


Figure 33.24 (a) A half-wave rectifier with an optional filter capacitor. (b) Current versus time in the resistor. The solid curve represents the current with no filter capacitor, and the dashed curve is the current when the circuit includes the capacitor.

being zero, as determined by the *RC* time constant of the circuit. As the current in the circuit begins to rise at $t = 0$ in Figure 33.24b, the capacitor charges up. When the current begins to fall, however, the capacitor discharges through the resistor, so that the current in the resistor does not fall as fast as the current from the transformer.

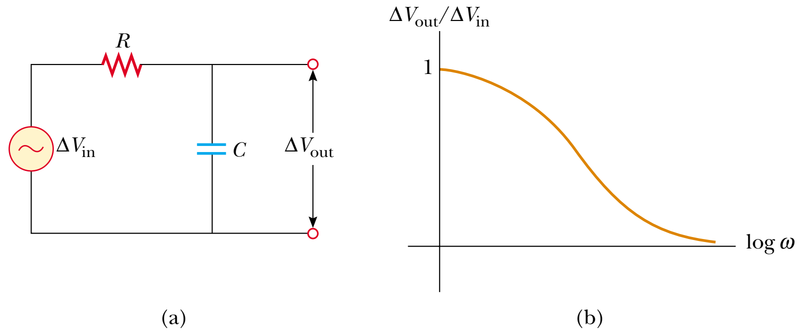
The *RC* circuit in Figure 33.24a is one example of a **filter circuit,** which is used to smooth out or eliminate a time-varying signal. For example, radios are usually powered by a 60-Hz alternating voltage. After rectification, the voltage still contains a small AC component at 60 Hz (sometimes called *ripple*), which must be filtered. By "filtered," we mean that the 60-Hz ripple must be reduced to a value much less than that of the audio signal to be amplified, because without filtering, the resulting audio signal includes an annoying hum at 60 Hz.

We can also design filters that will respond differently to different frequencies. Consider the simple series *RC* circuit shown in Figure 33.25a. The input voltage is across the series combination of the two elements. The output is the voltage across the resistor. A plot of the ratio of the output voltage to the input voltage as a function of the logarithm of angular frequency (see Fig. 33.25b) shows that at low frequencies ΔV_{out} is much smaller than ΔV_{in}, whereas at high frequencies the two voltages are equal. Because the circuit



Active Figure 33.25 (a) A simple *RC* high-pass filter. (b) Ratio of output voltage to input voltage for an *RC* high-pass filter as a function of the angular frequency of the AC source.

At the Active Figures link at http://www.pse6.com, you can adjust the resistance and the capacitance of the circuit in part (a). You can then determine the output voltage for a given frequency or sweep through the frequencies to generate a curve like that in part (b).



Active Figure 33.26 (a) A simple *RC* low-pass filter. (b) Ratio of output voltage to input voltage for an *RC* low-pass filter as a function of the angular frequency of the AC source.

At the Active Figures link at http://www.pse6.com, you can adjust the resistance and the capacitance of the circuit in part (a). You can then determine the output voltage for a given frequency or sweep through the frequencies to generate a curve like that in part (b).

preferentially passes signals of higher frequency while blocking low-frequency signals, the circuit is called an *RC* high-pass filter. (See Problem 51 for an analysis of this filter.)

Physically, a high-pass filter works because a capacitor "blocks out" direct current and AC current at low frequencies. At low frequencies, the capacitive reactance is large and much of the applied voltage appears across the capacitor rather than across the output resistor. As the frequency increases, the capacitive reactance drops and more of the applied voltage appears across the resistor.

Now consider the circuit shown in Figure 33.26a, where we have interchanged the resistor and capacitor and the output voltage is taken across the capacitor. At low frequencies, the reactance of the capacitor and the voltage across the capacitor is high. As the frequency increases, the voltage across the capacitor drops. Thus, this is an *RC* low-pass filter. The ratio of output voltage to input voltage (see Problem 52), plotted as a function of the logarithm of ω in Figure 33.26b, shows this behavior.

You may be familiar with crossover networks, which are an important part of the speaker systems for high-fidelity audio systems. These networks use low-pass filters to direct low frequencies to a special type of speaker, the "woofer," which is designed to reproduce the low notes accurately. The high frequencies are sent to the "tweeter" speaker.

Quick Quiz 33.11 Suppose you are designing a high-fidelity system containing both large loudspeakers (woofers) and small loudspeakers (tweeters). If you wish to deliver low-frequency signals to a woofer, what device would you place in series with it? (a) an inductor (b) a capacitor (c) a resistor. If you wish to deliver high-frequency signals to a tweeter, what device would you place in series with it? (d) an inductor (e) a capacitor (f) a resistor.

SUMMARY

Take a practice test for this chapter by clicking on the Practice Test link at http://www.pse6.com.

If an AC circuit consists of a source and a resistor, the current is in phase with the voltage. That is, the current and voltage reach their maximum values at the same time.

The **rms current** and **rms voltage** in an AC circuit in which the voltages and current vary sinusoidally are given by the expressions

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max} \qquad (33.4)$$

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\, \Delta V_{max} \qquad (33.5)$$

where I_{max} and ΔV_{max} are the maximum values.

If an AC circuit consists of a source and an inductor, the current lags behind the voltage by 90°. That is, the voltage reaches its maximum value one quarter of a period before the current reaches its maximum value.

If an AC circuit consists of a source and a capacitor, the current leads the voltage by 90°. That is, the current reaches its maximum value one quarter of a period before the voltage reaches its maximum value.

In AC circuits that contain inductors and capacitors, it is useful to define the **inductive reactance** X_L and the **capacitive reactance** X_C as

$$X_L \equiv \omega L \tag{33.10}$$

$$X_C \equiv \frac{1}{\omega C} \tag{33.18}$$

where ω is the angular frequency of the AC source. The SI unit of reactance is the ohm.

The **impedance** Z of an RLC series AC circuit is

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2} \tag{33.25}$$

This expression illustrates that we cannot simply add the resistance and reactances in a circuit. We must account for the fact that the applied voltage and current are out of phase, with the **phase angle** ϕ between the current and voltage being

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \tag{33.27}$$

The sign of ϕ can be positive or negative, depending on whether X_L is greater or less than X_C. The phase angle is zero when $X_L = X_C$.

The **average power** delivered by the source in an RLC circuit is

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}\,\Delta V_{\text{rms}} \cos\phi \tag{33.31}$$

An equivalent expression for the average power is

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}^2\, R \tag{33.32}$$

The average power delivered by the source results in increasing internal energy in the resistor. No power loss occurs in an ideal inductor or capacitor.

The rms current in a series RLC circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}} \tag{33.34}$$

A series RLC circuit is in resonance when the inductive reactance equals the capacitive reactance. When this condition is met, the current given by Equation 33.34 reaches its maximum value. The **resonance frequency** ω_0 of the circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{33.35}$$

The current in a series RLC circuit reaches its maximum value when the frequency of the source equals ω_0—that is, when the "driving" frequency matches the resonance frequency.

Transformers allow for easy changes in alternating voltage. Because energy (and therefore power) are conserved, we can write

$$I_1\,\Delta V_1 = I_2\,\Delta V_2 \tag{33.42}$$

to relate the currents and voltages in the primary and secondary windings of a transformer.

QUESTIONS

1. How can the average value of a current be zero and yet the square root of the average squared current not be zero?

2. What is the time average of the "square-wave" potential shown in Figure Q33.2? What is its rms voltage?

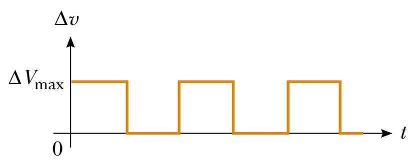


Figure Q33.2

3. Do AC ammeters and voltmeters read maximum, rms, or average values?

4. In the clearest terms you can, explain the statement, "The voltage across an inductor leads the current by 90°."

5. Some fluorescent lights flicker on and off 120 times every second. Explain what causes this. Why can't you see it happening?

6. Why does a capacitor act as a short circuit at high frequencies? Why does it act as an open circuit at low frequencies?

7. Explain how the mnemonic "ELI the ICE man" can be used to recall whether current leads voltage or voltage leads current in *RLC* circuits. Note that E represents emf $\mathcal{E}$.

8. Why is the sum of the maximum voltages across each of the elements in a series *RLC* circuit usually greater than the maximum applied voltage? Doesn't this violate Kirchhoff's loop rule?

9. Does the phase angle depend on frequency? What is the phase angle when the inductive reactance equals the capacitive reactance?

10. In a series *RLC* circuit, what is the possible range of values for the phase angle?

11. If the frequency is doubled in a series *RLC* circuit, what happens to the resistance, the inductive reactance, and the capacitive reactance?

12. Explain why the average power delivered to an *RLC* circuit by the source depends on the phase angle between the current and applied voltage.

13. As shown in Figure 7.5a, a person pulls a vacuum cleaner at speed v across a horizontal floor, exerting on it a force of magnitude F directed upward at an angle θ with the horizontal. At what rate is the person doing work on the cleaner? State as completely as you can the analogy between power in this situation and in an electric circuit.

14. A particular experiment requires a beam of light of very stable intensity. Why would an AC voltage be unsuitable for powering the light source?

15. Do some research to answer these questions: Who invented the metal detector? Why? Did it work?

16. What is the advantage of transmitting power at high voltages?

17. What determines the maximum voltage that can be used on a transmission line?

18. Will a transformer operate if a battery is used for the input voltage across the primary? Explain.

19. Someone argues that high-voltage power lines actually waste more energy. He points out that the rate at which internal energy is produced in a wire is given by $(\Delta V)^2/R$, where R is the resistance of the wire. Therefore, the higher the voltage, the higher the energy waste. What if anything is wrong with his argument?

20. Explain how the quality factor is related to the response characteristics of a radio receiver. Which variable most strongly influences the quality factor?

21. Why are the primary and secondary coils of a transformer wrapped on an iron core that passes through both coils?

22. With reference to Figure Q33.22, explain why the capacitor prevents a DC signal from passing between A and B, yet allows an AC signal to pass from A to B. (The circuits are said to be capacitively coupled.)

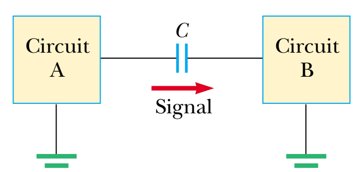


Figure Q33.22

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Note: Assume all AC voltages and currents are sinusoidal, unless stated otherwise.

Section 33.1 AC Sources
Section 33.2 Resistors in an AC Circuit

1. The rms output voltage of an AC source is 200 V and the operating frequency is 100 Hz. Write the equation giving the output voltage as a function of time.

2. (a) What is the resistance of a lightbulb that uses an average power of 75.0 W when connected to a 60.0-Hz power source having a maximum voltage of 170 V? (b) **What If?** What is the resistance of a 100-W bulb?

3. An AC power supply produces a maximum voltage $\Delta V_{\max} = 100$ V. This power supply is connected to a 24.0-Ω resistor, and the current and resistor voltage are measured with an ideal AC ammeter and voltmeter, as shown in Figure P33.3. What does each meter read? Note that an ideal ammeter has zero resistance and that an ideal voltmeter has infinite resistance.

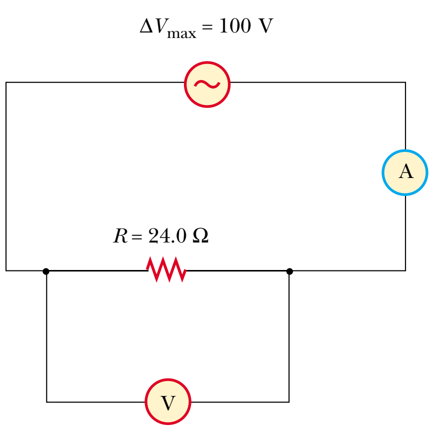


Figure P33.3

4. In the simple AC circuit shown in Figure 33.2, $R = 70.0\ \Omega$ and $\Delta v = \Delta V_{\max} \sin \omega t$. (a) If $\Delta v_R = 0.250\ \Delta V_{\max}$ for the first time at $t = 0.010\,0$ s, what is the angular frequency of the source? (b) What is the next value of t for which $\Delta v_R = 0.250\ \Delta V_{\max}$?

5. The current in the circuit shown in Figure 33.2 equals 60.0% of the peak current at $t = 7.00$ ms. What is the smallest frequency of the source that gives this current?

6. Figure P33.6 shows three lamps connected to a 120-V AC (rms) household supply voltage. Lamps 1 and 2 have 150-W bulbs; lamp 3 has a 100-W bulb. Find the rms current and resistance of each bulb.

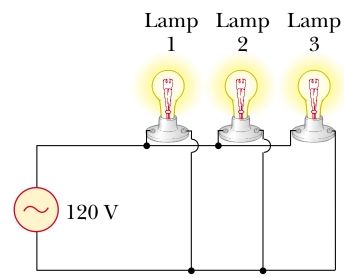


Figure P33.6

7. An audio amplifier, represented by the AC source and resistor in Figure P33.7, delivers to the speaker alternating voltage at audio frequencies. If the source voltage has an amplitude of 15.0 V, $R = 8.20\ \Omega$, and the speaker is equivalent to a resistance of 10.4 Ω, what is the time-averaged power transferred to it?

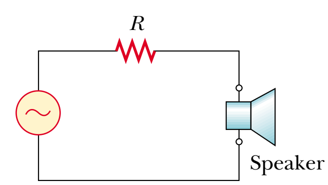


Figure P33.7

Section 33.3 Inductors in an AC Circuit

8. An inductor is connected to a 20.0-Hz power supply that produces a 50.0-V rms voltage. What inductance is needed to keep the instantaneous current in the circuit below 80.0 mA?

9. In a purely inductive AC circuit, as shown in Figure 33.6, $\Delta V_{\max} = 100$ V. (a) The maximum current is 7.50 A at 50.0 Hz. Calculate the inductance L. (b) **What If?** At what angular frequency ω is the maximum current 2.50 A?

10. An inductor has a 54.0-Ω reactance at 60.0 Hz. What is the maximum current if this inductor is connected to a 50.0-Hz source that produces a 100-V rms voltage?

11. For the circuit shown in Figure 33.6, $\Delta V_{\max} = 80.0$ V, $\omega = 65.0\pi$ rad/s, and $L = 70.0$ mH. Calculate the current in the inductor at $t = 15.5$ ms.

12. A 20.0-mH inductor is connected to a standard electrical outlet ($\Delta V_{\text{rms}} = 120$ V; $f = 60.0$ Hz). Determine the energy stored in the inductor at $t = (1/180)$ s, assuming that this energy is zero at $t = 0$.

13. **Review problem.** Determine the maximum magnetic flux through an inductor connected to a standard electrical outlet ($\Delta V_{\text{rms}} = 120$ V, $f = 60.0$ Hz).

Section 33.4 Capacitors in an AC Circuit

14. (a) For what frequencies does a 22.0-μF capacitor have a reactance below 175 Ω? (b) **What If?** Over this same frequency range, what is the reactance of a 44.0-μF capacitor?

15. What is the maximum current in a 2.20-μF capacitor when it is connected across (a) a North American electrical outlet having $\Delta V_{\text{rms}} = 120$ V, $f = 60.0$ Hz, and (b) **What If?** a European electrical outlet having $\Delta V_{\text{rms}} = 240$ V, $f = 50.0$ Hz?

16. A capacitor C is connected to a power supply that operates at a frequency f and produces an rms voltage ΔV. What is the maximum charge that appears on either of the capacitor plates?

17. What maximum current is delivered by an AC source with $\Delta V_{\max} = 48.0$ V and $f = 90.0$ Hz when connected across a 3.70-μF capacitor?

18. A 1.00-mF capacitor is connected to a standard electrical outlet ($\Delta V_{\text{rms}} = 120$ V; $f = 60.0$ Hz). Determine the current in the capacitor at $t = (1/180)$ s, assuming that at $t = 0$, the energy stored in the capacitor is zero.

Section 33.5 The *RLC* Series Circuit

19. An inductor ($L = 400$ mH), a capacitor ($C = 4.43$ μF), and a resistor ($R = 500$ Ω) are connected in series. A 50.0-Hz AC source produces a peak current of 250 mA in the circuit. (a) Calculate the required peak voltage $\Delta V_{\max}$. (b) Determine the phase angle by which the current leads or lags the applied voltage.

20. At what frequency does the inductive reactance of a 57.0-μH inductor equal the capacitive reactance of a 57.0-μF capacitor?

21. A series AC circuit contains the following components: $R = 150$ Ω, $L = 250$ mH, $C = 2.00$ μF and a source with $\Delta V_{\max} = 210$ V operating at 50.0 Hz. Calculate the (a) inductive reactance, (b) capacitive reactance, (c) impedance, (d) maximum current, and (e) phase angle between current and source voltage.

22. A sinusoidal voltage $\Delta v(t) = (40.0 \text{ V}) \sin(100t)$ is applied to a series *RLC* circuit with $L = 160$ mH, $C = 99.0$ μF, and $R = 68.0$ Ω. (a) What is the impedance of the circuit? (b) What is the maximum current? (c) Determine the numerical values for $I_{\max}$, ω, and ϕ in the equation $i(t) = I_{\max} \sin(\omega t - \phi)$.

23. An *RLC* circuit consists of a 150-Ω resistor, a 21.0-μF capacitor, and a 460-mH inductor, connected in series with a 120-V, 60.0-Hz power supply. (a) What is the phase angle between the current and the applied voltage? (b) Which reaches its maximum earlier, the current or the voltage?

24. Four circuit elements—a capacitor, an inductor, a resistor, and an AC source—are connected together in various ways. First the capacitor is connected to the source, and the rms current is found to be 25.1 mA. The capacitor is disconnected and discharged, and then connected in series with the resistor and the source, making the rms current 15.7 mA. The circuit is disconnected and the capacitor discharged. The capacitor is then connected in series with the inductor and the source, making the rms current 68.2 mA. After the circuit is disconnected and the capacitor discharged, all four circuit elements are connected together in a series loop. What is the rms current in the circuit?

25. A person is working near the secondary of a transformer, as shown in Figure P33.25. The primary voltage is 120 V at 60.0 Hz. The capacitance C_s, which is the stray capacitance between the hand and the secondary winding, is 20.0 pF. Assuming the person has a body resistance to ground $R_b = 50.0$ kΩ, determine the rms voltage across the body. (*Suggestion:* Redraw the circuit with the secondary of the transformer as a simple AC source.)

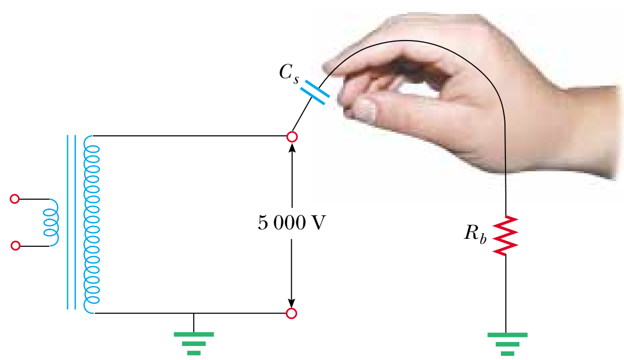


Figure P33.25

26. An AC source with $\Delta V_{\max} = 150$ V and $f = 50.0$ Hz is connected between points a and d in Figure P33.26. Calculate the maximum voltages between points (a) a and b, (b) b and c, (c) c and d, and (d) b and d.

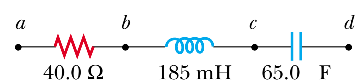


Figure P33.26 Problems 26 and 68.

27. Draw to scale a phasor diagram showing Z, X_L, X_C, and ϕ for an AC series circuit for which $R = 300$ Ω, $C = 11.0$ μF, $L = 0.200$ H, and $f = (500/\pi)$ Hz.

28. In an *RLC* series circuit that includes a source of alternating current operating at fixed frequency and voltage, the resistance R is equal to the inductive reactance. If the plate separation of the capacitor is reduced to half of its original value, the current in the circuit doubles. Find the initial capacitive reactance in terms of R.

29. A coil of resistance 35.0 Ω and inductance 20.5 H is in series with a capacitor and a 200-V (rms), 100-Hz source. The rms current in the circuit is 4.00 A. (a) Calculate the capacitance in the circuit. (b) What is ΔV_{rms} across the coil?

Section 33.6 Power in an AC Circuit

30. The voltage source in Figure P33.30 has an output of $\Delta V_{\text{rms}} = 100$ V at an angular frequency of $\omega = 1\,000$ rad/s. Determine (a) the current in the circuit and (b) the power supplied by the source. (c) Show that the power delivered to the resistor is equal to the power supplied by the source.

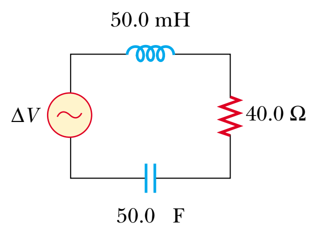


Figure P33.30

31. An AC voltage of the form $\Delta v = (100 \text{ V}) \sin(1\,000t)$ is applied to a series *RLC* circuit. Assume the resistance is 400 Ω, the capacitance is 5.00 μF, and the inductance is 0.500 H. Find the average power delivered to the circuit.

32. A series *RLC* circuit has a resistance of 45.0 Ω and an impedance of 75.0 Ω. What average power is delivered to this circuit when $\Delta V_{\text{rms}} = 210$ V?

33. In a certain series *RLC* circuit, $I_{\text{rms}} = 9.00$ A, $\Delta V_{\text{rms}} = 180$ V, and the current leads the voltage by 37.0°. (a) What is the total resistance of the circuit? (b) Calculate the reactance of the circuit ($X_L - X_C$).

34. Suppose you manage a factory that uses many electric motors. The motors create a large inductive load to the electric power line, as well as a resistive load. The electric company builds an extra-heavy distribution line to supply you with a component of current that is 90° out of phase with the voltage, as well as with current in phase with the voltage. The electric company charges you an extra fee for "reactive volt-amps," in addition to the amount you pay for the energy you use. You can avoid the extra fee by installing a capacitor between the power line and your factory. The following problem models this solution.

In an *RL* circuit, a 120-V (rms), 60.0-Hz source is in series with a 25.0-mH inductor and a 20.0-Ω resistor. What are (a) the rms current and (b) the power factor? (c) What capacitor must be added in series to make the power factor 1? (d) To what value can the supply voltage be reduced, if the power supplied is to be the same as before the capacitor was installed?

35. Suppose power $\mathcal{P}$ is to be transmitted over a distance d at a voltage ΔV with only 1.00% loss. Copper wire of what diameter should be used for each of the two conductors of the transmission line? Assume the current density in the conductors is uniform.

36. A diode is a device that allows current to be carried in only one direction (the direction indicated by the arrowhead in its circuit symbol). Find in terms of ΔV and R the average power delivered to the diode circuit of Figure P33.36.

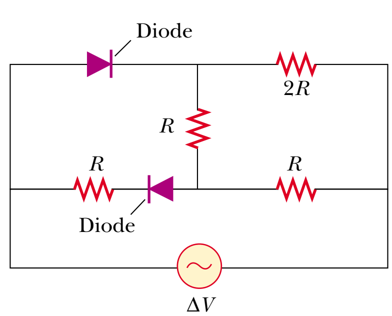


Figure P33.36

Section 33.7 Resonance in a Series *RLC* Circuit

37. An *RLC* circuit is used in a radio to tune into an FM station broadcasting at 99.7 MHz. The resistance in the circuit is 12.0 Ω, and the inductance is 1.40 μH. What capacitance should be used?

38. The tuning circuit of an AM radio contains an *LC* combination. The inductance is 0.200 mH, and the capacitor is variable, so that the circuit can resonate at any frequency between 550 kHz and 1 650 kHz. Find the range of values required for *C*.

39. A radar transmitter contains an *LC* circuit oscillating at 1.00×10^{10} Hz. (a) What capacitance will resonate with a one-turn loop of inductance 400 pH at this frequency? (b) If the capacitor has square parallel plates separated by 1.00 mm of air, what should the edge length of the plates be? (c) What is the common reactance of the loop and capacitor at resonance?

40. A series *RLC* circuit has components with following values: $L = 20.0$ mH, $C = 100$ nF, $R = 20.0$ Ω, and $\Delta V_{\text{max}} = 100$ V, with $\Delta v = \Delta V_{\text{max}} \sin \omega t$. Find (a) the resonant frequency, (b) the amplitude of the current at the resonant frequency, (c) the *Q* of the circuit, and (d) the amplitude of the voltage across the inductor at resonance.

41. A 10.0-Ω resistor, 10.0-mH inductor, and 100-μF capacitor are connected in series to a 50.0-V (rms) source having variable frequency. Find the energy that is delivered to the circuit during one period if the operating frequency is twice the resonance frequency.

42. A resistor *R*, inductor *L*, and capacitor *C* are connected in series to an AC source of rms voltage ΔV and variable frequency. Find the energy that is delivered to the circuit during one period if the operating frequency is twice the resonance frequency.

43. Compute the quality factor for the circuits described in Problems 22 and 23. Which circuit has the sharper resonance?

Section 33.8 The Transformer and Power Transmission

44. A step-down transformer is used for recharging the batteries of portable devices such as tape players. The turns ratio inside the transformer is 13:1 and it is used with 120-V (rms) household service. If a particular ideal transformer draws 0.350 A from the house outlet, what are (a) the voltage and (b) the current supplied to a tape player from the transformer? (c) How much power is delivered?

45. A transformer has $N_1 = 350$ turns and $N_2 = 2\,000$ turns. If the input voltage is $\Delta v(t) = (170 \text{ V}) \cos \omega t$, what rms voltage is developed across the secondary coil?

46. A step-up transformer is designed to have an output voltage of 2 200 V (rms) when the primary is connected across a 110-V (rms) source. (a) If the primary winding has 80 turns, how many turns are required on the secondary? (b) If a load resistor across the secondary draws a current of 1.50 A, what is the current in the primary, assuming ideal conditions? (c) **What If?** If the transformer actually has an efficiency of 95.0%, what is the current in the primary when the secondary current is 1.20 A?

47. In the transformer shown in Figure P33.47, the load resistor is 50.0 Ω. The turns ratio N_1:N_2 is 5:2, and the source voltage is 80.0 V (rms). If a voltmeter across the load measures 25.0 V (rms), what is the source resistance R_s?

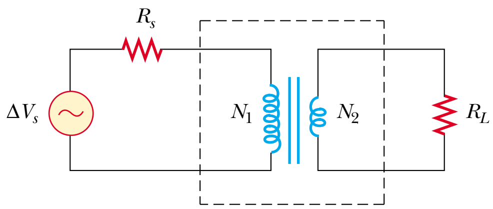


Figure P33.47

48. The secondary voltage of an ignition transformer in a furnace is 10.0 kV. When the primary operates at an rms voltage of 120 V, the primary impedance is 24.0 Ω and the transformer is 90.0% efficient. (a) What turns ratio is required? What are (b) the current in the secondary and (c) the impedance in the secondary?

49. A transmission line that has a resistance per unit length of 4.50×10^{-4} Ω/m is to be used to transmit 5.00 MW over 400 miles (6.44×10^5 m). The output voltage of the generator is 4.50 kV. (a) What is the line loss if a transformer is used to step up the voltage to 500 kV? (b) What fraction of the input power is lost to the line under these circumstances? (c) **What If?** What difficulties would be encountered in attempting to transmit the 5.00 MW at the generator voltage of 4.50 kV?

Section 33.9 Rectifiers and Filters

50. One particular plug-in power supply for a radio looks similar to the one shown in Figure 33.23 and is marked with the following information: Input 120 V AC 8 W Output 9 V DC 300 mA. Assume that these values are accurate to two digits. (a) Find the energy efficiency of the device when the radio is operating. (b) At what rate does the device produce wasted energy when the radio is operating? (c) Suppose that the input power to the transformer is 8.0 W when the radio is switched off and that energy costs \$0.135/kWh from the electric company. Find the cost of having six such transformers around the house, plugged in for thirty-one days.

51. Consider the filter circuit shown in Figure 33.25a. (a) Show that the ratio of the output voltage to the input voltage is

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{R}{\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}}$$

(b) What value does this ratio approach as the frequency decreases toward zero? What value does this ratio approach as the frequency increases without limit? (c) At what frequency is the ratio equal to one half?

52. Consider the filter circuit shown in Figure 33.26a. (a) Show that the ratio of the output voltage to the input voltage is

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{1/\omega C}{\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}}$$

(b) What value does this ratio approach as the frequency decreases toward zero? What value does this ratio approach as the frequency increases without limit? (c) At what frequency is the ratio equal to one half?

53. The RC high-pass filter shown in Figure 33.25 has a resistance $R = 0.500$ Ω. (a) What capacitance gives an output signal that has half the amplitude of a 300-Hz input signal? (b) What is the ratio $(\Delta V_{\text{out}}/\Delta V_{\text{in}})$ for a 600-Hz signal? You may use the result of Problem 51.

54. The RC low-pass filter shown in Figure 33.26 has a resistance $R = 90.0$ Ω and a capacitance $C = 8.00$ nF. Calculate the ratio $(\Delta V_{\text{out}}/\Delta V_{\text{in}})$ for an input frequency of (a) 600 Hz and (b) 600 kHz. You may use the result of Problem 52.

55. The resistor in Figure P33.55 represents the midrange speaker in a three-speaker system. Assume its resistance to be constant at 8.00 Ω. The source represents an audio amplifier producing signals of uniform amplitude $\Delta V_{\text{in}} =$ 10.0 V at all audio frequencies. The inductor and capacitor are to function as a bandpass filter with $\Delta V_{\text{out}}/\Delta V_{\text{in}} = 1/2$ at 200 Hz and at 4 000 Hz. (a) Determine the required values of L and C. (b) Find the maximum value of the ratio $\Delta V_{\text{out}}/\Delta V_{\text{in}}$. (c) Find the frequency f_0 at which the ratio has its maximum value. (d) Find the phase shift between ΔV_{in} and ΔV_{out} at 200 Hz, at f_0, and at 4 000 Hz. (e) Find the average power transferred to the speaker at 200 Hz, at f_0, and at 4 000 Hz. (f) Treating the filter as a resonant circuit, find its quality factor.

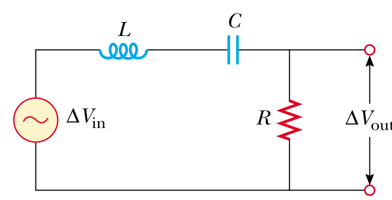


Figure P33.55

Additional Problems

56. Show that the rms value for the sawtooth voltage shown in Figure P33.56 is $\Delta V_{\max}/\sqrt{3}$.

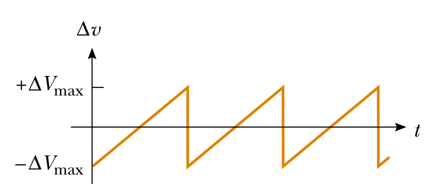


Figure P33.56

57. A series RLC circuit consists of an 8.00-Ω resistor, a 5.00-μF capacitor, and a 50.0-mH inductor. A variable frequency source applies an emf of 400 V (rms) across the combination. Determine the power delivered to the circuit when the frequency is equal to half the resonance frequency.

58. A capacitor, a coil, and two resistors of equal resistance are arranged in an AC circuit, as shown in Figure P33.58. An AC source provides an emf of 20.0 V (rms) at a frequency of 60.0 Hz. When the double-throw switch S is open, as shown in the figure, the rms current is 183 mA. When the switch is closed in position 1, the rms current is 298 mA. When the switch is closed in position 2, the rms current is 137 mA. Determine the values of *R*, *C*, and *L*. Is more than one set of values possible?

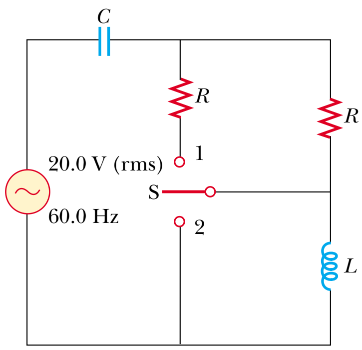


Figure P33.58

59. To determine the inductance of a coil used in a research project, a student first connects the coil to a 12.0-V battery and measures a current of 0.630 A. The student then connects the coil to a 24.0-V (rms), 60.0-Hz generator and measures an rms current of 0.570 A. What is the inductance?

60. **Review problem.** One insulated conductor from a household extension cord has mass per length 19.0 g/m. A section of this conductor is held under tension between two clamps. A subsection is located in a region of magnetic field of magnitude 15.3 mT perpendicular to the length of the cord. The wire carries an AC current of 9.00 A at 60.0 Hz. Determine some combination of values for the distance between the clamps and the tension in the cord so that the cord can vibrate in the lowest-frequency standing-wave vibrational state.

61. In Figure P33.61, find the rms current delivered by the 45.0-V (rms) power supply when (a) the frequency is very large and (b) the frequency is very small.

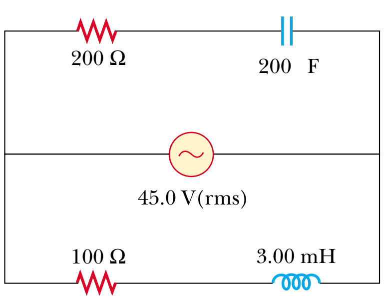


Figure P33.61

62. In the circuit shown in Figure P33.62, assume that all parameters except for *C* are given. (a) Find the current as a function of time. (b) Find the power delivered to the circuit. (c) Find the current as a function of time after *only* switch 1 is opened. (d) After switch 2 is *also* opened, the current and voltage are in phase. Find the capacitance *C*. (e) Find the impedance of the circuit when both switches are open. (f) Find the maximum energy stored in the capacitor during oscillations. (g) Find the maximum energy stored in the inductor during oscillations. (h) Now the frequency of the voltage source is doubled. Find the phase difference between the current and the voltage. (i) Find the frequency that makes the inductive reactance half the capacitive reactance.

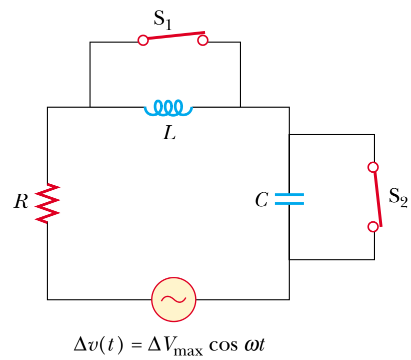


Figure P33.62

63. An 80.0-Ω resistor and a 200-mH inductor are connected in *parallel* across a 100-V (rms), 60.0-Hz source. (a) What is the rms current in the resistor? (b) By what angle does the total current lead or lag behind the voltage?

64. Make an order-of-magnitude estimate of the electric current that the electric company delivers to a town (Figure P33.64) from a remote generating station. State the data you measure or estimate. If you wish, you may consider a suburban bedroom community of 20 000 people.

Eddie Hironaka/Getty Images

Figure P33.64

65. Consider a series *RLC* circuit having the following circuit parameters: $R = 200\ \Omega$, $L = 663$ mH, and $C = 26.5\ \mu$F. The applied voltage has an amplitude of 50.0 V and a frequency of 60.0 Hz. Find the following amplitudes: (a) The current I_{max}, including its phase constant ϕ relative to the applied voltage Δv, (b) the voltage ΔV_R

across the resistor and its phase relative to the current, (c) the voltage ΔV_C across the capacitor and its phase relative to the current, and (d) the voltage ΔV_L across the inductor and its phase relative to the current.

66. A voltage $\Delta v = (100 \text{ V}) \sin \omega t$ (in SI units) is applied across a series combination of a 2.00-H inductor, a 10.0-μF capacitor, and a 10.0-Ω resistor. (a) Determine the angular frequency ω_0 at which the power delivered to the resistor is a maximum. (b) Calculate the power delivered at that frequency. (c) Determine the two angular frequencies ω_1 and ω_2 at which the power is half the maximum value. [The Q of the circuit is $\omega_0/(\omega_2 - \omega_1)$.]

67. *Impedance matching.* Example 28.2 showed that maximum power is transferred when the internal resistance of a DC source is equal to the resistance of the load. A transformer may be used to provide maximum power transfer between two AC circuits that have different impedances Z1 and Z2, where 1 and 2 are subscripts and the Z's are italic (as in the centered equation). (a) Show that the ratio of turns N_1/N_2 needed to meet this condition is

$$\frac{N_1}{N_2} = \sqrt{\frac{Z_1}{Z_2}}$$

(b) Suppose you want to use a transformer as an impedance-matching device between an audio amplifier that has an output impedance of 8.00 kΩ and a speaker that has an input impedance of 8.00 Ω. What should your N_1/N_2 ratio be?

68. A power supply with $\Delta V_{\text{rms}} = 120$ V is connected between points a and d in Figure P33.26. At what frequency will it deliver a power of 250 W?

69. Figure P33.69a shows a parallel RLC circuit, and the corresponding phasor diagram is given in Figure P33.69b. The instantaneous voltages (and rms voltages) across each of the three circuit elements are the same, and each is in phase with the current through the resistor. The currents in C and L lead or lag behind the current in the resistor, as shown in Figure P33.69b. (a) Show that the rms current delivered by the source is

$$I_{\text{rms}} = \Delta V_{\text{rms}} \left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L} \right)^2 \right]^{1/2}$$

(b) Show that the phase angle ϕ between ΔV_{rms} and I_{rms} is

$$\tan \phi = R \left(\frac{1}{X_C} - \frac{1}{X_L} \right)$$

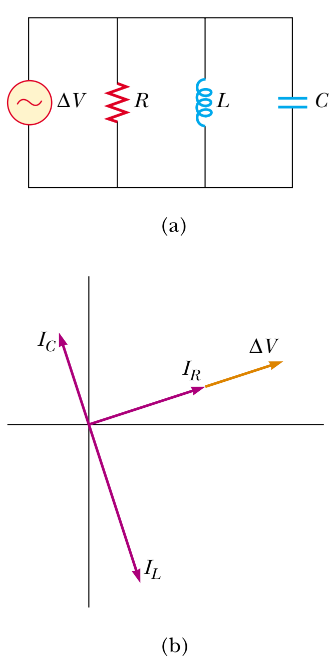


Figure P33.69

70. An 80.0-Ω resistor, a 200-mH inductor, and a 0.150-μF capacitor are connected *in parallel* across a 120-V (rms) source operating at 374 rad/s. (a) What is the resonant frequency of the circuit? (b) Calculate the rms current in the resistor, inductor, and capacitor. (c) What rms current is delivered by the source? (d) Is the current leading or lagging behind the voltage? By what angle?

71. A series RLC circuit is operating at 2 000 Hz. At this frequency, $X_L = X_C = 1\,884\ \Omega$. The resistance of the circuit is 40.0 Ω. (a) Prepare a table showing the values of X_L, X_C, and Z for f = 300, 600, 800, 1 000, 1 500, 2 000, 3 000, 4 000, 6 000, and 10 000 Hz. (b) Plot on the same set of axes X_L, X_C, and Z as a function of ln f.

72. A series RLC circuit in which $R = 1.00\ \Omega$, $L = 1.00$ mH, and $C = 1.00$ nF is connected to an AC source delivering 1.00 V (rms). Make a precise graph of the power delivered to the circuit as a function of the frequency and verify that the full width of the resonance peak at half-maximum is $R/2\pi L$.

73. Suppose the high-pass filter shown in Figure 33.25 has $R = 1\,000\ \Omega$ and $C = 0.050\,0\ \mu$F. (a) At what frequency does $\Delta V_{\text{out}}/\Delta V_{\text{in}} = \frac{1}{2}$? (b) Plot $\log_{10}(\Delta V_{\text{out}}/\Delta V_{\text{in}})$ versus $\log_{10}(f)$ over the frequency range from 1 Hz to 1 MHz. (This log–log plot of gain versus frequency is known as a *Bode plot.*)

Answers to Quick Quizzes

33.1 (a). The phasor in part (a) has the largest projection onto the vertical axis.

33.2 (b). The phasor in part (b) has the smallest-magnitude projection onto the vertical axis.

33.3 (c). The average power is proportional to the rms current, which, as Figure 33.5 shows, is nonzero even though the average current is zero. Condition (a) is valid only for an open circuit, and conditions (b) and (d) can not be true because $i_{\text{av}} = 0$ if the source is sinusoidal.

33.4 (b). For low frequencies, the reactance of the inductor is small so that the current is large. Most of the voltage from the source is across the bulb, so the power delivered to it is large.

33.5 (a). For high frequencies, the reactance of the capacitor is small so that the current is large. Most of the voltage from the source is across the bulb, so the power delivered to it is large.

33.6 (b). For low frequencies, the reactance of the capacitor is large so that very little current exists in the capacitor branch. The reactance of the inductor is small so that current exists in the inductor branch and the lightbulb glows. As the frequency increases, the inductive reactance increases and the capacitive reactance decreases. At high frequencies, more current exists in the capacitor branch than the inductor branch and the lightbulb glows more dimly.

33.7 (a) $X_L < X_C$. (b) $X_L = X_C$. (c) $X_L > X_C$.

33.8 (c). The cosine of $-\phi$ is the same as that of $+\phi$, so the $\cos\phi$ factor in Equation 33.31 is the same for both frequencies. The factor ΔV_{rms} is the same because the source voltage is fixed. According to Equation 33.27, changing $+\phi$ to $-\phi$ simply interchanges the values of X_L and X_C. Equation 33.25 tells us that such an interchange does not affect the impedance, so that the current I_{rms} in Equation 33.31 is the same for both frequencies.

33.9 (c). At resonance, $X_L = X_C$. According to Equation 33.25, this gives us $Z = R$.

33.10 (a). The higher the quality factor, the more sensitive the detector. As you can see from Figure 33.19, when $Q = \omega_0/\Delta\omega$ is high, a slight change in the resonance frequency (as might happen when a small piece of metal passes through the portal) causes a large change in current that can be detected easily.

33.11 (a) and (e). The current in an inductive circuit decreases with increasing frequency (see Eq. 33.9). Thus, an inductor connected in series with a woofer blocks high-frequency signals and passes low-frequency signals. The current in a capacitive circuit increases with increasing frequency (see Eq. 33.17). When a capacitor is connected in series with a tweeter, the capacitor blocks low-frequency signals and passes high-frequency signals.