Guide for Determining the Fire Endurance of Concrete Elements

Reported by ACI Committee 216

Melvin S. Abrams
Chairman

Stanley G. Barton
James E. Bihr
Richard W. Bletzacker
Merle E. Brander
Boris Bresler
John W. Dougill
Frank G. Erskine
Richard G. Gewain
A. H. Gustaferro
Tibor Z. Harmathy
Lionel Issen
Donald W. Lewis
Howard R. May
W. J. McCoy
Richard Muenow
George E. Troxell
G. M. Watson
Roger H. Wildt
N. G. Zoldners

The committee voting to revise this document was as follows:

Tibor Z. Harmathy
Chairman

Melvin S. Abrams
Stanley G. Barton
Richard W. Bletzacker
Paul C. Breeze
Boris Bresler
John W. Dougill
William L. Gamble
Richard G. Gewain
Armand H. Gustaferro
Tung D. Lin*
Howard R. May
Jaime Moreno
Richard A. Muenow
Harry C. Robinson
Thomas J. Rowe
F. R. Vollert

*Chairman of the editorial subcommittee who prepared this report

This Guide for determining the fire resistance of concrete elements is a summary of practical information intended for use by architects. engineers and building officials who must design concrete structures for particular fire resistances or evaluate structures as designed. The Guide contains information for determining the fire endurance of simply supported slabs and beams; continuous beams and slabs; floors and roofs in which restraint to thermal expansion occurs; walls; and reinforced concrete columns. Information is also given for determining the jire endurance of certain concrete members based on heat transmission criteria.

Also included is information on the properties of steel and concrete at high temperatures, temperature distributions within concrete members exposed to fire, and in the Appendix, a reliability-based technique for the calculation of fire endurance requirements.

Keywords: acceptability; beams (supports), columns (supports); compressive strength; concrete slabs, creep properties; heat transfer; fire ratings; fire resistance; fire tests; masonry walls; modulus of elasticity; normalized heat load; prestressed concrete; prestressing steels; reinforced concrete; reinforcing steels; reliability; stress-strain relationship; structural design; temperature distribution; thermal conductivity; thermal diffusivity; thermal expansion; thermal properties; walls.

CONTENTS

This report superceded ACI 216R-81 (Revised 1987). In the 1989 revisions, an appendix has been added outlining a reliability-based technique for the calculation of fire endurance requirements of building elements. along with new Example 7, which demonstrates the use of this technique. References have been added.

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CHAPTER 1-GENERAL

1.1-Scope

Building codes require that the resistance to fire be considered for most buildings. The type of occupancy, the size of building and its position on the property all affect the fire resistance ratings required of various building elements.

Higher fire resistance ratings often result in lower fire insurance rates, because insurance companies are concerned about fire resistance.

For the most part, fire resistance ratings have been determined by the results of standard fire tests. More recently, rational design methods have been developed which allow the fire resistance to be determined by calculations (Anderberg 1978; Becker and Bresler 1977; Bresler January 1976; Bresler September 1976; Bresler 198.5; Ehm and van Postel 1967; Gustaferro 1973; Gustaferro 1976; Gustaferro and Martin 1977; lding et al. 1977; Iding and Bresler 1984; Lie and Harmathy 1972; Nizamuddin and Bresler 1979; Pettersson 1976). The rational design concept makes use of study and research into the properties of materials at high temperatures, the behavior of structures during a fire, and basic structural engineering principles.

This guide illustrates the application of the structural engineering principles and information on properties of materials to determine the fire resistance of concrete construction.

Generally, the information in the Guide is applicable to flat slab floors and rectangular beams. Additional materials and techniques are required for applying the design procedure given in the Guide for structural members that have other geometries.

A technique for the calculation of fire endurance requirements is discussed in the Appendix.

1.2-Definitions and Notation

1.2.1-*Definitions*

Built-Up Roofing-Roof covering consisting of at least 3-ply 15 lb/100 ft^2 (0.75 kg/m^2) type felt and not having in excess of 1.20 lb/ft^2 (5.9 kg/m^2) of hot-mopped asphalt without gravel surfacing (see Section 7.3 of ASTM E 119-83).

Carbonate Aggregate Concrete-Concrete made with aggregates consisting mainly of calcium or magnesium carbonate, e.g., limestone or dolomite.

Cellular Concrete-A lightweight insulating concrete made by mixing a preformed foam with portland cement slurry and having a dry unit weight of about 30 pcf (480 kg/m^3).

Cold-Druwn Steel-Steel used in prestressing wire or strand. Note: Does not include high strength alloy steel bars used for post-tensioning tendons.

Critical Temperature-The temperature of the steel in unrestrained flexural members during fire exposure at which the nominal moment strength of the members is reduced to the applied moment due to service loads.

End Point Criteria-The conditions of acceptance for an ASTM E 119 fire test.

Fire Endurance-A measure of the elapsed time during which a material or assembly continues to exhibit fire resistance under specified conditions of test and performance; as applied to elements of buildings it shall be measured by the methods and to the criteria defined in ASTM E 119. (Defined in ASTM E 176)

Fire Resistance-The property of a material or assembly to withstand fire or to give protection from it; as applied to elements of buildings, it is characterized by the ability to confine a fire or to continue to perform a given structural function, or both. (Defined in ASTM E 176)

Fire Resistance Rating (sometimes called *fire rating, fire resistance classification* or *hourly rating)*-A legal term defined in building codes, usually based on fire endurance; fire resistance ratings are assigned by building codes for various types of construction and occupancies and are usually given in half-hour increments.

Fire Test-See standard fire test.

Glass Fiber Board-Fibrous glass roof insulation consisting of inorganic glass fibers formed into rigid boards using a binder; the board has a top surface faced with asphalt and kraft paper reinforced with glass fiber.

Gypsum Wallboard Type "X"-A mill-fabricated product made of a gypsum core containing special minerals and encased in a smooth, finished paper on the face side and liner paper on the back.

Heat Transmission End Point-An acceptance criterion of ASTM E 119 limiting the temperature rise of the unexposed surface to an average of 250 F (139 C) or a maximum of 325 F (181 C) at any one point.

High Strength Alloy Steel Bars—Bars used for post-tensioning conforming to the requirements of ASTM A 722.

Hot-Rolled Steel—Steel used in reinforcing bars or structural steel members.

Intumescent Mastic—A solvent-base spray-applied coating which reacts to heat at about 300 F (150 C) by foaming to a multicellular structure having 10 to 15 times its initial thickness.

Isotherm—A line drawn on the cross section of a member connecting points of the same temperature.

Lightweight Aggregate Concrete—Concrete made with aggregates of expanded clay, shale, slag, or slate or sintered fly ash, and weighing about 85 to 115 pcf (1360 to 1840 kg/m^3).

Mineral Board—A rigid felted thermal insulation board consisting of either felted mineral fiber or cellular beads of expanded aggregate formed into flat rectangular units.

Normalized Heat Load—Total heat absorbed by unit area of the compartment boundaries, such as the walls, floors, and ceiling of a room, during exposure to fire, divided by the thermal absorptivity of the boundaries.

Normal Weight Concrete—Any concrete made with natural aggregates or air-cooled blast furnace slag, cement, and water having a unit weight of about 135 to 155 pcf (2160 to 2480 kg/m^3).

Perlite Concrete—A lightweight insulating concrete having a dry unit weight of about 30 pcf (480 kg/m^3) made with perlite aggregate concrete. Note: Perlite aggregate is produced from a volcanic rock which, when heated, expands to form a glass-like material of cellular structure.

Restrained Assembly Classification—The classification derived from fire tests of floors, roofs, or beams in accordance with acceptance criteria of Sections 29 and 35 of ASTM E 119-83. Such a classification is considered to be applicable in buildings when (1) the surrounding or supporting structure is capable of resisting the thermal expansion induced by a standard fire (ASTM E 119), or (2) the assembly has structural continuity over supports or has structural continuity with its support.

Sand-Lightweight Concrete—Concrete made with a combination of expanded clay, shale, slag, or slate or sintered fly ash and natural sand. Its unit weight is generally between 105 and 120 pcf (1680 and 1920 kg/m^3).

Siliceous Aggregate Concrete—Concretes made with normal weight aggregates consisting mainly of silica or compounds other than calcium or magnesium carbonate.

Spray-Applied Coatings, Sprayed Insulation—See intumescent mastic, sprayed mineral fiber, or vermiculite cementitious material.

Sprayed Mineral Fiber—A blend of refined mineral fibers and inorganic binders. Water is added during the spraying operation, and the untamped unit weight is about 13 pcf (208 kg/m^3).

Standard Fire Exposure—The time-temperature relationship defined by ASTM E 119.

Standard Fire Test—The test prescribed by ASTM E 119.

Steel Temperature End Point—The acceptance criterion of ASTM E 119 defining the limiting steel temperatures for unrestrained assembly classifications, i.e., 1100 F (593 C) average or 1300 F (704 C) maximum for structural steel, 1100 F (593 C) average for reinforcing steel, and 800 F (427 C) for cold-drawn prestressing steel; for restrained classifications of beams spaced more than 4 ft (1.2 m) on centers, these limits must not be exceeded for the first half of the fire endurance period.

Structural End Point—The acceptance criterion of ASTM E 119 which states that the specimen shall sustain the applied load without collapse.

Thermal Absorptivity—A material property defined as $\sqrt{k\rho c}$, where k is thermal conductivity, ρ is density, and c is specific heat. Note: This property characterizes how readily a solid absorbs heat (after a transitional period) from a fluid at different temperature.

Unrestrained Assembly Classification—A classification derived from fire tests of floors, roofs, or beams in accordance with the acceptance criteria of Section 30 of ASTM E 119. Such a classification is considered applicable in buildings when the conditions for a restrained assembly classification are not met.

Vermiculite Cementitious Material—A cementitious mill-mixed material to which water is added to form a mixture suitable for spraying. The mixture has a wet unit weight of about 55 to 60 pcf (880 to 960 kg/m^3).

Vermiculite Concrete—A lightweight insulating concrete made with vermiculite aggregate concrete which is a laminated micaceous material produced by expanding the ore at high temperatures. When added to a portland cement slurry the resulting concrete has a dry unit weight of about 30 pcf (480 kg/m^3).

1.2.2—*Notation*

a = average web thickness of hollow masonry unit (Chapter 3)
a = depth of equivalent rectangular stress block
a_θ = depth of equivalent rectangular stress block at elevated temperatures
a_{1m} = minimum measured web thickness
A = gross cross-sectional area of slab resisting the thrust force
A_{ps} = area of prestressed reinforcement in tension zone
A_s = area of nonprestressed tension reinforcement
b = width of compression face of member
b = width of joist (Chapter 7)
b = average web spacing (Chapter 3)
b_E = width of compression face of column strip at external support
C = degrees Celsius
d = distance from extreme compression fiber to centroid of tension reinforcement
d = minimum length of top reinforcing bars
d' = distance from extreme compression fiber to centroid of tension reinforcement over the support
E = modulus of elasticity
f_c' = specified compressive strength of concrete
f_{ps} = stress in prestressed reinforcement at nominal strength
f_{pu} = specified tensile strength of prestressing tendons
f_{py} = specified yield stress of prestressing tendons
f_y = specified yield stress of nonprestressed reinforcement
$f_{y\theta}$ = specified yield stress of nonprestressed reinforcement at elevated temperatures
F = degrees Fahrenheit

h = overall thickness of member
h = distance between centroidal axis and line of thrust action [Fig. 2.3.2.1(b)]
h = height of unit (Fig. 3.3.2.2)
h_e = equivalent thickness
k = thermal conductivity (at room temperature)
K = Kelvins
l = length of unit (Chapter 3)
l = span length
l = average face shell thickness (Chapter 3)
l = length of span of two-way flat plates in direction parallel to that of the reinforcement being determined
l_d = bar development length
l_m = minimum measured shell thickness
m = fraction of weight loss of concrete
M = design moment
M_n = nominal moment strength at section
$M_{n\theta}$ = nominal moment strength at section at elevated temperatures
$M^+_{n\theta}$ = nominal positive moment strength at section at elevated temperatures
M_{x1} = moment due to service load at section x_1
R = universal gas constant
s = heated perimeter
T = thrust
t = time
t_θ = temperature compensated time
u = concrete cover over main reinforcing bar or average effective cover
V = volume of displaced water
w = applied load (dead + live)
w = unit weight of concrete
w_{sd} = service dead load
y_{b1} = distance from centroidal axis of flexural member to extreme bottom fiber
Z = Zener-Hollomon parameter
Z' = A/s
α = linear coefficient of thermal expansion
β = constant
Δ = deflection (Chapter 2)
ΔH = activation energy of creep
Δl = elongation of slab due to temperamre
ε_t = creep strain
ε_{to} = creep parameter
θ = temperature
θ_1 = temperature, F
θ_2 = temperature, C
κ = thermal diffusivity (at room temperature)
ρ = density of concrete
ρ_w = density of water
τ = fire resistance of concrete wall in natural moist condition
τ_o = fire resistance of masonry wall in dry condition
ϕ = volumetric moisture content
ω = $A_s f_y / bdf'_c$
ω_p = $A_{ps} f_{pu} / bdf'_c$

1.3-Standard fire tests of building construction and materials

ASTM E 119 specifies the test methods and procedures for determing the fire resistive properties of building components, and is a generally accepted standard for performing fire tests.

1.3.1-*Endpoint criteria of ASTM E I19*

1.3.1.1-The test assembly must sustain the applied load during the fire endurance test (structural end point).

1.3.1.2-Flame or gases hot enough to ignite cotton wasie must not pass through the test assembly (flame passage end point).

1.3.1.3-Transmission of heat through the test assembly shall not increase the temperature of the unexposed surface more than an average of 250 F (139 C) or 325 F (181 C) at any one point (heat transmission end point).

1.3.1.4-There are additional end point criteria for special cases. Those applicable to concrete are as follows:

1.3.1.4.1-Unrestrained concrete structural members: average temperature of the tension steel at any section must not exceed 1100 F (593 C) for reinforcing bars or 800 F (427 C) for cold-drawn prestressing steel.

1.3.1.4.2-Restrained concrete beams more than 4 ft (1.2m) on centers: the temperatures in1.3.1.4.1 must not be exceeded for classifications of 1 hr or less; for classifications longer than 1 hr, the above temperatures must not be exceeded for first half of the classification period or 1 hr, whichever is longer.

1.3.1.4.3-Restrained concrete beams spaced 4 ft (1.2 m) or less on centers and slabs are not subjected to the steel temperature limitations.

1.3.1.4.4-Walls and partitions must meet the same criteria as in1.3.1.1, 1.3.1.2, and 1.3.1.3. In addition, they must sustain a hose stream test.

1.4-Application of design principles

In the design of a structural member, the ratio of the load carrying capacity and the anticipated applied loads is often expressed in terms of a “factor of safety.” In designing for fire, the “factor of safety” is contained within the fire resistance rating. Thus for a given situation, a member with a 4 hr rating would have a greater “factor of safety” than one with a 2 hr rating. The introduction to ASTM E 119 states. “When a factor of safety exceeding that inherent in the test conditions is desired, a proportional increase should be made in the specified time-classification period.”

The design methods and examples in this Guide are consistent with the strength (ultimate) design principles of ACI 318. BCGWC the factors of safety in design for fire are included in the resistance ratings, the load factors and strength reduction factor (Sections 9.2 and 9.3) are equal to 1.0 when designing for fire resistance.

CHAPTER 2-FIRE ENDURANCE OF CONCRETE SLABS AND BEAMS

2.1-Simply supported (unrestrained) slabs and beams

2.1.1 *Structural behavior* -Fig. 2.1.1(a) and (b) illustrate a simply supported reinforced concrete slab. The rocker and roller supports indicate that the ends of the slab are free to rotate and expansion can occur without resistance. The reinforcement consists of straight bars locat d near the bottom of the slab. If the underside of the slab is exposed to fire, the bottom of the slab will expand more than the top, resulting in a deflection of the slab. The tensile strength of the concrete and steel near the bottom of the slab will decrease as the temperature increases. When the strength of the steel at elevated temperature reduces to that of the stress in the steel, flexural collapse will occur (Gustaferro and Selvaggio 1067).

Fig. 2.1.1(b) illustrates the behavior of a simply supported slab exposed to fire from beneath. If the reinforcement is straight and uniform throughout the length, the nominal moment strength will be constant throughout the length

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) \qquad (2\text{-}1)$$

where

A_s is the area of the reinforcing steel

f_y is the yield strength of the reinforcing steel

d is the distance from the centroid of the reinforcing steel to the extreme compressive fiber

a is the depth of the equivalent rectangular compressive stress block at ultimate load, and is equal to $A_s f_y/0.85 f_c' b$ where f_c' is the cylinder compressive strength of the concrete and b is the width of the slab

If the slab is uniformly loaded, the moment diagram will be parabolic with a maximum value at midspan

$$M = \frac{wl^2}{8} \qquad (2\text{-}2)$$

where w is dead plus live load per unit of length, and
l is span length.

It is generally assumed that during a fire the dead and live loads remain constant. However, the material strengths are reduced so that the retained nominal moment strength is

$$M_{n\theta} = A_s f_{y\theta} \left(d - \frac{a_\theta}{2} \right) \qquad (2\text{-}3)$$

in which θ signifies the effects of elevated temperatures. Note that A_s and d are not affected, but $f_{y\theta}$ is reduced. Similarly a_θ is reduced, but the concrete strength at the top of the slab f_c' is generally not reduced significantly. If, however. the compressive zone of the concrete is heated, an appropriate reduction should be assumed.

Flexural failure can be assumed to occur when $M_{n\theta}$ is reduced to M. From this statement. it can be noted that the fire endurance depends on the load intensity and the strength-temperature characteristics of steel. In turn, the duration of the fire until the "critical" steel temperature is reached depends upon the protection afforded to the reinforcement.

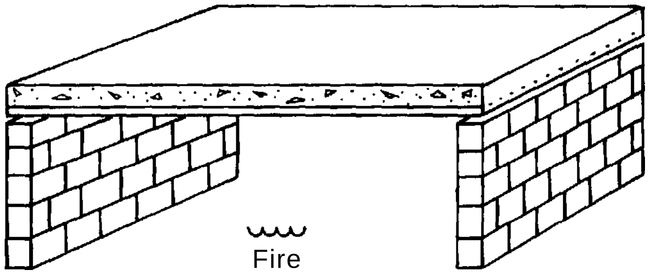


Fig. 2.1.1 (a)-Simply supported reinforced concrete slab subjected to fire from below

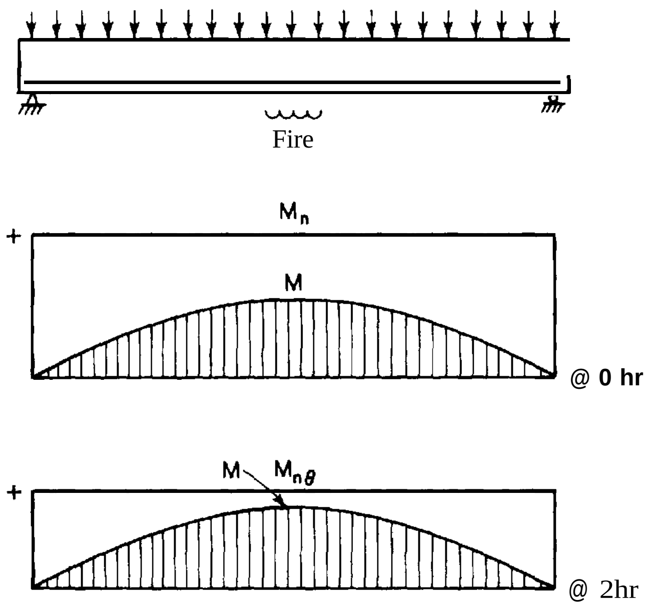


Fig. 2.1.1 (b)-Moment diagrams for simply supported beam or slab before and during fire exposure

Usually the protection consists of the concrete cover, i.e., the thickness of concrete between the fire exposed surface and the reinforcement. In some cases, additional protective layers of insulation or tnetnbrane ceilings might be present.

For prestressed concrete the nominal moment strength formulas must be modified by substituting f_{ps} for f_y and A_{ps} for A_s, where f_{ps} is the stress in the prestressing steel at ultimate load, and A_{ps} is the area of the prestressing steel. In lieu of an analysis based on strain compatibility the value of f_{ps} can be assumed to be

$$f_{ps} = f_{pu} \left(1 - \frac{0.5\, A_{ps} f_{pu}}{b d f_c'} \right) \qquad (2\text{-}4)$$

where f_{pu} is the ultimate tensile strength of the prestressing steel.

2.1.2 *Estimating structural fire endurance*-Fig. 2.1.2.1 shows the fire endurance of simply supported concrete slabs as affected by type of reinforcement (hot-rolled reinforcing bars and cold-drawn wire or strand), type of concrete (carbonate, siliceous, and lightweight aggregate), moment intensity, and the thickness of concrete between the center of the reinforcement and the fire exposed surface (referred to as "u"). If the reinforcement is distributed over the tensile zone

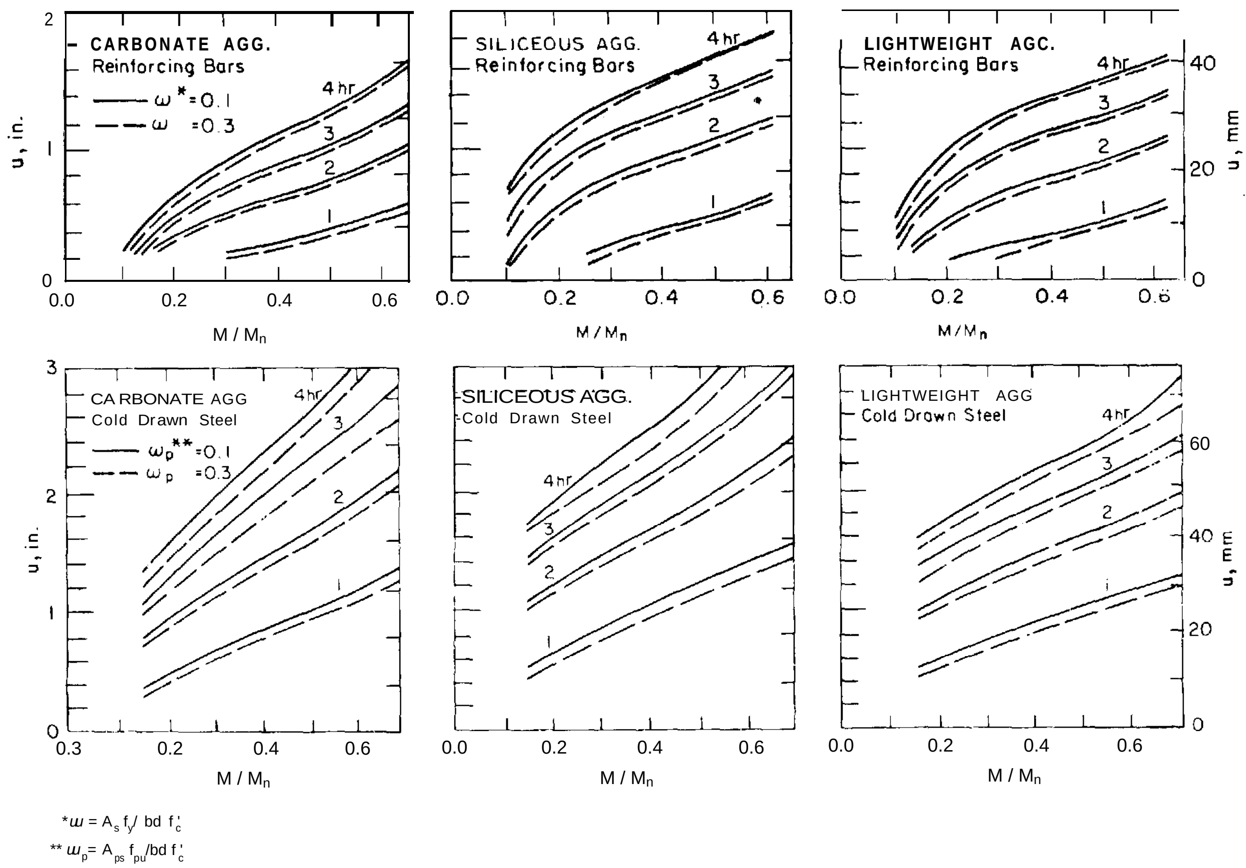


Fig. 2.1.2.1-Fire endurance of concrete slabs as influenced by aggregate type, reinforcing steel type, moment intensity and u *(defined in Section 2.1.2)*

of the cross section, the value of *u* is the average of the *u* distances of the individual bars or strands in the tensile zone. The curves are applicable to the bottom face shells of hollow-core slabs as well as to solid slabs.

The graphs in Fig. 2.1.2.1 can be used to estimate the fire endurance of simply supported concrete beams by using "effective *u*" rather than "*u*". Effective *u* accounts for beam width by assuming that the *u* values for corner bars or tendons are reduced by one-half for use in calculating the average *u*.

Examples 1 and 2 (in Chapter 8) illustrate the use of Fig. 2.1.2.1 in estimating the fire endurance of a slab and a beam. Note: Gustaferro and Martin (1977) present a variety of examples using prestressed concrete. The same principles are applicable to reinforced concrete.

2.2-Continuous beams and slabs

2.2.1 *Structural behavior*-Structures that are continuous or otherwise statically indeterminate undergo changes in stresses when subjected to fire (Abrams et al. 1976; Ehm and van Postel 1967; Gustaferro 1970; TN0 Institute for Structural Materials and Building Structures Report No. B l-59-22). Such changes in stress result from temperature gradients within structural members, or changes in strength of structural materials at high temperatures, or both.

Fig. 2.2.1 shows a continuous beam whose underside is exposed to fire. The bottom of the beam becomes hotter than the top and tends to expand more than the top. This differential heating causes the ends of the beam to tend to lift from their supports, thus increasing the reaction at the interior support. This action results in a redistribution of moments, i.e., the negative moment at the interior support increases while the positive moments decrease.

During the course of a fire, the negative moment reinforcement (Fig. 2.2.1) remains cooler than the positive moment reinforcement because it is better protected from the fire. Thus, the increase in negative moment can be accommodated. Generally, the redistribution that occurs is sufficient to cause yielding of the negative moment reinforcement. The resulting decrease in positive moment means that the positive moment reinforcement can be heated to a higher temperature before failure will occur. Thus, it is apparent that the fire endurance of a continuous reinforced concrete beam is generally significantly longer than that of a similar simply supported beam loaded to the same moment intensity.

2.2.2 *Detailing precautions*-It should be noted that the amount of redistribution that occurs is sufficient to cause yielding of the negative moment reinforcement. Since by increasing the amount of negative moment reinforcement, a greater negative moment will be attracted, care must be exercised in designing the member to assure that flexural tension will govern the design. To avoid a compressive failure in the negative moment region, the amount of negative moment reinforcement should be small enough so that ω, i.e., $A_s f_y / b d f'_c$ is less than about 0.30 even after reductions due to temperature in f_y, f'_c, b, and d are taken into account. Furthermore, the negative moment reinforcing bars must be long enough to accommodate the complete redistributed moment and change in the location of inflection points. It is recommended that at least 20 percent of the maximum negative moment reinforcement in the span extend throughout the span

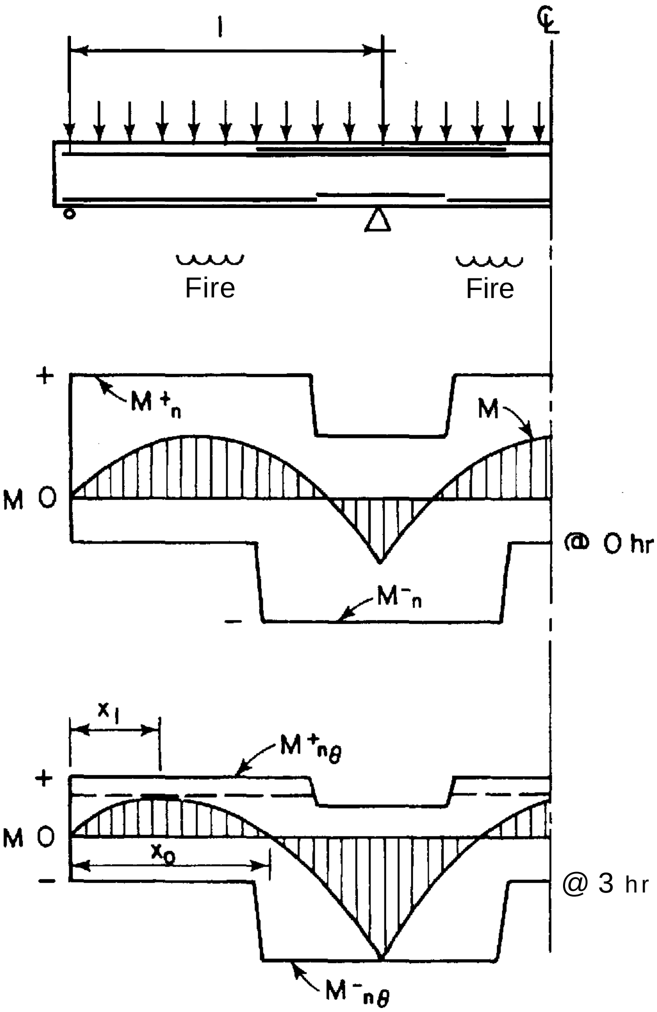


Fig.2.2.1-Moment diagrams for one-half of a continuous three-spun beam before and during fire exposure

(FIP /CEB Report on Methods of Assessment of Fire Resistance of Concrete Structural Members 1978).

2.2.3 *Estimating structural fire endurance*-The charts in Fig. 2.1.2.1 can be used to estimate the fire endurance of continuous beams and slabs. To use the charts, first estimate the negative moment at the supports taking into account the temperaturcs of the negative moment reinforcement and of the concrete in compressive zone near the supports (see Fig. 2.2.3). Then estimate the maximum positive moment after redistribution. By entering the appropriate chart with the ratio of that positive moment to the initial positive nominal moment strength, the fire endurance for the positive moment region can be estimated. If the resulting fire endurance is considerably different from that originally assumed in estimating the steel and concrete temperatures. a more accurate estimate can be made by trial and error. Usually such refinement is unnecessary.

It is also possible to design the reinforcement in a continuous beam or slab for a particular fire endurance period. Example 3 (in Chapter 8) illustrates this application of Fig. 2.1.2.1. From the lowermost diagram of Fig. 2.2.1, the beam can be expected to collapse when the positive nominal moment strength $M^+_{n\theta}$ is reduced to the value indicated by the

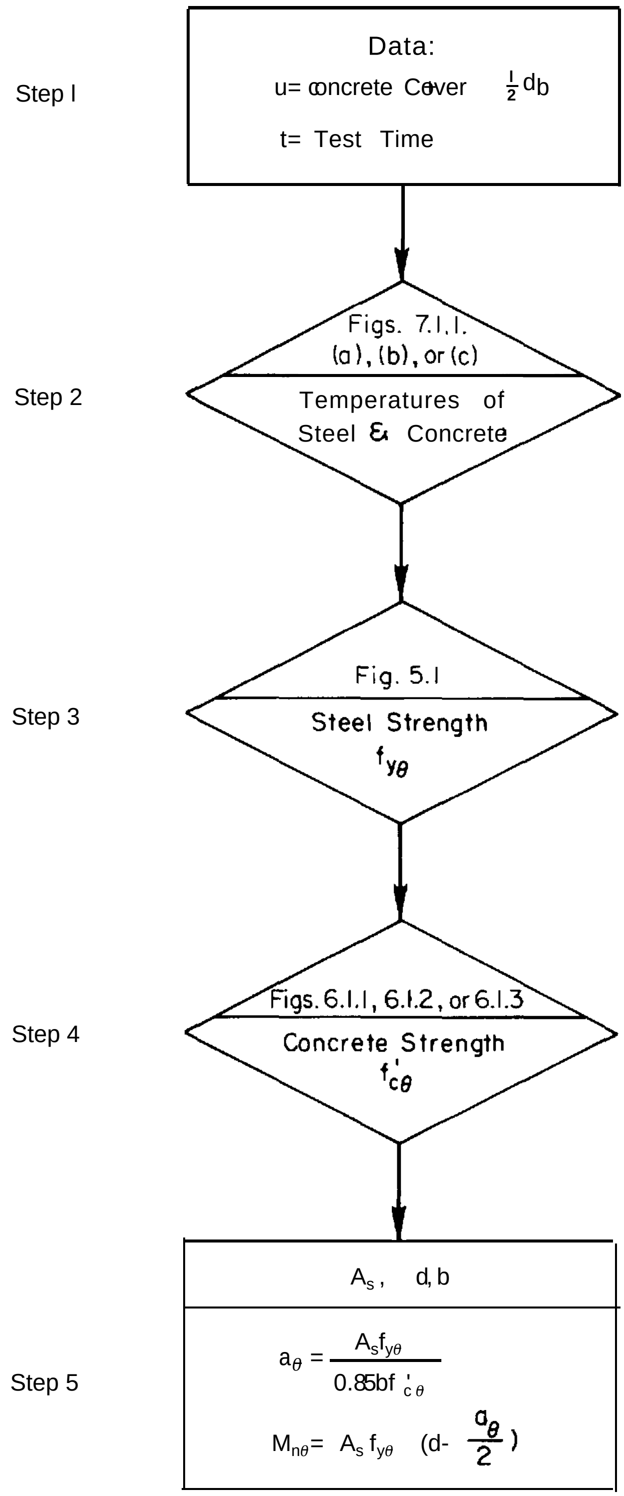


Fig. 2.2.3-Computational procedure for $M_{n\theta}$

dashed horizontal line, i.e., when the applied moment at a point x_1 from the outer support $M_{x1} = M^+_{n\theta}$

For a uniform applied load w

$$M_{x1} = \frac{wlx_1}{2} - \frac{wx_1^2}{2} - \frac{M^-_{n\theta}x_1}{l} = M^+_{n\theta}$$

$$x_1 = \frac{l}{2} - \frac{M^-_{n\theta}}{wl}$$

and

$$M^-_{n\theta} = \frac{wl^2}{2} - wl^2 \sqrt{\frac{2M^+_{n\theta}}{wl^2}}$$

Also

$$x_o = 2x_1$$

For a symmetrical interior bay

$$x_1 = l/2$$

$$M_{x1} = \frac{wl^2}{8} - M^-_{n\theta}$$

or

$$M^-_{n\theta} = \frac{wl^2}{8} - M^+_{n\theta}$$

2.3-Fire endurance of floors and roofs in which restraint to thermal expansion occurs

2.3.1 *Structural behavior*-If a fire occurs beneath a small interior portion of a large reinforced concrete slab, the heated portion will tend to expand and push against the surrounding part of the slab. In turn, the unheated part of the slab exerts compressive forces on the heated portion. The compressive force, or thrust, acts near the bottom of the slab when the fire first occurs, but as the fire progresses the line of action of the thrust rises (Selvaggio and Carlson 1967). If the surrounding slab is thick and heavily reinforced. the thrust forces that occur can be quite large, but considerably less than those calculated by use of elastic properties of concrete and steel together with appropriate coefficients of expansion. At high

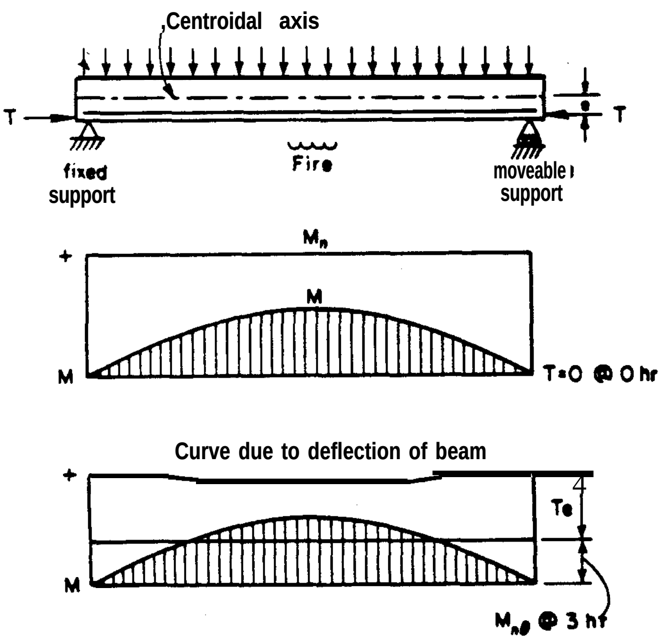


Fig. 2.3.1-Moment diagrams for axially restrained beam during fire exposure. Note that at 3 hr $M_{n\theta}$ is less than M and effects of axial restraint permit beam to continue to support load (Gustaferro 1970)

temperatures, creep and stress relaxation play an important role. Nevertheless, the thrust is generally great enough to increase the fire endurance significantly. In most fire tests of restrained assemblies (Lin and Abrams 1983), the fire endurance is determined by temperature rise of the unexposed surface rather than by structural considerations, even though the steel temperatures often exceed 1500 F (815 C) (Gustaferro 1970; Issen, Gustaferro, and Carlson 1970).

The effects of restraint to thermal expansion can be characterized as shown in Fig. 2.3.1. The thermal thrust acts in a manner similar to an external prestressing force, which, in effect, increases the positive nominal moment strength.

2.3.2 *Estimating structural fire endurance*-The increase in nominal moment strength is similar to the effect of "fictitious reinforcement" located along the line of action of the thrust (Salse and Gustaferro 1971; Salse and Lin 1976). It can be assumed that the "fictitious reinforcement" has a strength (force) equal to the thrust. By this approach, it is possible to determine the magnitude and location of the required thrust to provide a given fire endurance. The procedure for estimating thrust requirements is: (1) determine temperature distribution at the required fire test duration; (2) determine the retained nominal moment strength for that temperature distribution; (3) if the applied moment M is greater than the retained moment capacity $M_{n\theta}$, estimate the midspan deflection at the given fire test time (if $M_{n\theta}$ is greater than M no thrust is needed); (4) estimate the line of action of the thrust; (5) calculate the magnitude of the required thrust T; (6) calculate the "thrust parameter" T/AE where A is the gross cross-sectional area of the section resisting the thrust and E is the concrete modulus of elasticity prior to fire exposure (Issen, Gustaferro, and Carlson 1970); (7) calculate Z' defined as $Z' = A/s$ in which s is the "heated perimeter" defined as that portion of the perimeter of the cross section resisting the thrust exposed to fire; (8) enter Fig. 2.3.2 with the appropriate thrust parameter and Z' value and determine the "strain parameter" $\Delta l/l$; (9) calculate Δl by multiplying the strain parameter by the heated length of the member; and (10) determine if the surrounding or supporting structure can support the thrust T with a displacement no greater than Δl. Example 5 (in Chapter 8) illustrates this procedure.

The above explanation is greatly simplified because in reality restraint is quite complex, and can be likened to the behavior of a flexural member subjected to an axial force. Interaction diagrams (Abrams, Gustaferro, and Salse 1971; Gustaferro and Abrams 1971) can be constructed for a given cross section at a particular stage of a fire, e.g., 2 hr of a standard fire exposure.

The guidelines in ASTM E 119 for determining conditions of restraint are useful for preliminary design purposes. Basically, interior bays of multibay floors or roofs can be considered to be restrained and the magnitude and location of the thrust are generally of academic interest only.

2.4-Heat transmission

2.4.1 *Single course slab thickness requirements*-In addition to structural integrity, ASTM E 119 limits the average temperature rise of the unexposed (top) surface of floors or roofs to 250 F (139 C) during standard fire tests. For concrete slabs, the temperature rise of the top surface is dependent mainly upon the thickness, unit weight, moisture content,

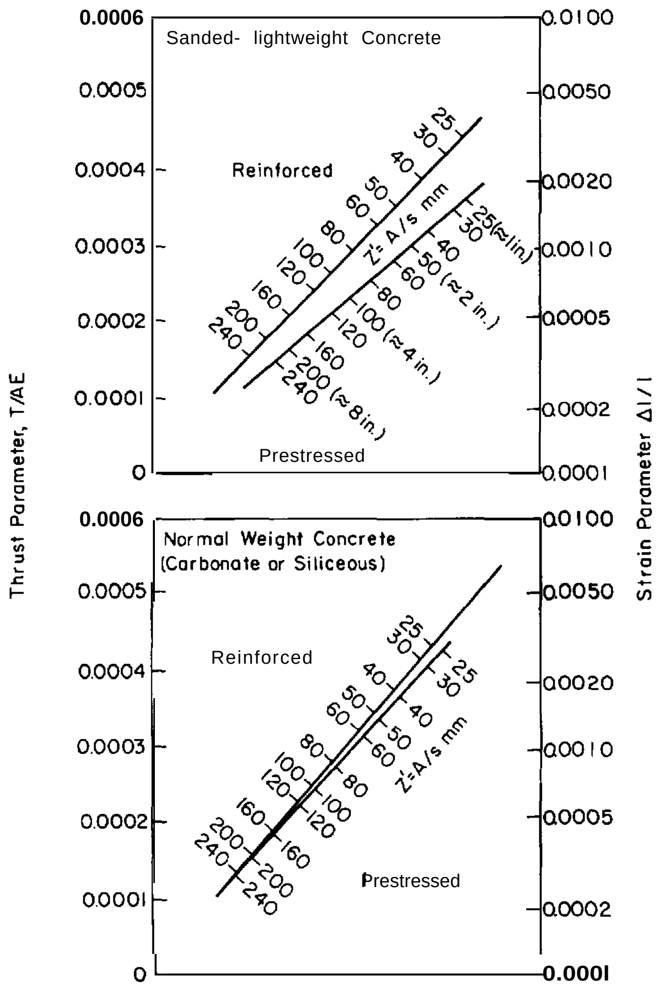


Fig. 2.3.2-Nomogrum relating thrust, strain, and Z' ratio (Issen, Gustaferro, and Carlson 1970)

and aggregate type. Other factors that affect temperature rise but to a lesser extent, include air content, aggregate moisture content at the time of mixing, maximum size of aggregate, water-cement ratio, cement content, and slump.

2.4.1.1 *Effect of slab thickness and aggregate type*-Fig. 2.4.1.1 shows the relationship between slab thickness and fire endurance for structural concretes made with a wide range of aggregates. The curves are for air-entrained concretes fire tested when the concrete was at the standard moisture condition (75 percent relative humidity at mid-depth), made with air-dry aggregates having a nominal maximum size of $^3/_4$ in. (19 mm). On the graph, lightweight aggregates include expanded clay, shale, slate, and fly ash that make concrete having a unit weight of about 9.5 to 105 pcf (1520 to 1680 kg/m^3) without sand replacement. The unit weight of air cooled blast-furnace slag aggregate was found to have little effect on the resulting fire endurance of the normal weight concretes in which it is used.

2.4.1.2 *Effect of unit weight*-Fire endurance generally increases with a decrease in unit weight. For structural concretes, the influence of aggregate type may overshadow the effect of unit weight. For low density concretes, a relationship exists between unit weight (oven-dry) and fire endurance, as shown in Fig. 2.4.1.2. The curves in Fig. 2.4.1.2 represent average values for concretes made with dry vermiculite or perlite, or with foam (cellular concrete), with or

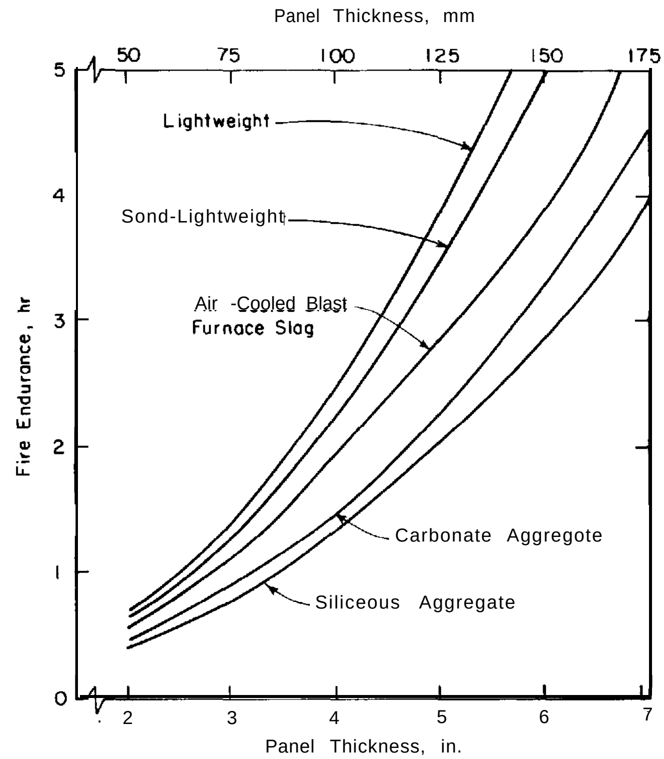


Fig. 2.4.1.1-Effect of slab thickness and aggregate type on fire endurance of concrete slabs. [Based on 250 F (139 C) rise in temperature of unexposed surface]

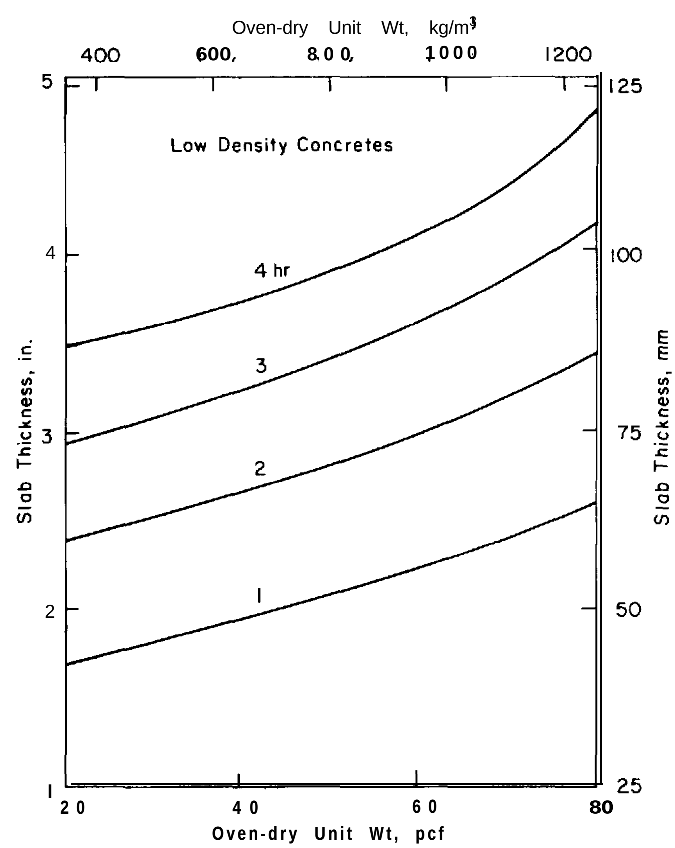


Fig. 2.4.1.2-Effect of dry unit weight and slab thickness on fire endurunce of low density concretes. [Based on 250 F (139 C) rise in temperature of unexposed surface]

Table 2.4.2.1(a)-Data on mixes

Symbol	Carb	Sil	LW			
Type of mix	Carbonate aggregate* concrete	Siliceous aggregate† concrete	Expanded shale aggregate‡ concrete	Vermiculite aggregate concrete	Perlite aggregate concrete	Cellular concrete
Cement. Type I. lb/yd³ (kg/m³)	374(222)	408(242)	446(265)	424(252)	424(252)	673§(399)
Coarse aggregate, lb/yd³ (kg/m³)	1785(1059)	1828(1085)	467(277)	—	—	
Medium aggregate, lb/yd³ (kg/m³)	—	—	248(147)	—		—
Fine aggregate, lb/yd³ (kg/m³)	—		344(204)	—		—
Sand, lb/yd³ (kg/m³)	1374(815)	1419(842)	1076(638)	—	—	
Vermiculite aggregate, lb/yd³ (kg/m³)	—		—	162(96)		
Perlite aggregate, lb/yd³ (kg/m³)	—	—	—	—	216(128)	
Water, lb/yd³ (kg/m³)	238**(141)	223**(132)	464††(275)	756(449)	454(269)	424‡‡(252)
Avg air content, percent	6.2	5.8	6.9	—		
Avg wet unit weight, pcf (kg/m³)	140(2240)	144(2310)	113(1810)	50(800)	41(660)	41(660)
Avg dry unit weight, pcf (kg/m³)			—	28(450)	29(465)	30(480)
Avg compressive strength at 28 days, psi (MPa)	4000(28)	4100(28)	3900(27)	270(1.9)	230(1.6)	420(2.9)

*¾ in. (9 mm) maximum size gravel and sand from Elgin, Ill.
†¾ in. (9 mm) maximum size gravel and sand from Eau Claire, Wis.
‡Rotary-kiln produced expanded shale from Ottawa, Ill., and sand from Elgin, Ill.
§Type III cement.
**Based on saturated surface-dry aggregates
††Based on oven dry aggregates
‡‡Includes weight of foam 54 lb/yd³ (32 kg/m³)

without masonry sand (Gustaferro, Abrams, and Litvin 1971).

2.4.1.3 *Effect of moisture condition*-The moisture content of the concrete at the time of test and the manner in which the concrete is dried affect fire endurance (Abrams and Gustaferro 1968). Generally, a lower moisture content or drying at elevated temperature 120 to 200 F (SO to 9.5 C) reduces the fire endurance. A method is available for adjusting fire endurance of concrete slabs for moisture level and drying environment (Appendix X4, ASTM E 119).

2.4.1.4 *Effect of air content*-The fire endurance of a concrete slab increases with an increase in air content, particularly for air contents above 10 percent. Also, the improvement is more pronounced for lightweight concrete.

2.4.1.5 *Effect of sand replacement in lightweight concrete*-As indicated in Fig. 2.4.1.1, replacement of lightweight aggregate fines with sand results in somewhat shorter fire endurance periods.

2.4.1.6 *Effect of aggregate moisture*-The influence on fire endurance of absorbed moisture in aggregates at the time of mixing is insignificant for normal weight aggregates but may be significant for lightweight aggregates. An increase in aggregate moisture increases the fire endurance. Thus, the fire endurances obtained from Fig. 2.4.1.1 represent minimum values.

2.4.1.7 *Effect of water-cement ratio, cement content, and slump*-Results of a few fire tests indicate that these factors, per se, within the normal range for structural concretes, have almost no influence on fire endurance.

2.4.1.8 *Effect of maximum aggregate size*-For normal weight concretes, fire endurance is improved by decreasing the maximum aggregate size.

2.4.2–*Two-course floors and roofs*

2.4.2.1–Floors or roofs may consist of base slabs of concrete with overlays or undercoatings of either insulating materials or other types of concrete. In addition, roofs generally have built-up roofing. Fig. 2.4.2.2 through 2.4.2.6 show fire endurances of various two-course floors and roofs (Abrams and Gustaferro 1969). Descriptions and symbols of the various concretes and insulating materials referred to in the figures are given in Tables 2.4.2.1(a) and 2.4.2.1(b).

2.4.2.2–Fig. 2.4.2.2 relates to various combinations of normal and lightweight concrete slabs. Note from Fig.

Table 2.4.2.1(b)-Descriptions of materials and mixes

Insulating concrete

Cellular Concrete-A lightweight insulating concrete made by mixing a preformed foam with portland cement slurry and having a dry unit weight of about 30 pcf (480 kg/m³). Foam was preformed in a commercial foam generator.

Vermiculite Concrete-A lightweight insulating concrete made with vermiculite concrete aggregate which is a laminated micaceous material produced by expanding the ore at elevated temperatures. When added to portland cement slurry, a plastic mix was formed having a dry unit weight of about 28 pcf (450 kg/m³).

Perlite Concrete-A lightweight insulating concrete made with perlite concrete aggregate. Perlite aggregate is produced from a volcanic rock which, when heated, expands to form a glass-like material of cellular structure. When mixed with water and portland cement a plastic mix was formed having a dry unit weight of about 29 pcf(460 kg/m³).

Undercoating materials

Vermiculite CM-A proprietary cementitious mill-mixed material to which water is added to form a mixture suitable for spraying. Material was mixed with 1.93 parts of water, by weight. and the mixture had a wet unit weight of 59 pcf (950 kg/m³).

Sprayed Mineral Fiber-A proprietary blend of virgin asbestos fibers, relined mineral fibers and inorganic binders. Water was added during the spraying operation.

Intumescent Mastic-A proprietary solvent-base spray-applied coating which reacts to heat at about 300 F (150C) by foaming to a multicellular structure having 10 to 15 times its initial thickness. The material had a unit weight of 75 pcf (1200 kg/m³) and was used as received.

Roof insulation

Mineral Board, Manufacturer A-A rigid. felted. mineral fiber insultion board; with a flame spread rating not over 20, a fuel contributed rating not over 20. and a smoke developed rating not over 0: conforming to Federal Specification HH-I-00526 b.

Mineral Board, Manufacturer B-Thermal insulation board composed of spherical cellular beads of expanded aggregate and fibers formed into rigid, flat rectangular units with an integral waterproofing treatment.

Glass Fiber Board-Fibrous glass roof insulation consisting of inorganic glass fibers formed into rigid boards using a binder. The board has a top surface faced with asphalt reinforced with glass fiber and kraft.

Miscellaneous

Standard Built-Up Roofing-Consist:, of 3-ply, 15 lb/100 ft² (0.73 kg/m²) felt and not in excess of 1.20 psf (5.86 kg/m²) of hot mopping asphalt without gravel surfacing (Defined in ASTM E 119).

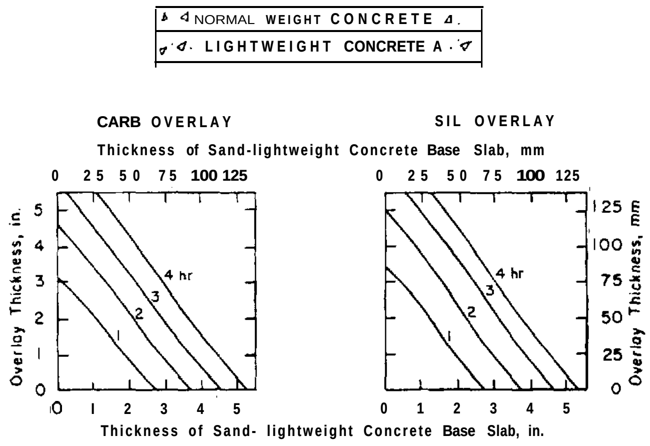


Fig. 2.4.2.2(a)Fire endurance of normal weight concrete overlays on lightweight concrete base slabs

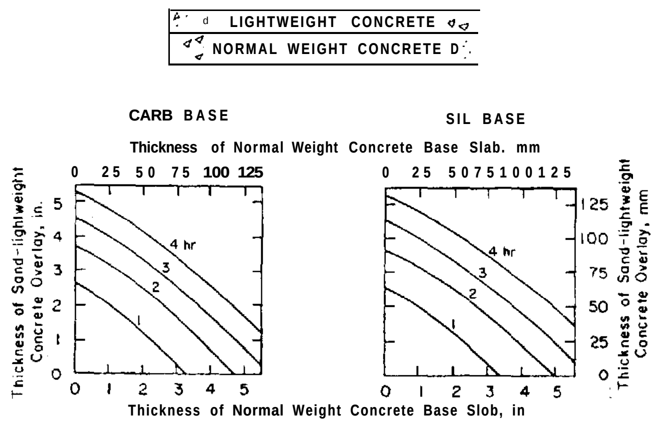


Fig. 2.4.2.2(b)-Fire endurance of lightweight concrete overlays on normal weight concrete base slabs

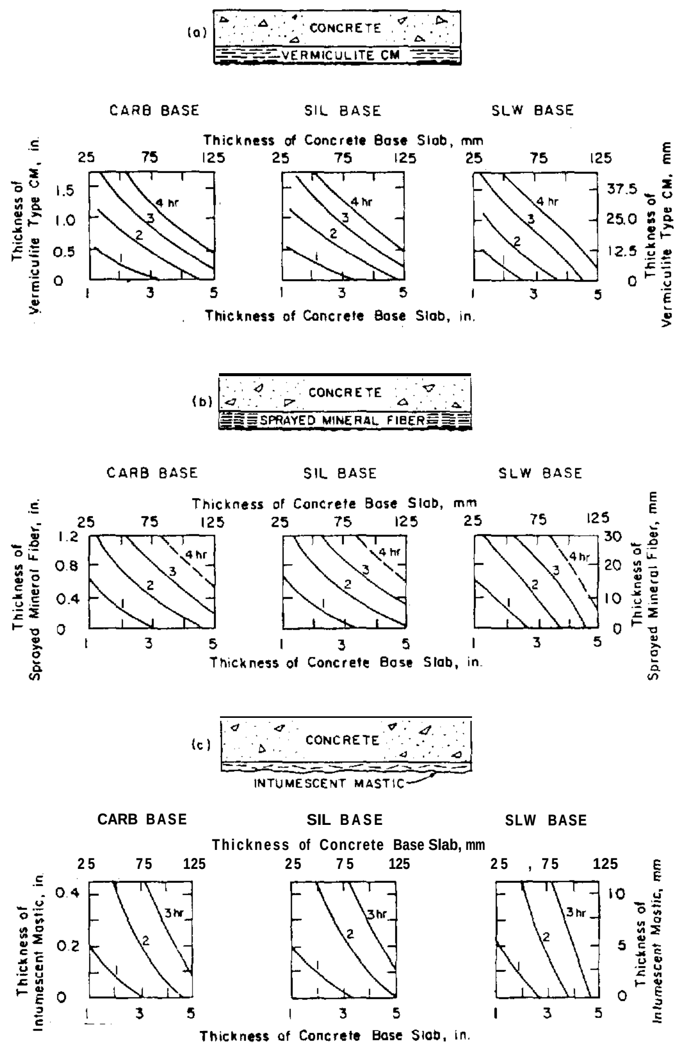


Fig. 2.4.2.3-Fire endurance of concrete slabs undercoated with vermiculite cementitious material, sprayed mineral fiber and intumescent mastic

2.4.2.2(a) that a floor consisting of a 3 in. (76 mm) base slab of lightweight concrete with a 2 in. (51 mm) overlay of carbonate aggregate concrete will have a fire endurance of about 3 hr. A method also exists for calculating the fire endurance of floors and roofs of lightweight and normal weight concretes (Lie 1978; Lin and Abrams 1983).

2.4.2.3-Fig. 2.4.2.3 shows fire endurances of concrete floor slabs undercoated with various thicknesses of (a) vermiculite CM, (b) sprayed mineral fiber, and (c) intumescent mastic.

2.4.2.4-Fig. 2.4.2.4 shows fire endurances of roof slabs (without built-up roofing) made of concrete base slabs and insulating concrete overlays. Each of the insulating concretes represented has a dry unit weight of about 30 pcf (480 kg/m^3). Standard built-up roofing will add about 10 to 20 min to the fire endurance values.

The graphs in Fig. 2.4.2.4 can be modified to include other types of concrete base slabs or concrete overlays. For example, Fig. 2.4.2.4(a) can be modified as shown in Fig. 2.4.2.4(d) to include an overlay having a dry unit weight of 50 pcf (800 kg/m^3) From Fig. 2.4.1.2, thicknesses of 50 pcf (800 kg/m^3) material required for 1, 2, 3, and 4 hr can be determined. For 1 hr, a thickness of about 2.1 in. (53 mm) is required. Thus, a curve for 1 hr representing a carbonate aggregate concrete base slab with an overlay of 50 pcf (800 kg/m^3) material (shown as a dashed line in Fig. 2.4.2.4(d)) must have an ordinate intercept of 2.1 in. (53 mm), an abscissa intercept of 3.25 in. (83 mm) as on the carbonate base curves in Fig. 2.4.2.4(a), (b), and (c), and the curve must be asymptotic at the abscissa intercept to the solid 1 hr curve in Fig. 2.4.2.4(d). A similar procedure can be used for 2,3, and 4 hr endurances and also for different concrete base slabs.

2.4.2.5-Fig. 2.4.2.5 shows the fire endurance of concrete roofs with rigid board insulation. Standard built-up roofing is included in the assemblies.

2.4.2.6-Fig. 2.4.2.6 shows the relationship between total slab thickness and fire endurance for three types of terrazzo floors. The "underbed" consists of one part cement and 4 to 5 parts sand with just enough water to permit molding. It can be noted that "monolithic" terrazzo has the same fire endurance as the base slab concrete of the same total thickness. "Bonded" and "sand cushion" terrazzos have somewhat longer fire endurances than concrete base slabs of the same total thickness.

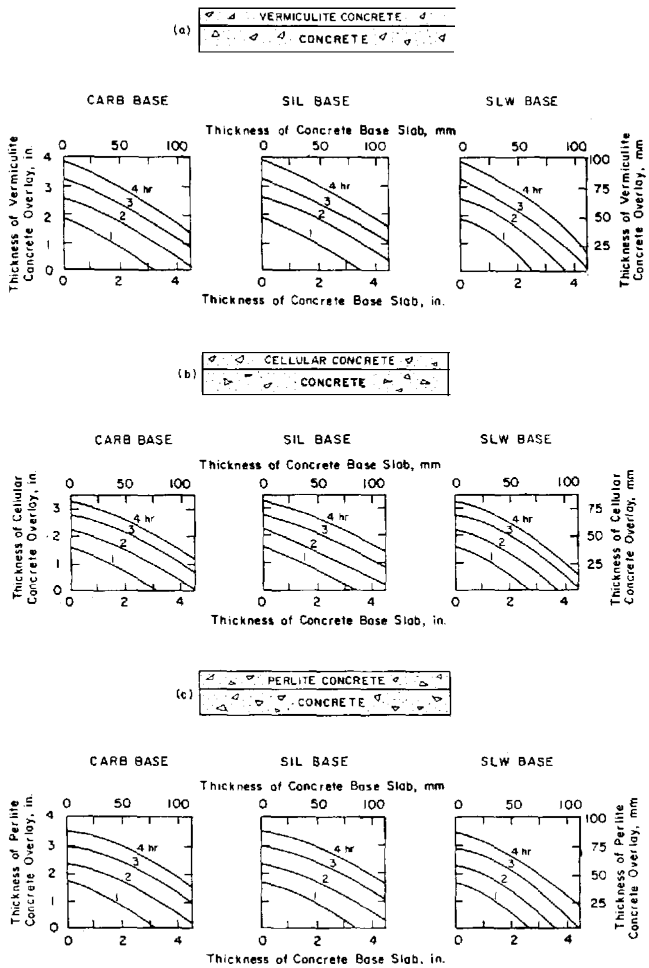


Fig. 2.4.2.4(a), (b), and (c)-Fire resistance of concrete base slabs with overlays of vermiculite, cellular, and perlite concretes

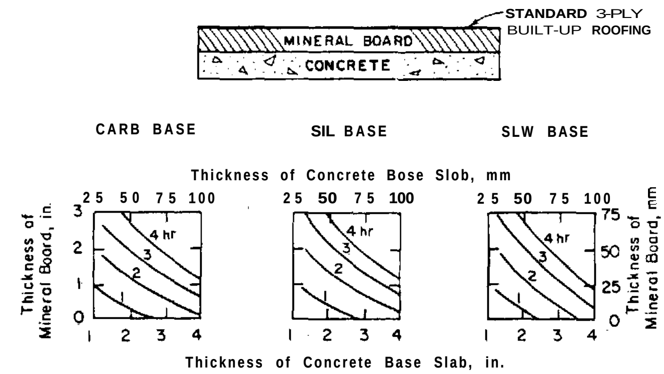


Fig. 2.4.2.5(a)-Mineral board insulation on concrete roofs

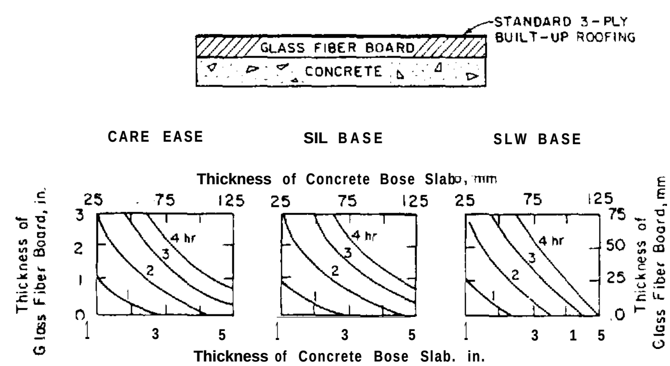


Fig. 2.4.2.5(b)-Glass fiber board insulation on concrete roofs

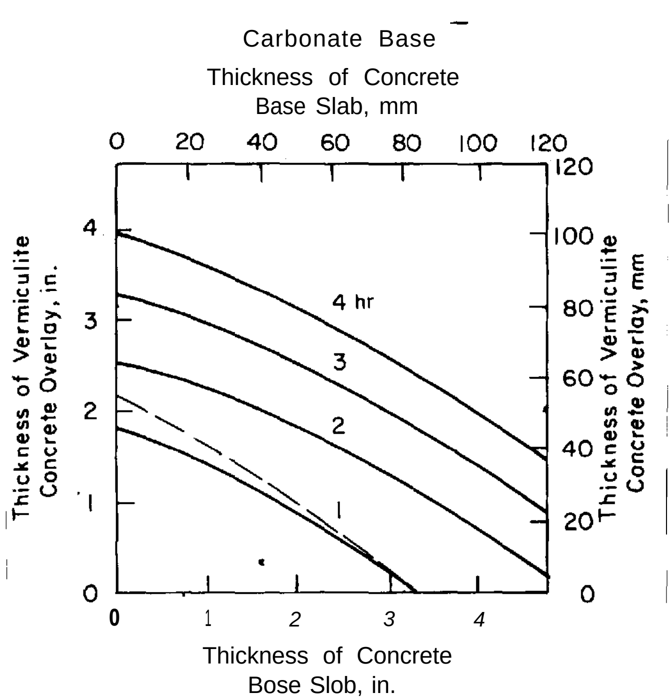


Fig. 2.4.2.4(d)-Dashed line indicates fire endurance of 1 hr for carbonate aggregate concrete base slabs with overlays of concrete having an oven-dry unit weight of 50 pcf (800 kg/m³)

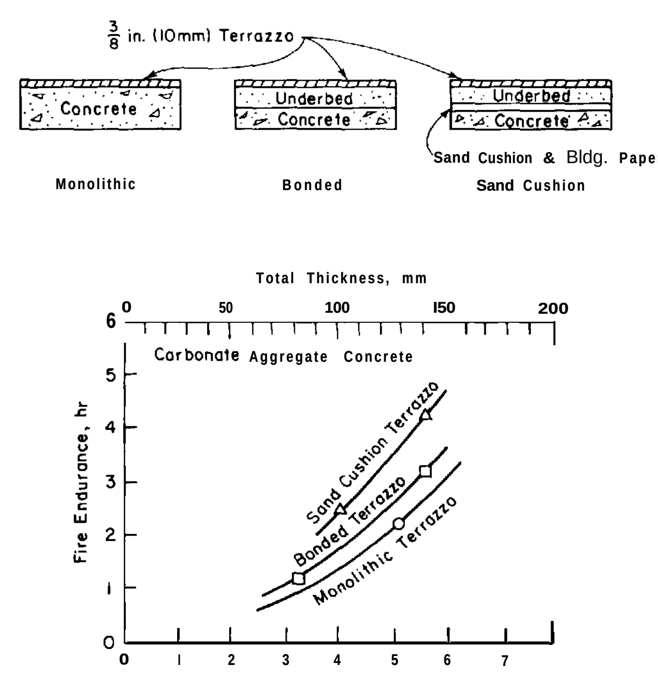


Fig. 2.4.2.6-Fire endurance of terrazzo floors

2.4.3 *Other unexposed surface temperature limits*-Although ASTM E 119 limits the temperature rise of the unexposed surface to 250 F (139 C), other temperatures may be appropriate for certain conditions. For example, vaults for storage of computer tapes are sometimes designed to keep the temperature within the vault below a certain temperature, such as 200 F (93 C) for a specified duration of a standard fire. To determine the required thickness of a concrete slab (or a two-course floor), it is necessary to have data on the temperature of the unexposed surface during fire tests of such slabs. Fig. 2.4.3 shows the unexposed surface temperatures during fire tests of slabs made of carbonate aggregate concrete. The dashed line in Fig. 2.4.3 indicates, for example, that a slab thickness of about 9.5 in. (241 mm) is required to limit the temperature of the unexposed surface to 200 F (93 C) for a 4 hr fire exposure period.

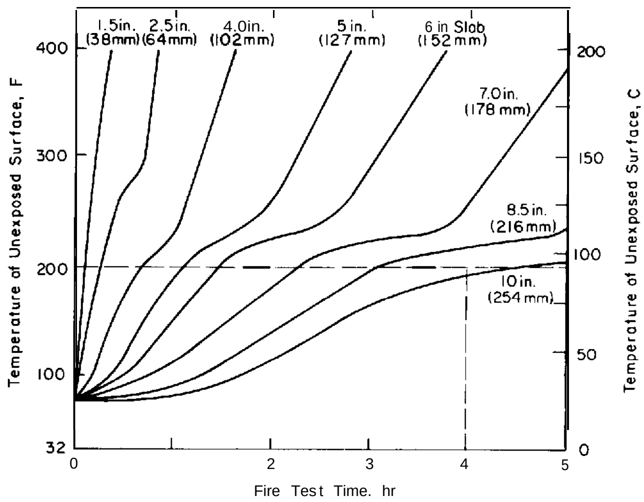


Fig. 2.4.3-Unexposed surface temperatures during fire tests of concrete slabs made with carbonate aggregates (1.5 in. = 38 mm, 2.5 in. = 64 mm, 4 in. = 102 mm, 5 in. = 127 mm, 6 in. = 152mm, 7 in. = 178mm, 8.5 in. = 216 mm, 10 in. = 254 mm)

CHAPTER 3–FIRE ENDURANCE OF WALLS

3.1–Scope

3.1.1-In fire tests of walls consisting of plain concrete, reinforced concrete. and concrete masonry units, the fire endurancc is generally governed by heat transmission rather than structural consideration assuming that the structural requirement of the building code has been satisfied. For that reason the material in Section 2.4 is basically applicable to this chapter.

3.1.2 *Fire tests of walls*-ASTM E 119 prescribes test methods for bearing walls and for nonbearing walls. The principal difference in the test methods is that the bearing wall is loaded to the working stress contemplated by design and the vertical edges arc not restrained whereas the nonbearing wall is not loaded and is restrained on ail four edges. An ASTM E 119 hose stream test, which is intended to simulate the cooling and abrading effect of a fireman's hose stream, is a condition of acceptance of fire test results of walls, ASTM E 119 allows the hose stream test to be performed on a duplicate specimen subjected to one-half of that indicated as the resistance period in the fire endurance test, but not for more than 1 hr or performed on the specimen subjected to the fire endurance test. The latter is more severe.

3.1.3 *Bearing and nonbearing walls*-Generally the fire endurance of concrete and concrete masonry walls is detertnincd by heat transmission with the differentiation between bearing and nonbearing walls being based on building code structural requirements.

3.2–Plain and reinforced concrete walls

3.2.1 *Determination of fire endurance*-Plain or reinforced concrete walls are similar to single course slabs. To find their fire endurance the reader is referred to Section 2.4.1 and Fig. 2.4.1.1.

Where other material is placed on one or both sides of a concrete wall, the fire endurance will bc increased. See Sections 2.4.2 and 3.3.6.

3.3–Concrete masonry walls

3.3.1 *Solid masonry units-Determination of fire endurance*-The fire endurance of solid concrete masonry unit walls can be determined as for plain and reinforced concrete walls. Sec Sections 2.4.1 and 3.2.

3.3.2 *Hollow masonry units-determination of fire endurance*-the fire endurance may be determined by any of the following:

— Fire Test-E 119.
— Interpolation or extrapolation from test results using the "Equivalent Thickness Method"-Section 3.3.2.1.
— Calculation by an "Empirical Method"-Section 3.3.2.2 and Example 6 (in Chapter 8).

The equivalent thickness method has been in use for a number of years. While it may have some shortcomings in a small number of cases, it provides an adequate accuracy for all practical situations.

The empirical method is new. It takes into account variations that may be desirable in evaluating small differences in similar constructions.

3.3.2.1 *Equivalent thickness*-Equivalent thickness is a term intended to quantify the solid contents of the wall. It is determined by dividing the solid volume of a unit by its face area.

3.3.2.1.1 *Underwriter's immersion method*-underwriters Laboratories, Inc. (U.L.) has published a "Procedure for Determining Equivalent Thickness" dated September 1979 which contains details of the procedure and a sketch of the immersion tank. The tank is approximately 8 x 12 in. (200 x 300 mm) and should be at least 24 in. (610 mm) deep. A 0.375 in. (10 mm) weep hole with a 3 in. (76 mm) section of pipe is inserted 17 in. (430 mm) from the bottom of the tank. The unit to be tested is soaked in water for 24 hr. It is then removed, allowed to drain on a screen rack for 1 min then sponged with a clean damp cloth. After 2 min the unit is gently lowered into the container (which has previously been filled with water), and the water from the drain hole is caught in another container. The volume of water displaced is con-

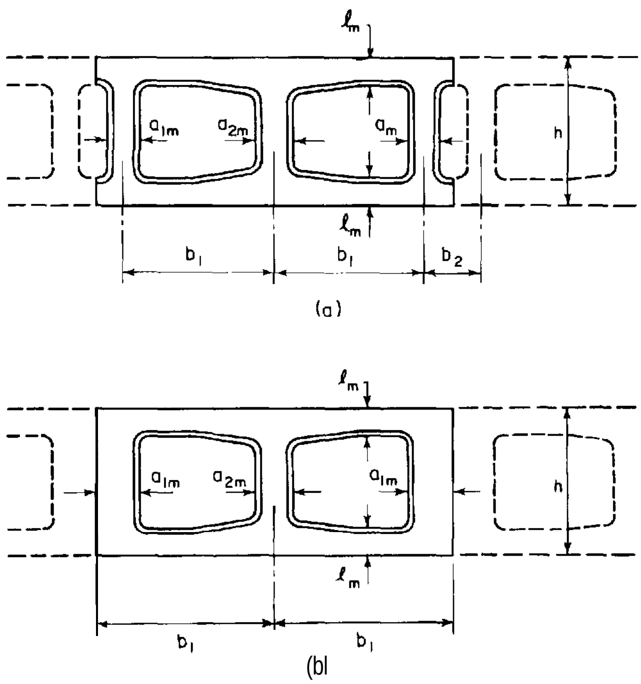


Fig. 3.3.2.2—Hollow masonry units

verted to cubic inches and the equivalent thickness is computed from the formula

$$h_e = \frac{V}{l \times h} \quad (3\text{-}1)$$

where

V = volume of displaced water
l = length of unit
h = height of unit

The disadvantage of this method is that lightweight open textured units continue to drain for much longer than 1 min consequently the absolute solid volume of the unit may not be accurately determined.

3.3.2.1.2 *ASTM C 140 Immersion Method*-In ASTM C 140, Section 10, the net volume of the concrete masonry unit is calculated by weighing dry, wet, and suspended. The net volume (A) replaces V in the U.L. formula to determine the equivalent thickness.

This method has the same disadvantages as the U.L. method.

3.3.2.1.3 *Sand or lead shot method*-In this method a fairly uniformly graded sand or No. 10 shot is used to fill the cores and recessed ends of the unit (Harmathy and Oracheski 1970). The solid volume of sand or shot is subtracted from the gross volume of the unit. Equivalent thickness is computed by the U.L. formula.

This method is more accurate than the U.L. Immersion Method because fine or coarse texture has little effect on the result and is recommended as the desirable method of determining equivalent thickness.

3.3.2.1.4 *Measurement*-In the case of hollow units, the thickness can bc computed from the block machine manufacturer's drawings.

This method has the advantage of eliminating variations due to aggregate type and gradation as well as compaction of the unit.

The disadvantage is that block molds wear with use. Consequently. block made with old molds do not have exactly the same dimensions as block made with new molds. Block manufactured for a fire test should always be made with new molds. If the equivalent thickness rating is assigned on the basis of fire test of units made with new molds. then the consumer is protected because as block are manufactured the molds wear and the equivalent thickness increases.

3.3.2.1.5 *Fire endurance determination*-After the equivalent thickness has been determined by one of the above methods, the fire endurance can be estimated from tables or graphs given in the American Insurance Association's "Fire Resistance Ratings" and the Expanded Shale. Clay and Slate Institute's Information Sheet No. 14 on "Fire Resistance of Expanded Shale, Clay and Slate Concrete Masonry." Such tables or graphs were developed from results of numerous fire tests.

3.3.2.2 *Empirical method*-It must be emphasized that the equivalent thickness is a geometric parameter, and can be used only for interpolation or extrapolation from already available fire test results, within a specific group of concretes of supposedly identical thermal properties. To ascertain whether a particular material indeed belongs to a particular group. one may have to determine its thermal properties. If, however, information on the thermal properties of1 the material is available, one can use an empirical method for the prediction of the fire resistance, which is more accurate than the technique of interpolation or extrapolation from available fire test data (Allen and Harmathy 1972; Harmathy 1973).

This empirical method can be employed whenever the following information is available:

Material properties (at room temperature):

thermal conductivity k	Btu/h ft F* (W/mK)
thermal diffusivity κ	ft^2/h (m^2/h)

(thermal diffusivity is thermal conductivity divided by product of density and specific heat)

Geometric variables (see Fig. 3.3.2.2):

overall thickness h	ft (m)
average face shell thickness l	ft (m)
average web thickness a	ft (m)
average web spacing b	ft (m)

Volumetric moisture content ϕ ft^3/ft^3 (m^3/m^3)

The value a is the average of a,, a_2, etc., and b is the average of b,, b_2, etc. As the figure shows, the inner cores of the masonry units are generally made with some slope, so that the effective values of I, a_1, a_2, etc., are not easily obtainable by simple measurements. The following formulas may be used

$$l = 1.1 l_m \quad (3\text{-}2)$$

$$a_1 = 1.15\ a_{1m} \text{ etc.} \quad (3\text{-}3)$$

*In practice k is oflen expressed in Btu in/h ft^{2}F; to obtain values in Btu/h ft F divide values in Btu in./h ft^{2}F by 12.

where l_m, a_{1m}, etc. are dimensions measured on the side of minimum thickness.

The values of a and b for the shape shown in Fig. 3.3.2.2(a) arc obtained as

$$a = \frac{1}{3}(2a_1 + a_2) \tag{3-4}$$

$$b = \frac{1}{3}(2b_1 + b_2) \tag{3-5}$$

and for the shape shown in Fig. 3.3.2.2(b) the average web thickness is expressed as

$$a = \frac{1}{2}(2a_1 + a_2) \tag{3-6}$$

The volumetric moisture content ϕ is obtained from the moisture content expressed as weight fraction m as

$$\phi = m\frac{\varrho}{\varrho_w} \tag{3-7}$$

where m is usually determined by measuring the weight loss of concrete after sufficiently long heating at 22 1 F (10.5 C), ϱ is the density of concrete, and ϱ_w is the density of water, both densities in pounds per cubic foot (kilograms per cubic meter).

The fire resistance of the masonry wall in dry (moistureless) condition, τ_o, can be calculated from the following expression:

$$\tau_o = \left(\frac{1}{\dfrac{a/b}{\tau_{1o}^{1/2}} + \dfrac{1-a/b}{\tau_{2o}^{1/2}}}\right)^2 \tag{3-8}$$

where

$$\tau_{1o} = C_{14}\left(\frac{k}{h}\right)^{0.55}\left(\frac{h^2}{\kappa}\right)^{1.2} \tag{3-9}$$

$$\tau_{2o} = C_{15}\left(\frac{k}{l}\right)^{0.60}\left(\frac{l^2}{\kappa}\right)^{1.1} \tag{3-10}$$

where

$$C_{14} = \begin{array}{l} 0.205\ \mathrm{ft^{1.1}h^{0.35}F^{0.55}/Btu^{0.55}} \\ 0.0153\ \mathrm{m^{1.1}s^{0.35}C^{0.55}/J^{0.55}} \end{array}$$

and

$$C_{15} = \begin{array}{l} 0.750\ \mathrm{ft^{1.2}h^{0.5}F^{0.6}/Btu^{0.6}} \\ 0.\text{[illegible]}7\ \mathrm{m^{1.2}s^{0.5}C^{0.6}/J^{0.6}} \end{array}$$

in the case of solid walls $\tau_o \cong \tau_{1o}$. The fire resistance of the concrete wall in natural (moist) condition, τ, can finally be obtained from the following formula:

$$\tau = \frac{\tau_o^2 + 4\tau_o(1+\beta\phi)}{4+\tau_o} \tag{3-11}$$

where β = 5.5 for normal weight concretes and β = 8 .O for lightweight concretes (ASTM E 119).

Example 6 (in Chapter 8) illustrates use of these equations.

3.3.3 *Moisture content versus relative humidity*-As is stated in Section 2.4.1.3, the amount of moisture in a specimen will affect the fire endurance. In practice, the moisture condition of the specimen is usually expressed in terms of equilibrium relative humidity (in the pores of the concrete). Appendix X4 of ASTM E 119 describes a method for calculating the moisture content from known values of the equilibrium relative humidity.

3.3.4 *Effect of aggregate type and aggregate moisture*-See Section 2.4.1.

3.3.5 *Effect of filling cores*-Fire tests show that filling the cores of hollow concrete masonry units with lightweight aggregate increases the fire endurance of the wall. In most cases a 2 or 3 hr rated wall would have its rating increased to 4 hr when the cores are filled with a lightweight aggregate. The aggregate in the cores increases the insulation value of the wall as well as provides additional moisture which absorbs heat during the fire.

3.3.6 *Effect of plaster or other material on face of walls*-Addition of a layer of plaster or other material to the wall increases the resistance to heat transmission, thus, increasing the fire endurance. The reader is referred to Section 2.4.2 and to UL 618 and the Expanded Shale, Clay and Slate Institute's Information Sheet No. 14 on "Fire Resistance of Expanded Shale, Clay and Slate Concrete Masonry."

CHAPTER 4–REINFORCED CONCRETE COLUMNS

4.1–General

Reinforced concrete columns have performed well during exposure to fire throughout the history of concrete construction.

Columns larger than 12 in. (305 mm) in diameter or 12 in. (305 mm) square are assigned 3 hr and 4 hr fire resistance classifications in most building codes in America.

It is suggested that the information in Table 4.1 be used for designing reinforced concrete columns for exposure to fire. This information is based on the results of a comprehensive series of fire tests on concrete columns (Lie, Lin, Allen, and Abrams 1984). The entire series of the test program consists of 38 full-size concrete columns.

Columns designed in accordance with the requirements of Table 4.1 have been used in concrete buildings for years. These ratings combined with requirements for structural adequacy have given economical column sizes that have performed well.

In the 1970s analytical procedures (Lie and Allen in NRC Technical Papers 378 and 416; Lie and Harmathy 1974) were developed for estimating temperature distributions in concrete columns during exposure to fire and for designing concrete columns for specific fire endurances and loads.

CHAPTER 5–PROPERTIES OF STEEL AT HIGH TEMPERATURES

Evaluating the fire endurance of concrete elements by calculations requires information on certain thermal and mechanical properties of concrete and reinforcing steel over a

Table 4.1-Load and performance of test columns*

Specimen no.	Load kips	Load kN	Length of test, hr: min	Mode of failure
Siliceous aggregate				
1	0	0	4:00	None
2	300	1300	2:50	Compression
3	180	800	3:38	"
4	160	710	3:40	"
5†	0	0	5:00	None
6‡	38	170	3:00	Buckling
7	240	1070	3:28	Compression
8	400	1800	2:26	"
9	300	1300	3:07	"
Carbonate aggregate				
10	180	800	8:30	"
11	240	1070	6:06	"
12	400	1800	3:36	"

*Cross section is 12 x 12 in. (305 x 305 mm) unless otherwise indicated.
†Cross section is 16 x 16 in. (406 x 406 mm).
‡Cross section is 8 x 8 in. (203 x 203 mm).
Notes:
1. Full design load for a 12 x 12 in. (305 x 305 mm) square column is 240 kips (1070) kN).
2. Concrete cover is 1 1/2 in. (38 mm) to ties.
3. More test data are available from National Research Council of Canada, Ottawa, or Construction Technology Laboratones of the Portland Cement Association, Skokie, IL.

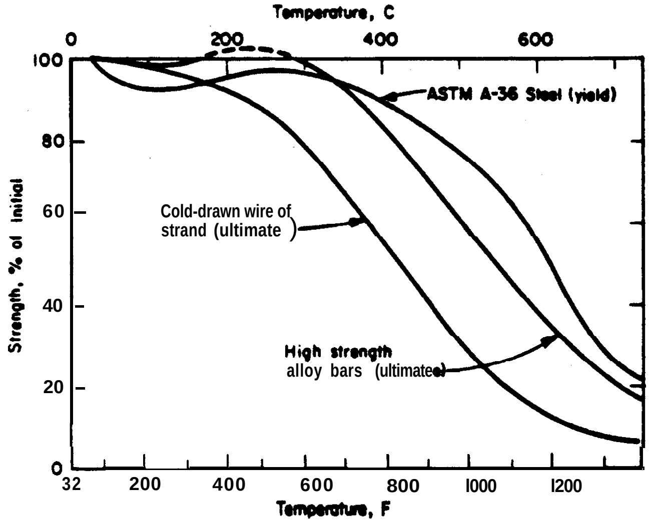


Fig. 5.1-Strength of certain steels at high temperatures

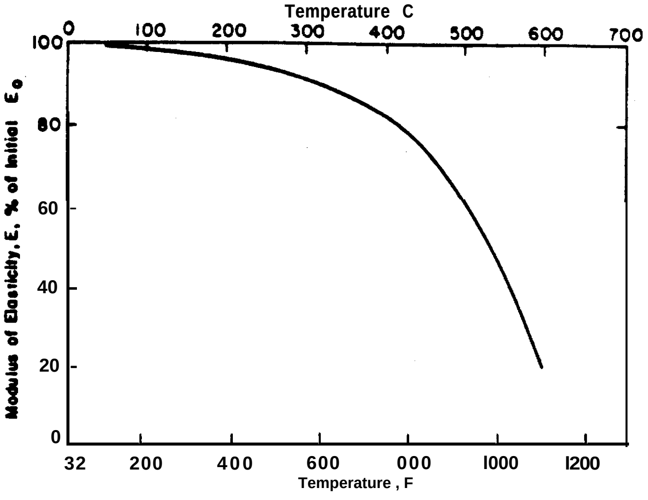


Fig. 5.2–Modulus of elasticity of steel at high temperatures. Note: This curve was developed for the European Convention for Construction of Steel Structures (ECCS) (Weigler and Fischer 1964)

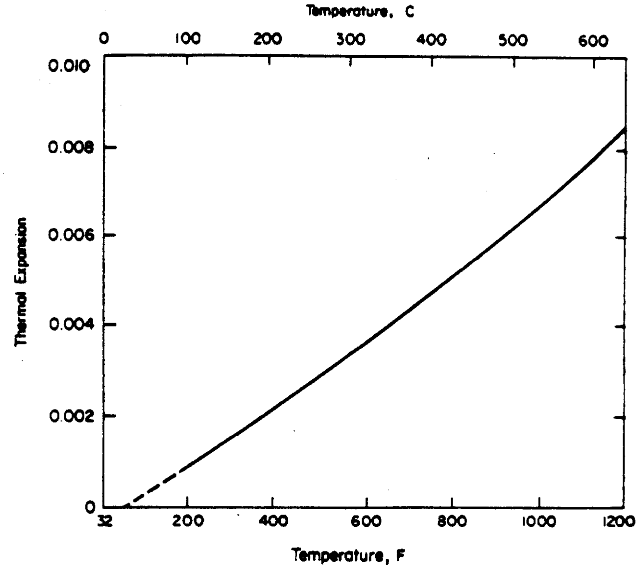


Fig. 5.3–Thermal expansion of ferritic steels at high temperatures

wide temperature range. The thermal properties of concrete form the input information for heat flow studies aimed at determining the temperature distribution in concrete elements exposed to fires. Together with information on the temperature distribution, the mechanical properties of steel and concrete provide the basis for the assessment of the structural performance of building elements during fire exposure.

This chapter contains data on the elevated-temperature properties of steel. It should be noted that most of the curves presented here and in Chapter 6 represent averages of many observations.

5.1–Strength

Fig. 5.1 shows the influence of temperature on the strength of certain steels. Included are data on the yield stress of structural steels (Brockenbrough and Johnston 1968) and ultimate strengths of cold-drawn steel (Abrams and Cruz 1961; Day, Jenkinson, and Smith 1960) and high strength alloy steel bars (Gustaferro, Abrams, and Salse 1971; Carlson, Selvaggio, and Gustaferro 1966) used in prestressed concrete. Generally; the strengths of steels decrease with increasing temperature but ultimate strengths of hot rolled steels are often slightly higher at temperatures up to about 500 F (260 C) than they are at room temperature.

5.2–Modulus of elasticity

The modulus of elasticity of steel decreases with increasing temperature as shown in Fig. 5.2 (Weigler and Fischer 1964). Modulus of elasticity for ferritic steels decreases linearly to about 750 F (400 C). Above 750 F (400 C) the modulus decreases at a higher rate. The curve in Fig. 5.2 is representative of the types of steels used in concrete construction.

5.3—Thermal expansion

The average linear thermal expansion of ferritic steels over a temperature range of 400 to 1200 F (200 to 650 C) is shown in Fig. 5.3 (U.S. Steel Corporation 1965). The coefficient of

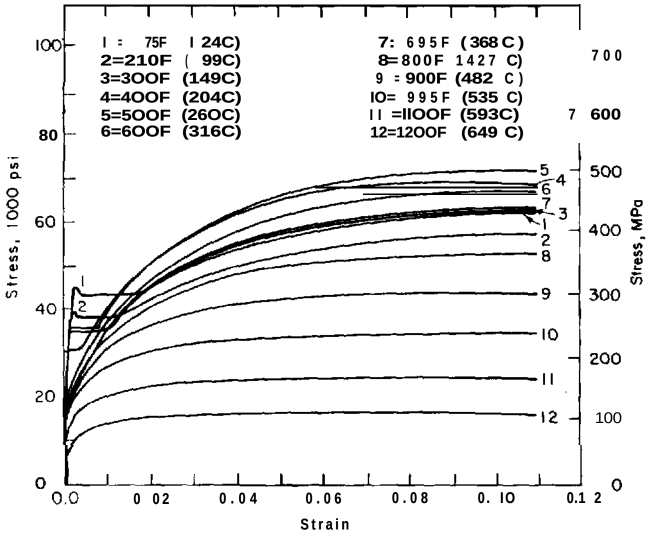


Fig. 5.4.1—Stress-strain curves for structural steels (ASTM A 36) at various high temperatures

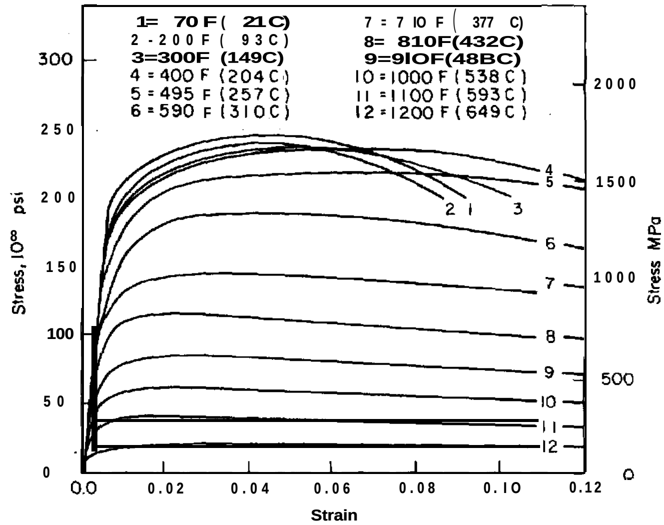


Fig. 5.4.2-Stress-strain curves for prestressing steel (ASTM A 421) at various high temperatures

thermal expansion is not constant over this temperature region but increases as temperature increases. The temperature dependence of the coefficient of thermal expansion α is approximated by the formula

$$\alpha = (6.1 + 0.0020\theta_1) \times 10^{-6}/F$$

or

$$\alpha = (11 + 0.0036\theta_2) \times 10^{-6}/C$$

in which $\theta_1(\theta_2)$ is temperature in deg F (C) (American Institute of Steel Construction 1980).

5.4–Stress-strain relationships

Stress-strain relationships for several types of steel have been reported by Harmathy and Stanzak (1970). Such curves for an ASTM A 36 steel are shown in Fig. 5.4.1. Fig. 5.4.2 shows a family of stress-strain curves for ASTM A 421 cold-drawn prestressing steel (Dorn 1954).

5.5–Creep

In high-temperature processes the time-dependent nonrecoverable (plastic) unit deformation of steel is referred to as creep strain. When dealing with fire problems, it is convenient to express the creep strain according to Dorn's concept, in terms of a "temperature-compensated time," defined as

$$t_\theta = \int_0^t e^{-\Delta H/R\theta} dt \qquad (5\text{-}1)$$

where

- t_θ = temperature-compensated time, hours
- t = time, hours
- ΔH = activation energy of creep, J/(kg · mole)
- R = gas constant, J/(kg · mole · K)
- θ = temperature, K

Harmathy (1967a, 1967b) showed that the creep strain can be satisfactorily described by the following equation

$$\varepsilon_t = \frac{\varepsilon_{to}}{\ln 2} \cosh^{-1} 2^{Z t_\theta/\varepsilon_{to}} \qquad (5\text{-}2)$$

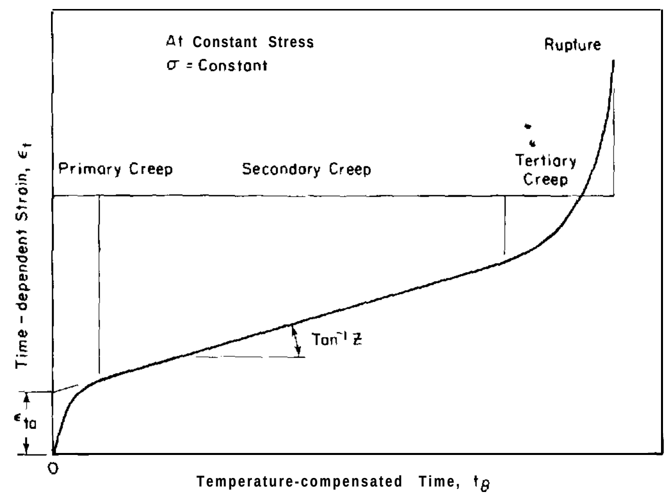


Fig. 5.5-Interpretation of creep parameters and three periods of creep

where

- ε_t = creep strain
- Z = Zener-Hollomon parameter, h^{-1}
- ε_{to} = (unnamed) creep parameter

Z and ε_{to} are dependent on the applied stress only (independent of temperature). Their meaning is explained in Fig. 5.5 which also shows the three periods of creep. From a practical point of view the secondary creep is the most important. (The equation given earlier for ε_t does not cover the tertiary creep.)

Empirical equations for Z and ε_{to} and the values of $\Delta H/R$ for three important steels are given by Harmathy and Stanzak (1970). Numerical techniques applying the creep information to the calculation of the deflection history of joints and beams during fire exposure have been reported (Harmathy 1967; Harmathy 1976; Pettersson, Magnusson, and Thor 1976).

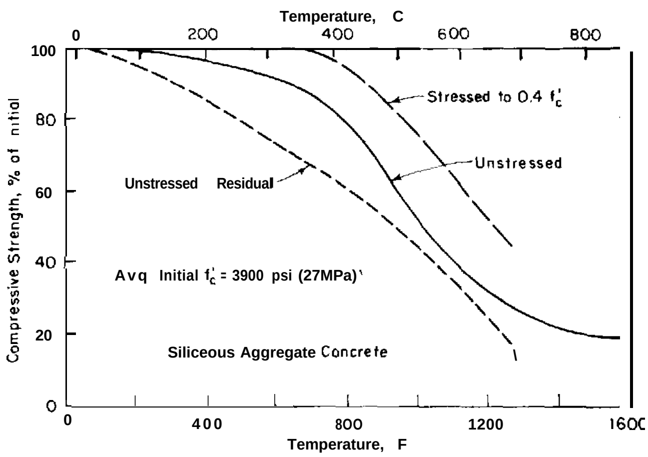


Fig. 6.1.1-Compressive strength of siliceous aggregate concrete at high temperature and after cooling

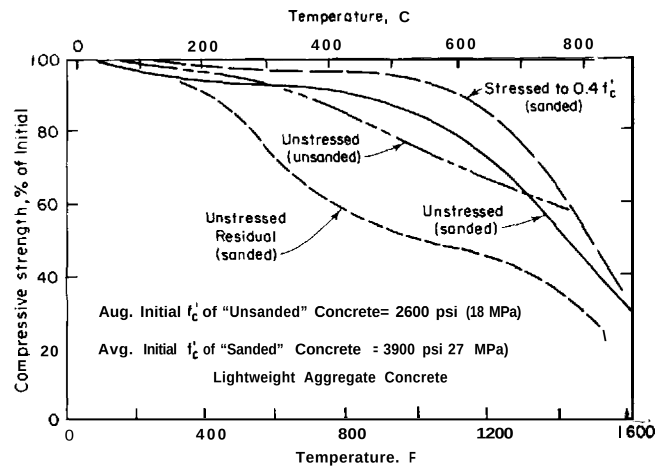


Fig. 6.1.3-Compressive strength of lightweight concrete at high temperature and after cooling

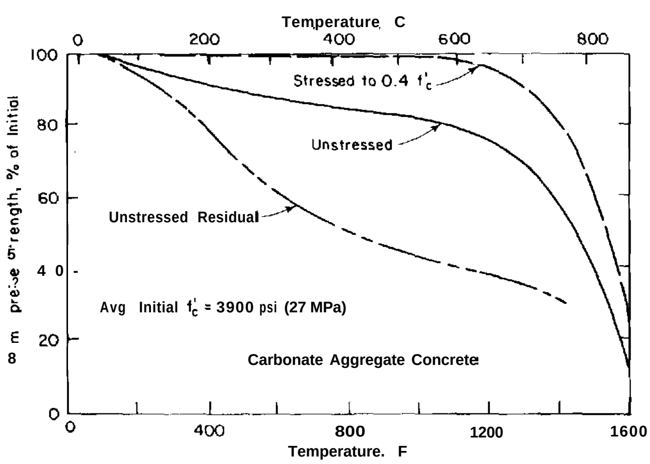


Fig. 6.1.2-Compressive strength of carbonate aggregate concrete at high temperature and after cooling

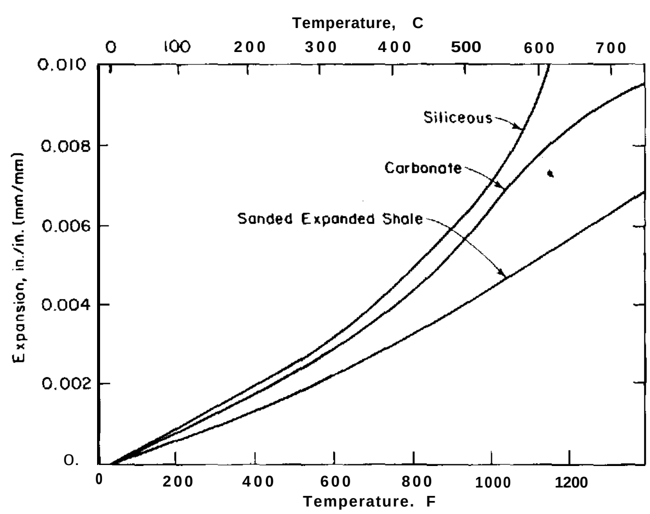


Fig. 6.2-Thermal expansion of concrete at high temperatures

CHAPTER 6–PROPERTIES OF CONCRETE AT HIGH TEMPERATURES

6.1–Compressive strength

Compressive strengths ofconcretes made with different types of aggregates are shown in Fig. 6.1.1, 6.1.2, and 6.1.3 (Abrams 1971). Curves designated "unstressed" are for specimens heated to test temperature with no superimposed load and tested hot. Strengths of specimens heated while stressed to $0.4f_c'$ and then tested hot are designated "stressed to $0.4f_c'$". The "unstressed residual" strengths were determined from specimens heated to test temperature, cooled to room temperature, stored in air at 75 percent relative humidity for six days and then tested in compression. Note that the "stressed" strengths are higher than the "unstressed" strengths. Abrams (1971) found that stress levels of 0.25 to $0.55f_c'$ had little effect on the strength obtained. The "unstressed residual" strengths were in all cases lower than the strengths determined by the other two procedures. Abrams also noted that original concrete strengths between 4000 and 6500 psi (28 and 4.5 MPa) have little effect on the percentage of strength retained at test temperature. In Fig. 6.1.3 the "sanded" specimens were made with sand replacing 60 percent of the lightweight fines, by volume.

The "unsanded" concrete was the kind used in masonry block manufacture. Harmathy and Berndt (1966) reported data on the compressive strength of cement paste and a lightweight concrete from tests performed on specimens held at the target temperature in no-load condition for a period of 1 to 24 hr.

Further data on the strength of concrete at high temperatures have been reported by Zoldners (1960); Malhotra (1956); Saemann and Washa (1957); Binner, Wilkie, and Miller (1949); and Weigler and Fischer (1964, 1968).

6.2–Linear thermal expansion

Fig. 6.2 shows data on linear thermal expansion of concretes made with different aggregates. The data were obtained by Cruz using a dilatometric method but the results have not yet been published. Harmathy and Allen (1973)

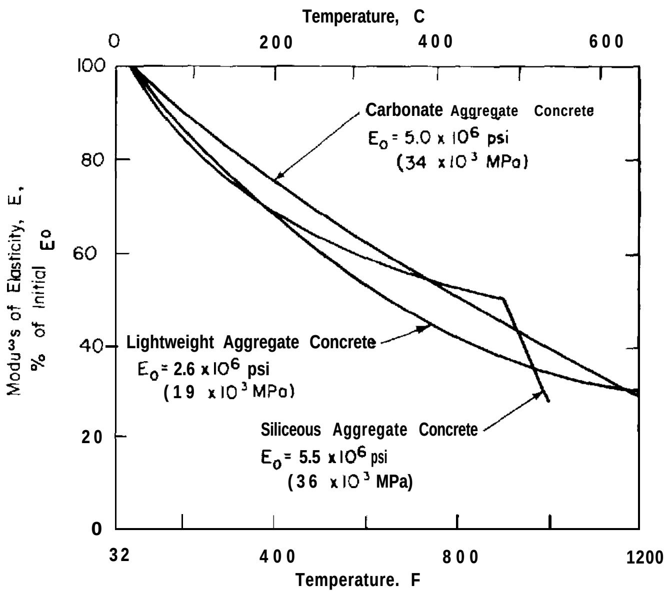


Fig. 6.3.1-Modulus of elasticity of concrete at high temperatures

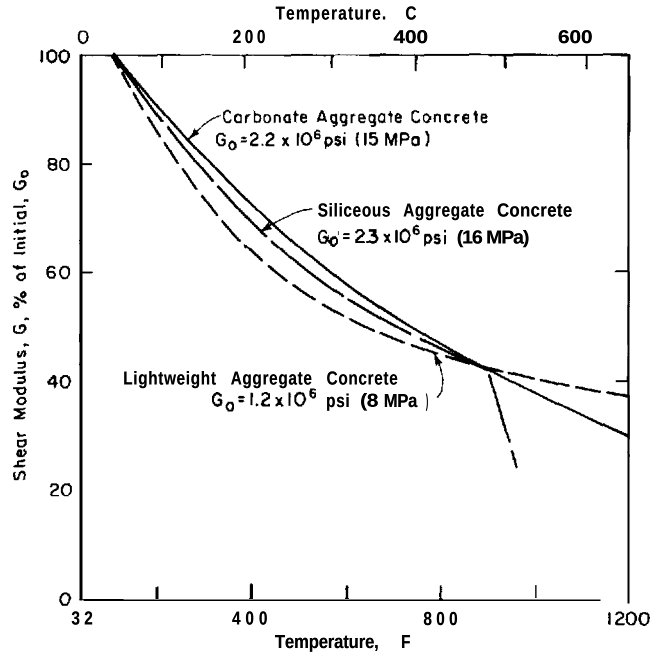


Fig. 6.3.2-Shear modulus of concrete at high temperatures

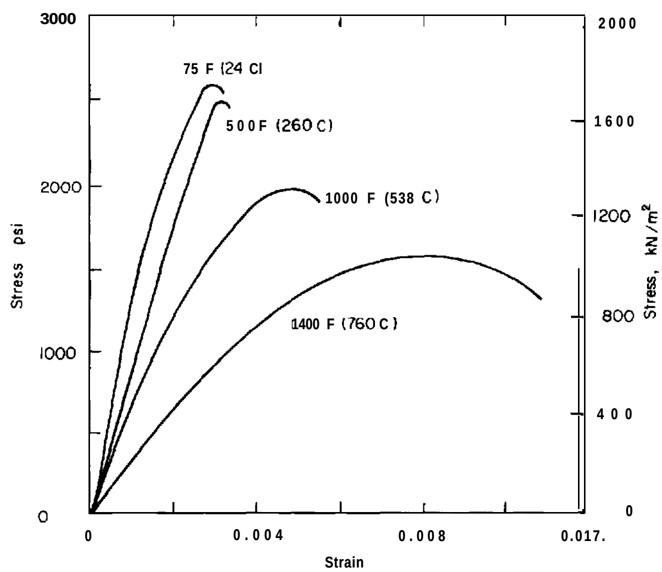


Fig. 6.5-Stress-strain curves for a lightweight masonry concrete at various high temperatures

studied the thermal expansion of 16 different concretes used in masonry units. Among these, pumice concretes were found to exhibit considerable shrinkage at temperatures above 600 F (3 15 C). Dettling (1964) pointed out that thermal expansion of concrete is influenced by aggregate type, cement content, water content, and age. Philleo (1958) performed tests on a carbonate aggregate concrete using a different technique. He obtained somewhat higher values than those obtained by Cruz at temperatures above 700 F (370 C).

6.3–Modulus of elasticity and shear modulus

Fig. 6.3.1 and 6.3.2 show the effect of high temperatures on the moduli of elasticity and shear of concretes made with three types of aggregate. The data were obtained by Cruz (1966) using an optical method. From Cruz's data, it appears that aggregate type and concrete strength do not significantly affect moduli at high temperatures.

Philleo (1958) obtained values for modulus of elasticity of a carbonate aggregate concrete using a dynamic method. His results agree closely with those obtained by Cruz up to about 700 F (370 C). From 700 to 1200 F (370 to 650 C), Philleo obtained higher values. Harmathy and Berndt (1966) and Saemann and Washa (1957) determined the modulus of elasticity in compression and found little change up to about 400 F (200 C).

6.4–Poisson's ratio

Philleo (1958) and Cruz (1966) reported data on Poisson's ratio of concrete at high temperatures. Even though Philleo indicated a decrease in Poisson's ratio, both he and Cruz pointed out that results were erratic and no general trend of the effect of temperature was clearly evident.

6.5-Stress-strain relationships

Rather complete data between 75 and 1400 F(24 and 760 C) on stress-strain relationships in compression of a lightweight masonry concrete (expanded shale aggregate) were reported by Harmathy and Berndt (1966). Fig. 6.5 shows some of the data. Kordina and Schneider (1975) studied the stress-strain response of normal weight concretes at variable temperatures under a number of loading conditions.

6.6–Stress relaxation and creep

Some data on stress relaxation and creep at high temperatures of a carbonate aggregate concrete were reported by Cruz (1968). Fig. 6.6.1 and 6.6.2 show the data graphically for a 5 hr test period. Nasser and Neville (1967) reported that age, moisture condition, type and strength of concrete, and stress-strength ratio affect creep of concrete at high temperatures. Mukaddam and Bresler (1972) and Mukaddam (1974) conducted studies on the creep of concrete at variable temperatures.

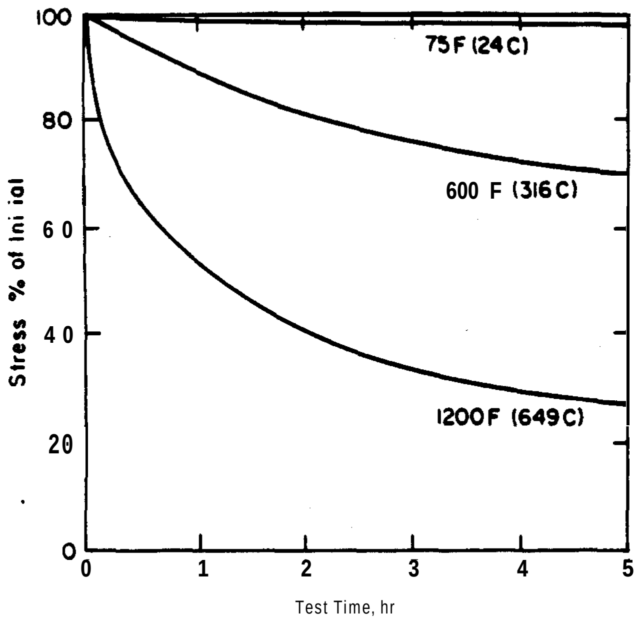


Fig. 6.6.1–Stress relaxation of a carbonate aggregate concrete

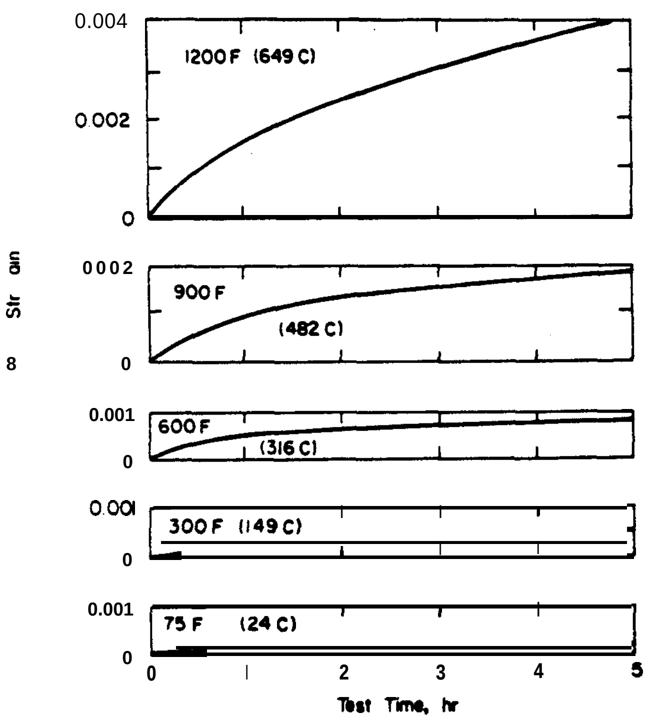


Fig. 6.6.2–Creep of a carbonate aggregate concrete at various temperatures [applied stress = 1800psi (12 MPa), f'_c = 4000 psi (28 MPa)]

6.7–Thermal conductivity, specific heat, and thermal diffusivity

Harmathy (1964) developed a variable-state method by which all three of these properties of building materials can be determined from a single measuremcnt. Harmathy (1970) also presented methods for the calculation of the thermal conductivity of all kinds of concrete up to 1800 F (980 C). He defined four concretes two (No. 1 and 2) representing limiting cases (from the point of view of thermal properties) among normal weight concretes, and two (No. 3 and 4) among lightweight concretes. The thermal conductivities of these four concretes. together with some experimental data. arc shown in Fig. 6.7.1. Harmathy and Allen (1973) published information on the thermal conductivity, thermal diffusivity. and specific heat of 16 masonry unit concretes for 70 to 1250 F (20 to 680 C) temperature range. Odeen (1968) studied the thermal conductivity of a concrete containing granitic aggregate. Carman and Nelson (1921) determined the thermal conductivity and diffusivity of a carbonate aggregate concrete between 120 and 390 F (50 and 200 C).

Research on the specific heat of various concretes has also been reported in papers by Harmathy (1970) and Harmathy and Allen (1973). Typical ranges for the "volumetric" specific heats (product of specific heat and density) for (non-autoclaved) normal weight and lightweight concretes are shown in Fig. 6.7.2, Odeen (1968) also studied the volumetric specific heat of concrete over a temperature range up to 1800 F (980 C).

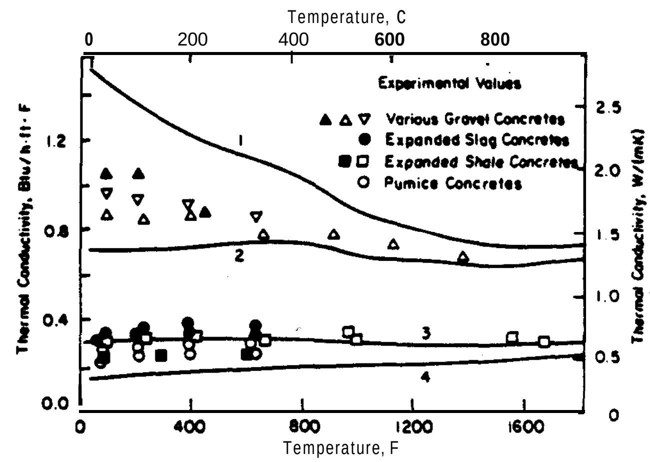


Fig. 6.7.1–Thermal conductivity of four "limiting" concretes and some experimental thermal conductivity data

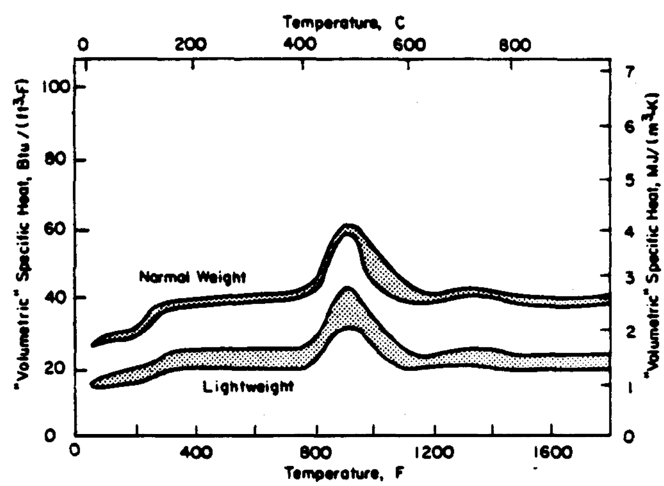


Fig. 6.7.2–Volumetric specific heats of normal weight and lightweight concretes

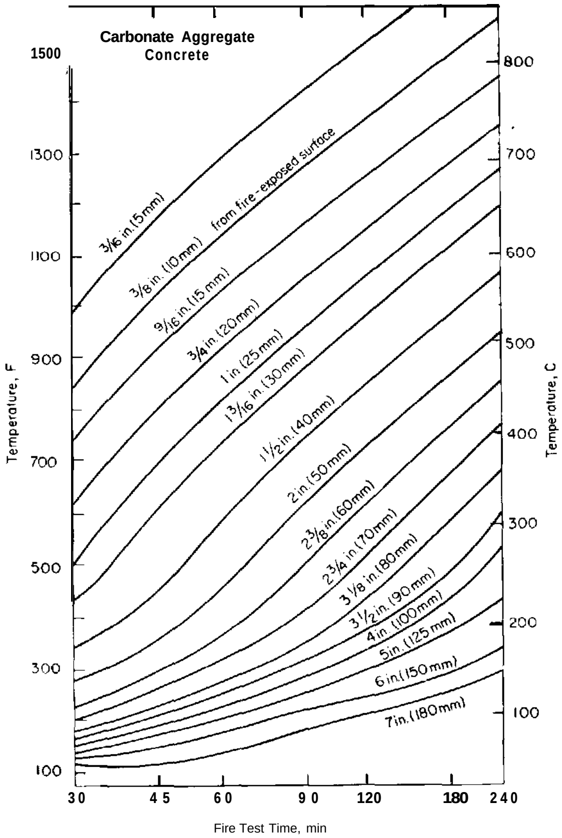


Fig. 7.1.1(a)-Temperature within slabs during, fire tests-carbonate aggregate concrete

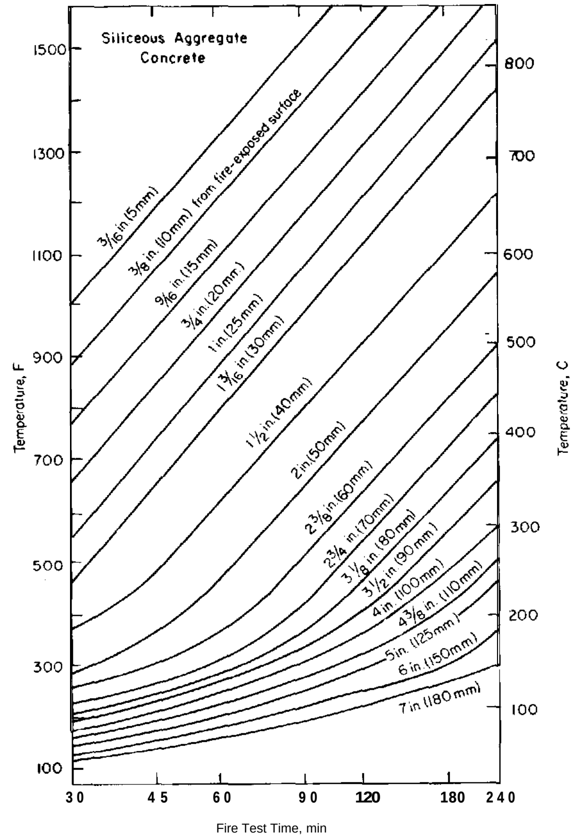


Fig. 7.1.1(b)-Temperatures within slabs during fire tests-siliceous aggregate concrete

CHAPTER 7–TEMPERATURE DISTRIBUTION WITHIN CONCRETE MEMBERS EXPOSED TO A STANDARD FIRE

This chapter provides information on the temperature distribution in a number of concrete shapes during fire exposure, and refers to calculation techniques to be used when experimental information is not available.

7.1–Slabs

Fig. 7.1.1(a), (b), and (c) show temperatures within concrete slabs during fire tests (Abrams and Gustaferro 1968). Slab thickness did not significantly affect the temperatures except for very thin slabs or when the temperatures were less than about 400 F (200 C). Fig. 7.1.2(a), (b), and (c) show similar data for lightweight insulating concretes (Gustaferro, Abrams, and Litvin 1971). Temperatures in slabs were obtained from specimens 3 x 3 ft (0.9 x 0.9 m) in plan with protected edges.

7.2–Rectangular and tapered joists

Computed and measured temperatures within rectangular beams made with quartzitic gravel have been reported (Ehm and van Postel 1967). Beam sizes tested ranged in size from 2.5 x 12 in. to 11 x 22 in. (64 x 305 mm to 280 x 560 mm).

Fig. 7.2.1 through 7.2.6 show temperature distributions along the center line at various distances from the bottom of the beam and for widths up to 10 in. (254 mm) for normal weight carbonate aggregate concrete and lightweight concrete for fire endurance periods of 1, 2, and 3 hr. The width ***b*** is the beam width for rectangular members and the width at a

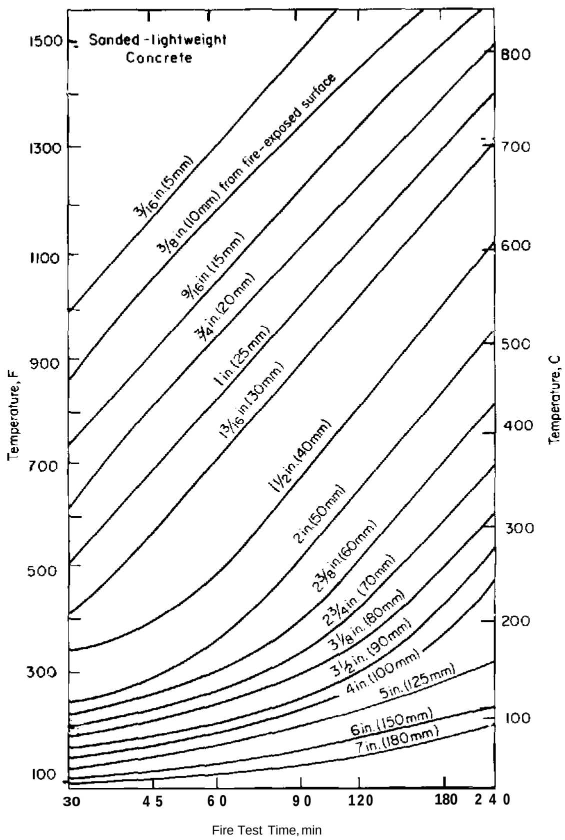


Fig. 7.1.1(c)-Temperatures within slabs during fire tests-sanded lightweight concrete

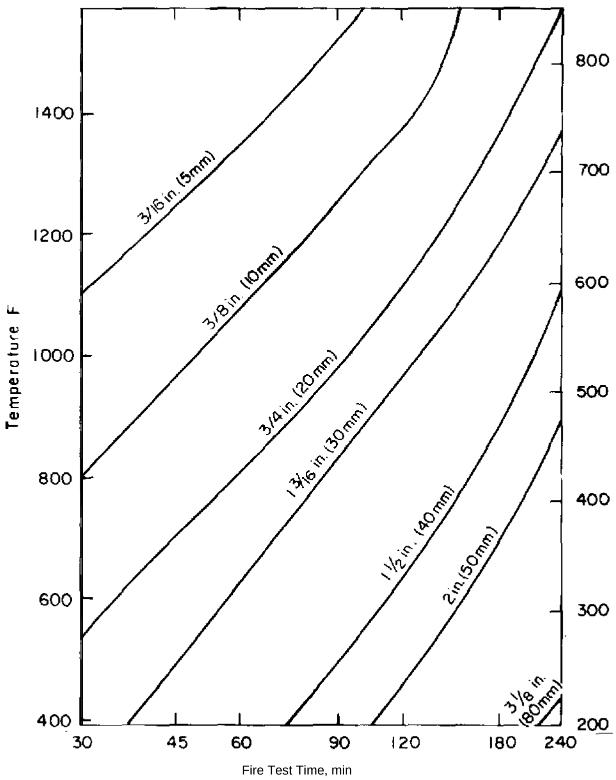


Fig. 7.1.2(a)-Temperatures within 20-30 pcf (320-480 kg/m³) lightweight insulating concrete slabs during fire tests

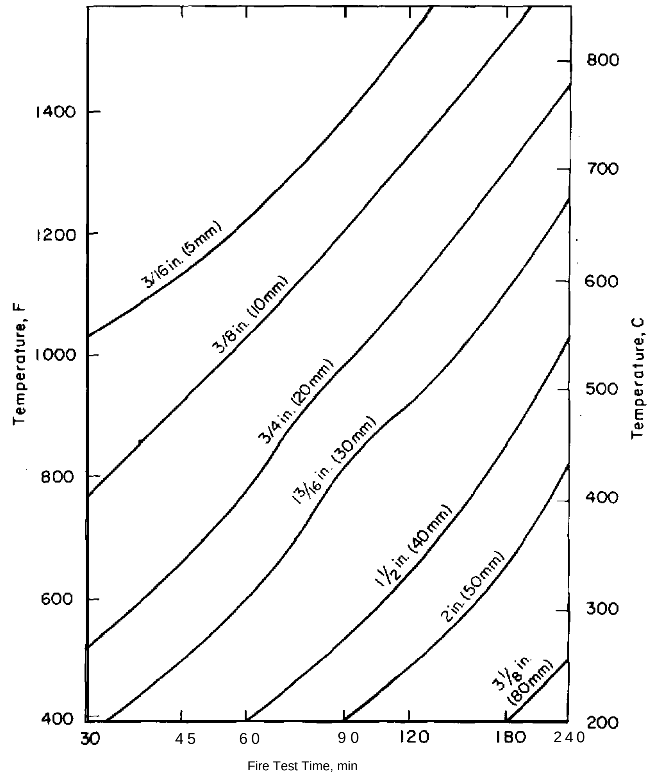


Fig. 7.1.2(c)-Temperatures within 70-80 pcf (1120-1280 kg/m³) lightweight insulating concrete slabs during fire tests

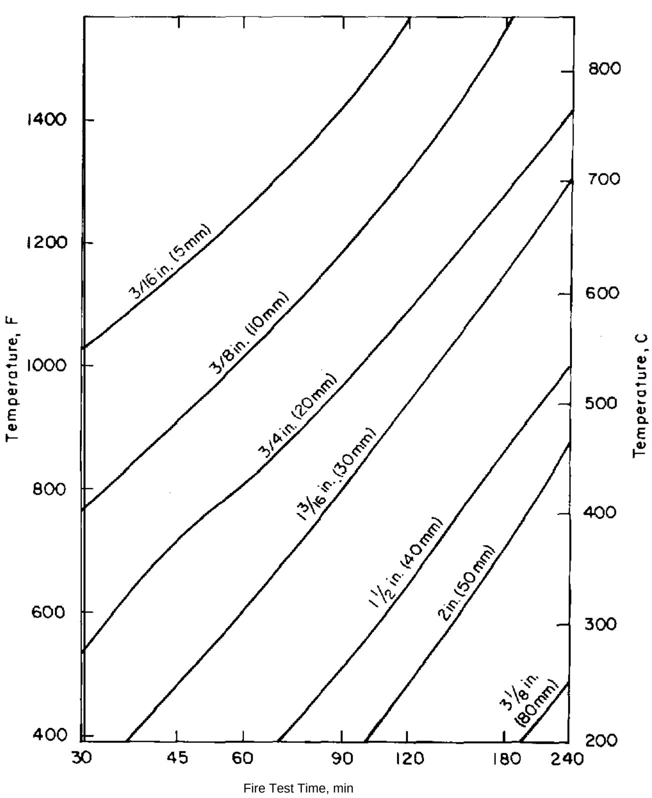


Fig 7.1.2(b)-Temperatures within 50-60 pcf (800-900 kg/m³) lightweight insulating concrete slabs during fire tests

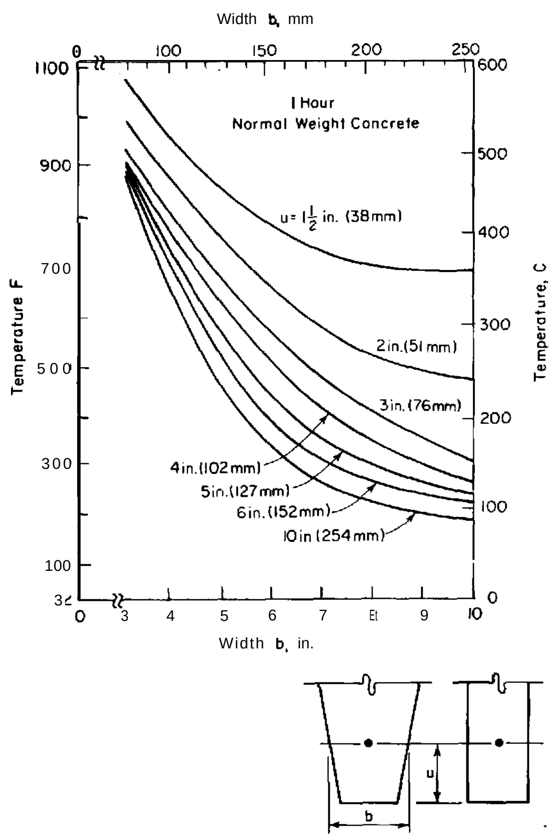


Fig. 7.2.1-Temperatures in normal weight concrete rectangular and tapered units at 1 hr of fire exposure

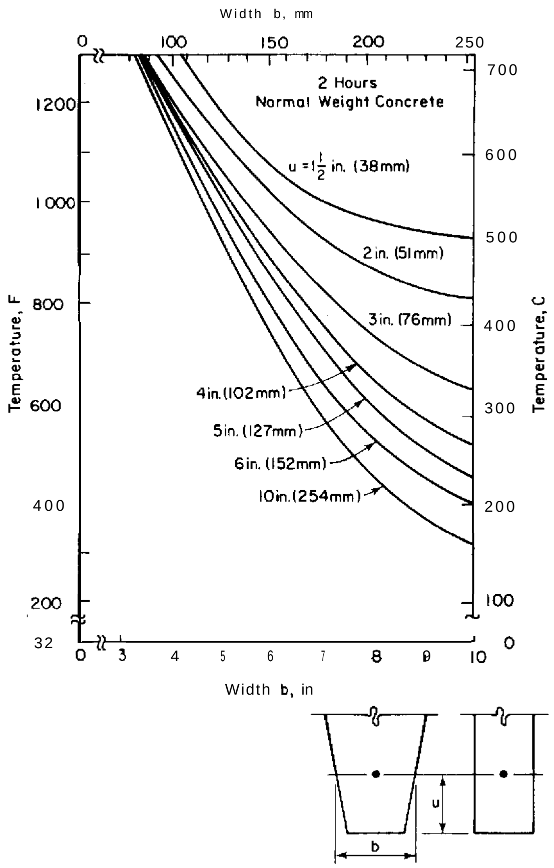


Fig. 7.2.2-Temperatures in normal weight concrete rectangular and tapered units at 2 hr of fire exposure

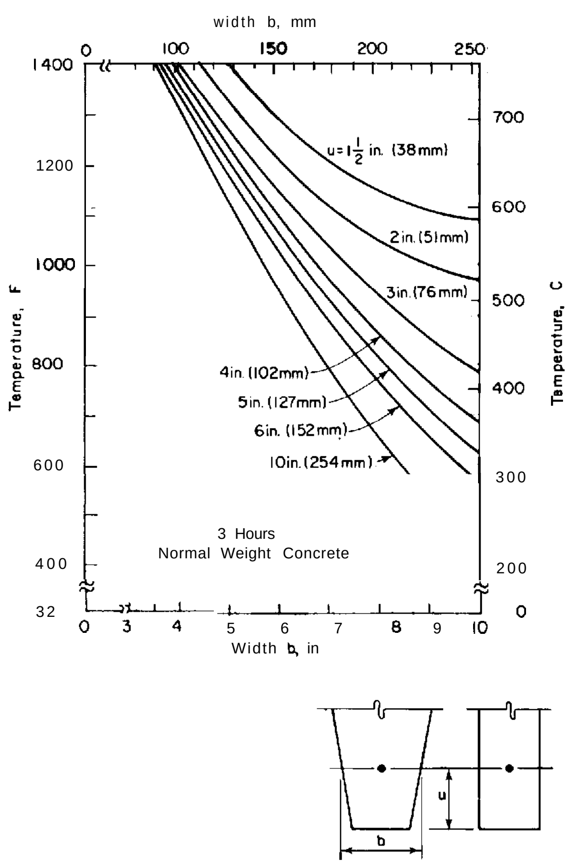


Fig. 7.2.3-Temperatures in normal weight concrete rectangular and tapered units at 3 hr of fire exposure

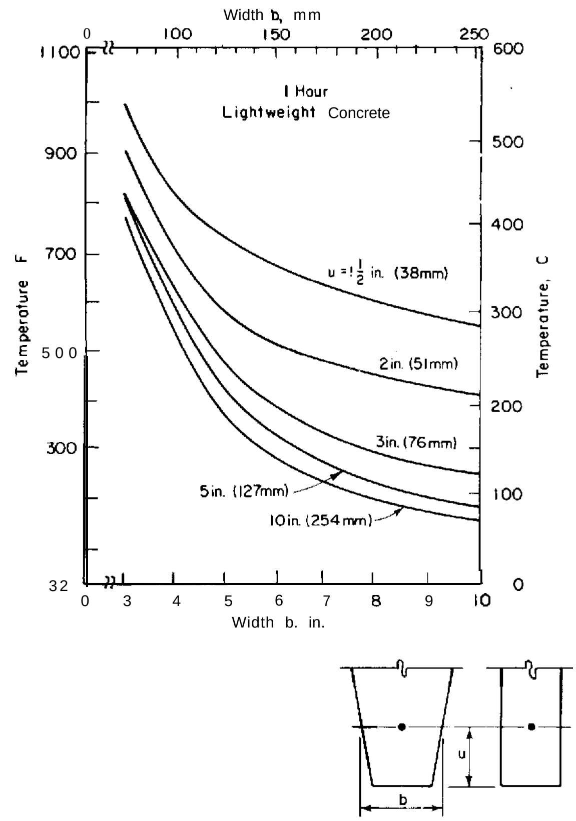


Fig. 7.2.4-Temperatures in lightweight concrete rectangular and tapered units at 1 hr of fire exposure

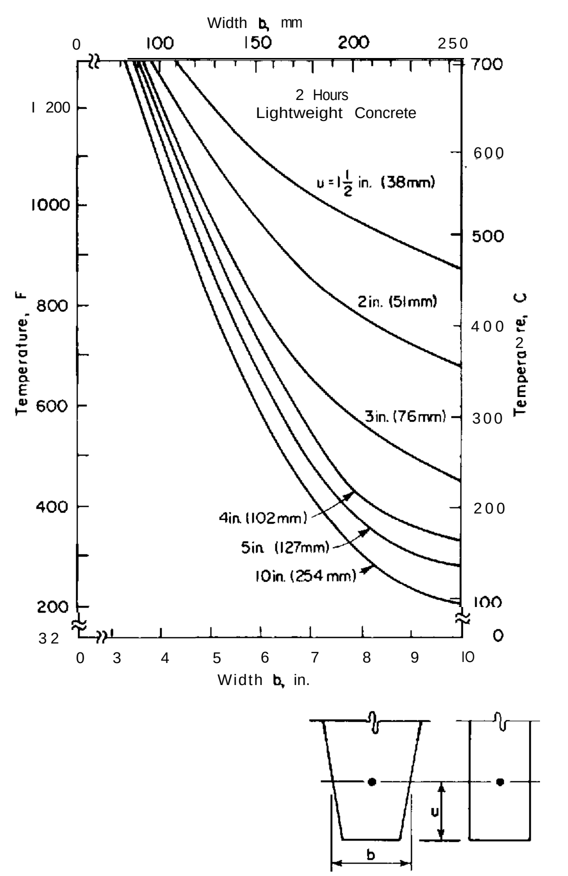


Fig. 7.2.5-Temperatures in lightweight concrete rectangular and tapered units at 2 hr of fire exposure

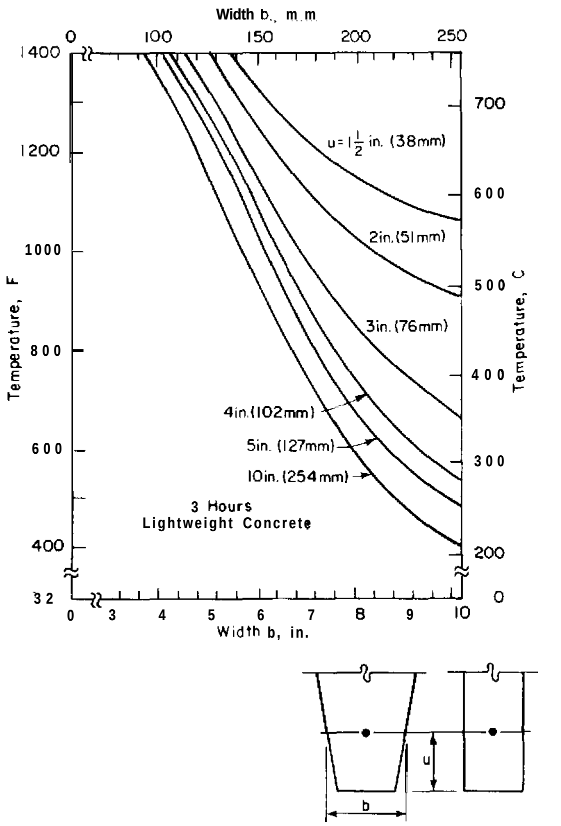


Fig. 7.2.6-Temperatures in lightweight concrete rectangular and tapered units at 3 hr of fire exposure

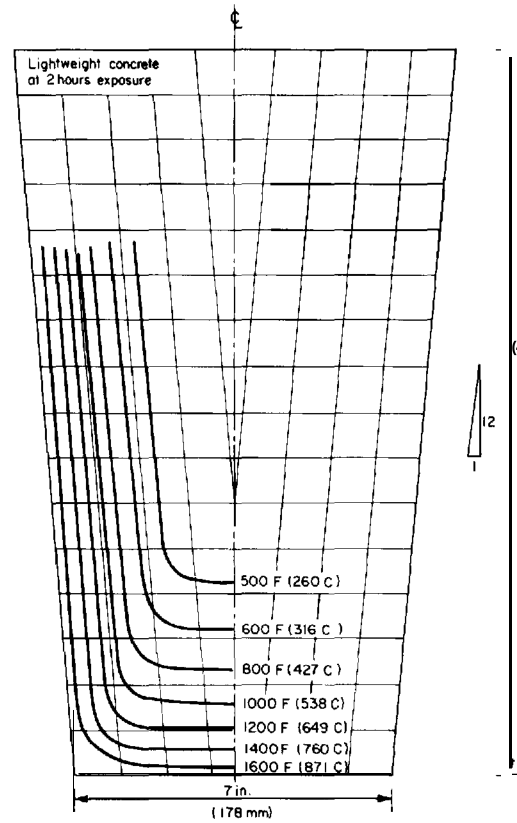


Fig. 7.2.8-Measured temperature distribution at 2 hr of fire exposure for lightweight concrete tapered unit

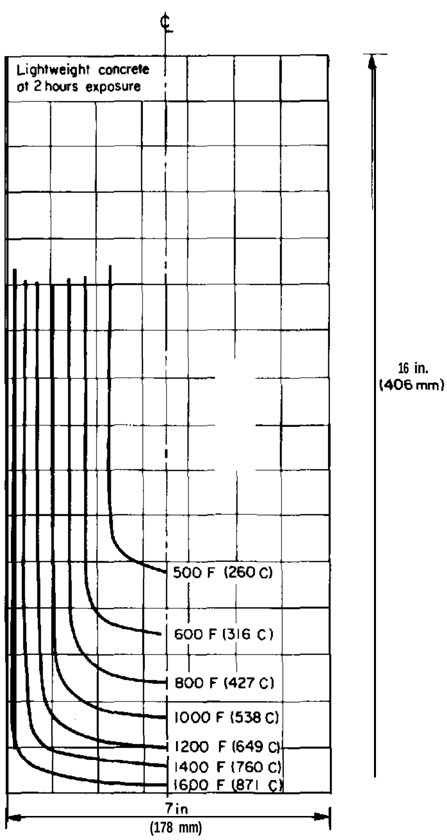


Fig. 7.2.7-Measured temperature distribution at 2 hr of fire exposure for lightweight concrete rectangular unit

distance "*u*" from the bottom for the tapered member. These charts were generated from test data obtained from tests of rectangular and tapered members. Tests were carried out in Underwriters' Laboratories Floor Furnace, Northbrook. Illinois, and Portland Cement Association's Beam Furnace, Skokie, Illinois. Temperature distributions obtained in other furnaces may differ from those shown due to differences in furnace size and design, furnace wall construction, and flame type.

The distributions shown in Fig. 7.2.1 through 7.2.6 were presented in this format because the chart conveniently relates the required design parameters of concrete cover, thickness, temperature, and fire endurance time. Should it be necessary to know the temperatures in the member at locations other than the center line, isotherms can be generated from the data given in Fig. 7.2.1 through 7.2.6 and from distributions obtained in test programs and computer studies completed at PCA (Lin and Abrams 1983). Sample isothermal distributions for a fire endurance period of 2 hr for lightweight aggregate concrete-rectangular and tapered sections 7 in. (178 mm) wide are shown in Fig. 7.2.7 and 7.2.8. Fig. 7.2.9 through 7.2.11 show temperature distributions in a 12 in. (305 mm) wide rectangular carbonate aggregate concrete beam. These curves were based on test temperatures developed at PCA. For members larger than 12 in. (305 mm) the temperature information shown in Fig. 7.1 for flat slabs can be used by considering the corner bars to have half the actual cover. For example, consider a 16 in. (406 mm) wide rectangular normal weight concrete beam having four equally spaced horizontal bars with 2 in. (51 mm) clear cover to the bars from the bottom of the beam and 2 in. (51 mm) clear side

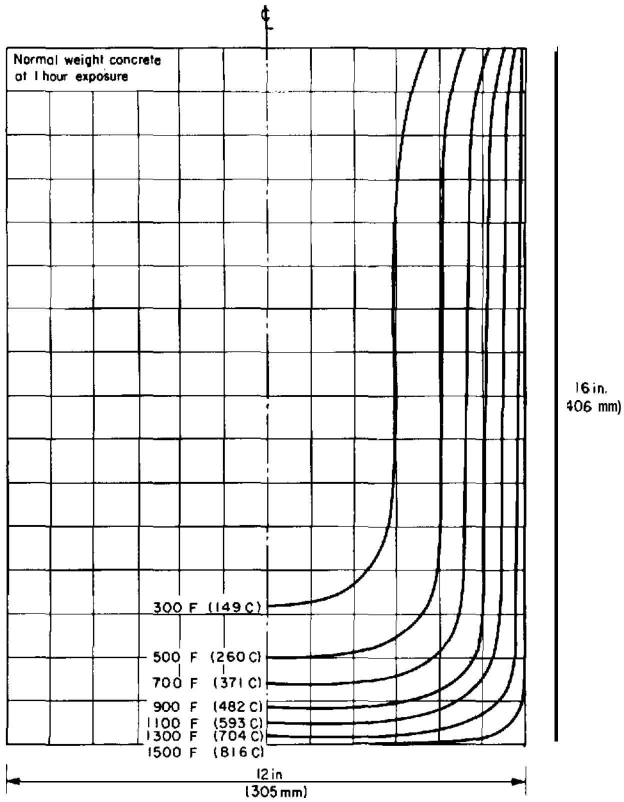


Fig. 7.2.9 -Temperature distribution in normal weight rectangular unit at 1 hr of fire exposure

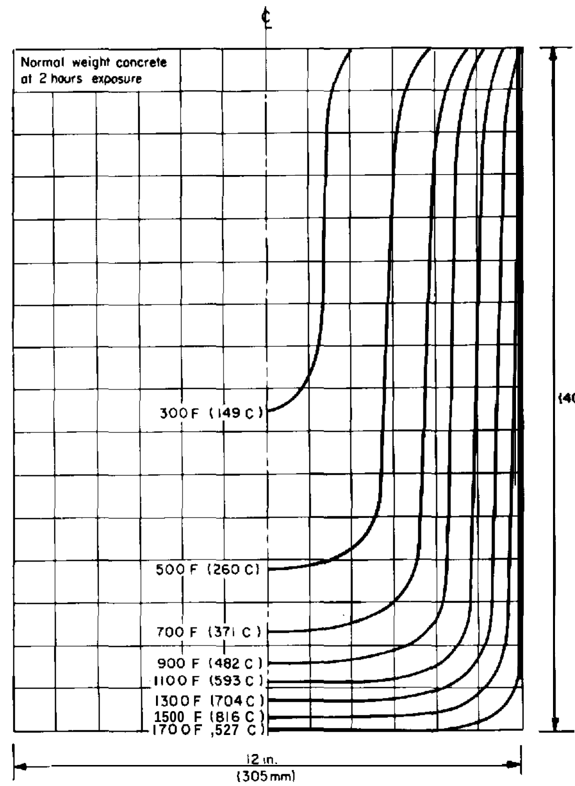


Fig. 7.2.10-Temperature distribution in normal weight concrete rectangular unit at 2 hr of fire exposure

cover to the corner bars. The average cover is calculated as [2 + 2 + ½(2) + ½(2)]/4 = 1½ in. [(51 – 51 – 25 – 25)/4 = 38 mm]. The temperature of the steel is taken from temperature distributions in the normal weight concrete shown in Fig. 7.1 for the 1.5 in. (38 mm) distance from the exposed surface.

Fig. 7.2.12 and 7.2.13 show temperature distributions in carbonate aggregate concrete joists 4 in. (102 mm) wide by 16 in. (406 mm) high coated with 0.5 in. (13 mm) or 1.25 in. (32 mm) of vermiculite CM (VCM) or sprayed mineral fiber (SMF). The tests were made at the Portland Cement Association laboratories and reported by Lin and Abrams (1983). The coating materials were described by Abrams and Gustaferro (1968).

7.3–Double T Units

Odeen (1968) investigated the temperature distribution in double Ts during fire tests. Temperatures within eight sizes of double Ts have been calculated at ½, 1, 1½, and 2 hr of fire exposure.

7.4–Masonry units

Harmathy (1970a) performed an extensive series of computer calculations to study the heat flow during fire tests of masonry walls. The calculations take into account the geometry of the unit, concrete type, and temperature dependent thermal properties and heat flow mechanisms.

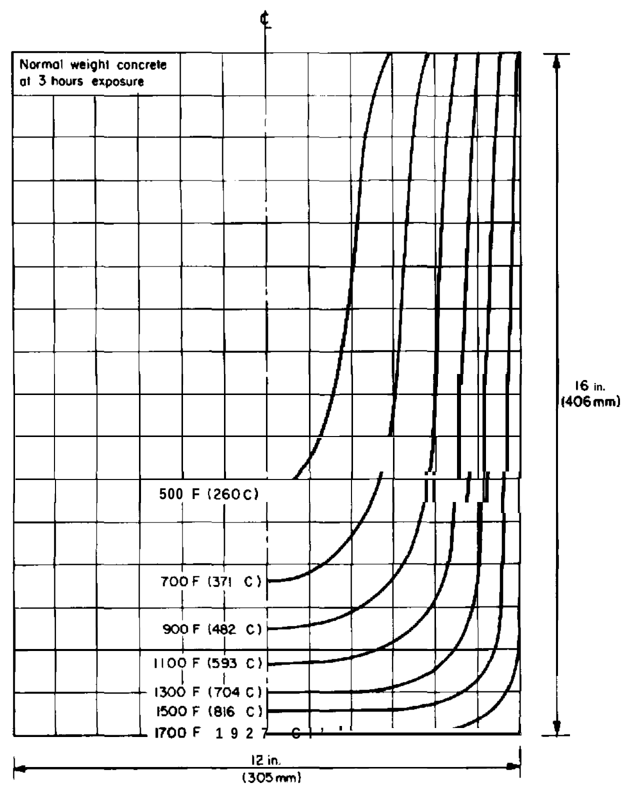


Fig. 7.2.11-Temperature distribution in normal weight rectangular unit at 3 hr of fire exposure

7.5-Columns

Based on a numerical technique developed by Lie and Harmathy (1972) the temperature distribution in concrete-protected steel columns was analyzed, and an empirical formula was derived for the calculation of the fire endurance of such columns (Lie and Harmathy 1974). Lie and Allen, in Technical Paper 378, studied the temperature distribution in solid concrete columns during fire. Lie and Lin conducted a series of 38 fire tests of full-sized reinforced concrete columns in the period from 1976 to 1986. These latter studies covered reinforced concrete beams as well.

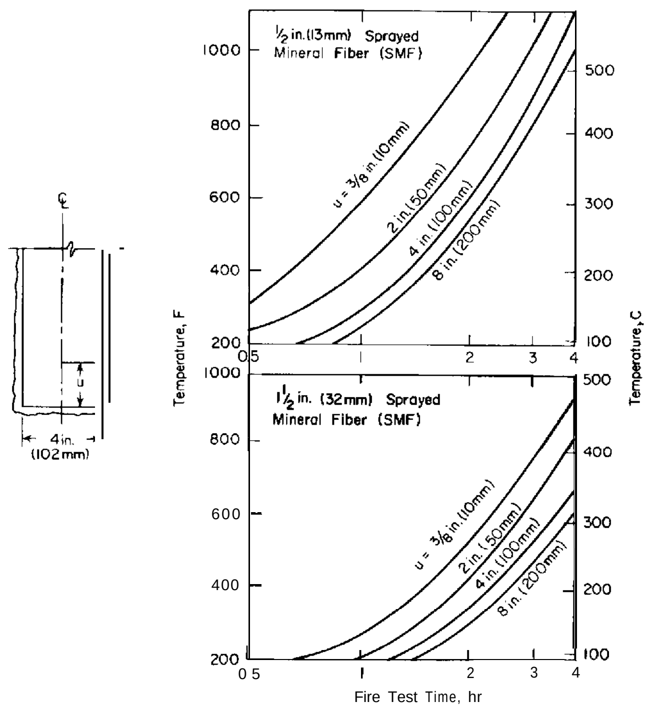


Fig. 7.2.12-Temperatures along vertical center lines at various fire exposures for 4.0 in. (100 mm) wide rectangular units coated with SMF

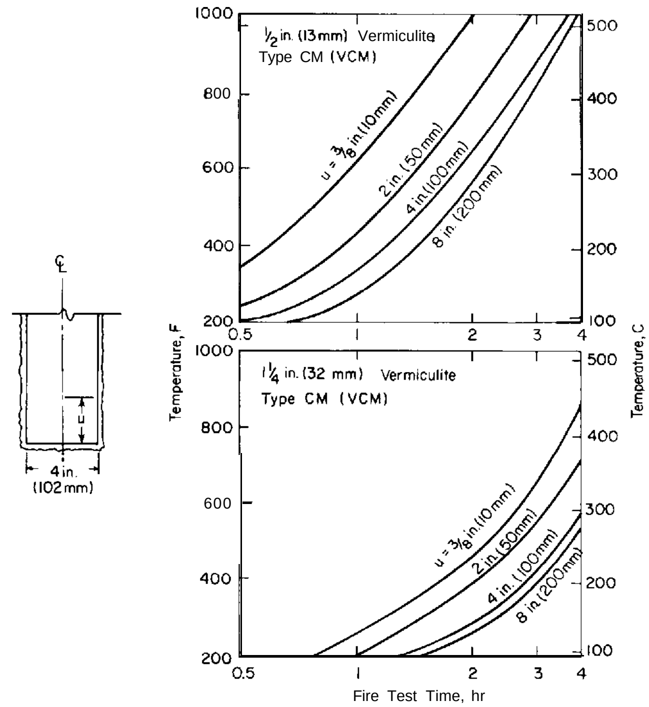


Fig. 7.2.13- Temperatures along vertical center lines at various fire exposures for 4.0 in. (100 mm) wide rectangular units coated with VCM

CHAPTER 8–EXAMPLES

Six examples illustrate the calculation techniques discussed in Chapters 2 and 3 and use the data presented in Chapters 5 and 6. Example 7 shows the preliminary assessmcnt of fire endurance requirement using the technique described in the Appendix.

Example 1–Determination of fire endurance of a simply supported one-way slab

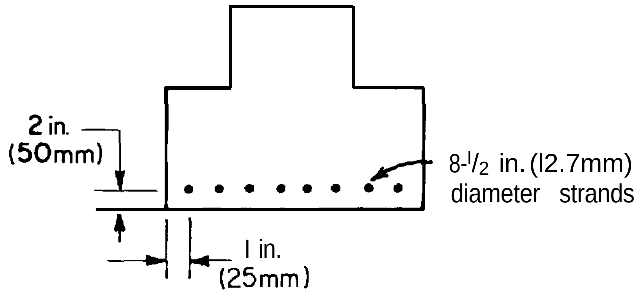
Given: A simply supported one-way slab reinforced with #4 Grade 60 bars on 6.0 in. (150 mm) centers. The slab is made of carbonate aggregate concrete with a density of 150 pcf (2400 kg/m³). Its spccitied compressive strength is 4000 psi (28 MPa). Cover is 0.75 in. (19 mm). The slab is 6.0 in. (150 mm) thick and its span is 14.8 ft (4.5 m). Live load is 100 psf (4.8 kPa).

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 1-List known values	Reinforcement $d_b = 0.500$ in. $A_b = 0.20$ in.² $A_s = \frac{12}{6}(0.20) = 0.40$ in.²/ft	Reinforcement $d_b = 12.7$ mm $A_b = 129$ mm² $A_s = \frac{1000}{150}(129) = 860$ mm²

Example 1-(Continued)

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 1–(Continued)	f_y = 60,000 psi cover = 0.75 in. u = 1.00 in. Slab h = 6.0 in. l = 14.8 ft f_c' = 4000 psi Loading w_l = 100 psf $w_d = \frac{6}{12}(150) = 75$psf w = 100 + 75 = 175 psf	f_y = 410 MPa cover = 19 mm u = 25 mm Slab h = 150 mm l = 4.51 m f_c' = 28 MPa Loading w_l = 4.8 kPa $w_d = \frac{150}{1000}\left(9.8\frac{\text{kg}}{\text{Nm}}\right)\left(\frac{2400}{1000}\right)$ = 3.5 kPa w = 4.8 + 3.5 = 8.3 kPa
Step 2–Calculate M from $M = \frac{wl^2}{8}$	$M = \frac{175\ (14.8)^2}{8}$ = 4790 lb-ft/ft width	$M = \frac{8.3\ (4.51)^2}{8}$ = 21 kN-m/m width
Step 3–Calculate M_n from $M_n = A_s f_y \left(d - \frac{a}{2}\right)$ To do so, d and a are needed $d = h - \text{cover} - ½\, d_b$ $a = \frac{A_s F_y}{0.85 f_c'\, b}$ $M_n = A_s f_y \left(d - \frac{a}{2}\right)$	 d = 6.0 – 0.75 – ½ (0.500) = 5.0 in. $a = \frac{0.40\ (60{,}000)}{0.85\ (4000)\ (12)}$ = 0.59 in. $M_n = \frac{0.40\,(60{,}000)\left(5.0 - \frac{0.59}{2}\right)}{12\ (\text{in./ft})}$ = 9400 lb-ft/ft width	 d = 150 – 19 – ½ (12.7) = 125 mm $a = \frac{860\ (410)}{0.85\ (28)\ (1000)}$ = 15 mm $M_n = \frac{860\ (410)\left(125 - \frac{15}{2}\right)}{(1000\ \text{kg/g})(1000\ \text{mm/m})}$ = 41.4 kN-m/m width
Step 4–Calculate M/M_n	$M/M_n = \frac{4790}{9400} = 0.51$	$M/M_n = \frac{21}{41.4} = 0.51$
Step 5–Calculate ω from $\omega = A_s f_y/(bdf_c')$	ω = 0.40 (60,000)/[12 (5.0) (4000)] = 0.10	ω = 860 (410)/[1000 (125) (28)] = 0.10
Step 6–From graph for carbonate aggregate concrete with reinforcing bars on Fig. 2.1.2. 1, read fire endurance for given u and calculated M/M_n and ω	For u = 1.0in., M/M_n = 0.51, and ω = 0.10. fire endurance is just over 2.5 hours	For u =25 mm, M/M_n = 0.51, and ω = 0.10, fire endurance is just over 2.5 hours

Example 2—Determination of fire endurance of a prestressed concrete beam

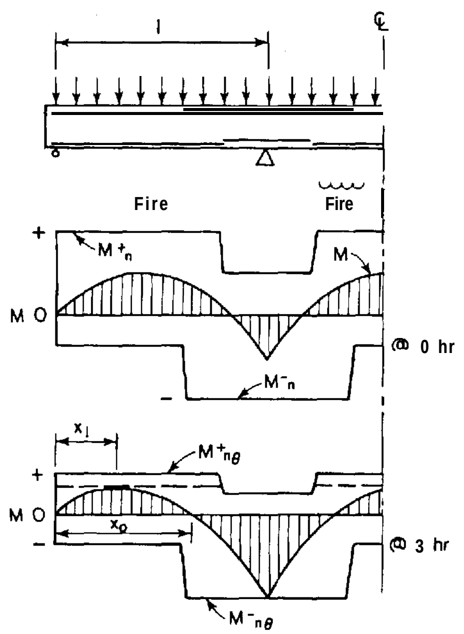


Given: The prestressed beam shown in the sketch. Beam is made of lightweight concrete. M/M_n is 0.50 and ω_p is 0.306.

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 1—List known values	$M/M_n = 0.50$ $\omega_p = 0.306$ cover + ½ d_b = 2 in	$M/M_n = 0.50$ $\omega_p = 0.306$ cover + ½ d_b = 50 mm
Step 2—Determine effective u where for interior bars, u = cover + ½ d_b and for corner bars, u = ½ (cover + ½ d_b) effective u = average u	 u = 2 in. u = ½ (2) in. effective $u = \frac{6\ (2.0) + 2\ (½)(2.0)}{8}$ = 1.8 in.	 u = 50 mm u = ½ (50) mm effective $u = \frac{6\ (50) + 2\ (½)(50)}{8}$ = 44 mm
Step 3—From graph for lightweight concrete on Fig. 2.1.2.1, read fire endurance for calculated effective u and given M/M_n and ω_p	For effective u = 1.8 in., M/M_n = 0.50, and ω_p = 0.306, fire endurance is just over 2% hours	For effective u = 44 mm, M/M_n = 0.50, and ω_p = 0.306, fire endurance is just over 2½ hours

Example 3 - Determination of cross sectional area and length of negative reinforcement required in a two-span slab to provide three-hour fire endurance

Given: A two-span siliceous aggregate concrete slab 6.0 in. (150 mm) thick, reinforced for positive moment with #4 Grade 60 bars on 6.0 in. (150 mm) centers with 0.75 in. (19 mm) cover. Each span is 18.0 ft (5.5 m) and superimposed load is 42 psf (2.0 kPa). Concrete has a unit weight of 150 pcf (2400 kg/m^3) and a specified compressive strength of 4000 psi (28 MPa).



Example 3-(Continued)

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 1—List known values	Reinforcement d_b = 0.500 in. A_b = 0.20 in.^{2} A_s = (12.0/6) (0.20) = 0.40 in.^{2} f_y = 60,000 psi cover = 0.75 in. u = 0.75 + ½ (0.500) = 1.00 in. Slab f_c' = 4000 psi h = 6.0 in. l = 18.0 ft Loading w_l = 42 psf w_d = 150 pcf w = (6/12) (150) + 42 = 117 lb/ft width	Reinforcement d_b = 12.7 mm A_b = 129 mm^2 A_s = (1000/ 150) (129) = 860 mm^2 f_y = 410 MPa cover = 19 mm u = 19 + ½ (12.7) = 25 mm Slab f_c' = 28 MPa h = 150 mm l = 5.5 m Loading w_l = 2.0 kPa w_d = 2400 kg/m^3 $w = \frac{(150/1000)\ (2400)\ (9.8)}{1000} + 2.0$ = 5.5 kN/m width
Step 2—Determine positive nominal moment strength $M_{n\theta}^+$ at 3 hours of fire exposure from $$M_{n\theta}^+ = A_s f_{y\theta}\left(d - \frac{a_\theta^+}{2}\right)$$ To do so, values of θ, $f_{y\theta}$, and a_θ^+ must be found		
Find θ, using Fig. 7.1.1 (b)	For u = 1.00 in. and t = 180 minutes, θ = 1380 F	For u = 25 mm and t = 180 minutes, θ = 750 c
Determine $f_{y\theta}$ using Fig. 5.1	For θ = 1380 F, $f_{y\theta}/f_y$ = 0.20 $f_{y\theta}$ = 0.20 (60,000) = 12,000 psi	For θ = 750 C, $f_{y\theta}/f_y$ = 0.20 $f_{y\theta}$ = 0.20 (410) = 82 MPa
Calculate d from $d = h -$ cover $- ½\, d_b$	d = 6.0 − 0.75 − ½ (0.500) = 5.0 in.	d = 150 − 19 − ½ (12.7) = 125 mm
Calculate a_θ^+ from $$a_\theta^+ = \frac{A_s f_{y\theta}}{0.85\, f_{c\theta}' b}$$ Note: $f_{c\theta}'$ is assumed to equal f_c' $$M_{n\theta}^+ = A_s f_{y\theta}\left(d - \frac{a_\theta^+}{2}\right)$$	$a_\theta^+ = \frac{0.40\ (12{,}000)}{0.85\ (4000)\ (12)} = 0.12$ in. $M_{n\theta}^+ = 0.40\ (12{,}000)\left(\frac{5.0 - \frac{0.12}{2}}{12}\right)$ = 2000 lb-ft/ft width	$a_\theta^+ = \frac{860\ (82)}{0.85\ (28)\ (1000)}$ = 3.0 mm $M_{n\theta}^+ = \frac{860\ (82)}{1000}\left(\frac{125 - \frac{3.0}{2}}{1000}\right)$ = 8.7 kN-m/m width

Example 3-(Continued)

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 3–Determine required negative nominal moment strength $M^-_{n\theta}$ at interior support from $M^-_{n\theta} = \frac{wl^2}{2} - wl^2\sqrt{\frac{2\,M^+_{n\theta}}{wl^2}}$	$M^-_{n\theta} = \frac{117\,(18.0)^2}{2} - 117\,(18.0)^2\sqrt{\frac{2\,(2000)}{117\,(18.0)^2}}$ = 6600 lb-ft/ft width	$M^-_{n\theta} = \frac{5.5\,(5.5)^2}{2} - 5.5\,(5.5)^2\sqrt{\frac{2\,(8.7)}{5.5\,(5.5)^2}}$ = 29 kN-m/m width
Step 4–Determine required cross sectional area A^-_s of negative moment reinforcement from req. $A^-_s = \frac{M^-_{n\theta}}{f_{y\theta}\left(d'_\theta - \frac{a^-_\theta}{2}\right)}$		
To do so, θ, $f_{y\theta}$, d'_θ, and a_θ must be found		
Location of negative movement reinforcement is $d' = h$ − cover − ½ d_b	d' = 6.0 − 0.75 − ½ (0.500) = 5.0 in.	d' = 150 − 19 − ½ (12.7) = 125 mm
Find θ, using Fig. 7.1.1 (b) for d'	For d' = 5.0 in. and t = 180 minutes. θ = 380 F	For d' = 125 mm and t = 180 minutes, θ = 190 C
Read $f_{y\theta}$ from Fig. 5.1	For θ = 380 F, $f_{y\theta}/f_y$ = 94 percent; $f_{y\theta}$ = 0.94 (60,000) = 56,000 psi	For θ = 190 C, $f_{y\theta}/f_y$ = 94 percent; $f_{y\theta}$ = 0.94 (410) = 390 MPa
Calculate d'_θ, neglecting concrete at 1400 F (760 C) or higher temperatures		
Determine thickness of slab at 1400 F (760 C) or higher, using Fig. 7.11.(b)	0.9 in. of concrete is at 1400 F or higher	23 mm of concrete is at 760 C or higher
	effective d' = 6.0 − 0.9 − 0.75 − ½ (0.500) = 4.1 in.	effective d' = 150 − 23 − 19 − ½(12.7) = 102 mm
To find required A_s, assume a value for a^-_θ and calculate A_s from req. $A_s = \frac{M_{n\theta}}{f_{y\theta}\left(\text{eff. } d' - \frac{a_\theta}{2}\right)}$	assume a_θ = 0.9 in. req. $A_s = \frac{6600\,(12\text{ in./ft})}{56{,}000\,\left(4.1 - \frac{0.9}{2}\right)}$ = 0.39 in.²/ft width	assume a_θ = 23 mm req. $A_s = \frac{29\,(1000\text{ N/kN})\,(1000\text{ mm/m})}{390\left(102 - \frac{23}{2}\right)}$ = 820 mm²/m width
Check assumed a^-_θ using $a^-_\theta = \frac{A_s f_{y\theta}}{0.85 f'_{c\theta} b}$		

Example 3-(Continued)

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 4—(Continued) To do so. A_s and $f_{c\theta}'$ must be found		
For $A_s \geq$ required A_s, try X4 Grade 60 bars on 5.5 in. (140 mm) centers	$A_s^- = \frac{12.0}{5.5}(0.20)$ $= 0.44$ in.^{2}/ft width > 0.39 in.^{2}/ft width	$A_s^- = \frac{1000}{140}(129)$ $= 921$ mm^2/m width > 820 mm^2/m width
To find $f_{c\theta}'$, assume that effective temperature of concrete is average of I) 1400 F (760 C) and 2) temperature of concrete at 0.38 (eff. d').	At 0.38 (4.1) + 0.9 = 2.5 in. and t = 180 minutes. θ = 800 F	At 0.38 (102) + 23 = 62 mm and t = 180 minutes, θ = 430 C
Find 2) using Fig 7.1. I (b)	average $\theta = \frac{1400 + 800}{2}$ $= 1100$ F	average $\theta = \frac{760 + 430}{2}$ $= 600$ C
Read $f_{c\theta}'/f_c'$ from Fig. 6.1.1 for concrete stressed to $0.4 f_c'$ and calculate $f_{c\theta}'$	For θ = 1100 F, $f_{c\theta}'/f_c'$ = 64 percent $f_{c\theta}' = 0.64(4000) = 2600$psi	For θ = 600 C, $f_{c\theta}'/f_c'$ = 64 percent $f_{c\theta}' = 0.64\ (28) = 18$ MPa
Calculate $a_\theta^- = \frac{A_s^- f_{y\theta}}{0.85 f_{c\theta}' b}$ and compare with assumed a_θ^-	$a_\theta^- = \frac{0.44\ (56{,}000)}{0.85\ (2600)\ (12)}$ $= 0.93$ in. ≈ 0.9 assumed	$a_\theta^- = \frac{921\ (390)}{0.85\ (18)\ (1000)}$ $= 23$ mm $= 23$ mm assumed
Step 5—Verify that $\omega_\theta \leq 0.30$ where $\omega_\theta = \frac{A_s f_{y\theta}}{b\ (\text{eff.}d')\ f_{c\theta}'}$	$\omega_\theta = \frac{0.44\ (56{,}000)}{12\ (4.1)\ (2600)}$ $= 0.19$ in. < 0.30 $\therefore$ assumed a_θ^- of 0.9 in. and A_s of 0.44 in.^{2} are satisfactory	$\omega_\theta = \frac{921(390)}{1000(102)(18)}$ $= 0.20 < 0.30$ $\therefore$assumed a_θ^- of 23 mm and A_s of 390 mm^2 are satisfactory
Step 6—Determine required length of top bars from $x_0 = 2x_1 = 2\left(\frac{l}{2} - \frac{M_{n\theta}}{wl}\right)$	$x_0 = 2\left(\frac{18.0}{2} - \frac{7300}{117\ (18.0)}\right)$ $= 11.1$ft	$x_0 = 2\left(\frac{5.5}{2} - \frac{32}{5.5(5.5)}\right)$ $= 3.4$m
Note: Theoretically bars could be cut a distance $l - x_0 + l_d$ on either side of intermediate support, where l_d is the development length of bars. However, it is recommended that only 40 percent of bars be cut off at $l - x_0 + l_d$ on either side of the intermediate support	$18 - 11.1 + l_d = 6.9 + l_d$	$5.5 - 3.4 + l_d = 2.1 + l_d$

Example 3-(Continued)

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 6-(Continued) 40 percent be cut at $l - \frac{1}{2}x_0 + l_d$ on either side of the intermediate support	$18 - \frac{1}{2}(11.1) + l_d = 12.5 \text{ ft} + l_d$	$5.5 - \frac{1}{2}(3.4) + l_d = 3.8 + l_d$
20 percent of bars extend to the external support		

Example 4–Verification that an exterior-bay floor panel qualifies for a two-hour fire endurance rating

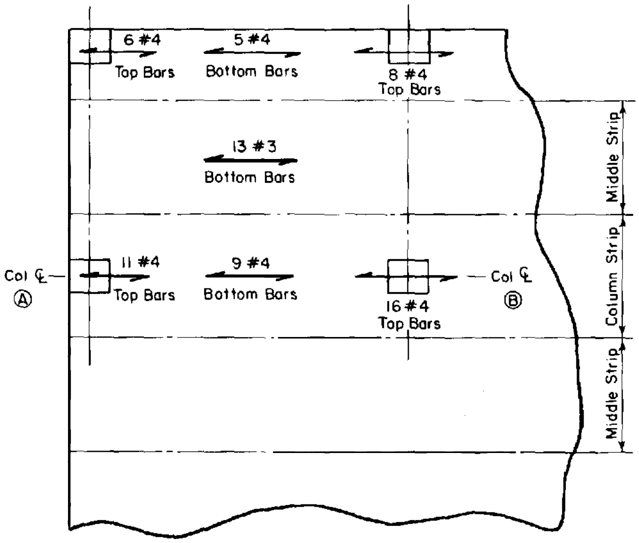
Given: A two-way multispan rectangular floor slab of 7.0 in. (180 mm) thick flat plate construction. Exterior panel in question is supported by 15.0 in. 380 mm) square tied columns. Panel spans are 18.0 ft (5.50 m).

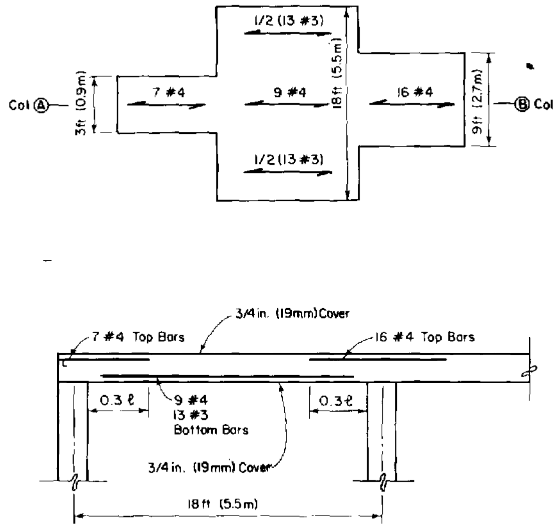
Reinforcement is #3 and #4 Grade 60 bars placed as shown in the sketches. Reinforcement is the same in both directions.

Concrete is made with siliceous aggregate and has a specified compressive strength of 4000 psi (28 MPa).

Dead load is 15 psf (0.72 kN/m^2) and live load is 40 psf (1.9) kN/m^2).

Note: Section 13.3.3.2 of ACI 318-83 requires that a fraction of the unbalanced moment be considered to be transferred by flexure over an effective slab width between lines that are 1½ times slab thickness outside opposite faces of the column. The design detail shown meets this requirement.





Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 1-given data	Reinforcement #4 bar d_b = 0.500 in. A_b = 0.20 in.^{2} #3 bar d_b = 0.375 in. A_b = 0.11 in.^{2} A_s^- = 1.40 in.^{2} (exterior support) A_s^- = 3.20 in.^{2} (interior support) A_s^+ = 3.23 in.^{2} (mid-span) f_y = 60,000 psi cover = 0.75 in.	Reinforcement #4 bar d_b = 12.7 mm A_b = 129 mm^2 #3 bar d_b = 9.5 mm A_b = 71 mm^2 A_s^- = 903 mm^2 (exterior support) A_s^- = 2060 mm^2 (interior support) A_s^+ = 2080 mm^2 (mid-span) f_y = 410 MPa cover = 19 mm

Example 4–(Continued)

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 1–(Continued)	$u = 0.75 + \dfrac{(13 \times 0.375) + (9 \times 0.500)}{2 \times 22}$ $= 0.96$ in. $d = 7.0 - 0.96 = 6.0$ in	$u = 19 + \dfrac{(13 \times 9.5) + (9 \times 12.7)}{2 \times 22}$ $= 24$mm $d = 152$ mm
	Slab $f_c' = 4000$ psi $b = 108$ in. $b_E = 36$ in. $h = 7.0$ in. $l = 18.0$ ft $l_n = 16.75$ ft	Slab $f_c' = 27.6$MPa $b = 2.75$ m $b_E = 0.91$ m $h = 180$ mm $l = 5.5$m $l_n = 5.1$ m
	Loading $w_l = 55$ psf $w_d = \dfrac{7.0}{12}(150) = 87$ psf $w = 55 + 87 = 142$ psf	Loading $w_l = 2.6$ kN/m^2 $w_d = \dfrac{(180/1000)\ (2400)\ (9.81)}{1000}$ $= 4.2$ kN/m^2 $w = 2.6 + 4.2 = 6.8$ kN/m^2
Step 2–Determine positive nominal moment strength $M_{n\theta}^+$ at 3 hours of fire exposure from $M_{n\theta}^+ = A_s f_{y\theta}\left(d - \dfrac{a_\theta^+}{2}\right)$ To do so, values of θ, $f_{y\theta}$, and a_θ^+ must be found		
Find θ, using Fig. 7.1.1 (b)	For $u = 0.96$ in. and $t = 120$ min. $\theta = 1200$ F	For u = 24.5 mm and $t = 120$ min. $\theta = 650$ C
Determine $f_{y\theta}$ using Fig. 5.1 $f_{y\theta} = f_y\,(f_{y\theta}/f_y)$	For $\theta = 1200$ F, $f_{y\theta}/f_y = 0.44$ $f_{y\theta} = 0.44\ (60{,}000) = 26{,}400$ psi	For $\theta = 650$ C, $f_{y\theta}/f_y = 0.44$ $f_{y\theta} = 0.44(410) = 180$ MPa
From Fig. 6.1.1 read $f_{c\theta}'$ of bottom concrete	$f_{c\theta}' = 0.81\ f_c' = 3200$ psi	$f_{c\theta}' = 0.81\ (27.6) = 22.4$ MPa
Calculate a_θ^+ from $a_\theta^+ = \dfrac{A_s f_{y\theta}}{0.85 f_{c\theta}' b}$	$a_\theta^+ = \dfrac{3.23\ (26{,}400)}{0.85\ (4000)\ (108)} = 0.23$ in.	$a_\theta^+ = \dfrac{2080(178)}{0.85\ (28)\ (2750)} = 5.7$ mm
Calculate $M_{n\theta}^+$ from $M_{n\theta}^+ = A_s f_{y\theta}\left(d - \dfrac{a_\theta^+}{2}\right)$	$M_{n\theta}^+ = 3.23\ (26{,}400)\left(\dfrac{6.0 - \dfrac{0.23}{2}}{12}\right)$ $= 41{,}800$ lb-ft/ft width $= 41.8$ kip-ft/ft width	$M_{n\theta}^+ = 2080\ (178)\left(152 - \dfrac{5.7}{2}\right)$ $= 55.8$ kN-m/m width

Example 4–(Continued)

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 3–Determine negative nominal moment strength over the exterior support from $$M_{n\theta} = A_s f_{y\theta}\left(\text{eff. } d' - \frac{a_\theta}{2}\right)$$		
To do so, values of θ, $f_{y\theta}$, eff. d', and a_θ must be found		
Find θ using Fig. 7.1.1 (b)	For u = 6 in. and t = 120 min. θ = 250 F	For u = 152 mm and t = 120 min, θ = 120 C
Determine $f_{y\theta}$ using Fig. 5.1	For θ = 250 F, $f_{y\theta}/f_y$ = 0.95 $f_{y\theta}$ = 0.95 (60,000) = 57,000 psi	For θ = 120 C, $f_{y\theta}/f_y$ = 0.95 $f_{y\theta}$ = 0.95 (410) = 390 MPa
Determine thickness of slab at 1400 F (760 C) or higher, using Fig. 7.1.1 (b)	0.6 in. concrete at exposed surface is at 1400 F or higher	15 mm concrete at exposed surface is at 760 C or higher
Determine effective d'	eff. d' = $7.0 - 0.6 - 0.75 - \frac{1}{2}\left(\frac{0.500 + 0.375}{2}\right)$ = 5.4 in.	eff. d' = $180 - 15 - 19 - \frac{1}{2}\left(\frac{12.7 + 9.5}{2}\right)$ = 140 mm
To find $f'_{c\theta}$, assume that the realistic temperature of concrete is the average of 1) 1400 F (760 C) and 2) temperature of concrete at 0.35 (eff. d')		
Find 2) using Fig. 7.1.1 (b)	At 0.35 (5.4) + 0.6 = 2.5 in. and t = 120 min, θ = 600 F	At 0.35 (140) + 15 = 65 mm and t = 120 min, θ = 320 C
Find average θ	average $\theta = \frac{1400 + 600}{2} = 1000$ F	average $\theta = \frac{760 + 320}{2} = 540$ C
From Fig. 6.1.1 $$f'_{c\theta} = (f'_{c\theta}/f'_c)(f'_c)$$	For θ = 1100 F, $f'_{c\theta}/f'_c$ = 0.75 $f'_{c\theta}$ = 0.75 (4000) = 3000 psi	For θ = 540 C, $f'_{c\theta}/f'_c$ = 0.75 $f'_{c\theta}$ = 0.75 (27.6) = 21 MPa
Determine negative nominal moment strength over the exterior support from $$a^-_\theta = \frac{A^-_s f_{y\theta}}{0.85 f'_{c\theta} b}$$	$a^-_\theta = \frac{1.4\ (57{,}000)}{0.85\ (3000)\ (36.0)} = 0.87$ in.	$a^-_\theta = \frac{903\ (390)}{0.85\ (21)\ (914)} = 22$ mm
and $$M^-_{n\theta} = A^-_s f_{y\theta}\left(d' - \frac{a^-_\theta}{2}\right)$$	$M^-_{n\theta} = \frac{1.40(57{,}000)}{1000}\left(\frac{5.4 - \frac{0.87}{2}}{12}\right)$ = 33.1 kip-ft/ft width	$M^-_{n\theta} = \frac{903\ (390)}{1000}\left(\frac{140 - \frac{22}{2}}{1000}\right)$ = 45 kN-m/m width

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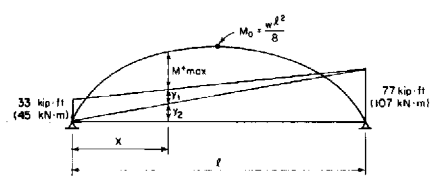
Example 4–(Continued)

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 4–Determine negative nominal moment strength $M^-_{n\theta}$ over the interior support using $a^-_\theta = \frac{A^-_s f_{y\theta}}{0.85 f_{c\theta}{}' b}$	$a_\theta = \frac{(3.23)\ (57{,}000)}{0.85\ (3000)\ (108)} = 0.67$ in.	$a_\theta = \frac{(2080)\ (390)}{0.85\ (21)\ (2740)} = 17$ mm
and $M^-_{n\theta} = A^-_s f_{y\theta}\left(d' - \frac{a^-_\theta}{2}\right)$	$M_{n\theta} = \frac{3.20\ (57{,}000)}{1000}\left(\frac{5.4 - \frac{0.67}{2}}{12}\right)$ = 77 kip-ft/ft width	$M_{n\theta} = \frac{2064\ (377.2)}{1000}\left(\frac{139 - \frac{16}{2}}{1000}\right)$ = 107 kN-m/m width
Step 5–Determine maximum positive bending movement M^+_{max} 	$w = 142\ (18.0) = 2.56$ kip/ft; $M_0 = \frac{wl^2}{8} = \frac{2.56\ (16.75)^2}{8}$ = 89.8 kip-ft/ft width	$w = 6.8\ (5.5) = 37$ kN/m; $M_0 = \frac{37\ (5.11)^2}{8}$ = 120 kN-m/m width
A general bending moment equation can be expressed:	$M = \frac{wx\ (l - x)}{2} - \frac{33\ (l - x)}{l}\ \frac{77\ x}{l}$	$M = \frac{wx\ (l - x)}{2} - \frac{45\ (l - x)}{l}\ \frac{107\ x}{l}$
The condition $dM/dx = 0$ is used to determine the location of the maximum positive bending moment	$\frac{dM}{dx} = 0$	$\frac{dM}{dx} = 0$
Differentiating, substituting w and l, and then solving for x	$\frac{d}{dx}\left(\frac{wxl}{2} - \frac{wx^2}{2} - 33 - \frac{44\ x}{l}\right) = 0$; $\frac{wl}{2} - wx - \frac{44}{l} = 0$; $x = 7.3$ ft	$\frac{d}{dx}\left(\frac{wxl}{2} - \frac{wx^2}{2} - 45 - \frac{62\ x}{l}\right) = 0$; $\frac{wl}{2} - wx - \frac{62}{l} = 0$; $x = 2.2$mm
Find M^+_{max} by substituting the value of x into the moment equation	$M^+_{max} = \frac{2.56\ (7.3)\ (16.75 - 7.3)}{2} - \frac{33\ (16.75 - 7.3)}{16.75} - \frac{77\ (7.3)}{16.75}$ = 88 – 19 – 34 = 35 kip-ft/ft width	$M^+_{max} = \frac{37\ (2.2)\ (5.11 - 2.2)}{2} - \frac{45\ (5.11 - 2.2)}{5.11} - \frac{107\ (2.2)}{5.11}$ = 120 – 26 – 46 = 48 kN-m/m width

Example 4–(Continued)

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 6–Compare maximum positive bending moment M^+_{max} with nominal positive moment strength $M^+_{m\theta}$ at 2 hr of fire exposure (from Step 2)	$M^+_{n\theta} = 41.8 > M^+_{max}$ $= 35$ kip-ft/ft width ∴ the floor panel qualifies for a 2-hr fire endurance rating	$M^+_{n\theta} = 55.8 > M^+_{max}$ $= 48$ kN-m/m width ∴ the floor panel qualifies for a 2-hr fire endurance rating
Step 7–Determine negative steel cut-off point and compare resulting bar length with minimum length required by ACI 318-83 Section 13.4.8		
Substitute values of w and l into the moment equation, and set M equal to 0	$M = \frac{2.56(16.75-x)}{2} - \frac{33(16.75-x)}{16.75} - \frac{77x}{16.75} = 0$ $x^2 - 14.7x + 25.8 = 0$	$M = \frac{37(5.11-x)}{2} - \frac{45(5.11-x)}{5.11} - \frac{107x}{5.11} = 0$ $x^2 - 4.47x + 2.41 = 0$
Solve the quadratic equation for location of zero moment $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$x_1 = 2.0$ ft $x_2 = 12.7$ ft (realistic value)	$x_1 = 0.62$ m $x_2 = 3.87$ m (realistic value)
Verify that ACI 318-83 Section 12.2.5, which requires that $l_d \geq 12$ in. (0.3 m), is satisfied	$l_d = 12$ in.	$l_d = 0.3$ m
Calculate the negative reinforcement cut-off point	$d = (16.75 - 12.7) + 1.0 = 5.0$ ft	$d = (5.11 - 3.87) + 0.3 = 1.54$m
Section 13.4.8 of ACI 318-83 gives the minimum required bar length from face of support as 0.30 l_n.	$d = 0.30(16.75) = 5.0$ ft OK ∴ negative reinforcement is adequate	$d = 0.30(5.11) = 1.53$ m OK ∴ negative reinforcement is adequate

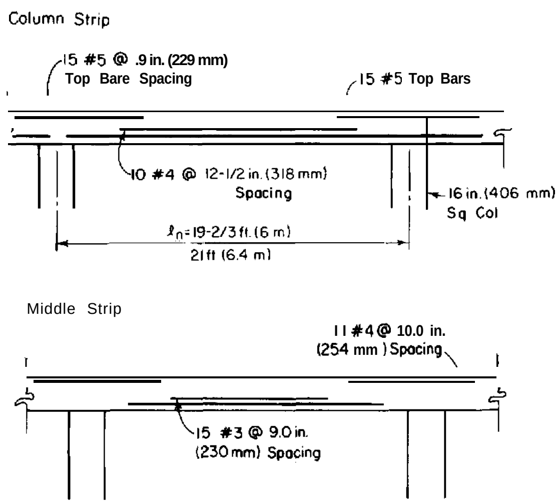
Example 5–Verification that an interior-bay floor panel qualifies for a three-hour fire endurance rating

Given: A two-way multispan rectangular floor slab of 7.0 in. (180 mm) thick flat plate construction. Interior panel in question is supported by 16.0 in. (406 mm) square tied columns. Panel spans are 21.0 ft (6.40 m).

Reinforcement is #4 and #5 Grade 60 bars placed as shown in the sketches.

Concrete is made with siliceous aggregate and has a specified compressive strength of 4000 psi (28 MPa)

Dead load is 15 psf (0.72 kN/m²) and live load is 40 psf (1.9 kN/m²).



Example 5—(Continued)

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 1-List given data	Reinforcement #4 bar d_b = 0.500 in. A_b = 0.20 in.^{2} #5 bar d_b = 0.625 in. A_b = 0.31 in.^{2} A_s^- = 4.65 in.^{2} (column strip) A_s^- = 2.20 in.^{2} (middle strip) A_s^+ = 2.00 in.^{2} (column strip) A_s^+ = 1.65 in.^{2} (middle strip) f_y = 60,000 psi cover = 0.75 in. $u = 0.75 + \dfrac{0.500}{2} = 1.0$ in $d = 7.0 - 1.0 = 6.0$ in. Slab f_c' = 4000 psi b = 126 in. h = 7.0 in. l = 21.0 ft l_n = 19.7 ft Loading $w_l = 40 + 15 = 55$ psf $w_d = \dfrac{7.0}{12}(150) = 87$ psf $w = 55 + 87 = 142$ psf	Reinforcement #4 bar d_b = 12.7 mm A_b = 129 mm #5 bar d_b = 15.9 mm A_b = 200 mm^2 A_s^- = 3000 mm^2 (column strip) A_s^- = 1420 mm^2 (middle strip) A_s^+ = 1290 mm^2 (column strip) A_s^+ = 1065 mm^2 (middle strip) f_y = 410 MPa cover = 19 mm $u = 19 + \dfrac{12.7}{2} = 25$ mm $d = 180 - 25.4 = 150$ mm Slab f_c' = 27.6 MPa b = 3.20 m h = 178 mm l = 6.4 m l_n = 6.0 m Loading $w_l = 1.92 + 0.72 = 2.64$ kN/m^2 w_d = 4.2 kN/m^2 $w = 2.64 + 4.2 = 6.8$ kN/m^2
Step 2-Determine positive nominal moment strength $M_{n\theta}^+$ at 3 hr of fire exposure from $$M_{n\theta}^+ = A_s f_{y\theta}\left(d - \frac{a_\theta^+}{2}\right)$$ To do so, values of θ, $f_{y\theta}$, and a_θ^+ are needed.		
Find θ, using Fig. 7.1.1 (b)	For u = 1.0 in. and t = 180 min, θ = 1380 F	For u = 25 mm and t = 180 min, θ = 750 C
Determine $f_{y\theta}$ using Fig. 5.1 $f_{y\theta} = (f_{y\theta}/f_y)(f_{y\theta})$	For θ = 1380 F, $f_{y\theta}/f_y$ = 0.22 $f_{y\theta}$ = 0.22 (60,000) = 13,000 psi	For θ = 750 C, $f_{y\theta}/f_y$ = 0.22 $f_{y\theta}$ = 0.22 (410) = 90 MPa
Since θ < 700 F (370 C), $f_{c\theta}' = f_c'$ for top concrete	$f_{c\theta}'$ = 4000 psi	$f_{c\theta}'$ = 27.6 MPa

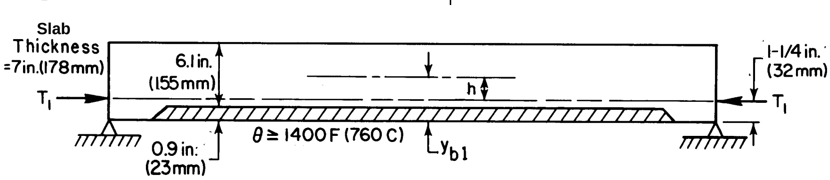
Example 5—(Continued)

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 2-(Continued) A) Column Strip Calculate $a_\theta^+ = \dfrac{A_s^+ f_{y\theta}}{0.85 f_{c\theta}' b}$	$a_\theta^+ = \dfrac{2.00\ (13{,}200)}{0.85\ (4000)\ (126)}$ $= 0.062$ in.	$a_\theta^+ = \dfrac{1290\ (90.2)}{0.85\ (27.6)\ (3200)}$ $= 1.5$ mm
Calculate	$M_{n\theta}^+ = \dfrac{2.00\ (13{,}200)}{1000}\left(\dfrac{6.0 - \frac{0.62}{2}}{12}\right)$ = 12.5 kip-ft/ft width	$M_{n\theta}^+ = \dfrac{1290\ (90.2)}{1000}\left(\dfrac{150 - \frac{1.5}{2}}{1000}\right)$ = 17.6 kN-m/m width
B) Middle Strip Calculate a_θ^+	$a_\theta^+ = \dfrac{1.65\ (13{,}200)}{0.85\ (4000)\ (126)} = 0.05$ in.	$a_\theta^+ = \dfrac{1065\ (90.2)}{0.85\ (27.6)\ (3200)} = 1.3$ mm
Calculate $M_{n\theta}^+$	$M_{n\theta}^+ = 1.65\dfrac{(13{,}200)}{1000}\left(\dfrac{6 - \frac{0.05}{2}}{12}\right)$ = 10.8 kip-ft/ft width	$M_{n\theta}^+ = \dfrac{1065\ (90.2)}{1000}\left(\dfrac{12 - \frac{1.3}{2}}{1000}\right)$ = 14.5 kN-m/m width
Step 3-Determine negative nominal moment strength $M_{n\theta}^-$ over the support To do so, values of θ, effective d', $f_{y\theta}$, $f_{c\theta}'$, and a_θ^- are needed		
Find θ using Fig. 7.1.1 (b)	For u = 6.0 in. and t = 180 min, θ = 325 F	For u = 150 mm and t = 180 min, θ = 160 C
Determine $f_{y\theta}$ using Fig. 5.1 $f_{y\theta} = (f_{y\theta}/f_y)\ (f_y)$	For θ = 325 F, $f_{y\theta}/f_y = 0.94$ $f_{y\theta} = 0.94\ (60{,}000) = 56{,}400$ psi	For θ = 160 C, $f_{y\theta}/f_y = 0.94$ $f_{y\theta} = 0.94\ (410) = 385$ MPa
Determine the thickness of slab at 1400 F (760 C) or higher temperature using Fig. 7.1.1 (b)	0.94 in. of concrete is at 1400 F or higher temperature	24 mm of concrete is at 760 C or higher temperature
Calculate effective d'	eff. d' = $7.0 - 0.94 - 0.75 - \left(\dfrac{0.625}{2}\right)$ = 5.0 in.	eff. d' = $180 - 24 - 19 - \dfrac{16}{2}$ = 130 mm
To find $f_{c\theta}'$, assume that the realistic temperature of concrete is the average of 1) 1400 F (760 C) and 2) temperature of concrete at 0.35 (eff. d') as read from Fig. 7.1.1 (b)	At 0.35 (5.0) + 0.94 = 2.7 in. and t = 180 min, θ = 730 F average $\theta = \dfrac{1400 + 730}{2} = 1065$ F For θ = 1065 F, $f_{c\theta}'/f_c' = 0.70$	At 0.35 (130) + 24 = 68 mm and t = 180 min, θ = 390 C average $\theta = \dfrac{760 + 390}{2} = 575$ C For θ = 575 C, $f_{c\theta}'/f_c' = 0.70$
$f_{c\theta}' = (f_{c\theta}'/f_c')\ (f_c')$	$f_{c\theta}' = 0.70\ (4000) = 2800$ psi	$f_{c\theta}' = 0.70\ (27.6) = 19$ MPa

Example 5-(Continued)

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 3-(Continued) A) Column Strip Calculate $a_\theta^- = \frac{A_s^- f_{y\theta}}{0.85 f_{c\theta}' b}$	$a_\theta^- = \frac{4.65\,(56{,}400)}{0.85\,(2800)\,(126)} = 0.87$ in.	$a_\theta^- = \frac{3000\,(385)}{0.85\,(19)\,(3200)} = 22$ mm
Calculate $M_{n\theta}^- = A_s^- f_{y\theta}\left(d' - \frac{a_\theta^-}{2}\right)$	$M_{n\theta}^- = \frac{4.65\,(56{,}400)}{1000}\left(\frac{5 - \frac{0.87}{2}}{12}\right)$ = 99.8 kip-ft/ft width	$M_{n\theta}^- = \frac{3000\,(385)}{1000}\left(\frac{127 - \frac{22}{2}}{1000}\right)$ = 134 kN-m/m width
B) Middle Strip Calculate a_θ^-	$a_\theta^- = \frac{2.20\,(56{,}400)}{0.85\,(2800)\,(126)} = 0.41$ in.	$a_\theta^- = \frac{1420\,(385)}{0.85\,(19.3)\,(3200)} = 10$ mm
Calculate $M_{n\theta}^-$	$M_{n\theta}^- = \frac{2.20\,(56{,}400)}{1000}\left(\frac{5 - \frac{0.41}{2}}{12}\right)$ = 49.6 kip-ft/ft width	$M_{n\theta}^- = \frac{1420\,(385)}{1000}\left(\frac{127 - \frac{10}{2}}{1000}\right)$ = 66.7 kN-m/m width
Step 4-Determine bending moment Find total moment M_o on the panel from $M_o = \frac{wl_n}{8}$	$M_o = \frac{142\,(19.67)(19.67)^2}{8}$ = 135 kip-ft/ft width	$M_o = \frac{6.8\,(6.0)\,(6.0)^2}{8}$ = 184 kN-m/m width
From Section 13.6.4.4 of ACI 318-83 $M_{middle}^+ = 0.32\ M_o$	$M_{middle}^+ = 0.32\ (135)$ = 43 kip-ft/ft width	$M_{middle}^+ = 0.32\ (184)$ = 59 kN-m/m width
$M_{col}^+ = 0.68\ M_o$	$M_{col}^+ = 0.68\ (135)$ = 92 kip-ft/ft width	$M_{col}^+ = 0.68\ (184)$ = 125 kN-m/m width

Step 5—Determine effect of restraint



Following Section 2.3 of this Guide and page 9-10 of "Reinforced Concrete Fire Resistance" published by CRSI, 1980, estimate the mid-span deflection Δ_1 of slab under minimal restraint as	$l_1 = 19.67$ ft $\Delta_o = 2.8$ in. $y_{b1} = 3.5$ in. (see figure)	$l_1 = 6000$ mm $\Delta_o = 70$mm $y_{b1} = 90$ mm (see figure)
$\Delta_1 = \frac{1}{5}\left(\frac{l_1^2 \cdot \Delta_o}{c\ y_{b1}}\right)$	$\Delta_1 = \frac{1}{5}\left(\frac{(19.67\ \times 12)^2\,(2.8)}{3500\ (3.5)}\right)$	$\Delta_1 = \frac{1}{5}\left(\frac{(6000)^2\ (70)}{89{,}000\ (90)}\right)$

Example 5—(Continued)

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 5-(Continued)		
where Δ_o is deflection of the flexural member in simply supported condition and c is 3500 for inches and 89,000 for millimeters	= 2.5 in .	= 63mm
However, the deflection Δ of slab under restraint greater than the minimal will be smaller than Δ_1	A_1 = 84 in.^{2} E_1 = 3.0 x 10^6 psi T_1 = 15,600 lb	A_1 = 5.4 x 10^{-2} m^2 E_1 = 2.1 x 10^{10} N/m^2 T_1 = 6.94 x 10^4 N
Estimate $\Delta = \Delta_1\left(0.3 + \frac{23 \text{ x } 10^{-6}}{T_1/A_1E_1}\right)$ (See Selvaggio, Carlson STP-422, 1967)	Δ = $2.5\left(0.3 + \frac{23 \text{ x } 10^{-6}}{15{,}600/[(84)(3.0\times10^6)]}\right)$ = 1.7 in.	Δ = 63 $\left(0.3 + \frac{23 \text{ x } 10^{-6}}{(6.94 \text{ x } 10^4)/[(5.4 \text{ x } 10^{-2})(2.1 \text{ x } 10^{10})]}\right)$ = 43mm
Since T_1 acts at 1¼ in. from slab bottom, then the distance h between centroidal axis and T_1 becomes:	$h = \left(\frac{7-0.94}{2} + 0.94\right) - 1¼ = 2.7$ in.	$h = \left(\frac{180-24}{2} + 24\right) - 32 = 70$ mm
Calculate moments due to T_1		
at mid span	$M_{T\text{-}mid} = \frac{15.6\ (2.7 - 1.7)\ (10.5)}{12}$ = 14 kip-ft/ft width	$M_{T\text{-}mid} = \frac{228\ (70 - 43)\ (3.2)}{1000}$ = 19 kN-m/m width
at support with T_1 acting at 1 1/4 in. from bottom of the slab	$M_{T\text{-}spt} = \frac{15.6\left(\frac{7}{2} - 1.25\right)(10.5)}{12}$ = 31 kip-ft/ft width	$M_{T\text{-}spt} = \frac{228\ (89 - 32)\ (3.2)}{1000}$ = 42 kN-m/m width
Step 6-Calculate moments assuming hinges are formed at supports and compare with nominal moment strength at 3-hr fire exposure		
A) Column Strip		
moment at midspan		
$M^+_{col} - M_{T\text{-}mid}$ $M^-_{n\theta} - M_{T\text{-}spt}$	92 − 14 = 78 kip-ft/ft width 100 − 31 = 69 kip-ft/ft width M = 78 − 69 = 9 kip-ft/ft width $< M^+_{n\theta}$ = 12 kip-ft/ft width OK	125 − 19 = 106 kN-m/m width 134 − 42 = 92 kN-m/m width M = 106 − 92 = 14 kN-m/m width $< M^+_{n\theta}$ = 17.6 kN-m/m width OK
B) Middle Strip		
moment at midspan		

Example 5—(Continued)

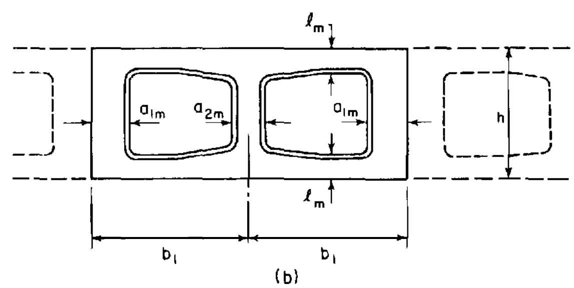
Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 6-(Continued) $M^+_{middle} - M_{T\text{-}mid}$ $M^-_{n\theta} - M_{T\text{-}spt}$	$43 - 14 = 29$ kip-ft/ft width $49.6 - 31 = 19$ kip-ftift width $M = 29 - 19 = 10$ kip-ft/ft width $< M^+_{n\theta} = 10.8$ kip-ftift width OK ∴ the interior-bay floor qualifies for a 3-hr fire endurance rating	$59 - 19 = 40$ kN-m/m width $66.7 - 42 = 25$ kN-m/m width $M = 40 - 25 = 15$ kN-m/m width $\approx M^+_{n\theta} = 14.5$ kN-m/m width OK ∴ the interior-bay floor qualifies for a 3-hr fire endurance rating

Example 6-Calculation of fire resistance of a concrete masonry wall in the naturally moist condition

Given: A concrete masonry wall made of the lightweight-concrete units shown and the properties listed.

Properties of concrete used

concrete density
$\rho = 90.5$ lb/ft^3 (1450 kg/m^3)
thermal conductivity
$k = 0.30$ Btu/(h•ft•F) [0.52 W/(m•K)]
thermal diffusivity
$\kappa = 0.016$ ft^2/h (4.13×10^{-7} m^2/s)
moisture content
$m = 0.05$
permeability factor
$\beta = 8.0$



Dimensions
$h = 5.6$ in. (142 mm)
$l_m = 1.0$ in. (25 mm)
$a_{1m} = 1.1$ in. (28 mm)
$a_{2m} = 1.0$ in. (25 mm)
$b_1 = 7.8$ in. (198 mm)

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step l-Calculate effective values of block dimensions and ratios		
$l = 1.1 l_m$ $a_1 = 1.15\, a_{1m}$ $a_2 = 1.15\, a_{2m}$ $a_1 = ⅓\,(2a_{1m} + a_{2m})$ $b = ⅓\,(2b_1)$	$1.1\,(1.0) = 1.1$ in. $1.15\,(1.1) = 1.3$ in. $1.15\,(1.0) = 1.2$ in. $⅓\,[2\,(1.3) + 1.2] = 1.3$ in. $⅓\,(2)\,(7.8) = 5.2$ in.	$1.1\,(25) = 28$ mm $1.15\,(28) = 32$ mm $1.15\,(25) = 29$ mm $⅓\,[2\,(32) + 29] = 31$ mm $⅓\,(2)\,(198) = 132$ mm
a/b	$\frac{1.3}{5.2} = 0.25$	$\frac{31}{132} = 0.23$
$\phi = m\frac{\rho}{\rho_w}$ where $\rho_w = 62.4$ lb/ft^3 (1000 kg/m^3)	$0.05\frac{90.5}{62.4} = 0.07$	$0.05\frac{1450}{1000} = 0.07$
Step 2-Calculate fire resistance of masonry in dry condition τ_o from $\tau_o = \left(\frac{1}{\frac{a/b}{\tau_{1o}^{1/2}} + \frac{1 - a/b}{\tau_{2o}^{1/2}}}\right)^2$ Eq. (3-8) To do so, values of τ_{1o} and τ_{2o} must be found		

Example 6-(Continued)

Procedure	Calculation in inch-pound units	Calculation in SI Metric units
Step 2-(Continued) Calculate τ_{1o} from		
$\tau_{1o} = C_{14}\left(\frac{k}{h}\right)^{0.55}\left(\frac{h^2}{\kappa}\right)^{1.2}$ Eq. (3-9)	$\tau_{1o} = 0.205\left(\frac{0.30}{5.6/12}\right)^{0.55}\left[\frac{(5.6/12)^2}{0.016}\right]^{1.2}$	$\tau_{1o} = 0.0153\left(\frac{0.52}{142/1000}\right)^{0.55}\left[\frac{(142/1000)^2}{4.13\times10^{-7}}\right]^{1.2}$
where C_{14} = 0.205 $ft^{1.1}\,h^{0.35}\,F^{0.55}/Btu^{0.55}$ (0.0153 $m^{1.1}\,s^{0.35}\,C^{0.55}/J^{0.55}$)	= 0.205 (0.78) (23) = 3.7 hours	= 0.0153 (2.04) (4.23 x 10^5) = 13,200 seconds = 3.7 hours
$\tau_{2o} = C_{15}\left(\frac{k}{l}\right)^{0.60}\left(\frac{l^2}{\kappa}\right)^{1.1}$ Eq. (3-10)	$\tau_{2o} = 0.750\left(\frac{0.30}{1.1/12}\right)^{0.60}\left[\frac{(1.1/12)^2}{0.016}\right]^{1.1}$	$\tau_{2o} = 0.117\left(\frac{0.52}{28/1000}\right)^{0.60}\left[\frac{(28/1000)^2}{4.13 \times 10^{-7}}\right]^{1.1}$
C_{15} = 0.750 $ft^{1.2}\,h^{0.5}\,F^{0.6}/Btu^{0.6}$ (0.117 $m^{1.2}\,s^{0.5}\,C^{0.6}/J^{0.6}$)	= 0.750 (2.0) (0.49) = 0.74 hour	= 0.117 (5.8) (4000) = 2700 seconds = 0.75 hour
$\tau_o = \dfrac{1}{\dfrac{a/b}{\tau_{1o}^{1/2}} + \dfrac{1 - a/b}{\tau_{2o}^{1/2}}}$	$\tau_o = \dfrac{1}{\dfrac{0.25}{3.7^{1/2}} + \dfrac{1 - 0.25}{0.74^{1/2}}}$ $= \dfrac{1}{0.13 + 0.87}$ = 1 hour	$\tau_o = \dfrac{1}{\dfrac{0.23}{3.7^{1/2}} + \dfrac{1 - 0.23}{0.75^{1/2}}}$ $= \dfrac{1}{0.12 + 0.89}$ = 1 hour
Step 3-Calculate fire resistance of masonry in naturally moist condition τ from		
$\tau = \dfrac{\tau_o^2 + 4\tau_o(1 + \beta\phi)}{4 + \tau_o}$ Eq. (3-11)	$\tau = \dfrac{(1)^2 + 4(1)[1 + (8.0)(0.07)]}{4 + 1}$ = 1.4 hours	$\tau = \dfrac{(1)^2 + 4(1)[1 + (8.0)(0.07)]}{4 + 1}$ = 1.4 hours
	Note: A standard fire test performed on this masonry yielded a fire resistance of 1.567 hours.	

Example 7 — Calculation of fire endurance requirement

Given: 10-story office building. Floor area per story is 4900 ft^2 (454m^2). There are 16 compartments on each story; they are 8.2 ft (2.5 m) tall and have an average floor area of 280 ft^2 (26 m^2). The compartments will be lined with conventional materials: walls and ceiling with gypsum-board and floor with wood parquet. The compartments have two windows, 5 ft (1.52 m) tall and 3.33 ft (1.02 m) wide.

Perform a preliminary assessment of the fire endurance requirement in such a way as to have the failure probability less than 0.05 (5 percent), given flashover (i.e., according to the final footnote in the Appendix, less than 1.05 percent given ignition).

Procedure	Calculations in conventional units	Calculations in SI units
Step 1 — List or calculate the input variables		
Average floor area	A_F = 280 ft^2	A_F = 26 m^2
Height of compartments	h_c = 8.2 ft	h_c = 2.5 m
Average surface area of compartments [Eq. (A-1)]	A_t = 2x280 + 4 x 8.2x $\sqrt{280}$ = 560 + 549 = 1109 ft^2	A_t = 2 x 26 + 4 x 2.5 x $\sqrt{26}$ = 52 + 51 = 103 m^2
Average thermal absorptivity of compartment	$\sqrt{k\rho c}$ = $\frac{1}{1109}$[(280 + 549) × 2.18 + 280 x 1.281 = 1.95 Btu $ft^{-2}h^{-1/2}R^{-1}$	$\sqrt{k\rho c}$ = $\frac{1}{103}$[(26 + 51) × 740 + 26 x 440] = 1665 J $m^{-2}s^{-1/2}K^{-1}$

Example 7—(Continued)

Procedure	Calculations in conventional units	Calculations in SI units
Step 1-(Continued) boundaries [Eq. (A-2), Table A.1] Minimum value of ventilation factor [Eq. (A-3)] Specific fire load for office buildings (Table A.2)	$\phi_{min} = 0.0755 \times 2 \times 5 \times 3.33 \sqrt{4.17 \times 10^2 \times 5}$ $= 11.5 \times 10^4 \text{ lb h}^{-1}$ $L_m = 5.08 \text{ lb ft}^{-2}$ $\sigma_L = 1.76 \text{ lb ft}^{-2}$	$\phi_{min} = 1.21 \times 2 \times 1.52 \times 1.02 \sqrt{9.8 \times 1.52}$ $= 14.5 \text{ kg s}^{-1}$ $L_m = 24.X \text{ kg m}^{-2}$ $\sigma_L = X.6 \text{ kg m}^{-2}$
Step 2 -Calculate the normalized heat load for real world fires, for $L = L_{ms}$ $\phi = \phi_{min}$ δ-factor [Eq. (A-5)] H' [Eq. (A-4)]	$\delta = 11.8 \sqrt{8.2^3 / 11.5 \times 10^4}$ $= 0.82$ $H' = 456 \dfrac{(11.0 \times 0.82 + 1.6)\, 280 \times 5.08}{1109 \times 1.95 + 0.223 \sqrt{11.5 \times 10^4 \times 280 \times 5.08}}$ $= 1370 \text{ h}^{1/2}\text{R}$	$\delta = 0.79 \sqrt{2.5^3 / 14.5}$ $= 0.82$ $H' = 1.06 \times 10^6 \dfrac{(11.0 \times 0.82 + 1.6)\, 26 \times 24.8}{103 \times 665 + 935 \sqrt{14.5 \times 26 \times 24.8}}$ $= 45{,}680 \text{ s}^{1/2}\text{K}$
Step 3 — Calculate required capacity for normalized heat load β from Fig. A.1 V_τ (all-purpose value) Ω_1 [Eq. (A-9)] Ω_2 [Eq. (A-10)] H''_d [Eq. (A-8)]	$\beta = 1.64$ $V_\tau = 0.1$ $\Omega_1 = \dfrac{1.76}{5.08} \cdot \dfrac{1109 \times 1.95 + 0.112 \sqrt{11.5 \times 10^4 \times 280 \times 5.08}}{1109 \times 1.95 + 0.223 \sqrt{11.5 \times 10^4 \times 280 \times 5.08}}$ $= 0.248$ $\Omega_2 = 0.9 \times 0.1 = 0.09$ $H''_d = 1370 \times \exp\left(1.64 \sqrt{0.248^2 + 0.09^2 + 0.101^2}\right)$ $= 2180 \text{ h}^{1/2}\text{R}$	$\beta = 1.64$ $V_\tau = 0.1$ $\Omega_1 = \dfrac{8.6}{24.8} \cdot \dfrac{103 \times 665 + 468 \sqrt{14.5 \times 26 \times 24.8}}{103 \times 665 + 935 \sqrt{14.5 \times 26 \times 24.8}}$ $= 0.248$ $\Omega_2 = 0.9 \times 0.1 = 0.09$ $H''_d = 45{,}680 \times \exp\left(1.64 \sqrt{0.248^2 + 0.09^2 + 0.101^2}\right)$ $= 72{,}600 \text{ s}^{1/2}\text{K}$
Step 4 — Calculate fire endurance requirement τ_d [Eq. (A-7)]	$\tau_d = 0.11 + 5.33 \times 10^{-3} \times 2180$ $+ 14.44 \times 10^{-8} \times 2180^2 = 1.96 \text{ h}$	$\tau_d = 0.11 + 0.16 \times 10^{-4} \times 72{,}600$ $+ 0.13 \times 10^{-8} \times 72{,}600^2 = 1.96 \text{ h}$

Conclusion: Using building elements of 2 h fire endurance will insure that the failure probability in fully-developed fires will not be more than 5 percent.

CHAPTER 9—REFERENCES

9.1-Documents of standards-producing organizations

The documents of the various standards-producing organizations referred to in this document are listed below with their serial designation, including year of adoption or revision. The documents listed were the latest effort at the time this document was revised. Since some of these documents are revised frequently, generally in minor detail only, the user of this document should check directly with the sponsoring group if it is desired to refer to the latest revision.

American Concrete Institute

318-83 Building Code Requirements for Reinforced Concrete

ASTM

A 36/ A 36M-84a Standard Specification for Structural Steel

A 421-80 Standard Specification for Uncoated Stress-Relieved Wire for Prestressed Concrete

A 722-75 (1981) Standard Specification for Uncoated High-Strength Steel Bar for Prestressing Concrete

C 140-75 (1980) Standard Methods of Sampling and Testing Concrete Masonry Units

E 119-83 Standard Methods of Fire Tests of Building Construction and Materials

E 152-81a Standard Methods of Fire Tests of Door Assemblies

E 176-83b Standard Terminology Relating to Fire Standards

Underwriters Laboratories, Inc.

618-79 Standard for Safety, Concrete Masonry Units

U.S. General Services Administration

HH-I-00526 Insulation Board, Thermal (Mineral Fiber)

The above publications may be obtained from the following organizations:

American Concrete Institute
P.O. Box 19150
Detroit, MI 48219

ASTM
1916 Race St.
Philadelphia, PA 19103

Underwriters Laboratories, Inc.
333 Pfingsten Rd.
Northbrook, IL 60062

U.S. General Services Administration
Regional Office
230 South Dearborn St.
Chicago, IL 60604

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This report was submitted to letter ballot of the committee and was approved in accordance with ACI balloting procedures.

APPENDIX-DESIGN OF BUILDING ELEMENTS FOR PRESCRIBED LEVEL OF FIRE SAFETY

The fire endurance of building elements (such as floors, beams, walls, and columns) is the length of time the elements are capable of functioning satisfactorily as structural members and as thermal barriers, as judged from fire tests conducted according to the ASTM E 119-83 standard. As shown in the previous sections of this Report, performing such tests is not necessary for a large number of concrete elements. Information on their fire endurances can be derived by the application of calculation procedures based on heat flow studies and structural analyses, and on the knowledge of the behavior of concrete and steel at elevated temperatures, rather than by fire tests.

The provision of fire safety in buildings consists largely of selecting or designing building elements of sufficient fire endurance. The fire endurance requirements are specified in the building codes. These requirements are based mainly on tradition or on rough estimates of the expected fire severities, assumed to be proportional to the amount of combustible materials present in the building.

It has been known since the mid-1950s that the amount of combustible materials in the building is only one of several factors on which the severity (destructive potential) of fires depends. Clearly specifying fire endurance requirements in terms of performance in test fires is justified only if (1) the severity of real-world fires can be described as a function of all significant parameters, and (2) the relationship between the performance of building elements in real-world fires and standard test fires is understood.

Several methods have been developed during the years for the calculation of this relationship. The best known among them are Ingberg's method (Ingberg 1928), Law's method (Law 1971), Pettersson's method (Pettersson 1975), the DIN mehtod (DIN 18230 1978), and a method based on the normalized heat load concept (Harmathy 1981; Harmathy and Mehaffey 1987). These methods have been reviewed and evaluated in the light of experimental information (Harmathy 1987).

The accuracy of the normalized heat load concept has been proved by a multitude of large-scale compartment burn-out tests (Mehaffey and Harmathy 1985). This concept is particularly well suited for providing an insight into the performance of building elements in real-world fires and standard test fires. The normalized heat load is the total heat absorbed by a unit area of the boundaries of an enclosure during an exposure to fire (real-world fire or test fire), divided by the thermal absorptivity (to be defined later) of the boundaries. It has been shown that, for a given structure, equal values of the normalized heat load represent equal harms done by any two fires, however dissimilar they are as to their temperature histories. The normalized heat load can, therefore, be regarded as a quantifier of the destructive potential of fires.

For real-world compartment fires, the normalized heat load depends largely on the following variables (Harmathy 1983):

A_F = floor area of the compartment* on fire, ft^2 (m')
A_t = surface area of the compartment, ft^2 (m^2)
h_c = height of the compartment, ft (m)
= thermal absorptivity of the compartment boundaries, Btu ft^{-2}h$^{-1/2}$R^{-1} (J m^{-2}s$^{-1/2}$K^{-1}) (k is thermal conductivity, ρ is density, c is specific heat)
ϕ = ventilation factor, characteristic of the rate of inflow of air into the compartment during fire, lb h^{-1} (kg s^{-1})
L = specific fire load (mass of combustibles† per unit floor area), lb ft^{-2} (kg m^{-2})

If the plans of the building are already available, the normalized heat load can be determined for each separate compartment. But it is possible even in the first phase of the design to make a preliminary assessment of the fire endurance requirement, based on information on the use of the building, its total floor area, number and height of the compartments, and the contemplated compartment lining materials.

In such a preliminary assessment of the fire endurance requirement, the compartments can be regarded as square in shape, and the area of the windows and doors (usually small in comparison with the total wall area) may be left out of consideration. A_F will now mean the average floor area of the compartments (total floor area of the building, divided by the number of compartments), and the average surface area of the

*A compartment is defined as a building space (enclosed by walls, floor, and ceiling) which is most of the time separated from other building spaces bv closed door(s).

†Only those combustibles are to be counted that are "accessible" to combustion during a fire. With noncellulosic combustible, multiply the mass of the material by the heat of combustion of the material and divide it by the heat of combustion of wood.

compartments can be calculated by the following approximation

$$A_1 = 2A_F + 4h_c\sqrt{A_F} \qquad \text{(A-1)}$$

If the boundaries of the compartments are formed by different building materials, the thermal absorptivity should be looked upon as a surface-averaged value, to be calculated as

$$\sqrt{k\rho c} = \frac{1}{A_t}\left(A_1\sqrt{k_1\rho_1 c_1} + A_2\sqrt{k_2\rho_2 c_2} + \ldots\right) \qquad \text{(A-2)}$$

in which the numerical subscripts refer to the various materials and the surface areas formed by them.

The thermal absorptivities of a number of common building materials are listed in Table A. 1 (Harmathy and Mehaffey, 1985).

Table A.1 — Thermal absorptivity of a number of common building materials*

Material	Btu ft^{-2} h$^{-1/2}$ R^{-1}	J m^{-2}s$^{-1/2}$K^{-1}
Marble	6.67	2270
Normal-weight concrete	6.44	2190
Brick	4.46	1520
Lightweight concrete density: 90.5 lb ft^{-3} (1450 kg m^{-3})	2.73	930
Gypsum board	2.18	740
Vermiculite plaster	1.96	670
Wood	1.28	440
Mineral wool	0.25	90

*Hatmathy and Mehaffey (1984).

In the case of composite structures, it is sufficiently accurate to use values of thermal absorptivity applicable to the surface-forming layers, provided those layers are at least ½-in. (13-mm) thick (Harmathy and Mehaffey 1987). (Note that the normalized heat load concept is not applicable to steel structures not provided with fire protection.)

The ventilation factor and the specific fire load are random variables. In the calculation scheme to be introduced, the ventilation factor will be taken into account with its most adverse value, which arises when the ventilation of the burning compartment is minimum, i.e., when the inflow of air is not augmented by drafts. This minimum value is

$$\phi_{min} = \rho_a A_v\sqrt{gh_v} \qquad \text{(A-3)}$$

in which

ρ_a = density of atmospheric air, ≅ 0.0755 lb ft^{-3} at 68 F (≅ 1.21 kg m^{-3} at 20 C)
A_v = area of the opening through which the burning compartment is ventilated [usually broken window(s), less frequently open door(s)], ft^2 (m^2)
h_v = height of the ventilation opening, ft (m)
g = gravitational constant ≅ 4.17 X 10^5 ft h^{-2} (≅ 9.8 m s^{-2})

The way of calculating ϕ when the compartment has vertical openings at various height levels has been described elsewhere (Magnusson and Thelandersson 1970).

Information on the specific fire load developed from some Swedish data (Pettersson et al. 1976) is presented in Table A.2 for a few occupancies. The arithmetic means L_m and the standard deviations σ_L are listed.

Table A.2 — Information on specific fire load, estimated from Swedish data*,†

	Specific fire load			
	Mean L_m		Standard deviation σ_L	
Occupancy	lb ft^{-2}	kg m^{-2}	lb ft^{-2}	kgm^{-2}
Dwelling	6.17	30.1	0.90	4.4
Office	5.08	24.8	1.76	8.6
School	3.59	17.5	1.05	5.1
Hospital	5.15	25.1	1.60	7.8
Hotel	2.99	14.6	1.86	4.2

*The tablulated values are, on the whole, representative of the occupancies shown. Discrepancies in specific fire load values reported in various publications are attributable mainly to differences in the sampling and evaluation techniques, and the estimation of that part of the fire load which is not accessible to combustion. A detailed listing of specific fire load for various occupancies and countries is available from a CIB report (Workshop CIB W14 1986).
†Pettersson et al. (1976).

The normalized heat load on the compartment boundaries (and on the compartment as a whole) can be expressed by the following semi-empirical equation (Mehaffey and Harmathy 1981)*

$$H' = C_1\frac{(11.0\delta + 1.6)\, A_F L_m}{A_1\sqrt{k\rho c} + C_2\sqrt{\phi_{min} A_F L_m}} \qquad \text{(A-4)}$$

in which

$$C_1 = \begin{cases} 4.56 \text{ Btu lb}^{-1} \\ 1.06 \times 10^6 \text{ Jkg}^{-1} \end{cases}$$

$$C_2 = \begin{cases} 0.223 \text{ Btu lb}^{-1}\text{ R}^{-1} \\ 935 \text{ J kg}^{-1}\text{ K}^{-1} \end{cases}$$

and

H' = normalized heat load for a compartment fire (for $L = L_m$ and $\phi = \phi_{min}$), h$^{1/2}$ R (s$^{1/2}$K)

δ = semi-empirical factor. dimensionless, as defined by Eq. (A-5)

$$\delta = \begin{cases} C_3\sqrt{h_c^3/\phi_{min}} \\ 1 \end{cases} \text{ whichever is less} \qquad \text{(A-5)}$$

in which

$$C_3 = \begin{cases} 11.8 \text{ lb}^{1/2}\text{ft}^{-1/2}\text{h}^{-1/2} \\ 0.79 \text{ kg}^{1/2}\text{m}^{-1/2}\text{s}^{-1/2} \end{cases}$$

*C_1 included an empirical correction to account for the fact that the experimentally determined normalized heat load values were found to be on an average 6 percent higher than the calculated values. The coefficient of variation for the error associated with use of Eq. (A-4) is 0.101, i.e., 10.1 percent (Mehaffey and Harmathy 1985; Harmathy and Mehaffey 1987).

The destructive potential of a standard test fire is also quantifiable by the normalized heat load. Since in a test fire the furnace temperature follows a prescribed course, the normalized heat load imposed on a building element in a test fire is a unique function of the duration of the test which, if the test is carried on up to the point of failure, is equal to the fire endurance of the building element. The relationship between the fire endurance and normalized heat load is

$$\tau = 0.11 + C_4 H'' + C_5 (H'')^2 \qquad \text{(A-6)}$$

in which

$$C_4 = \begin{cases} 5.33 \times 10^{-4}\ \mathrm{h}^{1/2}\ \mathrm{R}^{-1} \\ 0.16 \times 10^{4}\ \mathrm{h\ s}^{-1/2}\ \mathrm{K}^{-1} \end{cases}$$

$$C_5 = \begin{cases} 14.44 \times 10^{-8}\ \mathrm{R}^{-2} \\ 0.13 \times 10^{-9}\ \mathrm{h\ s}^{-1}\ \mathrm{K}^{-2} \end{cases}$$

and

τ = fire endurance (or duration of test fire), h

H'' = normalized heat load on the construction in standard test fire, $\mathrm{h}^{1/2}\mathrm{R}$ ($\mathrm{s}^{1/2}\mathrm{K}$)

The fire endurance in Eq. (A-6) is expressed in terms of the normalized heat load (rather than the normalized heat load in terms of the fire endurance) for convenience in determining the fire endurance requirements. The fire endurance requirement is the value of τ for which $H'' \geqslant H'$, in other words the value of τ at which the ability of the building element to absorb in a standard test fire (without ill effects) a normalized heat load equal to or larger than that imposed on the element in the real-world fire.

The fire endurance (i.e., the result yielded by a standard fire test) is also a random quantity. The coefficient of variation V_τ for τ is somewhere between 0.01 and 0.15 (i.e., 1 and 15 percent) (ASTM E 5 1982), dependent mainly on the type of construction. $V_\tau \simeq 0.1$ may be regarded as an all-purpose value.

Taking into consideration the random nature of the specific fire load L and the result of fire test τ, as well as the uncertainty associated with the use of Eq. (A-4), and employing well-known reliability-based design procedures (Cornell 1969; Zahn 1977). the following formulas have been derived (Harmathy and Mehaffey 1985) for the calculation of the fire endurance requirements

$$\tau_d = 0.11 + C_4 H''_d + C_5 (H''_d)^2 \qquad \text{(A-7)}$$

in which*

$$H''_d = H'' \exp\left(\beta \sqrt{\Omega_1^2 + \Omega_2^2 + 0.101^2}\right) \qquad \text{(A-8)}$$

and in Eq. (A-8)

$$\Omega_1 = \frac{\sigma_L}{L_m} \cdot \frac{A_t \sqrt{k\rho c} + \frac{C_2}{2}\sqrt{\phi_{min} A_F L_m}}{A_t \sqrt{k\rho c} + C_2 \sqrt{\phi_{min} A_F L_m}} \qquad \text{(A-9)}$$

and

$$\Omega_2 = 0.9\, V_\tau \qquad \text{(A-10)}$$

In these equations, the d subscripts indicate design values and j3 is a factor (dimensionless), function of the allowed failure probability P_f (dimensionless), as shown in Fig. A.l. The failure probability is, naturally, conditional on flashover.†

When deciding on the allowable failure probability, the expected magnitude of human and property losses resulting from the failure is the most important consideration. Prescribing its value applicable to various situations is a task for the writers of building regulations.

After selecting a value for P_f, the fire endurance requirement is calculated from Eq. (A-4), (A-5), and (A-7) through (A-10).

*0.101 under the square-root sign has been introduced to take account of the error associated with the use of Eq. (A-4). See footnote relating to Eq. (A-4).

†Flashover is a dramatic change in the course of the fire, when all items in the fire compartment become rather suddenly ignited. A recent survey of the statistics of the National Fire Incident Reporting System has indicated that, given ignition, the probability that flashover will follow is 21 percent.

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