The Common-Drain Amplifier

Basic Circuit

Fig. 1 shows the circuit diagram of a single stage common-drain amplifier. The object is to solve for the small-signal voltage gain, input resistance, and output resistance.

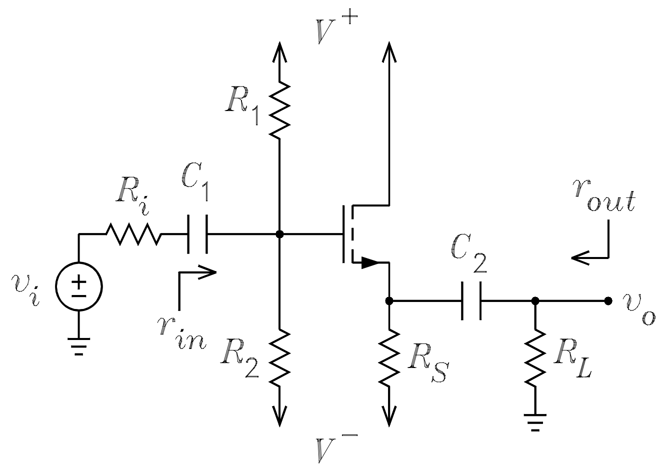

Figure 1: Common-drain amplifier.

DC Solution

(a) Replace the capacitors with open circuits. Look out of the 3 MOSFET terminals and make Thévenin equivalent circuits as shown in Fig. 2.

$$V_{GG} = \frac{V^+R_2 + V^-R_1}{R_1 + R_2} \qquad R_{GG} = R_1 \| R_2$$

$$V_{SS} = V^- \qquad R_{SS} = R_S \qquad V_{DD} = V^+ \qquad R_{DD} = 0$$

(b) Write the loop equation between the V_{GG} and the V_{SS} nodes.

$$V_{GG} - V_{SS} = V_{GS} + I_S R_{SS} = V_{GS} + I_D R_{SS}$$

(c) Use the equation for the drain current to solve for V_{GS}.

$$V_{GS} = \sqrt{\frac{I_D}{K}} + V_{TO}$$

(d) Solve the equations simultaneously.

$$I_D R_{SS} + \sqrt{\frac{I_D}{K}} + [(V_{GG} - V_{SS}) - V_{TO}] = 0$$

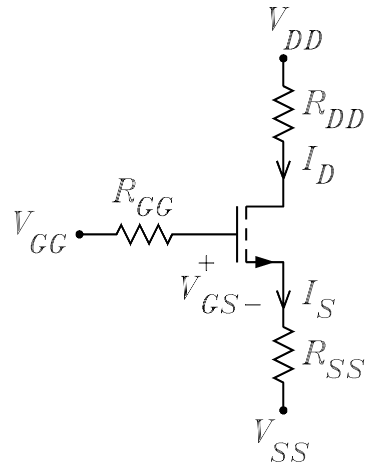

Figure 2: Bias circuit.

(e) Let $V_1 = (V_{GG} - V_{SS}) - V_{TO}$. Solve the quadratic for I_D.

$$I_D = \left(\frac{\sqrt{1 + 4KV_1R_{SS}} - 1}{2R_{SS}\sqrt{K}} \right)^2$$

(d) Verify that $V_{DS} > V_{GS} - V_{TO} = \sqrt{I_D/K}$ for the active mode.

$$V_{DS} = V_D - V_S = (V_{DD} - I_DR_{DD}) - V_{GG} = V_{DD} - V_{SS} - I_DR_{DD}$$

Small-Signal or AC Solutions

(a) Redraw the circuit with $V^+ = V^- = 0$ and all capacitors replaced with short circuits as shown in Fig. **??**.

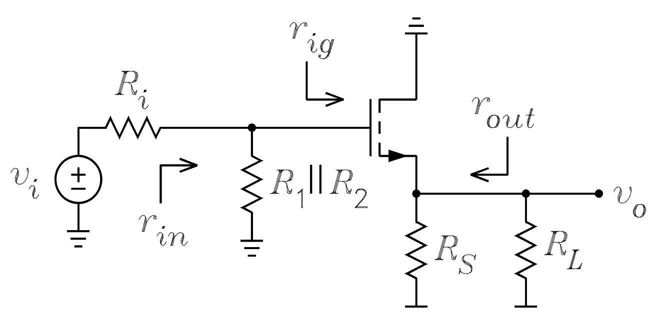

Figure 3: Signal circuit.

(b) Calculate g_m, r_s, and r_0 from the DC solution.

$$g_m = 2\sqrt{KI_D} \qquad r_s = \frac{1}{g_m} \qquad r_0 = \frac{V_A + V_{CE}}{I_C}$$

(c) Replace the circuits looking out of the gate with a Thévenin equivalent circuit as shown in Fig. 4.

$$v_{tg} = v_i \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \qquad R_{tg} = R_1 \| R_2$$

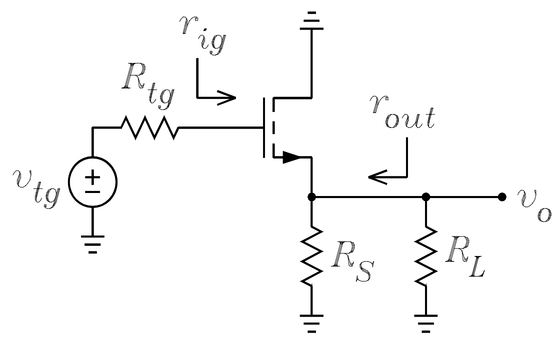

Figure 4: Signal circuit with Thévenin gate circuit.

Exact Solution

Note that the Thévenin resistance R_{td} seen looking out of the drain is zero. This exact solution is also valid for circuits where $R_{td} \neq 0$.

(a) Replace the circuit seen looking into the source with its Thévenin equivalent circuit as shown in Fig. 5. Solve for $v_{s(oc)}$.

$$v_{s(oc)} = v_{tg} \frac{r_0}{r_s + r_0} \qquad r_{is} = r_s \frac{r_0 + R_{td}}{r_s + r_0} \qquad R_{td} = 0$$

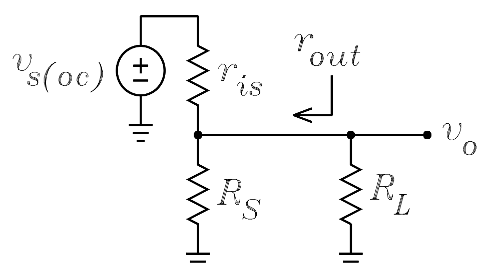

Figure 5: Thévenin source circuit.

(b) Solve for v_o.

$$v_o = v_{s(oc)} \frac{R_S \| R_L}{r_{is} + R_S \| R_L} = v_i \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \frac{R_S \| R_L}{r_{is} + R_S \| R_L}$$

(c) Solve for the voltage gain.

$$A_v = \frac{v_o}{v_i} = \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \frac{R_S \| R_L}{r_{is} + R_S \| R_L}$$

(d) Solve for r_{in}.

$$r_{in} = R_1 \| R_2$$

(e) Solve for r_{out}.

$$r_{out} = r_{is} \| R_S$$

(f) Special case for $r_0 = \infty$.

$$v_{s(oc)} = v_{tg} \qquad r_{is} = r_s$$

Example 1 *For the CS amplifier of Fig.* **??**, *it is given that* $R_i = 5\,\text{k}\Omega$, $R_1 = 5\,\text{M}\Omega$, $R_2 = 1\,\text{M}\Omega$, $R_D = 10\,\text{k}\Omega$, $R_S = 3\,\text{k}\Omega$, $R_3 = 50\,\Omega$, $R_L = 20\,\text{k}\Omega$, $V^+ = 24\,\text{V}$, $V^- = -24\,\text{V}$, $K_0 = 0.001\,\text{A}/\text{V}^2$, $V_{TO} = 1.75\,\text{V}$, $\lambda = 0.016\,\text{V}^{-1}$. *Solve for the gain* $A_v = v_o/v_i$, *the input resistance* r_{in}, *and the output resistance* r_{out}. *The capacitors can be assumed to be ac short circuits at the operating frequency.*

Solution. For the dc bias solution, replace all capacitors with open circuits. The Thévenin voltage and resistance seen looking out of the gate are

$$V_{GG} = \frac{V^+ R_2 + V^- R_1}{R_1 + R_2} = -16\,\text{V} \qquad R_{BB} = R_1 \| R_2 = 833.3\,\text{k}\Omega$$

The Thévenin voltage and resistance seen looking out of the source are $V_{SS} = V^-$ and $R_{SS} = R_S$. To calculate I_D, we neglect the Early effect by setting $K = K_0$. The bias equation for I_D is

$$I_D = \left(\frac{\sqrt{1 + 4KV_1 R_{SS}} - 1}{2\sqrt{K} R_{SS}} \right)^2 = 1.655\,\text{mA}$$

To test for the active mode, we calculate the drain-source voltage

$$V_{DS} = V_D - V_S = V^+ - \left(V^- + I_D R_{SS}\right) = 43.036\,\text{V}$$

This must be greater than $V_{GS} - V_{TO} = \sqrt{I_D/K} = 1.286\,\text{V}$. It follows that the MOSFET is biased in its active mode.

For the small-signal ac analysis, we need g_m, r_s, and r_0. When the Early effect is accounted for, the new value of K is given by

$$K = K_0 \left(1 + \lambda V_{DS}\right) = 1.689 \times 10^{-3}\,\text{A}/\text{V}^2$$

Note that this is an approximation because the Early effect was neglected in calculating V_{DS}. However, the approximation should be close to the true value. It follows that g_m, r_s, and r_0 are given by

$$g_m = 2\sqrt{KI_D} = 3.343 \times 10^{-3}\,\text{A}/\text{V} \qquad r_s = \frac{1}{g_m} = 299.135\,\Omega$$

$$r_0 = \frac{\lambda^{-1} + V_{DS}}{I_D} = 63.78\,\text{k}\Omega$$

For the small-signal analysis, V^+ and V^- are zeroed and the three capacitors are replaced with ac short circuits. The Thévenin voltage and resistance seen looking out of the gate are given by

$$v_{tg} = v_i \frac{R_1 \| R_2}{R_i + R_1 \| R_2} = 0.994 v_i \qquad R_{tg} = R_i \| R_1 \| R_2 = 4.97\,\mathrm{k\Omega}$$

The Thévenin voltage and resistance seen looking into the source are

$$v_{s(oc)} = v_{tg} \frac{r_0}{r_s + r_0} = 0.989 v_i \qquad R_{td} = 0 \qquad r_{is} = r_s \frac{r_0 + R_{td}}{r_s + r_0} = 297.738\,\Omega$$

The output voltage is given by

$$v_o = v_{s(oc)} \frac{R_S \| R_L}{r_{is} + R_S \| R_L} = 0.888 v_i$$

Thus the voltage gain is

$$A_v = \frac{v_o}{v_i} = 0.888$$

The input and output resistances are given by

$$r_{in} = R_1 \| R_2 = 833.3\,\mathrm{k\Omega} \qquad r_{out} = r_{is} \| R_S = 270.86\,\Omega$$

Alternate Solutions

Because the Thévenin resistance R_{td} seen looking out of the drain is zero, the drain-source resistance r_0 connects from source to ground. In this case, an exact solution can be obtained with r_0 in the circuit. In cases where $R_{td} > 0$, let $r_0 = \infty$ (an open circuit) to obtain approximate solutions.

Simplified T Model Solution

(a) After making the Thévenin equivalent circuits looking out of the gate, replace the MOSFET with the simplified T model as shown in Fig. 6.

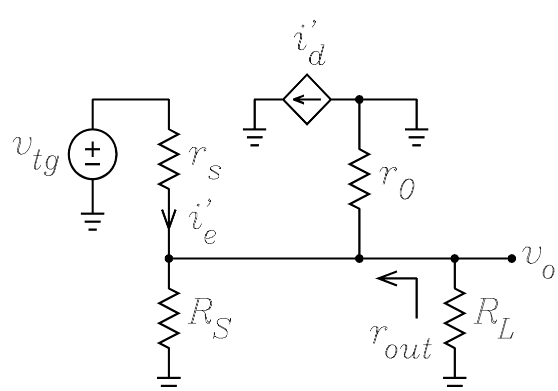

Figure 6: Simplified T model.

(b) Solve for v_o.

$$v_o = v_{tg}\frac{r_0\|R_S\|R_L}{r_s + r_0\|R_S\|R_L} = v_i\frac{R_1\|R_2}{R_s + R_1\|R_2}\frac{r_0\|R_S\|R_L}{r_s + r_0\|R_S\|R_L}$$

(c) Solve for the voltage gain.

$$A_v = \frac{v_o}{v_i} = \frac{R_1\|R_2}{R_s + R_1\|R_2}\frac{r_0\|R_S\|R_L}{r_s + r_0\|R_S\|R_L}$$

(d) Solve for r_{in}.

$$r_{in} = R_1\|R_2$$

(e) Solve for r_{out}.

$$r_{out} = r_0\|r_s\|R_S$$

Example 2 *Use the simplified T-model solutions to calculate the values of A_v, r_{in}, and r_{out} for Example 1.*

$$A_v = 0.994 \times 0.893 = 0.888$$

$$r_{in} = R_1\|R_2 = 833.3\,\mathrm{k\Omega} \qquad r_{out} = 270.86\,\Omega$$

π Model Solution

(a) After making the Thévenin equivalent circuits looking out of the gate and source, replace the MOSFET with the π model as shown in Fig. 7.

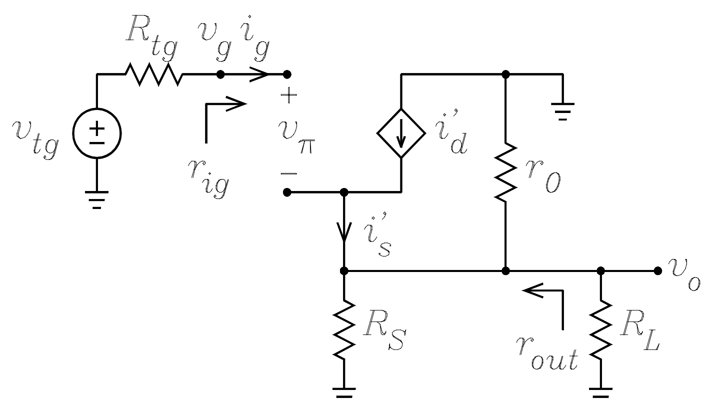

Figure 7: Hybrid-π model.

(b) Solve for i'_s.

$$\begin{aligned} v_{tg} &= v_\pi + i'_s r_0\|R_S\|R_L = \frac{i'_d}{g_m} + i'_s r_0\|R_S\|R_L = \frac{i'_s}{g_m} + i'_s r_0\|R_S\|R_L \\ \implies i'_s &= \frac{v_{tg}}{\dfrac{1}{g_m} + r_0\|R_S\|R_L} \end{aligned}$$

(c) Solve for v_o.

$$\begin{aligned}
v_o &= i_s' r_0 \| R_S \| R_L = \frac{v_{tg}}{\dfrac{1}{g_m} + r_0 \| R_S \| R_L} r_0 \| R_S \| R_L \\
&= v_i \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \frac{r_0 \| R_S \| R_L}{\dfrac{1}{g_m} + r_0 \| R_S \| R_L}
\end{aligned}$$

(d) Solve for the voltage gain.

$$A_v = \frac{v_o}{v_i} = \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \frac{r_0 \| R_S \| R_L}{\dfrac{1}{g_m} + r_0 \| R_S \| R_L}$$

(e) Solve for r_{in}.

$$r_{in} = R_1 \| R_2$$

(f) Solve for r_{out}.First, solve for the open-circuit output voltage. This is the output voltage with $R_L = \infty$.

$$v_{o(oc)} = v_{tg} \frac{r_0 \| R_S}{\dfrac{1}{g_m} + r_0 \| R_S}$$

Next, solve for the short-circuit output current. This is the output current with $R_L = 0$. The output current is given by

$$i_o = \frac{v_o}{R_L} = \frac{v_{tg}}{\dfrac{1}{g_m} + r_0 \| R_S \| R_L} \frac{r_0 \| R_S \| R_L}{R_L} = \frac{v_{tg}}{\dfrac{1}{g_m} + r_0 \| R_S \| R_L} \frac{r_0 \| R_S}{R_L + r_0 \| R_S}$$

Now, let $R_L = 0$ to obtain

$$i_{o(sc)} = \frac{v_{tg}}{\dfrac{1}{g_m}} = g_m v_{tg}$$

The output resistance is given by

$$r_{out} = \frac{v_{o(oc)}}{i_{o(sc)}} = \frac{r_0 \| R_S}{\dfrac{1}{g_m} + r_0 \| R_S} \frac{1}{g_m} = \frac{1}{g_m} \| r_0 \| R_S$$

Note this is simply $r_s \| r_0 \| R_S$, an answer that is obvious using the simplified T model.

Example 3 *Use the π model solutions to calculate the values of A_v, r_{in}, and r_{out} for Example 1.*

$$A_v = 0.994 \times 0.893 = 0.888$$

$$r_{in} = R_1 \| R_2 = 833.3\,\mathrm{k}\Omega \qquad r_{out} = 270.86\,\Omega$$

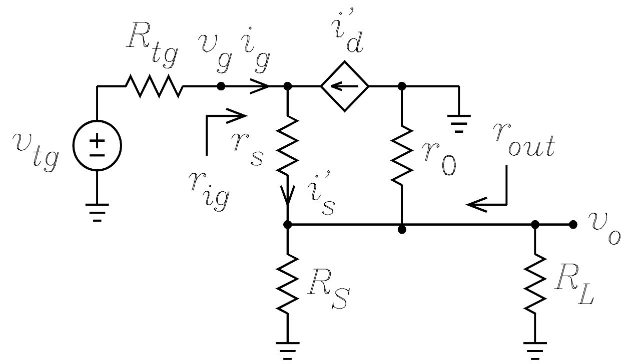

Figure 8: T model circuit.

T Model Solution

(a) After making the Thévenin equivalent circuits looking out of the gate and source, replace the MOSFET with the T model as shown in Fig. 8.

(b) Solve for i'_s.

$$\begin{aligned} v_{tg} &= i'_s \left(r_s + r_0 \| R_S \| R_L \right) \\ \implies & i'_s = \frac{v_{tg}}{r_s + r_0 \| R_S \| R_L} \end{aligned}$$

(c) Solve for v_o.

$$\begin{aligned} v_o &= i'_s r_0 \| R_S \| R_L = \frac{v_{tg}}{r_s + r_0 \| R_S \| R_L} r_0 \| R_S \| R_L \\ &= v_i \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \frac{r_0 \| R_S \| R_L}{r_s + r_0 \| R_S \| R_L} \end{aligned}$$

(d) Solve for the voltage gain.

$$A_v = \frac{v_o}{v_i} = \frac{R_1 \| R_2}{R_s + R_1 \| R_2} \frac{r_0 \| R_S \| R_L}{r_s + r_0 \| R_S \| R_L}$$

(e) Solve for r_{in}.

$$r_{in} = R_1 \| R_2$$

(f) Solve for r_{out}. First, solve for the open-circuit output voltage. This is the output voltage with $R_L = \infty$.

$$v_{o(oc)} = v_{tg} \frac{r_0 \| R_S}{r_s + r_0 \| R_S}$$

Next, solve for the short-circuit output current. This is the output current with $R_L = 0$. The output current is given by

$$i_o = \frac{v_o}{R_L} = \frac{v_{tg}}{r_s + r_0 \| R_S \| R_L} \frac{r_0 \| R_S \| R_L}{R_L} = \frac{v_{tg}}{r_s + r_0 \| R_S \| R_L} \frac{r_0 \| R_S}{R_L + r_0 \| R_S}$$

Now, let $R_L = 0$ to obtain

$$i_{o(sc)} = \frac{v_{tg}}{r_s}$$

The output resistance is given by

$$r_{out} = \frac{v_{o(oc)}}{i_{o(sc)}} = \frac{r_0 \| R_S}{r_s + r_0 \| R_S} r_s = r_s \| r_0 \| R_S$$

This is the same answer obtained from the simplified T model.

Example 4 *Use the T-model solutions to calculate the values of A_v, r_{in}, and r_{out} for Example 1.*

$$A_v = 0.994 \times 0.893 = 0.888$$

$$r_{in} = R_1 \| R_2 = 833.3\,\mathrm{k}\Omega \qquad r_{out} = 270.86\,\Omega$$